Finite size scaling of the density of states in photonic band gap crystals

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Photonic crystals are tailored periodic dielectric media that allow for an unprecedented control in the manipulation of light-matter interactions. One of their outstanding features is the realization of a complete photonic band gap that drastically inhibits light propagation in all directions and for all polarizations. A band gap is associated with a complete vanishing of the density of optical states (DOS) in the crystal. As a necessary corollary, it implies a vanishing of the local DOS (LDOS) too, which leads to a complete inhibition of spontaneous emission everywhere inside such a crystal.

The vanishing of the DOS – in other words the volume-averaged LDOS - in a photonic bandgap pertains to infinitely large periodic crystals. However, any practical realization implies a finite support, meaning that the crystal is surrounded by vacuum. As a result, the DOS in the bandgap is not zero anymore and LDOS does not vanish everywhere anymore. It is known that the LDOS in a crystal with finite support scales exponentially with crystal size [1]. This naturally gives rise to the question: how does the DOS in a crystal with finite support scales with crystal size? Is it exponential, or not at all?

Unfortunately, the problem is too challenging to be solved using *ab initio* analytical or numerical methods. Hence, we address this question by a phenomenological approach [2] that describes the modes in a finite size crystal as Lorentzians of finite linewidth [Fig. 1(a)]. Hence, the non-zero DOS in the bandgap is described as the spilling-over of Lorentzians across the bandgap edge due to their non-zero linewidths. In the limit of infinite crystal size, the Lorentzians become Dirac peaks zero linewidths and therefore no longer spill into the bandgap, leading to the well-known vanishing DOS [2].

In our paper, we will present our approach for the DOS in finite support crystals in both 2D and 3D spatial dimensions. We will show the surprising result that the DOS inside the bandgap decreases *linearly* with size irrespective of the crystal dimensionality as shown in Fig. 1(b) for the example of inverse woodpile crystals that are being pursued in our group [3]. Our work sets design rules for the sizes of photonic bandgap crystals for practical applications in cavity QED and quantum information processing (vacuum noise shielding). It also has the potential of establishing new methods for engineering finite crystals to enhance the suppression of DOS.

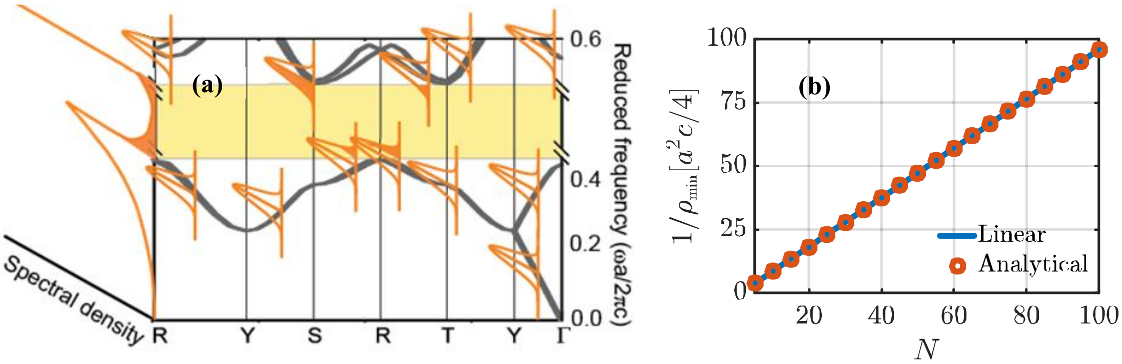

Fig. 1. (a) Schematic showing Lorentzian modes with finite linewidth in a photonic bandgap crystal with finite support. The left pane shows DOS calculated by summing all the Lorentzian modes. (b) Minimum of DOS in the bandgap of a 3D inverse woodpile crystal [3] as a function of the number of unit cells ***N*** in each direction. Circles represent the DOS calculated using our approach, and the solid blue line is a linear fit.

References

[1] A. A. Asatryan, K. Busch, R. C. McPhedran, L. C. Botten, C. Martijn de Sterke, and N. A. Nicorovici, “Two-dimensional Green’s function and local density of states in photonic crystals consisting of a finite number of cylinders of infinite length,” Phys. Rev. E **63**, 046612 (2001).
[2] E. Yeganegi, A. Lagendijk, A. P. Mosk, and W. L. Vos, "Local density of optical states in the band gap of a finite one-dimensional photonic crystal," Phys. Rev. B **89**, 045123 (2014).
[3] J. M. Van Den Broek, L. A. Woldering, R. W. Tjerkstra, F. B. Segerink, I. D. Setija, and W. L. Vos, "Inverse-woodpile photonic band gap crystals with a cubic diamond-like structure made from single-crystalline silicon," Adv. Funct. Mater. **22**, 25–31 (2012).

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978-1-5090-6736-7/17/$31.00 ©2017 European Union