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The Chebyshev center as an alternative to the analytic center in the feasibility pump

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Abstract
As a heuristic for obtaining feasible points of mixed integer linear problems, the feasibility pump (FP) generates two sequences of points: one of feasible solutions for the relaxed linear problem; and another of integer points obtained by rounding the linear solutions. In a previous work, the present authors proposed a variant of FP, named analytic center FP, which obtains integer solutions by rounding points in the segment between the linear solution and the analytic center of the polyhedron of the relaxed problem. This work introduces a new FP variant that replaces the analytic center with the Chebyshev center. Two of the benefits of using the Chebyshev center are: (i) it requires the solution of a linear optimization problem (unlike the analytic center, which involves a convex nonlinear optimization problem for its exact solution); and (ii) it is invariant to redundant constraints (unlike the analytic center, which may not be well centered within the polyhedron for problems with highly rank-deficient matrices). The computational results obtained with a set of more than 200 MIPLIB2003 and MIPLIB2010 instances show that the Chebyshev center FP is competitive and can serve as an alternative to other FP variants.

Keywords Chebyshev center · Analytic center · Interior-point methods · Mixed-integer linear programming · Feasibility pump · Feasibility problem · Primal heuristics · Large-scale optimization

1 Introduction

Given the mixed-integer linear problem (MILP)

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$$\begin{aligned} \min_x \quad & c^T x \\ \text{s. to} \quad & Ax = b \\ & x \geq 0 \\ & x_j \quad \textit{integer} \quad \forall j \in \mathcal{I}, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c \in \mathbb{R}^n, \mathcal{I} \subseteq \mathcal{N} = \{1, \dots, n\}$ and $\mathcal{P} = \{x \in \mathbb{R}^n : Ax = b, x \geq 0\}$ (that is, $\mathcal{P}$ is the feasible region of the linear relaxation of (1)), finding a feasible point of (1) is a challenging (NP-hard) problem. Many heuristics have been developed for obtaining feasible (hopefully good) solutions of (1). In this paper, we focus on the *feasibility pump* (FP) [5, 12], which has proven to be a successful heuristic, not only for linear problems but also for nonconvex nonlinear problems [4, 7, 9].

Briefly, FP alternates between two sequences of points: one of feasible solutions for the linear relaxation of (1), and another of integer points, that hopefully converge to a feasible integer solution. The integer point is obtained by applying some rounding procedure to the feasible solution of the linear relaxation.

Several strides have been made to further develop the original FP. In [1], the authors take the objective function of the MILP into account at each iteration of the algorithm in order to find better quality solutions. This approach was named *objective FP*. In [13], a new improved rounding scheme based on constraint propagation was introduced. Interior-point methods were applied to primal heuristics in [3] (an approach named *analytic center FP* or AC-FP) and [14] (resulting in the analytic center feasibility method or ACFM). Although both AC-FP and ACFM used the analytic center of $\mathcal{P}$, they are significantly different. In particular, AC-FP (which is briefly outlined in Sect. 3.1) relies on FP and it computes only one analytic center, while ACFM is based on a cutting plane method and it computes an analytic center at each iteration of the algorithm.

AC-FP explores non-integer points in the segment where the feasible point of the linear relaxation of (1) joins the analytic center. The motivation behind using the analytic center lies in the fact that rounding an interior point increases the chances of finding a feasible integer solution. AC-FP was proven in [3] to be a successful heuristic, namely by improving the standard FP in several tested MILP instances. In [6] the authors extended the AC-FP idea by enhancing the rounding procedure.

However, in both practice and theory, using the analytic center has two downsides. First, computing the exact analytic center of $\mathcal{P}$ means solving the convex nonlinear optimization problem $\min -\sum_{i=1}^{n} \ln x_i : Ax = b$. An approximate solution to this problem was suggested in [3] by applying a path-following (or barrier) interior-point algorithm to the problem $\min 0 : Ax = b, x \geq 0$. This excludes using the simplex method for computing the analytic center. The second drawback is that, theoretically, a large number of redundant constraints (in problems with highly rank deficient matrices A) may change the location of the analytic center [10]. In this paper, we consider an alternative to the analytic center, named the Chebyshev center. Using the Chebyshev center within FP overcomes the two above drawbacks: the center of $\mathcal{P}$ is not affected by redundant constraints in A (this is clearly shown below in Sect. 3.2); and the center can be computed using either the simplex or barrier algorithms. As this work shows, the *Chebyshev center FP* (CC-FP), for some instances, provides better solutions than objective FP and AC-FP variants.

The paper is organized as follows. Section 2 outlines the FP heuristic. Section 3 describes the use of a generic center point (analytic or Chebyshev center) in the FP heuristic, while Sect. 3.2 focuses on the computation of the Chebyshev center. Section 4 presents extensive computational results on a subset of MIPLIB2003 [2] and MIPLIB2010 [11] instances, in order to compare the objective FP, AC-FP and CC-FP, as well as to show the effectiveness of the CC-FP approach. Finally, we present some closing remarks in Sect. 5.

2 The feasibility pump heuristic

The original feasibility pump heuristic [12] works iteratively with two points: one (x^*) is feasible for the continuous relaxation of (1), although it is possibly integer infeasible, and the other ($\tilde{x}$) is integral but might not be in $\mathcal{P}$. The point x^* is set to the optimal solution of the linear programming relaxation of (1) while $\tilde{x}$ is obtained by rounding x^* to the closest integer point, as follows:

$$\tilde{x}_j = \begin{cases} [x_j^*] & \textit{if } j \in \mathcal{I} \\ x_j^* & \text{otherwise}, \end{cases} \tag{2}$$

where $[\cdot]$ represents scalar rounding to the nearest integer. Note that the continuous variables x_j, $j \notin \mathcal{I}$, do not play any role. At each iteration of the FP method, x^* is updated by minimizing the following linear optimization problem

$$\begin{aligned} \min_x \ & \Delta(x, \tilde{x}) \\ \text{s. to } & Ax = b \\ & x \geq 0 \end{aligned} \tag{3}$$

where $\Delta(x, \tilde{x})$ is the distance between x and $\tilde{x}$ using the L_1 norm:

$$\Delta(x, \tilde{x}) = \sum_{j \in \mathcal{I}} |x_j - \tilde{x}_j|. \tag{4}$$

Figure 1 illustrates one iteration of the FP method, where t represents the iteration number, and the final point $\tilde{x}_{t+1}$ is integer feasible. FP ends when the distance $\Delta(x, \tilde{x})$ is 0 (meaning we have obtained an integer feasible solution) or when a predefined termination criterion is reached. One of the main drawbacks of the FP heuristic is the possibility of visiting integer points already visited in previous iterations, thereby causing a cycle. To avoid this, a restart procedure is proposed in [12].

The FP implementation has three stages. In the first stage, the method considers only the set of binary variables by relaxing the integrality conditions on the general integer variables. In the second stage, FP takes all integer variables into account and uses the best point $\tilde{x}$ obtained at *Stage 1* as a starting point. Both stages terminate as soon as a feasible solution is found or when some termination criterion is reached (e.g., the best $\Delta(x, \tilde{x})$ is not updated during a certain number of iterations, or the maximum number of

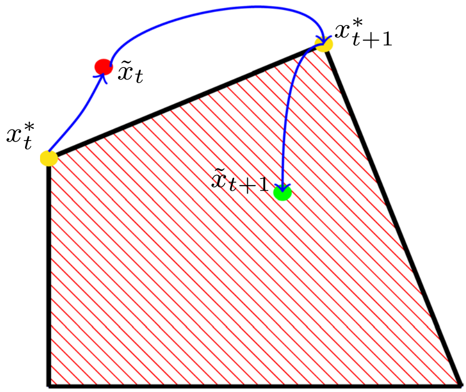


Fig. 1 Graphical representation of the FP method

iterations is reached—just to name two). The last stage (*Stage 3*) starts when FP cannot find a feasible solution to (1) within the established time limit. In this stage, a commercial solver is applied to (1) (CPLEX 12.7 is used in this work), for which the best point obtained from *Stage 2* is used as a starting point. *Stage 3* stops as soon as a feasible solution is found. An outline of the FP algorithm is shown in Fig. 2, and further details can be found in [5, 12].

Despite the successful results obtained by the original FP heuristic for finding feasible solutions of MILPs in a short computational time, using the objective function of (1) only at the beginning of the procedure often leads to a rather poor solution. To avoid this, a modified FP heuristic called objective FP was proposed in [1], which considers a convex combination of $\Delta(x, \tilde{x})$ and the objective function of (1). The idea is to focus the search for feasible solutions near the region of high-quality points. The modified objective function $\Delta_\alpha(x, \tilde{x})$ is defined as

$$\Delta_\alpha(x, \tilde{x}) := (1-\alpha)\Delta(x, \tilde{x}) + \alpha \frac{\|\Delta\|}{\|c\|} c^T x, \quad \alpha \in [0, 1], \tag{5}$$

initialize $t := 0$ and $x^* := \arg\min\{c^T x : Ax = b, x \geq 0\}$
if $x^*_{\mathcal{I}}$ is integer **then** return(x^*) **end if**
$\tilde{x} := [x^*]$ (rounding of x^*)
while time $<$ TimeLimit **do**
    $x^* := \arg\min\{\Delta(x, \tilde{x}) : Ax = b, x \geq 0\}$
    if $x^*_{\mathcal{I}}$ is integer **then** return(x^*) **end if**
    if $\exists j \in \mathcal{I} : [x^*_j] \neq \tilde{x}_j$ **then**
        $\tilde{x} := [x^*]$
    else
        restart
    end if
    $t := t + 1$
end while
return(FP failed)

Fig. 2 The feasibility pump heuristic (original version) [5, 12] where $\|\cdot\|$ is the Euclidean norm of a vector, and Δ is the objective function vector of $\Delta(x, \tilde{x})$ (i.e., the number of either binary variables at Stage 1, or both integer and binary variables at Stage 2). The weight α is reduced at every iteration. When $\alpha = 0$, the original FP heuristic is obtained. Note that the objective FP algorithm is nearly identical to the original FP algorithm in Fig. 2; it simply replaces $\Delta(x, \tilde{x})$ with $\Delta_{\alpha_t}(x, \tilde{x})$ in line 5 and adds the proper initialization and updating of α. Further details can be found in [1].

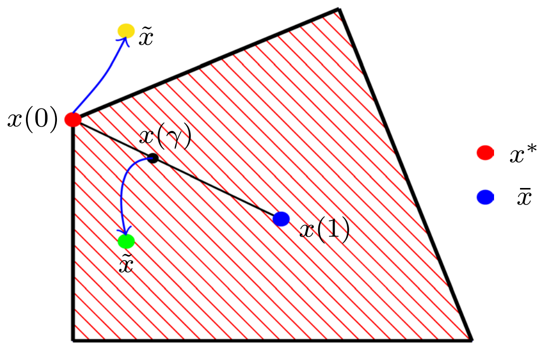
3 Using a center point in the feasibility pump

Let $\bar{x} \in \mathbb{R}^n$ be an interior point of polyhedron $\mathcal{P}$, that is, $A\bar{x} = b$ and $\bar{x} > 0$ (strictly positive components). All the points in the segment $\overline{\bar{x}x^*}$ are feasible, since they are a convex combination of two feasible points, x^* and $\bar{x}$, and therefore candidates to be rounded. In addition, all the points except x^* in the segment $\overline{\bar{x}x^*}$ are interior, thus increasing the chances for the rounded point to be feasible for $\mathcal{P}$. Of all interior points $\bar{x}$, those that are "well centered" inside $\mathcal{P}$ (let them be the *center points*) are the best choices. The above generic approach is named the *center point feasibility pump* (CP-FP) in this work.

At each iteration CP-FP considers points $x(\gamma) = \gamma\bar{x} + (1-\gamma)x^*$ for several $\gamma \in [0,1]$. Each $x(\gamma)$ is rounded to $\tilde{x}(\gamma)$ following (2). If $\tilde{x}(\gamma)$ is feasible, then a feasible integer solution is found and the procedure is stopped. Otherwise, CP-FP continues to consider the new integral point to be the one that is closest to $\mathcal{P}$ from all the points $\tilde{x}(\gamma)$ (the ℓ_∞ distance between $\mathcal{P}$ and $\tilde{x}(\gamma)$ is used to measure closeness). If more than one integer point $\tilde{x}(\gamma)$ is feasible for $\mathcal{P}$, CP-FP selects the one closest to x^* (which may probably have a better objective value). Figure 3 illustrates the behaviour of CP-FP: while the standard objective FP would provide the infeasible yellow point, CP-FP could deliver the feasible green point. An outline of the algorithm is shown in Fig. 4. Note that if $x(\gamma = 0)$ is selected at each iteration, CP-FP behaves exactly as the objective FP. Further details are given in [3].

There are several ways to get the center point. One option is the *analytic center* of the polyhedron, which was used in [3] with promising results. The main drawback of the analytic center is that redundant constraints can push it near the boundary of

Fig. 3 Illustration of CP-FP



```
initialize t := 0, α_0 ∈ [0,1], φ ∈ [0,1], and x* := arg min{c^T x : Ax = b, x ≥ 0}
compute the center point x̄
{ Beginning of stage 0}
for γ ∈ [0,1] do
    x(γ) := γx̄ + (1 − γ)x*
    x̃(γ) := [x(γ)] (rounding of x(γ))
    if x̃(γ) is feasible then return(x̃(γ)) end if
end for
{ End of stage 0}
select x̃ from the set {x̃(γ)}
while time < TimeLimit do
    x* := arg min{Δ_{α_t}(x, x̃) : Ax = b, x ≥ 0}
    for γ ∈ [0,1] do
        x(γ) := γx̄ + (1 − γ)x*
        x̃(γ) := [x(γ)] (rounding of x(γ))
        if x̃(γ) is feasible then return(x̃(γ)) end if
    end for
    select x̂ from the set {x̃(γ)}
    if x̂_I ≠ x̃_I then
        x̃ := x̂
    else
        restart
    end if
    α_{t+1} := φα_t
    t := t + 1
end while
return(FP failed)
```

Fig. 4 The center point feasibility pump heuristic (CP-FP) [3]

the polyhedron [10], as is shown in Sect. 3.2. To overcome this issue, we suggest in this paper using the *Chebyshev center*. Both center points are briefly outlined below.

3.1 The analytic center

The analytic center of $\mathcal{P}$ is defined as the point $\bar{x}$ that minimizes the *primal potential function* $-\sum_{i=1}^{n} \ln x_i$, i.e.,

$$\begin{array}{ll} \bar{x} = \arg\min\limits_{x} & -\sum_{i=1}^{n} \ln x_i \\ & \text{s. to } Ax = b \quad [\text{ and } x > 0]. \end{array} \tag{6}$$

Constraints $x > 0$ can be avoided, since the domain of ln are the positive numbers, and then (6) is an equality constrained strictly convex optimization problem. It is easily seen that $\bar{x}$ is also the solution of $\max \prod_{i=1}^{n} x_i \,:\, Ax = b$; that is, the analytic center attempts to maximize the distance to the hyperplanes $x_i = 0, i = 1, \dots, n$, and it is thus expected to be well centered in the interior of $\mathcal{P}$. Note that the analytic center is not a topological property of a polytope, and it depends on how $\mathcal{P}$ is defined through $Ax = b$ [15].

The analytic center solves the KKT conditions of (6), which can be recast as

$$\begin{aligned} Ax &= b \\ A^T y + s &= 0 \\ x_i s_i &= 1 \quad i = 1, \dots, n \\ (x, s) &> 0, \end{aligned} \tag{7}$$

$y \in \mathbb{R}^m$ and $s \in \mathbb{R}^n$ being, respectively, the Lagrange multipliers of $Ax = b$ and an auxiliary vector (associated to $x > 0$). Alternatively, we can make use of an available highly efficient implementation, in which we compute the analytic center by applying a primal-dual path-following interior-point algorithm to the barrier problem of the linear relaxation of (1) after setting $c = 0$, that is,

$$\begin{aligned} \min_x \quad & -\mu \textstyle\sum_{i=1}^n \ln x_i \\ \text{s. to} \quad & Ax = b, \quad x > 0, \end{aligned} \tag{8}$$

where μ is a positive parameter (the parameter of the barrier) that tends to zero. The arc of solutions of the barrier problem for every $\mu > 0$ is named the central path. The central path converges to the analytic center of the optimal set of a linear optimization problem. When $c = 0$ (as in (8)) the central path converges to the analytic center of the feasible set $\mathcal{P}$ [15]. The use of the analytic center in the feasibility pump, introduced in [3], was named the *analytic center FP* (AC-FP).

3.2 The Chebyshev center

Given a convex polyhedron described by linear inequalities

$$\mathcal{Q} = \{x \in \mathbb{R}^n : a_i^T x \le b_i, i = 1, \dots, q\}, \tag{9}$$

the Chebyshev center $\bar{x}$ is the center of the largest inscribed Euclidean ball in $\mathcal{Q}$. A Euclidean ball of center $\bar{x} \in \mathbb{R}^n$ and radius r is the set of all points of distance less than or equal to r from $\bar{x}$, i.e., $\mathcal{B}(\bar{x}, r) = \{\bar{x} + u : \|u\|_2 \le r\}$. The optimization problem that finds the Chebyshev center is [8]

$$\begin{aligned} \max_{\bar{x}, r} \quad & r \\ \text{s. to} \quad & \mathcal{B}(\bar{x}, r) \subseteq \mathcal{Q}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mathcal{B}(\bar{x}, r) \subseteq \mathcal{Q} \quad & \Leftrightarrow \bar{x} + u \subseteq \mathcal{Q} \ \forall u \in \mathcal{B}(0, r) \\ & \Leftrightarrow a_i^T(\bar{x} + u) \le b_i \ \forall u : \|u\|_2 \le r, i = 1, \dots, q. \end{aligned} \tag{11}$$

Since $a_i^T u \le \|a_i\|_2 \|u\|_2 \le \|a_i\|_2 r$, we can then write (10) as the following linear optimization problem:

$$\begin{aligned} \max_{\bar{x}, r} \quad & r \\ \text{s. to} \quad & a_i^T \bar{x} + \|a_i\|_2 r \le b_i \quad \forall i = i, \dots, q. \end{aligned} \tag{12}$$

For the polyhedron $\mathcal{P}$ of the linear relaxation of (1), the Chebyshev center is defined only in terms of the inequalities $x \geq 0$ and is restricted to $Ax = b$, which results in the following problem:

$$\begin{aligned} \max_{\bar{x},r} \ & r \\ \text{s. to } & A\bar{x} = b \\ & -\bar{x}_i + r \leq 0 \ \ \forall i = i, \dots, n. \end{aligned} \tag{13}$$

The Chebyshev center does not change in the presence of redundant constraints, and it is then always well located in a central position inside the polyhedron, thus making it an effective choice for CP-FP. On the other hand, the analytic center can be pushed out near the boundary of $\mathcal{P}$ by redundant constraints. Figures 5 and 6 illustrate this situation with the convex polyhedron $\mathcal{Q}$ described by the following linear inequalities:

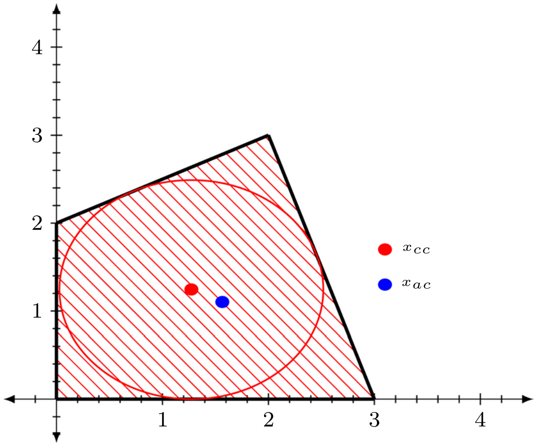


Fig. 5 The Chebyshev (x_{cc}) and analytic (x_{ac}) centers of polyhedron $\mathcal{Q}$ represented by $Qx \leq b$ (without redundant constraints). The largest inscribed Euclidean ball centered at x_{cc} is also shown

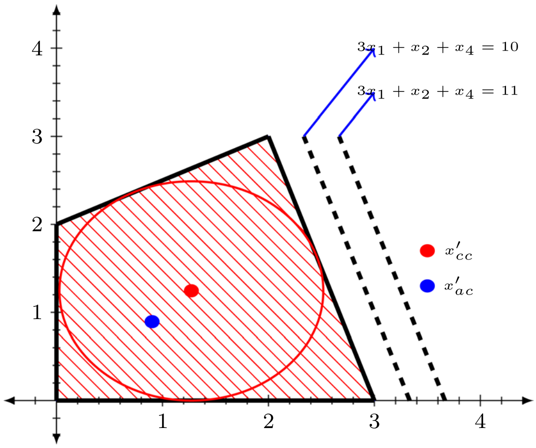


Fig. 6 The Chebyshev (x'_{cc}) and analytic (x'_{ac}) centers of the polyhedron $\mathcal{Q}$ represented by $Q'x \leq b'$ (with redundant constraints). Note that $x'_{ac} \neq x_{ac}$, whereas $x'_{cc} = x_{cc}$

$$\mathcal{Q} = \left\{x \in \mathbb{R}^3 \,:\, Qx \leq b\right\} = \left\{ x \in \mathbb{R}^3 \,:\, \begin{array}{rl} -x_1 + 2x_2 + x_3 & \leq 4 \\ 3x_1 + x_2 + x_4 & \leq 9 \\ -(x_1, x_2) & \leq 0 \end{array} \right\}.$$

Fig. 5 shows the analytic (x_{ac}) and the Chebyshev (x_{cc}) centers of $\mathcal{Q}$, as well as the largest inscribed Euclidean ball centered at x_{cc}. Let us consider an alternative representation of the polyhedron $\mathcal{Q} = \left\{x \in \mathbb{R}^3 \,:\, Q'x \leq b'\right\}$, which is obtained by adding the two redundant constraints $\{3x_1 + x_2 + x_4 \leq 10, 3x_1 + x_2 + x_4 \leq 11\}$ to $Qx \leq b$. Figure 6 shows the analytic (x'_{ac}) and the Chebyshev (x'_{cc}) centers of the new representation $Q'x \leq b'$. Note that $x_{cc} = x'_{cc}$, while x'_{ac} has been pushed out towards the boundary opposite to the redundant constraints. The FP variant based on Chebyshev centers introduced in this work is named *Chebyshev center FP* (CC-FP).

4 Computational experiments

4.1 Implementation and instances

Both AC-FP and CC-FP are implemented in C++ using the base code of the objective FP, which is freely available from https://site.unibo.it/operations-research/en/research/library-of-codes-and-instances-1. The optimization solver CPLEX (version 12.7) is used to solve the linear optimization subproblems. All the runs are carried out on a Fujitsu Primergy RX2540 M1 4X server with two 2.6 GHz Intel Xeon E5-2690v3 CPUs (48 cores) and 192 Gigabytes of RAM, under a GNU/Linux operating system (openSuse 13.2), without exploitation of the multithreading capabilities. A one-hour time limit is imposed on all the runs.

AC-FP, CC-FP and objective FP are tested on a subset of MIPLIB2003 [2] and MIPLIB2010 [11] instances, whose dimensions are shown in Tables 1, 2 and 3. The columns "rows", "cols", "nnz", "int", "bin" and "con" provide respectively the numbers of constraints, variables, nonzeros, general integer variables, binary variables and continuous variables of the instances. The column "objective" shows the optimal objective function.

4.2 Results

We first analyze the results for the subset of MIPLIB2003 instances. Table 4 presents the results obtained. For AC-FP and CC-FP we report the total CPU time spent on stages 0 to 3 ("tFP"); the time for computing the analytic/Chebyshev center ("tAC/tCC"); the stage in which the feasible point is found ("stage"); and the gap between the feasible and the optimal solution ("gap%"). For the objective FP, we report columns "gap%", "stage" and "tFP" with the same meaning as before. The primary goal of this preliminary study is to assess the benefits, if any, of using the Chebyshev center as an alternative to the analytic center. We start by comparing AC-FP with CC-FP. Looking at Table 4, we see 15 instances where AC-FP fails

Table 1 Characteristics of the subset of MILP instances from MIPLIB 2003

Instance_name	rows	cols	nnz	int	bin	con	Objective
10teams	230	2025	12,150	0	1800	225	924.00
a1c1s1	3312	3648	10,178	0	192	3456	11,503.40
aflow30a	479	842	2091	0	421	421	1158.00
aflow40b	1442	2728	6783	0	1364	1364	1168.00
air04	823	8904	72,965	0	8904	0	56,137.00
air05	426	7195	52,121	0	7195	0	26,374.00
arki001	1048	1388	20,439	96	415	877	7,580,810.00
atlanta-ip	21,732	48,738	257,532	106	46,667	1965	90.00
cap6000	2176	6000	48,243	0	6000	0	– 2,451,380.00
danoint	664	521	3232	0	56	465	65.66
disctom	399	10,000	30,000	0	10,000	0	– 5000.00
ds	656	67,732	1,024,059	0	67,732	0	93.52
fast0507	507	63,009	409,349	0	63,009	0	174.00
fiber	363	1298	2944	0	1254	44	405,935.00
fixnet6	478	878	1756	0	378	500	3983.00
gesa2-o	1248	1224	3672	336	384	504	25,779,900.00
gesa2	1392	1224	5064	168	240	816	25,779,900.00
glass4	396	322	1815	0	302	20	1,200,010,000.00
harp2	112	2993	5840	0	2993	0	– 73,899,800.00
manna81	6480	3321	12,960	3303	18	0	– 13,164.00
markshare1	6	62	312	0	50	12	1.00
markshare2	7	74	434	0	60	14	1.00
mas74	13	151	1706	0	150	1	11,801.20
mas76	12	151	1640	0	150	1	40,005.10
misc07	212	260	8619	0	259	1	2810.00
mkc	3411	5325	17,038	0	5323	2	– 563.84
mod011	4480	10,958	22,254	0	96	10,862	– 54,558,500.00
modglob	291	422	968	0	98	324	20,740,500.00
msc98-ip	15,850	21,143	92,918	53	20,237	853	19,839,500.00
mzzv11	9499	10,240	134,603	251	9989	0	– 21,718.00
mzzv42z	10,460	11,717	151,261	235	11,482	0	– 20,540.00
net12	14,021	14,115	80,384	0	1603	12,512	214.00
noswot	182	128	735	25	75	28	– 41.00
nsrand-ipx	735	6621	223,261	0	6620	1	51,200.00
nw04	36	87,482	636,666	0	87,482	0	16,862.00
opt1217	64	769	1542	0	768	1	– 16.00
p2756	755	2756	8937	0	2756	0	3124.00
pk1	45	86	915	0	55	31	11.00
pp08aCUTS	246	240	839	0	64	176	7350.00
pp08a	136	240	480	0	64	176	7350.00
protfold	2112	1835	23,491	0	1835	0	– 31.00
qiu	1192	840	3432	0	48	792	– 132.87

Table 1 (continued)

Instance_name	rows	cols	nnz	int	bin	con	Objective
roll3000	2295	1166	29,386	492	246	428	12,890.00
rout	291	556	2431	15	300	241	1077.56
set1ch	492	712	1412	0	240	472	54,537.80
seymour	4944	1372	33,549	0	1372	0	423.00
sp97ar	1761	14,101	290,968	0	14,101	0	660,706,000.00
swath	884	6805	34,965	0	6724	81	467.40
timtab1	171	397	829	94	64	239	764,772.00
timtab2	294	675	1482	164	113	398	1,096,560.00
tr12-30	750	1080	2508	0	360	720	130,596.00
vpm2	234	378	917	0	168	210	13.75

Table 2 Characteristics of the subset of MILP instances from MIPLIB 2010 (Part I)

Instance_name	rows	cols	nnz	int	bin	con	Objective
30n20b8	576	18,380	109,706	62	18,318	0	302.00
50v-10	233	2013	2745	183	1464	366	3311.18
CMS750_4	16,381	11,697	44,903	0	7196	4501	252.00
app1-1	4926	2480	18,275	0	1225	1255	− 3.00
app1-2	53,467	26,871	199,175	0	13,300	13,571	− 41.00
assign1-5-8	161	156	3720	0	130	26	212.00
b1c1s1	3904	3872	11,408	0	288	3584	24,544.25
bab2	17,245	147,912	2,027,730	0	147,912	0	− 357,544.31
bab6	29,904	114,240	1,283,180	0	114,240	0	− 284,248.23
beasleyC3	1750	2500	5000	0	1250	1250	754.00
binkar10_1	1026	2298	4496	0	170	2128	6742.20
blp-ar98	1128	16,021	200,601	0	15,806	215	6205.21
blp-ic98	717	13,640	191,947	0	13,550	90	4491.45
bppc4-08	111	1456	23,964	0	1454	2	53.00
cbs-cta	10,112	24,793	64,388	0	2467	22,326	0.00
chromaticindex1024-7	67,583	73,728	270,324	0	73,728	0	4.00
chromaticindex512-7	33,791	36,864	135,156	0	36,864	0	4.00
cmflsp50-24-8-8	3520	16,392	158,622	0	1392	15,000	55,789,389.89
co-100	2187	48,417	1,995,820	0	48,417	0	2,639,942.06
cod105	1024	1024	57,344	0	1024	0	− 12.00
comp07-2idx	21,235	17,264	86,577	109	17,155	0	6.00
comp21-2idx	14,038	10,863	57,301	71	10,792	0	74.00
cost266-UUE	1446	4161	12,312	0	171	3990	25,148,940.56
csched007	351	1758	6379	0	1457	301	351.00
csched008	351	1536	5687	0	1284	252	173.00
cvs16r128-89	4633	3472	12,528	0	3472	0	− 97.00
dano3_3	3202	13,873	79,655	0	69	13,804	576.34
dano3_5	3202	13,873	79,655	0	115	13,758	576.92

Table 2 (continued)

Instance_name	rows	cols	nnz	int	bin	con	Objective
decomp2	10,765	14,387	64,073	0	14,387	0	– 160.00
drayage-100-23	4630	11,090	41,550	0	11,025	65	103,333.87
drayage-25-23	4630	11,090	41,550	0	11,025	65	101,282.65
dws008-01	6064	11,096	56,400	0	6608	4488	37,412.60
eil33-2	32	4516	44,243	0	4516	0	934.01
eilA101-2	100	65,832	959,373	0	65,832	0	880.92
enlight_hard	100	200	560	100	100	0	37.00
exp-1-500-5-5	550	990	1980	0	250	740	65,887.00
fastxgemm-n2r6s0t2	5998	784	19,376	0	48	736	230.00
fiball	3707	34,219	104,792	258	33,960	1	138.00
gen-ip002	24	41	922	41	0	0	– 4783.73
gen-ip054	27	30	532	30	0	0	6840.97
germanrr	10,779	10,813	175,547	5251	5323	239	47,095,869.65
glass-sc	6119	214	63,918	0	214	0	23.00
gmu-35-40	424	1205	4843	0	1200	5	– 2,406,733.37
gmu-35-50	435	1919	8643	0	1914	5	– 2,607,958.33
graph20-20-1rand	5587	2183	19,277	0	2183	0	– 9.00
graphdraw-domain	865	254	2600	20	180	54	19,686.00
h80x6320d	6558	12,640	31,521	0	6320	6320	6382.10
hypothyroid-k1	5195	2602	433,884	1	2601	0	– 2851.00
ic97_potential	1046	728	3138	73	450	205	3942.00
icir97_tension	1203	2494	22,333	573	262	1659	6375.00
irp	39	20,315	98,254	0	20,315	0	12,159.49
istanbul-no-cutoff	20,346	5282	71,477	0	30	5252	204.08
k1mushroom	16,419	8211	1,697,950	1	8210	0	– 3288.00
leo1	593	6731	131,218	0	6730	1	404,227,536.16
leo2	593	11,100	219,959	0	11,099	1	404,077,441.12
lotsize	1920	2985	6565	0	1195	1790	1,480,195.00
mad	51	220	2808	0	200	20	0.03
map10	328,818	164,547	549,920	0	146	164,401	– 495.00
map16715-04	328,818	164,547	549,920	0	146	164,401	– 111.00
markshare_4_0	4	34	123	0	30	4	1.00
mc11	1920	3040	6080	0	1520	1520	11,689.00
mcsched	2107	1747	8088	0	1745	2	211,913.00
mik-250-20-75-4	195	270	9270	175	75	20	– 52,301.00
milo-v12-6-r2-40-1	5628	2688	14,604	0	840	1848	326,481.14
momentum1	42,680	5174	103,191	0	2349	2825	109,143.49
mushroom-best	8580	8468	188,735	118	8237	113	0.06
n2seq36q	2565	22,480	183,292	0	22,480	0	52,200.00
n3div36	4484	22,120	340,740	0	22,120	0	130,800.00
n5-3	1062	2550	9900	150	0	2400	8105.00
neos-1171448	13,206	4914	131,859	0	2457	2457	– 309.00

Table 2 (continued)

Instance_name	rows	cols	nnz	int	bin	con	Objective
neos-1171737	4179	2340	58,620	0	1170	1170	– 195.00
neos-1354092	3135	13,702	187,187	420	13,282	0	46.00
neos-1445765	2147	20,617	40,230	0	2150	18,467	– 17,783.00
neos-1456979	6770	4605	36,440	180	4245	180	176.00
neos-1582420	10,180	10,100	24,814	100	10,000	0	91.00
neos-2657525-crna	342	524	1690	378	146	0	1.81
neos-2978193-inde	396	20,800	41,600	0	64	20,736	– 2.39
neos-2987310-joes	29,015	27,837	580,291	0	3051	24,786	– 607,702,988.30
neos-3024952-loue	3705	3255	17,310	3255	0	0	26,756.00
neos-3046615-murg	498	274	1266	16	240	18	1600.00
neos-3083819-nubu	4725	8644	24,048	8644	0	0	6,307,996.00
neos-3216931-puriri	5989	3555	91,691	0	3268	287	71,320.00
neos-3381206-awhea	479	2375	4275	1900	475	0	453.00
neos-3402294-bobin	591,076	2904	2,034,890	0	2616	288	0.07
neos-3555904-turama	146,493	37,461	793,605	0	37,461	0	– 34.70
neos-3627168-kasai	1655	1462	5158	0	535	927	988,585.62
neos-3754480-nidda	402	253	1488	0	50	203	12,941.74
neos-4300652-rahue	76,992	33,003	183,616	0	20,900	12,103	2.14
neos-4338804-snowy	1701	1344	6342	42	1260	42	1471.00
neos-4387871-tavua	4554	4004	23,496	0	2000	2004	33.38
neos-4413714-turia	2303	190,402	761,756	0	190,201	201	45.37
neos-4532248-waihi	167,322	86,842	525,339	0	86,841	1	61.60
neos-4647030-tutaki	8382	12,600	3,953,390	0	7000	5600	27,265.71
neos-4722843-widden	113,555	77,723	311,529	20	73,349	4354	25,009.66
neos-4738912-atrato	1947	6216	19,521	5096	1120	0	283,627,956.60

Table 3 Characteristics of the subset of MILP instances from MIPLIB 2010 (Part II)

Instance_name	rows	cols	nnz	int	bin	con	Objective
neos-4763324-toguru	106,954	53,593	266,805	0	53,592	1	1613.04
neos-4954672-berkel	1848	1533	8007	0	630	903	2,612,710.00
neos-5093327-huahum	51,840	40,640	784,768	0	64	40,576	6260.00
neos-5107597-kakapo	6498	3114	19,392	0	2976	138	3645.00
neos-5188808-nattai	29,452	14,544	133,686	0	288	14,256	0.11
neos-5195221-niemur	42,256	14,546	176,586	0	9792	4754	0.00
neos-631710	169,576	167,056	834,166	0	167,056	0	203.00
neos-662469	1085	18,235	200,055	328	17,907	0	184,380.00
neos-787933	1897	236,376	298,320	0	236,376	0	30.00
neos-827175	14,187	32,504	110,790	0	21,350	11,154	112.00
neos-848589	1484	550,539	1,101,080	0	747	549,792	2351.40

Table 3 (continued)

Instance_name	rows	cols	nnz	int	bin	con	Objective
neos-860300	850	1385	384,329	0	1384	1	3201.00
neos-873061	93,360	175,288	350,576	0	87,644	87,644	113.66
neos-911970	107	888	3408	0	840	48	54.76
neos-933966	12,047	31,762	180,618	0	27,982	3780	318.00
neos-957323	3757	57,756	499,656	0	57,756	0	− 237.76
neos-960392	4744	59,376	189,503	0	59,376	0	− 238.00
neos17	486	535	4931	0	300	235	0.15
neos5	63	63	2016	0	53	10	15.00
neos8	46,324	23,228	313,180	4	23,224	0	− 3719.00
netdiversion	119,589	129,180	615,282	0	129,180	0	242.00
nexp-150-20-8-5	4620	20,115	42,465	0	17,880	2235	231.00
ns1208400	4289	2883	81,746	0	2880	3	2.00
ns1830653	2932	1629	100,933	0	1458	171	20,622.00
nu25-pr12	2313	5868	17,712	36	5832	0	53,905.00
nursesched-sprint02	3522	10,250	204,000	20	10,230	0	58.00
opm2-z10-s4	160,633	6250	371,240	0	6250	0	− 33,269.00
p200x1188c	1388	2376	4752	0	1188	1188	15,078.00
pg	125	2700	5200	0	100	2600	− 8674.34
pg5_34	225	2600	7700	0	100	2500	− 14,339.35
piperout-08	14,589	10,399	44,959	130	10,245	24	125,055.00
piperout-27	18,442	11,659	54,662	121	11,514	24	8124.00
proteindesign-121hz512p9	301	159,145	629,449	91	159,054	0	1473.00
proteindesign-122trx11p8	254	127,326	503,427	78	127,248	0	1747.00
qap10	1820	4150	18,200	0	4150	0	340.00
radiationm18-12-05	40,935	40,623	96,149	11,247	14,688	14,688	17,566.00
rail01	46,843	117,527	392,086	0	117,527	0	− 70.57
rail507	509	63,019	468,878	0	63,009	10	174.00
ran14x18-disj-8	447	504	10,277	0	252	252	3712.00
rd-rplusc-21	125,899	622	852,384	0	457	165	165,395.28
reblock115	4735	1150	13,724	0	1150	0	− 36,800,603.23
rmatr100-p10	7260	7359	21,877	0	100	7259	423.00
rmatr200-p5	37,617	37,816	113,048	0	200	37,616	4521.00
rocI-4-11	10,883	6839	27,383	1016	5192	631	− 6,020,203.00
rocII-5-11	26,897	11,523	303,291	0	11,341	182	− 6.68
rococoB10-011000	1667	4456	16,517	136	4320	0	19,449.00
rococoC10-001000	1293	3117	11,751	124	2993	0	11,460.00
roi2alpha3n4	1251	6816	878,812	0	6642	174	− 63.21
roi5alpha10n8	4665	106,150	2,370,220	0	105,950	200	− 52.32
satellites2-40	20,916	35,378	283,668	0	34,324	1054	− 19.00

Table 3 (continued)

Instance_name	rows	cols	nnz	int	bin	con	Objective
satellites2-60-fs	16,516	35,378	125,048	0	34,324	1054	– 19.00
savsched1	295,989	328,575	1,770,510	0	252,731	75,844	3217.70
sct2	2151	5885	23,643	0	2872	3013	– 230.99
seymour1	4944	1372	33,549	0	451	921	410.76
sing326	50,781	55,156	268,173	0	40,010	15,146	7,753,674.85
sing44	54,745	59,708	281,260	0	43,524	16,184	8,128,831.18
snp-02-004-104	126,512	228,350	463,941	167	167	228,016	586,803,238.66
sorrell3	169,162	1024	338,324	0	1024	0	– 16.00
sp150x300d	450	600	1200	0	300	300	69.00
sp98ar	1435	15,085	426,148	0	15,085	0	529,740,623.20
supportcase12	166,781	799,616	2,334,440	200	0	799,416	– 7559.53
supportcase18	240	13,410	28,920	0	13,410	0	48.00
supportcase26	870	436	2492	0	396	40	1745.12
supportcase33	20,489	20,203	211,915	101	20,102	0	– 345.00
supportcase40	38,192	16,440	104,420	0	2000	14,440	24,256.31
supportcase42	18,439	19,466	435,653	1026	0	18,440	7.76
supportcase6	771	130,052	584,976	1	130,051	0	51,906.48
supportcase7	6532	138,844	2,845,540	14	451	138,379	– 1132.22
swath1	884	6805	34,965	0	2306	4499	379.07
swath3	884	6805	34,965	0	2706	4099	397.76
tbfp-network	2436	72,747	215,837	0	72,747	0	24.16
thor50dday	53,360	106,261	212,060	0	53,131	53,130	40,417.00
traininstance2	15,603	12,890	41,531	2602	5278	5010	71,820.00
traininstance6	12,309	10,218	32,785	2056	4154	4008	28,290.00
trento1	1265	7687	93,571	0	6415	1272	5,189,487.00
triptim1	15,706	30,055	515,436	9592	20,456	7	22.87
uccase12	121,161	62,529	419,447	0	9072	53,457	11,507.41
uccase9	49,565	33,242	332,316	0	8064	25,178	10,993.13
uct-subprob	1973	2256	10,147	0	379	1877	314.00
unitcal_7	48,939	25,755	127,595	0	2856	22,899	19,635,558.24
var-smallemery-m6j6	13,416	5608	850,621	0	5606	2	– 149.38
wachplan	1553	3361	89,361	1	3360	0	– 8.00

(i.e., it requires stage 3). In five of those 15 instances (33.3%), CC-FP finds a feasible solution ("aflow40b", "harp2", "nsrand-ipx", "protfold" and "tr12-30"). In contrast, CC-FP fails in 12 instances. In two of those instances (16.7%), AC-FP finds a feasible solution ("ds" and "nw04"). Finally, in 14 of the 35 instances (40%) where both methods find a feasible solution, CC-FP obtains a solution with a lower gap than AC-FP. In another eight instances CC-FP obtains the same gap as AC-FP. Given that the total computational time is really low in both methods (less than one minute on average), CC-FP proves to be a good alternative to AC-FP.

Table 4 Computational results using AC-FP, CC-FP and objective FP for a subset of MILP instances from MIPLIB 2003

	AC-FP				CC-FP				Objective FP		
Instance	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
10teams	15	0	3	12.76	16	0	3	12.76	2	1	4.54
a1c1s1	0	0	0	234.18	1	1	0	200.16	2	2	86.73
aflow30a	0	0	2	238.40	1	0	2	294.13	0	1	103.28
aflow40b	3	0	3	610.09	6	0	2	450.81	0	1	77.16
air04	354	1	3	2.48	357	0	3	2.48	10	1	3.66
air05	82	1	3	13.32	80	0	3	13.32	0	1	1.35
arki001	29	0	3	2.17	53	0	3	2.70	0	3	2.47
atlanta-ip	256	5	3	61.56	516	4	3	50.56	8	3	68.15
cap6000	0	0	0	0.35	1	1	0	0.35	0	1	0.38
danoint	3	0	1	24.51	2	0	1	11.51	1	1	14.01
disctom	1	0	1	0.00	1	0	1	0.00	2	1	0.00
ds	1	1	0	5633.77	1037	5	3	5633.77	495	2	283.65
fast0507	0	0	0	56.57	52	52	0	56.57	11	1	2.29
fiber	0	0	1	1442.76	0	0	1	1442.76	0	1	1442.76
fixnet6	0	0	0	2342.14	0	0	0	2369.27	0	1	936.77
gesa2	0	0	2	13.64	0	0	2	53.43	0	2	9.32
gesa2-o	0	0	2	68.85	1	0	2	497.13	0	2	40.44
glass4	1	0	3	279.17	0	0	3	279.17	0	3	621.80
harp2	2	0	3	45.02	1	0	2	29.73	0	2	16.11
manna81	0	0	0	2.20	0	0	0	1.96	0	2	1.70
markshare1	0	0	0	3.6e5	0	0	0	3.6e5	0	1	3.6e4
markshare2	0	0	0	5.2e5	0	0	0	5.2e5	0	1	4.8e4
mas74	0	0	0	4.2e8	0	0	0	2.2e8	0	1	45.36
mas76	0	0	0	1.2e8	0	0	0	4.4e7	0	1	15.59
misc07	2	0	2	53.54	1	0	1	34.33	0	2	25.61
mkc	1	1	1	66.28	1	0	1	69.88	0	1	78.36
mod011	2	0	1	34.85	2	0	1	46.71	0	1	16.36
modglob	0	0	0	299.75	0	0	0	615.10	0	1	10.87
msc98-ip	52	43	1	51.09	12	5	1	55.89	2	1	51.86
mzzv11	194	3	3	100.00	181	12	3	100.00	3	3	100.00
mzzv42z	10	3	1	36.02	25	7	1	35.24	1	1	27.83
net12	11	8	1	57.21	7	4	1	57.21	2	2	57.21
noswot	1	0	2	73.81	0	0	2	35.71	0	2	2.38
nsrand-ipx	197	0	3	352.49	177	0	2	495.30	0	2	424.68
nw04	6	1	1	20.39	255	2	3	16.58	1	1	5.91
opt1217	0	0	0	94.12	0	0	1	0.00	0	1	0.00
p2756	4	0	3	1542.85	8	0	3	1542.85	0	3	1542.85
pk1	0	0	0	6000.00	0	0	1	616.67	0	1	208.33
pp08a	0	0	0	150.77	0	0	0	180.73	0	1	63.39
pp08aCUTS	0	0	0	192.83	0	0	0	332.28	0	1	13.74

Table 4 (continued)

	AC-FP				CC-FP				Objective FP		
Instance	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
protfold	456	1	3	31.25	5	0	1	34.38	27	3	37.50
qiu	0	0	0	2818.73	0	0	0	2539.89	1	1	638.67
roll3000	8	0	2	66.15	7	0	2	28.67	2	3	201.60
rout	0	0	1	49.20	1	0	1	62.76	0	2	22.52
set1ch	0	0	0	296.92	0	0	0	261.59	0	1	75.74
seymour	0	0	0	40.09	0	0	0	40.09	0	1	5.66
sp97ar	27	0	1	1757.00	37	1	1	1715.56	1	1	40.31
swath	20	1	2	167.22	37	1	2	192.32	0	3	6852.36
timtab1	2	0	3	96.55	2	0	3	88.05	0	2	95.03
timtab2	5	0	3	82.84	7	0	3	64.97	0	3	94.07
tr12-30	3	0	3	112.68	1	0	1	53.21	0	1	25.68
vpm2	0	0	1	42.37	0	0	1	57.63	0	1	32.20

Next, we focus on comparing the objective FP heuristic with CP-FP (choosing the best option between AC-FP and CC-FP). Table 5 presents the results obtained, with the column "tCP" showing the time needed for computing either AC or CC. In 39 instances, both CP-FP and objective FP find a feasible solution. In six instances ("cap6000", "danoint", "mkc", "msc98-ip", "roll3000" and "swath"), representing a 15.4% of the cases, CP-FP improves the quality of the feasible solution achieved. Furthermore, when the objective FP fails in nine instances ("arki001", "atlanta-ip", "glass4", "mzzv11", "p2756", "protfold", "roll3000", "swath" and "timtab2"), CP-FP finds a feasible solution in three of them ("protfold", "roll3000" and "swath"). It is noteworthy that CC-FP efficiently solves the instance "protfold" when both AC-FP and objective FP fail. We also observe that in the six instances where all methods fail ("arki001", "atlanta-ip", "glass4", "mzzv11", "p2756" and "timtab2"), CP-FP obtains a better feasible solution in two of them ("atlanta-ip" and " timtab2"); and in another two ("mzzv11" and "p2756") it provides the same solution as objective FP.

Second, we provide a similar comparison between AC-FP, CC-FP and objective FP for a subset of MIPLIB2010 instances. The original subset contains 215 instances, but 38 of them are removed because either (i) none of the methods find a feasible solution within the one hour time limit; or (ii) they exhaust the available memory. Tables 6 and 7 show the results obtained with the remaining 177 instances. Comparing AC-FP against CC-FP, we note that in four of the 70 instances (5.7%) where AC-FP fails, CC-FP finds a feasible solution. On the other hand, in seven of the 73 instances (9.6%) where CC-AC fails, AC-FP is able to find a feasible solution. It is worth noting that CC-FP gives a higher quality solution in 36 of the 100 instances (36%) in which both methods successfully find a feasible point. In another 37 instances, AC-FP and CC-FP find points with the same objective function. Therefore, in a total of 73 out of 100 instances

Table 5 Computational results using the best option between AC-FP and CC-FP (CP-FP) against objective FP for a subset of MILP instances from MIPLIB 2003

Instance	CP-FP				Objective FP		
	tFP	tCP	Stage	gap%	tFP	Stage	gap%
10teams	15	0	3	12.76	2	1	4.54
a1c1s1	1	1	0	200.16	2	2	86.73
aflow30a	0	0	2	238.40	0	1	103.28
aflow40b	6	0	2	450.81	0	1	77.16
air04	354	1	3	2.48	10	1	3.66
air05	80	0	3	13.32	0	1	1.35
arki001	29	0	3	2.17	0	3	2.47
atlanta-ip	516	4	3	50.56	8	3	68.15
cap6000	0	0	0	0.35	0	1	0.38
danoint	2	0	1	11.51	1	1	14.01
disctom	1	0	1	0.00	2	1	0.00
ds	1	1	0	5633.77	495	2	283.65
fast0507	0	0	0	56.57	11	1	2.29
fiber	0	0	1	1442.76	0	1	1442.76
fixnet6	0	0	0	2342.14	0	1	936.77
gesa2	0	0	2	13.64	0	2	9.32
gesa2-o	0	0	2	68.85	0	2	40.44
glass4	0	0	3	279.17	0	3	621.80
harp2	1	0	2	29.73	0	2	16.11
manna81	0	0	0	1.96	0	2	1.70
markshare1	0	0	0	3.6e5	0	1	3.6e4
markshare2	0	0	0	5.2e5	0	1	4.8e4
mas74	0	0	0	2.2e8	0	1	45.36
mas76	0	0	0	4.4e7	0	1	15.59
misc07	1	0	1	34.33	0	2	25.61
mkc	1	1	1	66.28	0	1	78.36
mod011	2	0	1	34.85	0	1	16.36
modglob	0	0	0	299.75	0	1	10.87
msc98-ip	52	43	1	51.09	2	1	51.86
mzzv11	181	12	3	100.00	3	3	100.00
mzzv42z	25	7	1	35.24	1	1	27.83
net12	11	8	1	57.21	2	2	57.21
noswot	0	0	2	35.71	0	2	2.38
nsrand-ipx	177	0	2	495.30	0	2	424.68
nw04	6	1	1	20.39	1	1	5.91
opt1217	0	0	1	0.00	0	1	0.00
p2756	4	0	3	1542.85	0	3	1542.85
pk1	0	0	1	616.67	0	1	208.33
pp08a	0	0	0	150.77	0	1	63.39
pp08aCUTS	0	0	0	192.83	0	1	13.74
protfold	5	0	1	34.38	27	3	37.50
qiu	0	0	0	2539.89	1	1	638.67

Table 5 (continued)

Instance	CP-FP				Objective FP		
	tFP	tCP	Stage	gap%	tFP	Stage	gap%
roll3000	7	0	2	28.67	2	3	201.60
rout	0	0	1	49.20	0	2	22.52
set1ch	0	0	0	261.59	0	1	75.74
seymour	0	0	0	40.09	0	1	5.66
sp97ar	37	1	1	1715.56	1	1	40.31
swath	20	1	2	167.22	0	3	6852.36
timtab1	2	0	3	88.05	0	2	95.03
timtab2	7	0	3	64.97	0	3	94.07
tr12-30	1	0	1	53.21	0	1	25.68
vpm2	0	0	1	42.37	0	1	32.20

(73%) CC-FP obtains an equal or better result than AC-FP. These results show that the Chebyshev center can be a good alternative to the analytic center for FP variants. Finally, Tables 8 and 9 show results comparing objective FP with the best option between CC-FP or AC-FP (named CP-FP in these tables). From Tables 8 and 9 we can state that: (i) in 19 of the instances where objective FP fails, CP-FP finds a feasible solution; and (ii) CP-FP obtains a better solution than objective FP in 26% of the cases where all methods successfully end (and in 5.4% of the cases, the solutions have the same objective function).

Table 10 summarizes the overall results for all the MIPLIB 2003 and 2010 instances, comparing AC-FP vs CC-FP in subtable (a), and the best between AC-FP and CC-FP (referred to as CP-FP) vs objective FP in subtable (b). The first two rows of each subtable provide, for each method, the percentage of successfully solved instances (i.e., a feasible solution is obtained by the heuristic before stage 3) and failures (i.e., stage 3 is reached). Looking at subtable (a) we notice that both AC-FP and CC-FP solve the same number of instances, although the particular set of instances solved by each method is different. In subtable (b) we see that CP-FP (either AC or CC) solves 4% more of instances than objective FP. The last two rows of subtable (a) show that CC-FP provides a solution of better gap than AC-FP in 3.5% more instances; in 19.65% of the instances both AC-FP and CC-FP report a solution of same gap. This information is also given in the last two rows of subtable (b) comparing CP-FP vs objective FP: it is seen that objective FP provides better gaps than CP-FP in many more cases. However, CP-FP is able to compute a solution in 26% of the instances that are not solved by objective FP.

Table 6 Computational results using AC-FP, CC-FP and objective FP for a subset of MILP instances from MIPLIB 2010 (Part I)

	AC-FP				CC-FP				Objective FP		
Instance	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
30n20b8	102	0	3	1725.41	95	0	3	1725.41	1	3	1725.41
50v-10	0	0	0	21,919.49	0	0	0	21,919.49	0	2	7288.95
CMS750_4	1	1	0	295.65	1	1	0	295.65	2	1	133.99
app1-1	11	0	3	75.00	11	0	3	75.00	0	3	75.00
app1-2	155	1	3	0.00	157	5	3	0.00	2	3	42.86
assign1-5-8	0	0	1	8.45	0	0	1	6.57	0	1	1.41
b1c1s1	0	0	0	373.64	0	0	0	343.00	2	2	208.34
bab2	2073	10	3	62.46	2469	30	3	60.45	108	3	48.40
bab6	1274	7	3	81.10	1394	22	3	82.93	108	3	57.95
beasleyC3	0	0	0	806.62	0	0	0	806.62	0	3	806.62
binkar10_1	0	0	1	75.81	0	0	1	11.17	0	1	5.42
blp-ar98	246	0	3	240.37	262	0	3	277.78	4	3	256.42
blp-ic98	30	0	1	235.24	31	1	1	235.24	2	1	186.79
bppc4-08	0	0	0	438.89	0	0	0	438.89	1	1	35.19
cbs-cta	26	0	1	9.5e7	14	0	1	1.5e7	3	1	1.5e7
chromaticindex1024-7	1898	2	3	0.00	794	10	2	0.00	52	1	0.00
chromaticindex512-7	601	1	1	0.00	179	2	1	0.00	18	3	0.00
cmflsp50-24-8-8	396	1	3	18.82	279	1	3	12.23	7	3	18.39
co-100	757	3	3	3544.80	776	1	3	3544.80	47	3	837.32
cod105	1	1	0	92.31	0	0	0	92.31	24	1	69.23
comp07-2idx	17	2	1	11,1357.14	37	1	2	124,728.57	6	3	268,157.14
comp21-2idx	13	2	1	10,412.00	67	0	3	22,596.00	7	3	22,596.00
cost266-UUE	0	0	0	129.34	0	0	0	108.86	0	1	41.63
csched007	9	0	3	93.47	8	0	3	89.20	0	3	75.57
csched008	9	0	3	8.62	7	0	3	8.62	0	1	8.62

Table 6 (continued)

Instance	AC-FP				CC-FP				Objective FP		
	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
cvs16r128-89	0	0	0	96.94	0	0	0	96.94	2	1	73.47
dano3_3	58	2	1	1.04	37	5	1	0.00	8	1	0.00
dano3_5	252	2	1	1.34	50	6	1	0.10	36	1	0.01
decomp2	57	1	3	101.86	53	1	3	101.86	1	3	101.86
drayage-100-23	30	2	3	854.01	29	1	3	854.01	3	3	854.01
drayage-25-23	30	2	3	873.33	30	1	3	873.33	4	3	873.33
dws008-01	82	0	3	109.96	69	0	3	74.79	1	3	106.57
eil33-2	0	0	0	440.23	1	0	1	326.31	0	2	77.99
eilA101-2	1	1	0	465.86	603	4	3	465.86	36	3	465.86
enlight_hard	0	0	3	0.00	1	0	3	0.00	0	3	0.00
exp-1-500-5-5	0	0	1	180.86	0	0	2	173.38	0	1	64.32
fastxgemm-n2r6s0t2	11	0	1	1428.57	6	1	1	272.73	1	1	650.65
fiball	128	0	3	1447.48	52	1	2	87.77	1	2	26.62
gen-ip002	0	0	0	6.76	0	0	0	6.76	0	2	4.55
gen-ip054	0	0	2	6.71	0	0	0	1.5e16	0	2	3.47
germanrr	262	0	3	25.70	255	1	3	25.70	0	3	25.70
glass-sc	0	0	0	58.33	1	1	0	795.83	1	1	20.83
gmu-35-40	0	0	1	38.43	1	0	1	52.20	0	1	27.70
gmu-35-50	0	0	1	43.18	1	0	1	25.83	0	1	18.82
graph20-20-1rand	2	0	1	70.00	3	1	1	80.00	1	1	30.00
graphdraw-domain	2	0	3	534.90	2	0	3	534.90	0	2	19.19
h80x6320d	15	0	3	213.63	16	1	3	213.63	3	1	296.40
hypothyroid-k1	87	39	1	0.00	69	22	1	0.00	12	1	0.00

Table 6 (continued)

Instance	AC-FP				CC-FP				Objective FP		
	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
ic97_potential	5	0	3	15.80	7	0	3	14.58	0	3	15.37
icir97_tension	66	0	3	16.11	47	0	3	16.58	0	2	14.05
irp	1	1	0	49.20	31	1	3	49.20	0	1	0.40
istanbul-no-cutoff	92	0	1	45.26	107	0	1	61.56	36	1	13.23
k1mushroom	3477	8	3	99.97	2715	42	3	99.97	209	3	99.97
leo1	37	0	2	49.53	37	1	2	103.17	0	2	42.93
leo2	64	0	2	101.61	65	0	2	52.37	0	2	64.76
lotsize	4	0	3	915.82	4	0	3	981.53	0	3	322.01
mad	0	0	1	416.24	0	0	1	305.26	0	1	504.64
map10	1188	4	1	12.50	1688	3	1	14.31	980	1	88.10
map16715-04	2142	4	1	99.11	1840	3	1	99.11	1998	1	99.11
markshare_4_0	0	0	0	154,050.00	0	0	1	12,050.00	0	1	21,550.00
mc11	0	0	0	1003.17	0	0	1	1003.17	1	3	1003.17
mcsched	2	0	1	4.69	2	0	1	7.97	1	1	4.23
mik-250-20-75-4	0	0	0	94.64	0	0	0	76.93	0	2	356.93
milo-v12-6-r2-40-1	23	0	3	199.18	24	0	3	190.94	3	3	110.14
momentum1	260	1	3	389.15	256	1	3	389.15	51	3	389.15
mushroom-best	0	0	0	769,883.90	218	1	3	1896.64	7	3	2286.15
n2seq36q	23	1	1	87.35	14	1	1	87.35	1	1	2868.53
n3div36	1	1	0	6698.11	4	0	1	65.14	0	1	113.30
n5-3	0	0	0	22,892.73	0	0	0	10,022.09	0	2	38.92
neos-1171448	0	0	0	99.68	9	2	1	99.68	11	1	14.27
neos-1171737	0	0	0	99.49	3	1	1	96.94	2	1	15.82

Table 6 (continued)

Instance	AC-FP				CC-FP				Objective FP		
	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
neos-1354092	90	1	1	27.66	44	1	1	10.64	55	1	0.00
neos-1445765	19	0	3	99.99	20	2	3	99.99	1	3	99.99
neos-1456979	37	1	3	183.62	38	0	3	192.66	5	3	82.49
neos-1582420	36	1	3	129.35	38	1	3	153.26	0	3	1834.78
neos-2657525-crna	3	0	3	672.75	10	0	3	369.58	0	3	890.68
neos-2978193-inde	0	0	1	3.44	6	2	1	83.69	0	1	3.44
neos-2987310-joes	26	22	1	0.12	69	61	1	0.18	1	1	0.00
neos-3024952-loue	85	1	3	2371.73	77	0	3	2314.80	0	3	2447.38
neos-3046615-murg	1	0	3	7156.09	2	0	3	1.8e13	0	3	61,170.08
neos-3083819-nubu	3	0	2	6.49	6	0	2	5.64	0	2	1.08
neos-3216931-puriri	242	1	3	59.66	144	1	3	43.33	7	3	128.56
neos-3381206-awhea	12	0	3	4.85	13	0	3	4.85	0	3	4.85
neos-3402294-bobin	1385	5	2	19.63	366	15	1	27.36	135	3	368.49
neos-3555904-turama	840	5	3	4.20	772	5	3	4.20	29	3	4.20
neos-3627168-kasai	1	0	1	3.59	1	0	1	3.59	0	1	3.59
neos-3754480-nidda	0	0	1	23.89	0	0	1	32.22	1	1	33.60
neos-4300652-rahue	2977	2	3	149.30	2211	2	3	178.74	363	3	173.90
neos-4338804-snowy	5	0	3	1067.46	5	0	3	1067.46	0	3	57,268.21
neos-4387871-tavua	11	0	3	41.63	14	0	3	37.52	0	3	325.97
neos-4413714-turia	1121	3	2	979.58	519	13	1	973.29	55	1	977.86
neos-4532248-waihi	1790	6	3	787.86	1882	3	3	628.59	22	3	499.68
neos-4647030-tutaki	1854	94	2	175.30	1545	155	3	114.39	25	3	101.49
neos-4722843-widden	956	2	3	474.22	602	2	3	474.22	12	3	554.19
neos-4738912-atrato	1	0	1	755.47	63	0	3	761.49	1	3	761.49

Table 7 Computational results using AC-FP, CC-FP and objective FP for a subset of MILP instances from MIPLIB 2010 (Part II)

	AC-FP				CC-FP				objective FP		
Instance	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
neos-4763324-toguru	1060	1	2	131.65	940	2	2	138.71	8	1	53.18
neos-4954672-berkel	0	0	0	608.76	1	0	1	303.50	0	1	153.99
neos-5093327-huahum	119	14	1	20.28	97	7	1	17.79	8	1	20.60
neos-5107597-kakapo	32	0	3	43,486.59	19	0	3	17,651.70	1	3	40,022.85
neos-5188808-nattai	18	3	1	436.96	25	1	1	741.44	4	1	550.45
neos-5195221-niemur	501	3	3	11.57	37	1	1	23.47	10	3	16.61
neos-631710	968	116	3	173.04	1323	4	3	173.04	330	3	173.04
neos-662469	19	0	1	1099.90	14	0	1	779.73	15	1	265.89
neos-787933	1	1	0	5593.55	187	144	1	5593.55	12	3	5593.55
neos-827175	9	1	1	8.85	9	1	1	8.85	5	3	8.86
neos-848589	850	5	1	526,174.64	1332	241	2	675,719.59	45	1	603,687.73
neos-860300	10	0	1	99.47	19	0	1	126.95	4	1	59.96
neos-873061	2	2	0	225,483.02	70	70	0	225,483.02	626	1	104,838.27
neos-911970	1	0	3	434.43	1	0	3	434.43	0	1	294.96
neos-933966	1	1	0	151,624.45	28	8	1	7568.65	19	1	2200.63
neos-957323	1	1	0	99.58	28	28	0	99.58	1	1	1.25
neos-960392	34	1	1	46.44	38	1	1	65.69	20	3	99.58
neos17	0	0	0	114.77	0	0	0	114.77	0	3	131.71
neos5	0	0	0	147.89	0	0	0	268.75	0	1	62.50
neos8	615	1	3	341.91	428	2	3	341.80	21	3	103.87
netdiversion	445	8	1	7,870,558.85	407	15	1	5,030,435.39	117	1	289,555.97
nexp-150-20-8-5	4	0	1	2400.86	4	1	1	2273.71	1	1	2243.97
ns1208400	241	2	3	0.00	200	0	3	0.00	75	3	0.00
ns1830653	87	0	3	616.72	93	0	3	616.72	7	3	606.12
nu25-pr12	30	0	3	270.57	0	0	0	7.5e15	0	2	2.34

Table 7 (continued)

Instance	AC-FP				CC-FP				objective FP		
	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
nursesched-sprint02	372	0	3	1916.95	203	1	3	1823.73	1	1	264.41
opm2-z10-s4	1	1	0	94.36	20	20	0	94.36	368	1	94.11
p200x1188c	0	0	0	23,671.01	0	0	0	23,671.01	0	1	6712.39
pg	0	0	1	52.48	1	0	1	17.49	0	1	17.49
pg5_34	1	0	1	11.91	1	0	1	11.91	0	1	11.91
piperout-08	100	0	3	85.44	94	0	3	27.61	2	3	100.46
piperout-27	121	1	3	81.85	118	1	3	109.51	2	3	81.85
proteindesign121hz512p9	642	2	3	2.44	655	10	3	2.44	8	3	2.44
proteindesign122trx11p8	712	1	3	1.14	721	11	3	1.14	5	3	1.14
qap10	24	0	1	11.14	7	0	1	20.53	6	1	13.49
radiationm18-12-05	197	1	3	501.27	192	1	3	501.27	8	3	179.63
rail01	2330	12	3	88.94	2365	14	3	87.82	1172	3	28.41
rail507	0	0	0	9577.71	0	0	0	9577.71	12	1	8.57
ran14x18-disj-8	0	0	0	1059.36	0	0	0	1063.58	0	1	15.62
rd-rplusc-21	765	1	3	13.53	505	0	3	12.92	7	3	5.78
reblock115	22	0	3	100.00	38	1	3	100.00	1	2	21.89
rmatr100-p10	1	1	0	152.83	1	0	1	40.80	1	1	1.42
rmatr200-p5	1	1	0	52.83	332	5	1	16.19	182	1	48.61
rocI-4-11	112	1	3	16.28	97	0	3	100.00	1	3	66.44
rocII-5-11	229	1	3	79.83	187	1	3	80.19	7	3	79.94
rococoB10-011000	24	0	3	504.77	7	0	3	549.75	0	3	493.73
rococoC10-001000	14	0	3	1428.03	17	0	3	2093.52	0	3	1329.32
roi2alpha3n4	99	1	1	98.44	101	9	1	98.44	5	3	98.44

Table 7 (continued)

Instance	AC-FP				CC-FP				objective FP		
	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
roi5alpha10n8	685	5	1	98.12	1283	533	1	98.12	49	3	98.12
satellites2-40	296	195	2	340.00	420	218	3	335.00	18	3	530.00
satellites2-60-fs	709	18	3	535.00	736	20	3	535.00	34	3	535.00
savsched1	4	4	0	11,284.78	857	714	1	305.79	973	1	762.09
sct2	0	0	0	589.98	0	0	0	589.98	1	1	52.20
seymour1	1	1	0	73.27	0	0	0	73.87	1	1	0.88
sing326	34	2	1	58.53	40	4	1	54.01	4	1	30.27
sing44	31	2	1	11.22	33	4	1	11.22	4	1	8.77
snp-02-004-104	1813	4	3	1282.83	1345	3	3	1282.83	14	3	1282.83
sorrell3	1	1	0	94.12	50	50	0	94.12	4	1	94.12
sp150x300d	0	0	0	330.00	0	0	0	330.00	1	3	330.00
sp98ar	38	1	1	1639.32	49	2	1	1388.39	1	1	1173.42
supportcase12	271	7	2	63.74	269	7	2	49.67	0	2	3.20
supportcase18	12	0	3	146.94	12	0	3	146.94	1	2	59.18
supportcase26	1	0	3	45.38	1	0	3	62.23	0	3	9667.77
supportcase33	484	1	3	99.71	317	1	3	99.71	120	3	99.71
supportcase40	24	1	1	10.05	24	0	1	10.05	22	1	11.85
supportcase42	0	0	0	368.11	0	0	0	1.3e20	0	2	25.59
supportcase6	145	2	1	39.57	180	5	1	155.33	24	1	19.75
supportcase7	750	6	1	4.51	763	2	1	9.71	102	1	10.11
swath1	5	0	1	8.64	5	1	1	8.64	2	2	660.27
swath3	11	0	2	1139.01	10	0	2	1139.01	1	1	18.62
tbfp-network	572	1	3	333.50	563	5	3	333.50	118	1	46.54

Table 7 (continued)

Instance	AC-FP				CC-FP				objective FP		
	tFP	tAC	Stage	gap%	tFP	tCC	Stage	gap%	tFP	Stage	gap%
thor50dday	1	1	0	1,053,987.29	1	1	0	1,053,987.29	138	1	195.06
traininstance2	65	0	3	26.12	60	0	3	26.12	0	3	31.24
traininstance6	61	0	3	46.98	33	0	3	368.21	0	3	31.71
trento1	25	1	1	613.54	23	2	1	405.96	30	1	224.53
triptim1	107	18	1	12.45	102	13	1	12.45	3	1	0.00
uccase12	6	3	1	0.00	4	2	1	0.00	1	1	0.00
uccase9	128	4	2	138.87	128	1	2	173.09	7	2	858.03
uct-subprob	0	0	0	601.90	0	0	0	606.35	0	1	23.81
unitcal_7	499	1	3	44.58	296	1	3	44.58	4	3	44.58
var-smallemery-m6j6	2	2	0	62.22	0	0	0	49.56	6	1	173.79
wachplan	9	0	1	0.00	16	0	1	0.00	9	1	0.00

Table 8 Computational results using the best option between AC-FP and CC-FP (CP-FP) against objective FP for a subset of MILP instances from MIPLIB 2010 (Part I)

	CP-FP				Objective FP		
Instance	tFP	tCP	Stage	gap%	tFP	Stage	gap%
30n20b8	95	0	3	1725.41	1	3	1725.41
50v-10	0	0	0	21,919.49	0	2	7288.95
CMS750_4	1	1	0	295.65	2	1	133.99
app1-1	11	0	3	75.00	0	3	75.00
app1-2	155	1	3	0.00	2	3	42.86
assign1-5-8	0	0	1	6.57	0	1	1.41
b1c1s1	0	0	0	343.00	2	2	208.34
bab2	2469	30	3	60.45	108	3	48.40
bab6	1274	7	3	81.10	108	3	57.95
beasleyC3	0	0	0	806.62	0	3	806.62
binkar10_1	0	0	1	11.17	0	1	5.42
blp-ar98	246	0	3	240.37	4	3	256.42
blp-ic98	30	0	1	235.24	2	1	186.79
bppc4-08	0	0	0	438.89	1	1	35.19
cbs-cta	14	0	1	1.5e7	3	1	1.5e7
chromaticindex1024-7	794	10	2	0.00	52	1	0.00
chromaticindex512-7	601	1	1	0.00	18	3	0.00
cmflsp50-24-8-8	279	1	3	12.23	7	3	18.39
co-100	757	3	3	3544.80	47	3	837.32
cod105	1	1	0	92.31	24	1	69.23
comp07-2idx	17	2	1	111,357.14	6	3	268,157.14
comp21-2idx	13	2	1	10,412.00	7	3	22,596.00
cost266-UUE	0	0	0	108.86	0	1	41.63
csched007	8	0	3	89.20	0	3	75.57
csched008	7	0	3	8.62	0	1	8.62
cvs16r128-89	0	0	0	96.94	2	1	73.47
dano3_3	37	5	1	0.00	8	1	0.00
dano3_5	50	6	1	0.10	36	1	0.01
decomp2	53	1	3	101.86	1	3	101.86
drayage-100-23	29	1	3	854.01	3	3	854.01
drayage-25-23	30	1	3	873.33	4	3	873.33
dws008-01	69	0	3	74.79	1	3	106.57
eil33-2	1	0	1	326.31	0	2	77.99
eilA101-2	1	1	0	465.86	36	3	465.86
enlight_hard	0	0	3	0.00	0	3	0.00
exp-1-500-5-5	0	0	2	173.38	0	1	64.32
fastxgemm-n2r6s0t2	6	1	1	272.73	1	1	650.65
fiball	52	1	2	87.77	1	2	26.62
gen-ip002	0	0	0	6.76	0	2	4.55
gen-ip054	0	0	2	6.71	0	2	3.47

Table 8 (continued)

	CP-FP				Objective FP		
Instance	tFP	tCP	Stage	gap%	tFP	Stage	gap%
germanrr	255	1	3	25.70	0	3	25.70
glass-sc	0	0	0	58.33	1	1	20.83
gmu-35-40	0	0	1	38.43	0	1	27.70
gmu-35-50	1	0	1	25.83	0	1	18.82
graph20-20-1rand	2	0	1	70.00	1	1	30.00
graphdraw-domain	2	0	3	534.90	0	2	19.19
h80x6320d	15	0	3	213.63	3	1	296.40
hypothyroid-k1	87	39	1	0.00	12	1	0.00
ic97_potential	7	0	3	14.58	0	3	15.37
icir97_tension	66	0	3	16.11	0	2	14.05
irp	1	1	0	49.20	0	1	0.40
istanbul-no-cutoff	92	0	1	45.26	36	1	13.23
k1mushroom	2715	42	3	99.97	209	3	99.97
leo1	37	0	2	49.53	0	2	42.93
leo2	65	0	2	52.37	0	2	64.76
lotsize	4	0	3	915.82	0	3	322.01
mad	0	0	1	305.26	0	1	504.64
map10	1188	4	1	12.50	980	1	88.10
map16715-04	2142	4	1	99.11	1998	1	99.11
markshare_4_0	0	0	1	12,050.00	0	1	21,550.00
mc11	0	0	0	1003.17	1	3	1003.17
mcsched	2	0	1	4.69	1	1	4.23
mik-250-20-75-4	0	0	0	76.93	0	2	356.93
milo-v12-6-r2-40-1	24	0	3	190.94	3	3	110.14
momentum1	256	1	3	389.15	51	3	389.15
mushroom-best	0	0	0	769,883.90	7	3	2286.15
n2seq36q	23	1	1	87.35	1	1	2868.53
n3div36	4	0	1	65.14	0	1	113.30
n5-3	0	0	0	10,022.09	0	2	38.92
neos-1171448	0	0	0	99.68	11	1	14.27
neos-1171737	3	1	1	96.94	2	1	15.82
neos-1354092	44	1	1	10.64	55	1	0.00
neos-1445765	19	0	3	99.99	1	3	99.99
neos-1456979	37	1	3	183.62	5	3	82.49
neos-1582420	36	1	3	129.35	0	3	1834.78
neos-2657525-crna	10	0	3	369.58	0	3	890.68
neos-2978193-inde	0	0	1	3.44	0	1	3.44
neos-2987310-joes	26	22	1	0.12	1	1	0.00
neos-3024952-loue	77	0	3	2314.80	0	3	2447.38
neos-3046615-murg	1	0	3	7156.09	0	3	61,170.08
neos-3083819-nubu	6	0	2	5.64	0	2	1.08

Table 8 (continued)

Instance	CP-FP				Objective FP		
	tFP	tCP	Stage	gap%	tFP	Stage	gap%
neos-3216931-puriri	144	1	3	43.33	7	3	128.56
neos-3381206-awhea	12	0	3	4.85	0	3	4.85
neos-3402294-bobin	1385	5	2	19.63	135	3	368.49
neos-3555904-turama	772	5	3	4.20	29	3	4.20
neos-3627168-kasai	1	0	1	3.59	0	1	3.59
neos-3754480-nidda	0	0	1	23.89	1	1	33.60
neos-4300652-rahue	2977	2	3	149.30	363	3	173.90
neos-4338804-snowy	5	0	3	1067.46	0	3	57,268.21
neos-4387871-tavua	14	0	3	37.52	0	3	325.97
neos-4413714-turia	519	13	1	973.29	55	1	977.86
neos-4532248-waihi	1882	3	3	628.59	22	3	499.68
neos-4647030-tutaki	1854	94	2	175.30	25	3	101.49
neos-4722843-widden	602	2	3	474.22	12	3	554.19
neos-4738912-atrato	1	0	1	755.47	1	3	761.49

Table 9 Computational results using the best option between AC-FP and CC-FP (CP-FP) against objective FP for a subset of MILP instances from MIPLIB 2010 (Part II)

Instance	CP-FP				Objective FP		
	tFP	tCP	Stage	gap%	tFP	Stage	gap%
neos-4763324-toguru	1060	1	2	131.65	8	1	53.18
neos-4954672-berkel	1	0	1	303.50	0	1	153.99
neos-5093327-huahum	97	7	1	17.79	8	1	20.60
neos-5107597-kakapo	19	0	3	17651.70	1	3	40,022.85
neos-5188808-nattai	18	3	1	436.96	4	1	550.45
neos-5195221-niemur	37	1	1	23.47	10	3	16.61
neos-631710	968	116	3	173.04	330	3	173.04
neos-662469	14	0	1	779.73	15	1	265.89
neos-787933	1	1	0	5593.55	12	3	5593.55
neos-827175	9	1	1	8.85	5	3	8.86
neos-848589	850	5	1	52,6174.64	45	1	603,687.73
neos-860300	10	0	1	99.47	4	1	59.96
neos-873061	2	2	0	225,483.02	626	1	104,838.27
neos-911970	1	0	3	434.43	0	1	294.96
neos-933966	28	8	1	7568.65	19	1	2200.63
neos-957323	1	1	0	99.58	1	1	1.25
neos-960392	34	1	1	46.44	20	3	99.58
neos17	0	0	0	114.77	0	3	131.71
neos5	0	0	0	147.89	0	1	62.50
neos8	428	2	3	341.80	21	3	103.87

Table 9 (continued)

	CP-FP				Objective FP		
Instance	tFP	tCP	Stage	gap%	tFP	Stage	gap%
netdiversion	407	15	1	50,30,435.39	117	1	289,555.97
nexp-150-20-8-5	4	1	1	2273.71	1	1	2243.97
ns1208400	200	0	3	0.00	75	3	0.00
ns1830653	87	0	3	616.72	7	3	606.12
nu25-pr12	0	0	0	7.5e15	0	2	2.34
nursesched-sprint02	203	1	3	1823.73	1	1	264.41
opm2-z10-s4	1	1	0	94.36	368	1	94.11
p200x1188c	0	0	0	23,671.01	0	1	6712.39
pg	1	0	1	17.49	0	1	17.49
pg5_34	1	0	1	11.91	0	1	11.91
piperout-08	94	0	3	27.61	2	3	100.46
piperout-27	121	1	3	81.85	2	3	81.85
proteindesign121hz512p9	642	2	3	2.44	8	3	2.44
proteindesign122trx11p8	712	1	3	1.14	5	3	1.14
qap10	24	0	1	11.14	6	1	13.49
radiationm18-12-05	192	1	3	501.27	8	3	179.63
rail01	2365	14	3	87.82	1172	3	28.41
rail507	0	0	0	9577.71	12	1	8.57
ran14x18-disj-8	0	0	0	1059.36	0	1	15.62
rd-rplusc-21	505	0	3	12.92	7	3	5.78
reblock115	22	0	3	100.00	1	2	21.89
rmatr100-p10	1	0	1	40.80	1	1	1.42
rmatr200-p5	332	5	1	16.19	182	1	48.61
rocI-4-11	112	1	3	16.28	1	3	66.44
rocII-5-11	229	1	3	79.83	7	3	79.94
rococoB10-011000	24	0	3	504.77	0	3	493.73
rococoC10-001000	14	0	3	1428.03	0	3	1329.32
roi2alpha3n4	99	1	1	98.44	5	3	98.44
roi5alpha10n8	685	5	1	98.12	49	3	98.12
satellites2-40	296	195	2	340.00	18	3	530.00
satellites2-60-fs	709	18	3	535.00	34	3	535.00
savsched1	857	714	1	305.79	973	1	762.09
sct2	0	0	0	589.98	1	1	52.20
seymour1	1	1	0	73.27	1	1	0.88
sing326	40	4	1	54.01	4	1	30.27
sing44	31	2	1	11.22	4	1	8.77
snp-02-004-104	1345	3	3	1282.83	14	3	1282.83
sorrell3	1	1	0	94.12	4	1	94.12
sp150x300d	0	0	0	330.00	1	3	330.00
sp98ar	49	2	1	1388.39	1	1	1173.42
supportcase12	269	7	2	49.67	0	2	3.20

Table 9 (continued)

	CP-FP				Objective FP		
Instance	tFP	tCP	Stage	gap%	tFP	Stage	gap%
supportcase18	12	0	3	146.94	1	2	59.18
supportcase26	1	0	3	45.38	0	3	9667.77
supportcase33	317	1	3	99.71	120	3	99.71
supportcase40	24	1	1	10.05	22	1	11.85
supportcase42	0	0	0	368.11	0	2	25.59
supportcase6	145	2	1	39.57	24	1	19.75
supportcase7	750	6	1	4.51	102	1	10.11
swath1	5	0	1	8.64	2	2	660.27
swath3	11	0	2	1139.01	1	1	18.62
tbfp-network	563	5	3	333.50	118	1	46.54
thor50dday	1	1	0	1,053,987.29	138	1	195.06
traininstance2	60	0	3	26.12	0	3	31.24
traininstance6	61	0	3	46.98	0	3	31.71
trento1	23	2	1	405.96	30	1	224.53
triptim1	107	18	1	12.45	3	1	0.00
uccase12	6	3	1	0.00	1	1	0.00
uccase9	128	4	2	138.87	7	2	858.03
uct-subprob	0	0	0	601.90	0	1	23.81
unitcal_7	296	1	3	44.58	4	3	44.58
var-smallemery-m6j6	0	0	0	49.56	6	1	173.79
wachplan	9	0	1	0.00	9	1	0.00

Table 10 Summary tables for all MIPLIB 2003 and 2010 instances

(a)

	AC-FP	CC-FP
Finds feasible solution	62.88%	62.88%
Fails (reaches stage 3)	37.12%	37.12%
Provides better gap	20.09%	23.58%
Provides equal gap	19.65%	

(b)

	CP-FP	oFP
Finds feasible solution	66.81%	62.88%
Fails (reaches stage 3)	33.19%	37.12%
CP-FP finds feasible solution when oFP fails	26.00%	
Provides better gap	15.72%	42.79%
Provides equal gap	6.55%	

Subtable (a) compares AC-FP vs CC-FP. Subtable (b) compares the best between AC-FP and CC-FP vs objective FP

Table cells give percentage of instances

5 Conclusions

We propose using the Chebyshev center as an alternative center point to the analytic center in the successful FP heuristic. Our extensive computational results show that the CC-FP variant is competitive in some instances. Furthermore we have also shown that, in theory, the Chebyshev center might provide important benefits when the MILP problem has many redundant constraints. Although CP-FP does not always outperform objective FP, using a center point within FP has been shown to provide a competitive advantage in other FP variants that complement CP-FP, such as in [6]. In those cases, using CC instead of AC can provide better and faster feasible points. Developing a decision tool to choose a priori the best center point to use within FP could form a part of further work to be done in this field.

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