

1 **Rigorous performance assessment of the algorithms for resolving motor unit**
2 **action potential superpositions**

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33 **Abstract**

34

35 **Background:**

36 It is necessary to decompose the intra-muscular EMG signal to extract motor unit action potential
37 (MUAP) waveforms and their firing times. Some algorithms were proposed in the literature to
38 resolve superimposed MUAPs, including Peel-Off (PO), branch and bound (BB), genetic
39 algorithm (GA), and particle swarm optimization (PSO). This study aimed to compare these
40 algorithms in terms of accuracy and efficiency.

41

42 **Methods:**

43 Two sets of two-to-five MUAP templates (set1: a wide range of energies, and set2: a high degree
44 of similarity) were used. Such templates were time-shifted, and white Gaussian noise was added.
45 A total of 1000 superpositions were simulated for each template and were resolved using PO (also,
46 POI: interpolated PO), BB, GA, and PSO algorithms. The overall accuracy and running time of
47 each resolution were measured. The Generalized Estimating Equation and the overall rank product
48 were then used.

49

50 **Results:**

51 The PSO outperformed the other algorithms following by BB, GA, PO, and POI in the first dataset,
52 while the ranking of the algorithm was BB, PSO, GA, PO, POI in the second set. The overall
53 ranking was BB, PSO, GA, PO, and POI in the entire datasets.

54

55 **Conclusion:**

56 Although the BB algorithm is generally fast, there are some cases in which the BB algorithm
57 spends too much time. The BB, PSO, and GA algorithms have a high enough accuracy in full
58 offline decomposition, but too slow for real-time applications.

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61 **Keywords**— Resolving Superposition; EMG decomposition; Motor unit action potentials;

62 **Introduction**

63

64 The electrical signal produced in a muscle is known as the electromyogram or the EMG
65 signal. It is the result of the algebraic summation of motor unit action potentials (MUAP's). EMG
66 signals can be recorded either by surface or intramuscular electrodes. The former is known as
67 surface EMG (sEMG), while the latter is known as intramuscular EMG (iEMG). Both sEMG and
68 iEMG consist of different motor unit action potential trains (MUAPTs) with the firing of motor
69 units repetitively during muscle contraction (LeFever and De Luca, 1982). EMG signals have a
70 variety of applications in rehabilitation, sports science, geriatrics, ergonomics, neuromuscular
71 disorders, and medicine (Gazzoni et al. , 2004). EMG signals can also be used as Human-Machine
72 interfaces to control external prosthetic devices (Farina and Holobar, 2015, Karimimehr et al.,
73 2017). In some applications, it is necessary to decompose the EMG signal to extract MUAP
74 waveforms and their firing times (Lateva and McGill, 2001a). There are many algorithms for
75 iEMG decomposition in the literature (De Luca et al., 1982a, De Luca et al., 1982b, LeFever and
76 De Luca, 1982, Marateb et al., 2011, McGill et al., 2005).

77

78 Most of the iEMG decomposition algorithms involve similar signal processing steps
79 (Marateb et al., 2016). The iEMG signal is first high-pass filtered to remove baseline fluctuations
80 and to select potentials arising from muscle fibers near the recording electrode. The segments of
81 the signal that contain potentials that rise significantly above the baseline noise (active segments,
82 AcS's) are then identified. Then, the AcS's are clustered to identify the similarly shaped MUAPs
83 that correspond to the discharges of distinct active MUs, and then every AcS is classified as being
84 a discharge of one or more MU. It is possible that during muscle contraction, two or more MUs
85 fire at the same time or in close temporal succession, and their action potentials overlap with each

86 other to form a superposition (Fang et al., 1999). Some algorithms attempt to resolve
87 superpositions into their constituent MUAPs (full decomposition). A full decomposition is an
88 essential tool for the study of muscle architecture (Lateva and McGill, 2001b), MU coordination
89 (De Luca and Erim, 1994), MU synchronization (Datta and Stephens, 1990) and discharge
90 irregularities (Lateva et al. , 2002).

91

92 There are some algorithms proposed in the literature to resolve superimposed MUAPs.
93 They either identify continuous or discrete-time shifts of the involved MUs. One of these
94 algorithms, the exhaustive search algorithm, considers all of the possible discrete-time shifts. It is
95 very time-consuming, especially when a large number of MUs are involved in a superposition.
96 Accordingly, this algorithm is not usually used in practice (Marateb, McGill, 2016). Other sub-
97 optimal algorithms were therefore proposed: Peel-Off (PO) (Christodoulou and Pattichis, 1999,
98 Etawil and Stashuk, 1996, Fang, Agarwal, 1999, LeFever and De Luca, 1982), branch and bound
99 (BB) (Lateva, McGill, 2002), genetic algorithm (GA) (Florestal et al., 2007a), and particle swarm
100 optimization (PSO) (Marateb and McGill, 2009a).

101 The PO algorithm is a primary and suboptimal algorithm that uses the correlation between
102 the MUAP templates and the superimposed signal to find the best match (**supplementary**
103 **material; Fig. s1**). Partial superpositions, in which the MUAPs overlap without peaks being
104 obscured, can be efficiently resolved using this algorithm.

105 The other algorithms are based on the minimization of a cost function, e.g., the squared
106 error of the residual between the given superposition and the reconstructed waveform based on the
107 estimated time shifts of the involved MUAPs. The BB algorithm is a constructive exact
108 optimization algorithm that smartly considers all possible discrete time-shift cases

109 **(supplementary material; Fig. s2)**. After finding the best discrete-time shifts, it then uses
110 interpolation, and Newton's algorithm to find the best continuous-time shifts. On the other hand,
111 GA and PSO are population-based probabilistic meta-heuristic optimization solution algorithms.
112 These two algorithms are inspired by nature. GA is inspired by the process of natural selection in
113 Genetics (Florestal et al., 2007b) **(supplementary material; Fig. s3)**, while PSO simulates the
114 social behavior of a flock of birds (Marateb and McGill, 2009b) **(supplementary material; Fig.**
115 **s4)**. These algorithms were used in the literature to minimize the superposition cost function.

116 In this manuscript, we aimed to compare these algorithms in terms of accuracy and
117 efficiency. We focused on the known-constituent case, in which the MUAPs involved in the
118 superposition are assumed to be known a-priori. We simulated a large number of partial,
119 constructive, and destructive superpositions and resolved them using the PO, POI (Peel-Off with
120 Interpolation factor of 10), BB, GA, and PSO algorithms. We then used proper statistical tests to
121 analyze and compare their performances rigorously.

122

123 **Materials and Methods**

124 The performance of the algorithms was evaluated using simulated superpositions. Two sets
125 of MUAP templates were used. The templates were taken from iEMG signals from the public
126 domain database at www.emglab.net. The templates were sampled at 10 kHz and high-pass
127 filtered at 10 kHz. Set 1 consisted of ten templates with a wide range of energies (338-1624), while
128 set 2 consisted of six templates with a high degree of similarity (correlation coefficients
129 0.59 ± 0.28).

130 Each superposition was involved from $N=2$ to $N=5$ templates. The appropriate number of
131 templates were chosen at random from one of the two sets. Each template was time-shifted by a

132 continuous random amount within ± 1.0 ms using a high-resolution alignment algorithm (McGill
133 and Dorfman, 1984). Then the time-shifted templates were added together along with white
134 Gaussian noise with the standard deviation equal to 0.05 times the mean peak-to-peak template
135 amplitude (Marateb and McGill, 2009b). A total of 1000 superpositions were formed for each
136 value of N for each template set, for a total of 8000 superpositions in all. The simulated datasets
137 and also the MATLAB code of the BB algorithm is available (**supplementary material;**
138 **data_code s1**). Figs. 1 and 2 show the templates of the sets 1 and 2 with some examples of
139 superpositions.

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[Figs 1 and 2 are included here].

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143 Each superposition was resolved using the PO, BB, GA, and PSO algorithms. Also, in
144 order to test whether the PO algorithm would be improved by interpolation, it was also resolved
145 using the PO algorithm after up-sampling the waveforms by a factor of 10, which we will refer to
146 as the POI algorithm. In each case, it was assumed that the identities of the templates involved in
147 the superposition were known. For each resolution, the time shifts estimated by the algorithm were
148 compared with the actual time shifts, and the offset errors were stratified into three classes: $< \pm$
149 0.1 ms (essentially correct), $\leq \pm 0.5$ ms (close), $> \pm 0.5$ ms (incorrect). The accuracy of each
150 resolution was measured by the overall accuracy ($I_d = \frac{n_c}{n_i + N}$, where n_c is the number of essentially
151 correct time shifts, n_i is the number of incorrect time shifts, and N is the total number of templates
152 involved in the superposition).

153

154 Also, the average and maximum running time of the superposition resolving algorithms
was recorded. The simulations were performed on an Intel dual-core 1.83 GHz CPU with 2 GB of

155 RAM. To identify the rank of the analyzed algorithms on the entire datasets, and number of
156 templates based on the performance indices I_d and running time, the rank of each protocol was
157 identified and then combined using the Rank Product (RP) (Breitling et al. , 2004). Each criterion
158 was considered as a ranker ($k = 2$). The overall RP for each superimposed MUAP resolution
159 method (g) was estimated as below:

$$160 \quad RP(g) = \left(\prod_{i=1}^k r_{g,i} \right)^{1/k} \quad (\text{Eq.1})$$

161 where $r_{g,i}$ is the rank of the method g of the $i - th$ ranker. RP values were then ordered in the
162 ascending order.

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164 **Statistical Methods**

165 Continuous variables were reported as MEAN \pm SD. Generalized Estimating Equation
166 (GEE) method was used for modeling factors associated with repeated responses (i.e., 1000
167 random realizations of superpositions) (Hardin and Hilbe, 2007). GEE was used to find significant
168 factors (MUAP set and the number of constituent templates) affecting the performance/efficiency
169 of the superposition resolution algorithms. Also, Friedman Post hoc statistical test was used to
170 identify which algorithm significantly outperformed the others. The level of statistical significance
171 of $p = 0.05$ was used in our study. The statistical analysis was performed using SPSS Statistics for
172 Windows version 22 (IBM Corp. Released 2013. Armonk, NY: IBM Corp.).

173

174 **Results**

175 The performance of the algorithms (BB, GA, PSO, PO, and POI) was assessed on sets 1
176 and 2 with superpositions involving $N = 2$ to $N = 5$ constituent templates) (Table 1).

177 **[Table 1 is included here].**

178 I_d analysis:

179 Data set, algorithm, and the number of templates were statistically significant ($p < 0.05$).

180 PSO algorithm was statistically better than the other algorithms ($p < 0.05$) followed by BB, GA,

181 POI, and finally, PO.

182

183 Running Time Analysis:

184 Data set and algorithm were statistically significant ($p < 0.05$). The PO algorithm was

185 significantly better than the other algorithms ($p < 0.05$) followed by POI, GA, BB, and finally

186 PSO.

187 To better analyze the algorithms, the cases in which the PO was not successful ($I_d < 30\%$),

188 i.e., cases involving constructive or destructive superposition, were re-analyzed separately (which

189 we will refer to as the hard set). The results for BB, GA, PSO are shown in Table 2.

190

191 **[Table 2 is included here].**

192

193 I_d analysis on the hard set:

194 Data set, algorithms, and the number of templates were statistically significant. BB

195 algorithm was statistically better than others ($p < 0.05$), followed by PSO, GA.

196

197 Running Time analysis on the hard set:

198 Data set, algorithm, and the number of templates were statistically significant. The POI
199 algorithm was significantly better than the other algorithms ($p < 0.05$) followed by GA, BB, and
200 finally PSO.

201

202 The overall results using Rank Product:

203 When combining the I_d and running time based on the Rank Product formula (Eq. 1), the
204 PSO outperformed the other algorithms following by BB, GA, PO, and POI in the first dataset.
205 For the second dataset, the ranking of the algorithm was BB, PSO, GA, PO, POI. In the first hard
206 dataset, the ranking was PSO, BB, and GA, while in the second hard dataset, the ranking was BB,
207 PSO, GA.

208 Combining the results of the first and second datasets, the overall ranking was BB, PSO,
209 GA, PO, and POI. In the hard datasets, the ranking was BB, PSO, and GA. Thus, we conclude
210 that the BB algorithm outperformed the other algorithms, considering different normal/hard
211 datasets.

212 It was shown in the literature that the alignment error is affected by the SNR of the
213 superposition. Thus, the overall identification of the BB algorithm for superpositions involving 2
214 constituent templates of the first set at different noise levels is shown in Fig. 3. The performance
215 of the algorithm was identical from the no-noise condition to the noise level of 0.05, which was
216 used in the current study. The identification rate was higher than 90% when the noise level was
217 0.1 or less.

218

219 **[Fig. 3 is included here].**

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223 **Discussion**

224 In this article, we analyze the most important and popular algorithms that are used for
225 resolving superposition in iEMG decomposition. This study aimed to compare all of these
226 algorithms in hard conditions in terms of both accuracy and computation time.

227 For creating a hard condition, we used two datasets, one of which included MUAPs with a
228 variety of energies and one of which included MUAPs with similar shapes. We simulated 8000
229 superpositions that covered all of the possible types of superpositions (constructive, destructive,
230 and partial).

231 The PO algorithm is the fastest and easiest algorithm for resolving superpositions, but
232 because it is based on finding the highest similarity between the templates and the superposition,
233 it does not do well when it encounters constructive and destructive superpositions. The
234 performance did not improve when signals were upsampled by a factor of 10 (POI), showing that
235 the difficulty is not merely a result of time quantization, but reflects the difficulty of resolving
236 superpositions in which none of the constituent templates are immediately recognizable. PSO and
237 GA algorithms are population-based algorithms, and there is no guarantee that they can reach the
238 global minimum. Also, their speed is not fast enough for real-time applications. The BB algorithm
239 intelligently searches for the global minimum and, in many cases, finds it quickly. However,
240 because resolving a superposition is a non-convex problem and because the search space may be
241 quite large, it sometimes takes many iterations to find the global minimum and prove that it is
242 indeed the global minimum.

243 Because the PO algorithm was not able to resolve many of the constructive and destructive
244 superpositions, we decided to look at those cases more closely (hard set). The analysis showed
245 that all the other algorithms had a higher I_d on the hard set than on the full set. It seems that BB,
246 PSO, and GA also have some problems in resolving the partial superpositions, and when these
247 cases were removed, the accuracy of these algorithms increased.

248 Although the BB algorithm is fast in some cases, there are other cases in which the BB
249 algorithm spends too much time. For example, for 5 constituent templates from set number 2, the
250 BB algorithm had a maximum computation time of 21.88 s, which is a massive time for resolving
251 superpositions of MUAPs.

252 The BB, PSO, and GA algorithms have a high enough accuracy to be suitable for
253 applications that require accurate full decomposition. However, at present, their use is limited to
254 offline applications, since they are too slow for real-time use.

255

256 **Declaration of Competing Interest**

257 The authors declared that there is no conflict of interest.

258

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262 **References**

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Table I: The performance of the analyzed MUAP superimposed resolution algorithms on sets 1

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and 2 in MEAN±SD [minimum,maximum].

Number of Templates (N)	algorithm	Set 1		Set 2	
		$I_d\%$ + std [max]	Time(ms) + std [max]	$I_d\%$ + std [max]	Time(ms) + std [max]
2	BB	0.99±0.02 [1.00]	17 ± 3 [35]	1.00±0.00 [1.00]	19±4 [35]
	GA	0.99±0.08 [1.00]	43.11 ± 40.31 [930.51]	1.00±0.03 [1.00]	41±48 [672]
	PSO	1.00±0.00 [1.00]	1564 ± 236 [5565]	1.00±0.00 [1.00]	1561±196 [4018]
	PO	0.49±0.32 [1.00]	0.23±0.09 [1.99]	0.47±0.37 [1.00]	0.15±0.05 [0.51]
	POI	0.52±0.34 [1.00]	1.80±0.15 [4.07]	0.49±0.38 [1.00]	1.80±0.15 [3.03]
3	BB	0.99±0.07 [1.00]	43 ± 18 [166]	1.00±0.00 [1.00]	67 ± 25 [189]
	GA	0.97±0.12 [1.00]	157 ± 142 [1294]	0.99±0.11 [1.00]	108 ± 131 [1476]
	PSO	0.99±0.05 [1.00]	3070 ± 488 [6598]	1.00±0.00 [1.00]	3078 ± 496 [6523]
	PO	0.32±0.21 [1.00]	0.23±0.09 [1.11]	0.30±0.23 [1.00]	0.19±0.06 [0.56]
	POI	0.33±0.22 [1.00]	2.57±0.48 [6.42]	0.32±0.25 [1.00]	2.32±0.14 [3.38]
4	BB	0.98±0.10 [1.00]	144 ± 107 [1317]	1.00±0.02 [1.00]	384 ± 230 [1724]
	GA	0.93±0.17 [1.00]	272 ± 268 [3346]	0.97±0.14 [1.00]	198 ± 182 [2119]
	PSO	0.98±0.10 [1.00]	7113 ± 557 [10076]	0.99±0.07 [1.00]	7210 ± 591 [12663]
	PO	0.26±0.17 [1.00]	0.21±0.06 [0.48]	0.25±0.18 [1.00]	0.23±0.06 [0.45]
	POI	0.27±0.17 [1.00]	3.02±0.42 [6.94]	0.26±0.19 [1.00]	2.86±0.16 [3.92]
5	BB	0.94±0.16 [1.00]	748 ± 1092 [18307]	0.99±0.06 [1.00]	3633 ± 3001 [21888]
	GA	0.84±0.25 [1.00]	326 ± 169 [2375]	0.87±0.28 [1.00]	328 ± 179 [1785]
	PSO	0.95±0.15 [1.00]	10940 ± 1132 [16762]	0.94±0.20 [1.00]	11012 ± 1002 [16405]
	PO	0.21±0.14 [1.00]	0.24±0.06 [0.56]	0.21±0.14 [0.80]	0.24±0.07 [0.50]
	POI	0.21±0.15 [1.00]	3.47±0.29 [6.49]	0.21±0.15 [0.80]	3.41±0.27 [8.38]

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BB: Branch and Bound; GA: Genetic Algorithm; PSO: Particle Swarm Optimization; PO: Peel-Off; POI: The

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interpolation of the signals by the factor of 10 and then running Peel-off.

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341 **Table II:** The performance of the analyzed MUAP superimposed resolution algorithms on
 342 selected hard sets 1 and 2 where the Peel-off algorithm was not successful in MEAN±SD
 343 [minimum,maximum].

Number of Templates (N)	algorithm	Hard Set 1		Hard Set 2	
		$I_d\%$ + std [max]	Time(ms) + std [max]	$I_d\%$ + std [max]	Time(ms) + std [max]
2	BB	1.00±0.00 [1.00]	17 ± 4 [35]	1.00±0.00 [1.00]	19 ± 4 [33]
	GA	1.00±0.00 [1.00]	26 ± 10 [82]	0.99±0.06 [1.00]	31 ± 34 [495]
	PSO	1.00±0.00 [1.00]	1553 ± 150 [2485]	1.00±0.00 [1.00]	1568 ± 221 [3825]
3	BB	0.99±0.05 [1.00]	41 ± 17 [166]	1.00±0.00 [1.00]	65 ± 25 [189]
	GA	0.98±0.11 [1.00]	146 ± 138 [1294]	0.99±0.11 [1.00]	93 ± 93 [1237]
	PSO	1.00±0.03 [1.00]	3079 ± 504 [5620]	1.00±0.00 [1.00]	3089 ± 496 [6482]
4	BB	0.98±0.09 [1.00]	141 ± 109 [1317]	1.00±0.00 [1.00]	390 ± 237 [1724]
	GA	0.94±0.16 [1.00]	274 ± 260 [2717]	0.97±0.14 [1.00]	188 ± 164 [2119]
	PSO	0.99±0.08 [1.00]	7120 ± 577 [9882]	0.99±0.08 [1.00]	7200 ± 579 [9805]
5	BB	0.94±0.16 [1.00]	723 ± 1144 [18307]	0.99±0.06 [1.00]	3626 ± 2970 [21888]
	GA	0.85±0.25 [1.00]	320 ± 164 [2375]	0.88±0.27 [1.00]	314 ± 139 [1600]
	PSO	0.95±0.16 [1.00]	10938 ± 1148 [16762]	0.95±0.20 [1.00]	11011 ± 1019 [16405]

344 BB: Branch and Bound; GA: Genetic Algorithm; PSO: Particle Swarm Optimization.

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355 **Captions to illustrations**

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357 **Fig. 1:** Ten MUAP templates in set 1 (top) and some examples of superpositions with 2-5
358 constituent MUAPs (bottom).

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362 **Fig. 2:** Six MUAP templates in set 2 (top) and some examples of superpositions with 2-5
363 constituent MUAPs (bottom).

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366 **Fig. 3:** The overall identification of the BB algorithm for 2-constituent superpositions from the
367 first set at different noise levels. The standard deviation of the added noise was the noise level
368 multiplied by the peak-to-peak of the superimposed signal.

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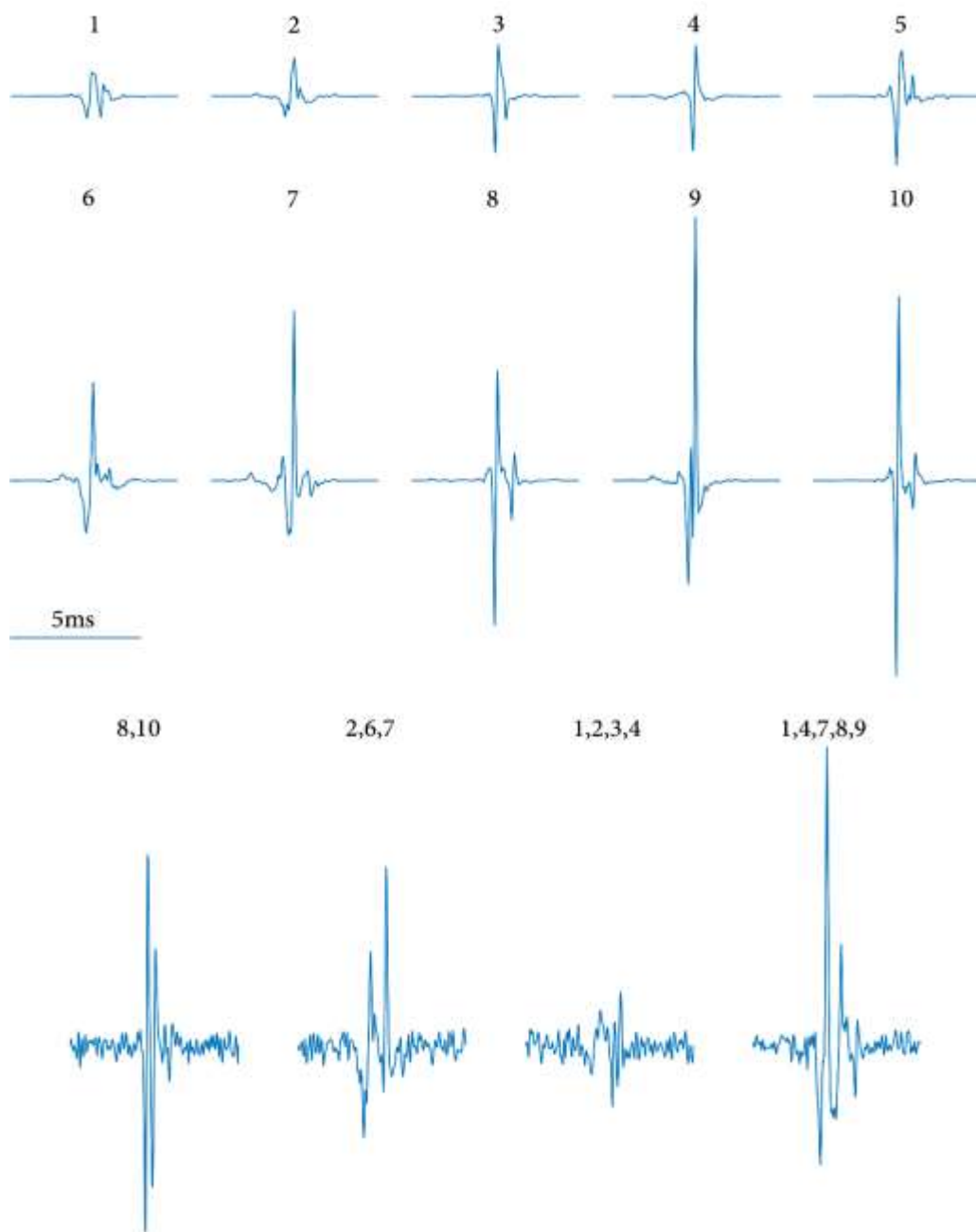
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382 **Fig.1**

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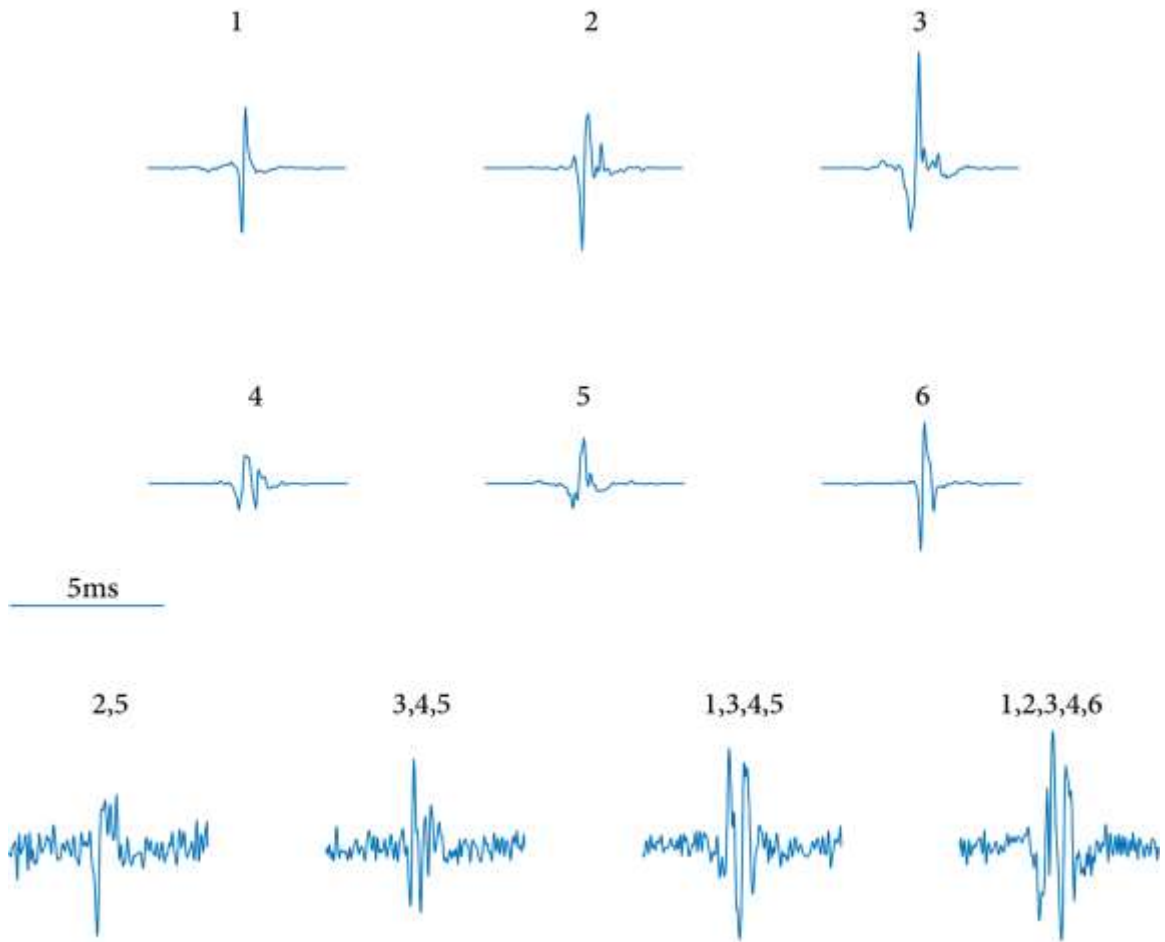
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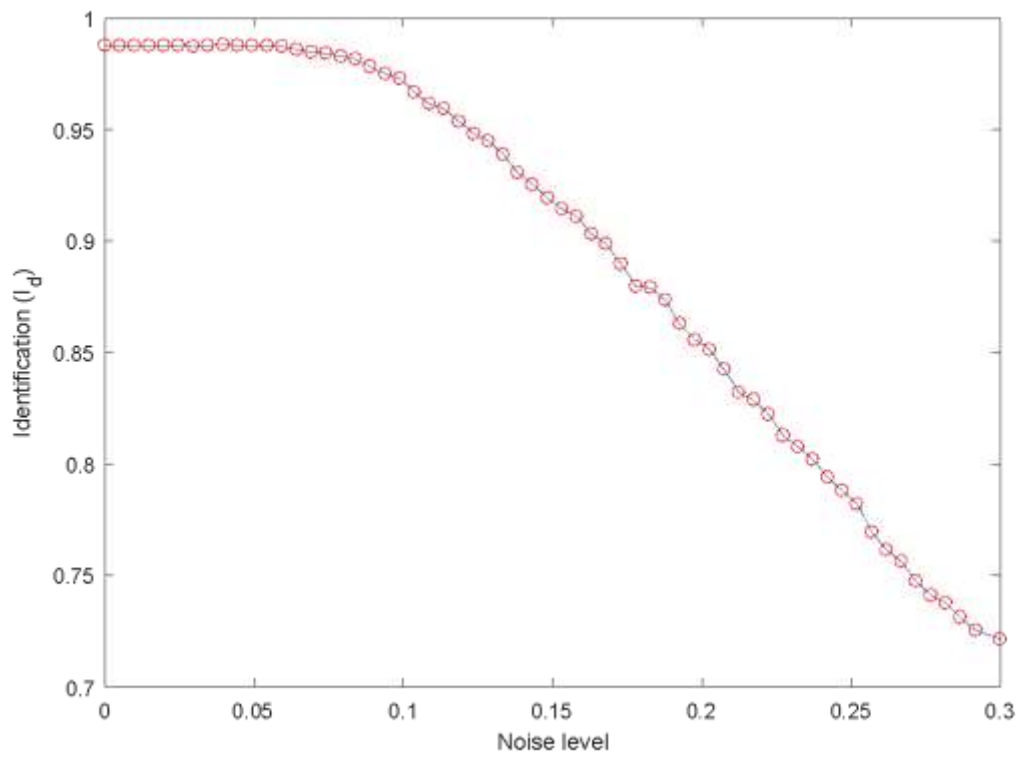
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390 **Fig.2**



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404 **Fig. 3**



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