Three Essays on Conglomerate Mergers(Dissertation_全文)

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Three Essays on Conglomerate Mergers

(コングロマリット合併をめぐる三つのエッセイ)

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Abstract

Conglomerate mergers are defined as mergers that are neither horizontal nor vertical. This dissertation theoretically investigates conglomerate effects on competition that are usually neglected by competition authorities like the European Commission. The dissertation consists of three essays about conglomerate mergers.

First, *Conglomerate Mergers and Competition: A Game Theoretic Approach With Research and Development Investments*, studies conglomerate mergers' effects on competition with a theoretical model where oligopolistic firms in technologically related markets choose to merge in conglomerates to shift research and development capabilities. It fully characterizes the equilibrium market outcomes and the underlying merger decisions. It finds that policy implications regarding conglomerate mergers are intricate.

Second, *Agency Problems in a Competitive Conglomerate with Production Constraints*, develops an adverse selection model of a competing conglomerate with production constraints. It fully characterizes the optimal contracts. It finds the instances where a contract improves the welfare in comparison to a symmetric information benchmark.

Third, *Conglomerate Merger and Divestment Dynamics*, constructs a discrete-time, infinite horizon theoretical model to analyze the diversifying and divesting behavior of a monopolist. It finds an approximate solution using numerical methods. It finds that the conglomerate acquires a firm and stays merged in periods where the value of the demand of the new market remains high. In periods where the value of the demand of the new market is low, the conglomerate will merge and divest intermittently.

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Chapter 1

Introduction

Conglomerate mergers are defined as mergers that are neither horizontal nor vertical. The E.U competition law and the U.S antitrust law have jurisdiction over conglomerate mergers only when these have the potential to lessen the competition (Markovits, 2014). Under the current policies, conglomerates can expand indefinitely to other markets as long as competition concerns are not raised. For instance, The Walt Disney Company has been able to increase its participation in the media industry through acquisitions that have been approved by the competition authorities in the U.S. As a particular example, the Federal Trade Commission approved the Disney's acquisition of MLB Media Holdings in 2017. The acquisition allowed Disney to enter the market of video streaming with Disney+ and ESPN+. This dissertation theoretically investigates conglomerate effects on markets that are usually neglected by competition authorities such as the European Commission.

Chapter 2 employs a theoretical model where oligopolistic firms in technologically related markets choose to engage in conglomerate mergers to shift research and development (R&D) capabilities. The two markets involved are not related horizontally or vertically but technologically. Each market has a duopoly structure where the firms engage in Cournot competition. The firms cannot merge with a firm in the same market but can merge with a firm in a different market. Assumedly, each firm in only one of the markets owns an R&D lab that can be used to reduce the cost of production; it can be used by a firm in the other market but only through a conglomerate merger. We fully characterize the equilibrium market outcomes and the underlying merger decisions. When the markets have similar sizes, any equilibrium has an asymmetric outcome, where only one firm invests in each market. The total profits are always maximized in the asymmetric outcome because the firms avoid R&D competition. However, the asymmetric outcome sometimes is the best scenario for the consumers as they benefit from the R&D investments. Policy implications regarding conglomerate mergers are, therefore, intricate, given that they may be outside the jurisdiction of competition authorities.

Chapter 3 explores the reciprocal effects between agency problems and market competition. We develop an adverse selection model of a competing conglomerate with production constraints. The conglomerate participates as the leader in two different duopolistic markets with a Stackelberg-Cournot framework and heterogeneous goods. The conglomerate is run by its headquarters and two division managers. The agency problem arises because the market demand size is a manager's private information, which the headquarters try to elicit by a contract mechanism. We fully characterize a first and a second-best contract. When the production constraints make the first best outcome unattainable, the second-best contract is either separating or pooling, depending on the severity of the constraints. The separating second-best contract sometimes improves the ex-ante welfare in comparison to a symmetric information benchmark. The pooling second-best contract never improves the ex-ante welfare. We also find that at an intermediate level of substitutability, the second-best contract is most likely to coincide with the first-best one, and any departure from that level toward either substitutability or complementarity makes the attainment of the first-best outcome less likely.

Chapter 4 employs a discrete-time, infinite horizon theoretical model to analyze the diversifying and divesting behavior of a monopolist. The monopolist participates in its core market and can merge by acquiring another monopolist firm in a new market. Thereafter, the firm in the core market can separate from the new firm by selling it for a one-period reward. The monopolist has a stock of capital which is used to reduce the cost of production. Capital is obtained through two channels: the external capital market or with a merger. The capital stock is reduced in two ways: by depreciation or with a separation. We find an approximate solution using numerical methods. We found that the conglomerate acquires a firm and stays merged in periods where the value of the demand of the new market remains high. In periods where the value of the demand of the new market is low, the conglomerate will merge and divest intermittently.

Chapter 2

Conglomerate Mergers and Competition: A Game Theoretic Approach with Research and Development Investments

2.1 Introduction

Conglomerate mergers are mergers that are neither horizontal nor vertical (i.e., the merging firms' products do not compete in the same market or do not have an input-output relationship) (Narver, 1967, p. 2-3). The E.U. competition law and the U.S antitrust law have jurisdiction over conglomerate mergers only when they have the potential to lessen competition (Markovits, 2014). Thus, this chapter examines a mechanism by which conglomerate mergers might harm competition and further analyzes their effects on consumer and social welfare.

In the E.U., there is a section dedicated to conglomerate mergers in the "Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings" (2008), where the conglomerate's effects are categorized into coordinated and non-coordinated. Coordinated effects might occur when a conglomerate merger reduces the number of competitors in a market or creates multimarket contact. Non-coordinated effects are more specific: only foreclosure (mainly through tying and bundling) is mentioned in the guidelines. However, the competition authorities in the U.S do not have guidelines for conglomerates mergers, as they consider that the effects of conglomerates recognized in the E.U. can be understood in the U.S within the context of horizontal or vertical mergers (Antitrust Division of the U.S. Department of Justice & U.S. Federal Trade Commission, 2020).

Conglomerate merger theory remains lacking. Thus, competition authorities usually neglect less straightforward conglomerate mergers' effects. One such non-coordinated effect is the conglomerate's ability to shift resources between its markets. Competition could be affected by the resources a conglomerate has at its disposal, such as research and development (R&D), computer facilities, legal services, and access to capital markets, but standalone firms do not (Goldberg, 1973). Using this pool of resources to, for example, improve the quality of the products, a conglomerate firm might have a more advantageous position in the market than its standalone competitors.

Nevertheless, competition authorities might allow mergers with potential anticompetitive effects if they have positive effects on the consumers, which also applies to conglomerates mergers, as in the E.U. non-horizontal merger guidelines. This situation might be one reason to underestimate sharing resources as a harmful effect, as it does not have a clear connection with competition but expands the capabilities of the firms, increasing the chances of benefiting consumers through, for example, innovation. This chapter develops a theoretical model where oligopolistic firms in technologically related markets choose to engage in conglomerate mergers to shift R&D capabilities. It finds an equilibrium outcome where competition is harmed, but more consumers enjoy the positive effects of

R&D. This chapter performs welfare analysis and finds that when the market sizes are uneven, this equilibrium outcome negatively affects the overall consumer surplus. Thus, even with the positive effects of R&D, regulation might be required in such cases.

The acquisition of DeepMind by Google in 2014 (Gannes, 2014) illustrates a case of technology transfer through a conglomerate merger. Currently, Google and DeepMind are part of the conglomerate Alphabet Inc. Google provides internet-related services, while DeepMind's business is artificial intelligence (AI). After the acquisition, DeepMind's AI research has been used in favor of Google and others businesses of Alphabet Inc. For example, with the help of DeepMind, machine learning techniques were implemented in Google Play (a digital store for mobile applications) to improve its recommendation system (DeepMind, n.d.).

There are various instances where conglomerate mergers of technologically related markets raised competitive concerns. For example, in 2010 the European Commission had concerns about the proposed acquisition of McAfee by Intel for possible conglomerate effects. Intel might have used its strong market position in the market of computer chips and chipsets to preclude competitors from the market of computer security. Given that Intel might have embedded McAfee's security into its chips, the interoperability of Intel's chips with other security software might have not been guaranteed (European Commission, 2011).

Another example is the proposed acquisition of LinkedIn by Microsoft in 2016. Microsoft, having a strong market position in operating systems (Windows) and productivity software (Office), might have given a competitive advantage to LinkedIn in the market of professional social networks, which might have been achieved by pre-installing LinkedIn in Windows or including it in the Office package (European Commission, 2016). Eventually, the Intel/McAfee and Microsoft/LinkedIn mergers were approved by the European Commission.

The products in both mergers had a strong complementarity; thus, potentially harmful practices like tying or bundling were foreseeable. However, even if the Google/DeepMind merger might have exhibited conglomerate effects like transferring technology, such effect is generally not of interest to competition authorities; thus, no competitive concerns were raised. Even so, this chapter's model suggests that such effects might depress competition considerably. The decisions and opinions on the three merger cases might have been different if the competition authorities had considered the effect of the transfer of technology.

The model presents two markets that are not related horizontally or vertically but share similarities in their production processes. Each market has a duopoly structure where firms engage in Cournot competition. The firms cannot merge with a firm in the same market but can merge with a firm in a different market. Only firms in one of the markets own an R&D lab that can be used to reduce the cost of production, though investing in R&D is costly. A firm in the other market can access and use one of these R&D labs only through a conglomerate merger. Assumedly, the technology of the R&D lab is not fully compatible with the firms that initially do not have access to the R&D lab. When a firm operates in two markets, there is an opportunity cost to use the R&D lab capabilities across markets. Thus, the firm must choose strategically how much to invest in each market in anticipation of its rival's R&D effort.

This chapter considers a three-stage game. In the first stage, the firms take merger decisions. In this stage, this chapter establishes two players: two teams each, one comprising two firms from different markets. A team decides whether to merge depending on both firms' total profits. If a team merges, in the subsequent stages, it plays the role of a conglomerate and maximizes the joint profit of its two firms. Otherwise, the team plays as two standalone firms, where each firm maximizes its profit. In the second stage, the firms with access to an R&D lab set their R&D effort. In the third stage, in each market, the firms engage Cournot competition.

Assumedly, the cost of R&D is quadratic. With this simple form, the model predicts that the firms invest in R&D in only one market. This chapter first develops a monopoly benchmark. It demonstrates that if the market with initial access to the R&D lab is the most profitable, no merger occurs. Otherwise, a conglomerate is formed, and the R&D lab is used only in the market without initial access to it.

In the duopoly case, this chapter finds two patterns of R&D investment behavior: symmetric and asymmetric patterns. In symmetric equilibria, all firms invest only in one market. Relative to the monopoly benchmark, this kind of equilibrium exists only if one market is much more profitable than the other. If the market with initial access to the R&D labs is relatively profitable, none of the firms merge and, hence, the firms only invest in that market. Conversely, if the market without the initial access to the R&D labs is relatively profitable, all the firms merge and the two conglomerates invest only in that market.

When the market sizes are similar, there exist asymmetric equilibria where only one conglomerate is formed. The standalone firm that owns an R&D lab invests in its market, while the conglomerate focuses its investments on the market without the initial access to the R&D lab. Both R&D labs focus on different markets; thus, the firms effectively avoid competition. However, unlike the symmetric equilibria, consumers in all the markets benefit from the R&D. Thus, competition authorities might allow such asymmetric equilibria even though competition is reduced.

The one-merger scenario maximizes the total producer surplus in that if all firms were to make their merger decisions to maximize the total profits, they would form one conglomerate. This result is expected as competition is reduced. While the R&D has positive effects, in equilibrium, a one-merger scenario does not maximize the total consumer surplus when the market sizes are quite different. In those cases, the anticompetitive effects offset the positive R&D effects; thus, the competition authority should prevent the conglomerate merger.

Accordingly, a situation resembling the merger paradox emerges: if the market with the initial access to the R&D labs is adequately large but not so large relative to the other market, in equilibrium, there is one merger such that the conglomerate earns a smaller total profit than the non-merged firms. Furthermore, the results offer one explanation for the conglomerate discount: if the market without the initial access to the R&D labs is adequately large, two conglomerates are formed in equilibrium, but the profit of the conglomerate is smaller than the sum of the profits of its individual firms in the non-merger scenario. The cause of the conglomerate discount in our model is the excess R&D competition.

The rest of this chapter is organized as follows. Section 2.2 reviews the related literature. Section 2.3 introduces the model. Section 2.4 presents the model results. Section 2.5 derives social welfare and discusses policy implications. Section 2.6 concludes.

2.2 Literature review

This chapter relates to several strands of literature, such as the merger literature. A notable study is Salant, Switzer, and Reynolds (1983). Their model predicts that horizontal mergers are not usually profitable for the merging parties but can be beneficial for firms excluded from the merger. This theory is known as the merger paradox. In this chapter's model, a similar phenomenon occurs as a conglomerate merger might be more profitable to outside parties than to the conglomerate itself. This phenomenon is observed when one conglomerate is formed to avoid R&D competition but the market the conglomerate invests in is the least profitable.

The theoretical literature on conglomerate mergers is scarce. They mainly focus on conglomerates participating in industries with complementary goods, thus they contribute to the body of knowledge of the non-coordinated foreclosure effect. Granier and Podesta (2010) propose a theoretical model where an electrical and a gas supplier merge endogenously. The merger allows the conglomerate to engage in price discrimination by selling both products in a bundle. Tan and Yuan (2003) study divestitures by assuming two competing conglomerates, each one supplying a group of goods. Within the conglomerate, the goods are complements; across, they are substitutes. Evidently, no theoretical study on conglomerate mergers examines the non-coordinated effect of shifting resources.

The resource-based view of the corporate diversification literature is also relevant to this chapter. One of the main hypotheses therein is that the diversification of the firm can be explained by its assets. Relevant to this chapter's model, there is a strand in the resource-based view that focuses on the diversification of firms in related industries. Such relatedness can take the form of technological capabilities. Jovanovic and Gilbert (1993) theoretically predict that firms diversify in related-technology industries to seek profits from cross-products spillovers; they provide empirical evidence to back their hypothesis. Silverman (1999) shows that firms diversify in markets where their current technological resources can be exploited. In this chapter's model, shifting R&D capabilities across markets is possible for technologically related firms. However, the effect of the R&D effort varies across markets because their production processes are similar but not equal. Thus, there is a degree of technological relatedness, which this chapter measures with a parameter of technological compatibility.

The resource-based view literature has also focused on the allocation of resources. Matsusaka (2001) proposes a theoretical model where firms try different industries searching for a good match for their capabilities. The model predicts that a firm with a bad match will exit the original industry and find a new activity; with a very good match, the firm will specialize. In intermediate cases, the firm will diversify, entering new markets without leaving the old ones. In this chapter's model, firms diversify to a new market with a conglomerate merger to change their R&D focus from the old market to the new market. In a duopoly, asymmetric diversification reduces competition as the firms focus their R&D on different markets.

In the theoretical study of Levinthal and Wu (2010), profit-maximizing firms take diversification decisions based on the opportunity cost of sharing a finite resource across industries. The competition and resource allocation stages of this chapter's model are similar to Levinthal and Wu (2010).[1] They find equilibria where a market is effectively monopolized. However, they do not highlight the policy implications of this result as the topic of their study was neither competition policy nor mergers. A major factor is that the firm can only produce in a market if the resources allocated to that market are positive. If no resources are allocated to that market, the firm will abandon the market,

[1]The authors assume two multimarket firms engaging in Cournot competition in two markets. These firms can relocate a fixed amount of resources across markets. The resource is not perfectly fungible, an assumption that is analogous to the imperfect technological compatibility in this chapter's model. The sequence of events is as follows. First, the firms decide how to allocate the resource. Second, they produce the output. Allocating the resource to one market reduces the marginal cost of that market. Given that the resource is finite, there is an opportunity cost in transferring the resource from one market to the other one.

and its rival will become a monopolist.

The greatest departure of this chapter's model from Levinthal and Wu (2010) is in the approach to resource allocation. This chapter assumes that a conglomerate decides how to use a single R&D lab to reduce the marginal cost in the two markets in which it potentially participates. The R&D elements of this chapter's model are based on Zhao (2015). There is an additional cost to invest in R&D effort; this cost is assumed to be quadratic, reflecting the decreasing returns in R&D investments and capturing the idea of opportunity cost by Levinthal and Wu. With this change, a conglomerate can participate in a market even if no R&D is invested in that market. Thus, the strategy of the rival firm takes a larger role in explaining the equilibrium with depressed competition.

Innovation is usually positively associated with economic growth and consumer welfare. Regulating mergers that might reduce innovation is in the interest of competition authorities (Federico, Langus, & Valletti, 2018). The models of Federico, Langus, and Valletti (2017); Federico et al. (2018) analyze the effects of the horizontal merger on innovation incentives.[2] Following Federico et al. (2017), this chapter assumes that the R&D capabilities are not intrinsically modified by a merger, and, hence, the results of the R&D efforts are explained by changes in the market structure caused by mergers. The models of horizontal mergers are not directly comparable with this chapter's model as conglomerate mergers do not always reduce competition. Furthermore, horizontal mergers do not exhibit the idea of opportunity cost across markets. In this chapter's model, even

[2]The result from Federico et al. (2017, 2018) is that the overall merger effect reduces innovation. Mergers affect innovation through two channels. The first channel is the competitive (price or production) channel, which relates to the reduction of competition after a merger. Less competition increases profits with and without innovation; thus, the effect of the merger on innovation through this channel is ambiguous and depends on the assumptions of the model. Second, the innovation externality channel relates to the negative effect of the firm's innovation on the profits of its rivals. The innovation externality is internalized with a merger; thus, innovation is reduced with a merger through this channel. Denicolò and Polo (2018) contends Federico et al. (2017) and asserts that in some cases, mergers increase innovation.

if a conglomerate merger reduces the R&D effort in one market, the R&D effort of the other market will increase given the opportunity cost. Thus, the effect of a conglomerate merger on the overall R&D effort of the economy is ambiguous and depends on the value of the parameters.

This chapter is also related to the literature on capital allocation efficiency, which asserts that the firm allocates financial resources by prioritizing more profitable endeavors over lesser ones (see, e.g., Stein (1997), Maksimovic and Phillips (2002), Brusco and Panunzi (2005)). Several other studies oppose this idea and affirm that the allocation of capital is inefficient. A usual explanation is that this inefficiency is caused by agency problems (see, e.g., Rajan, Servaes, and Zingales (2000), Stein and Scharfstein (2000), Wulf (2009), Arrfelt, Wiseman, and Hult (2013)). The literature on both perspectives of capital allocation concentrates on the internal dynamics of the firms, such that they do not consider competition or merger decisions as in this chapter. The results reconcile the opposing views regarding the efficiency of capital allocation. The symmetric equilibria are consistent with the efficiency literature because all firms invest only in the best market. The asymmetric equilibria are relevant to the inefficiency literature, as there is always a firm investing in the worst market. This result is not "inefficient" in the usual sense as the firm is still maximizing profits. That firm chooses to invest in the worst market, as it is the best it can do given its rival strategy.

Capital allocation inefficiency has been proposed as the explanation of the conglomerate discount (Busenbark, Wiseman, Arrfelt, & Woo, 2017). Broadly speaking, the conglomerate discount theory claims that the conglomerate is less than the sum of the values of its individual parts (Berger & Ofek, 1995). Matsusaka (2001) connects the conglomerate discount with mismatching organizational capabilities in a particular industry.

Levinthal and Wu (2010) propose that spreading imperfectly fungible resources across various divisions is an alternative way to explain the conglomerate discount, which manifests in this chapter's model as a consequence of excess R&D competition. Interestingly, such excess competition only occurs in a symmetric outcome, which this chapter labels as "efficient" from the perspective of capital allocation.

2.3 Model

This chapter presents two markets that are not related horizontally or vertically, denoted by $k \in \{A, B\}$. In each market, two firms sell a homogeneous good, denoted by $i \in \{1, 2\}$. A representative consumer in market k has a quasi-linear utility function with the form $U_k(q_0, q_{k1}, q_{k2}) = q_{k0} + v_k(q_{k1}, q_{k2})$, where q_{k0} is the quantity of the numeraire good, q_{ki} is the output of firm i, and $v_k(q_{k1}, q_{k2})$ is given by

$$v_k(q_{k1}, q_{k2}) = D_k \cdot (q_{k1} + q_{k2}) - \frac{1}{2}(q_{k1} + q_{k2})^2,$$

where D_k is a positive constant. The utility function generates the following inverse demand function faced by firm i in market k:

$$p_{ki}(q_{ki}, q_{kj}) = D_k - q_{ki} - q_{kj},$$

where $j \in \{1, 2\}$ for $j \neq i$.

This chapter assumes that mergers between firms in the same market are forbidden by law. However, a firm in A can merge with a firm in B; that is, conglomerate mergers are allowed. To avoid coordination problems, this chapter assumes that firm $A1$ ($A2$) can potentially merge only with firm $B1$ ($B2$). This chapter constructs the model such that the results do not depend on the identity of the firms; thus, another combination of firms would not change the results. These pairs of firms are *teams*.

Initially, all the firms face the same constant marginal cost c, normalized to zero for simplicity. Each firm in market B owns an R&D lab that can be used to reduce its marginal cost. Assumedly, firm Bi's effective marginal cost is $-x_{Bi}$, where x_{Bi} is the R&D effort.

Although the markets are not related horizontally or vertically, this chapter assumes that markets A and B share some similarities in their production processes. This situation is the case, for example, in consumer electronics markets or pharmaceutical markets. Thus, the R&D lab can somewhat be used to reduce the marginal cost of the firms in A. Still, market A and B are not the same industry; there must be a process to adapt the R&D lab to market A to use it at its full potential.[3] Thus, in this single period model, this chapter assumes that market A is not perfectly compatible with the technology of the R&D lab. The cost reduction in market A is βx_{Ai} with $\beta \in (0, 1)$, where β is the technological compatibility.

Assumedly, the firms in A can only access an R&D lab in B with a conglomerate merger. In practice, cost reduction technology might be acquired externally without the need for a merger. However, with imperfect compatibility, the technology cannot be simply bought and applied to the production process. Long-term cooperation among firms A and B might be necessary to initially integrate the R&D lab's operations to market A and eventually make the technology completely compatible. A merger might greatly facilitate this cooperation.[4]

[3]The adoption of new technology is not instantaneous (Hoppe, 2002). Moreover, accelerating the development of new technologies increases the associated costs (Pacheco de Almeida & Zemsky, 2007).

[4]This situation might be the case with the Google/DeepMind merger. AI technology cannot be easily

The cost of investment in R&D is quadratic and given by $\frac{1}{2}{x_{Bi}}^2$ in the case without a merger and $\frac{1}{2}\left(x_{Ai}+x_{Bi}\right)^2$ in the conglomerate case, thus reflecting the decreasing returns in R&D investments and in the conglomerate case embodying the opportunity cost of investing in one market or the other. These cost functions imply that there are no spillovers across markets. There is imperfect compatibility with market A; thus, the R&D effort cannot be freely copied from market B. It is necessary to invest again to reduce the marginal cost in market A. Furthermore, it is also implied that the R&D lab does not expand with the merger.

The R&D cost of conglomerate i is equal to $x_{Ai}^2+2x_{Ai}x_{Bi}+x_{Bi}^2$. The cross term $2x_{Ai}x_{Bi}$ is an extra cost required when the R&D lab operates in both markets simultaneously. Knudsen, Levinthal, and Winter (2014) assert that there is a rate at which the firms reliably adjust their scale of operations. A fast adjustment might disrupt some operating practices of the firm. A conglomerate merger increases the scale of operations of a firm, as it now participates in multiple markets. This expansion does not match the original single-industry capabilities of the R&D lab in the model.

This chapters then considers a three-stage game. In the first stage, each team simultaneously and independently decides whether to merge. In the second stage, the firms that are capable of investing in R&D set their R&D efforts simultaneously and independently. Here, this chapter assumes that the R&D effort is perfectly observable. In the third stage, in each market, the firms engage in Cournot competition by simultaneously and independently setting their output. The solution concept utilized is subgame-perfect Nash equilibrium in pure strategies. This chapter employs backward induction to solve the game.

assembled into existing products. It is reasonable to claim that a deeply coordinated effort between DeepMind and other subsidiaries of Alphabet Inc. was required to integrate the AI technology into other products.

In the first stage, each team maximizes its total profits. Thus, a team of firms chooses to merge when the joint profit of the merged firm is larger than the sum of the separated firms' profits. The set of actions is to merge (M) or not (DM). Multiple equilibria might appear when a player is indifferent between (M) and (DM). In such cases this chapter considers only the equilibria where such player chooses (DM). Mergers have additional costs (e.g. establishing contracts) that are not considered in the model. Thus, if the total operating profit of a team is the same regardless of the merger decision, the reasonable choice is not to merge. This equilibrium refinement is formally stated as follows.

Equilibrium Refinement 1 ER1. *Given a rival's strategy, a player will choose DM if (M) and (DM) yield the same payoff.*

The interpretation of ER1 is that there is an infinitesimal cost of the merger that is not explicitly stated in the calculations.

The objective functions in the subsequent stages depend on the decisions of the first stage. If a team chooses to merge, in the second and third stages, it assumes the role of a single conglomerate, and, thus, maximizes its joint profit. Otherwise, it plays the role of two standalone firms, and each firm independently maximizes its standalone profit. With two firms in each market, at most two conglomerates can be formed. Accordingly, there are three kinds of possible subgames starting from the second stage: 1) Zero-merger subgame, where none of the firms merge, and, thus, four standalone firms participate in the subgame; 2) Two-merger subgame, where all firms merge and two conglomerates are created; and 3) One-merger subgame, where only one conglomerate is formed, and two standalone firms remain.

In the second stage, this chapter considers only equilibria where both R&D labs are operative, and, hence, the overall R&D effort of each team is strictly positive. The teams are always symmetric in at least one market in any subgame. In the two and zero-merger subgames, the teams are symmetric. Thus, it seems reasonable that both teams can use their R&D lab in equilibrium. In the one-merger subgame, the conglomerate has an advantage only in market A, as it is the only team that can invest in that market. Hence, it is reasonable that the conglomerate can use its R&D lab. Furthermore, as there is no advantage in market B, it is also reasonable that the standalone firm in market B can use its R&D lab. This equilibrium refinement is formally stated as follows.

Equilibrium Refinement 2 ER2. *For any i, $x_{Ai} + x_{Bi} > 0$ holds in equilibrium.*

In the third stage, when firms Ai and Bi are not merged, their profit functions are as displayed in (2.1) and (2.2), respectively.

$$(D_A - q_{Ai} - q_{Aj})\, q_{Ai}, \tag{2.1}$$

$$(D_B - q_{Bi} - q_{Bj} + x_{Bi})\, q_{Bi} - \frac{1}{2} x_{Bi}^2. \tag{2.2}$$

When firms Ai and Bi are merged, the joint profit of the conglomerate i is as given in (2.3).

$$(D_A - q_{Ai} - q_{Aj} + \beta x_{Ai})\, q_{Ai} + (D_B - q_{Bi} - q_{Bj} + x_{Bi})\, q_{Bi} - \frac{1}{2}\left(x_{Ai} + x_{Bi}\right)^2. \tag{2.3}$$

2.4 Results

This section presents the equilibrium results of the model. It begins with the results of a monopoly benchmark. Assumedly, each market is a monopoly. Figure 2.1 illustrates the solution of the monopoly benchmark in Proposition 1. Appendix 2.1 presents the proof.

Proposition 1. *Define a threshold* $\theta_M = \frac{\sqrt{2-\beta^2}}{\beta}$. *The solution of the monopoly benchmark is characterized as follows:*

a) Define the MA outcome as the set of R&D efforts and outputs $x_A^{MA} = \frac{\beta D_A}{2-\beta^2}$, $x_B^{MA} = 0$, $q_A^{MA} = \frac{D_A}{2-\beta^2}$, *and* $q_B^{MA} = \frac{D_B}{2}$, *where the team invests only in market A. The team merges and obtains the MA outcome if and only if* $\frac{D_A}{D_B} > \theta_M$.

b) Define the MB outcome as the set of R&D efforts and outputs $x_A^{MA} = 0$, $x_B^{MB} = D_B$, $q_A^{MB} = \frac{D_A}{2}$, *and* $q_B^{MB} = D_B$, *where the team invests only in market B. With ER1, the team does not merge and obtains the MB outcome if and only if* $\frac{D_A}{D_B} \leq \theta_M$.

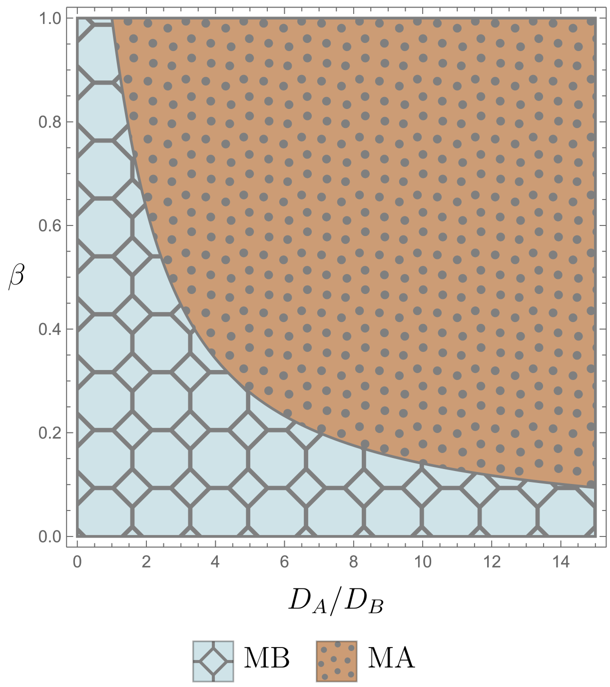

Figure 2.1: Monopoly benchmark's solution

The solution depends on $\frac{D_A}{D_B}$. This chapter refers to this quotient between the intercepts of the demand functions of the markets A and B as the *market ratio*. Thus, if the market ratio is adequately high, the two monopoly firms merge and choose to invest only in the more profitable market A. Otherwise, the monopoly firms do not merge and the standalone firm in B invests only in market B. Hence, the incentive to merge is to use the R&D lab in market A.

The form of the solution where the conglomerate invests only in one market is a consequence of the R&D cost assumptions. The conglomerate would incur extra costs if it invests in both markets given that the R&D lab is not fully prepared to operate in them. Thus, it is more profitable to invest in only one market.

Notice that if $\beta = 1$, the condition to form a conglomerate and invest only in A would simply be $D_A > D_B$. Hence, if there is full technological compatibility in both markets, the decision of which market to invest in is simply based on its relative size. As in Figure 2.1, the market ratio must be higher as β decreases for the team to form a conglomerate to invest in market A. Its size must be much greater than market B to compensate for the lack of full compatibility in market A.

This chapter presents the equilibrium results of the duopoly model in Proposition 2, illustrated in Figure 2.2. Appendix 2.2 presents the proof.

Proposition 2. *Define the thresholds* $\theta_A = \frac{(9-4\beta^2)\sqrt{7}}{4\beta\sqrt{9-8\beta^2}}$ *and*

$$\hat{\theta}_B = \begin{cases} \frac{3\sqrt{9-8\beta^2}}{10\sqrt{2}\beta} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{2}\beta}{5\sqrt{16\beta^2-9}} & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}.$$

For $\beta \in (0, \sqrt{3}/2)$*, define also* $\hat{\theta}_A = \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$*. The thresholds satisfy* $\hat{\theta}_B < \theta_A$ *and* $\theta_A < \hat{\theta}_A$ *for any* $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$*. With ER2, all the equilibria in duopoly are characterized as follows:*

(a) Define the A-outcome as the set of R&D efforts and outputs $x_{Ai}^A = \frac{4\beta D_A}{9-4\beta^2}$*,* $x_{Bi}^A = 0$*,* $q_{Ai}^A = \frac{3D_A}{9-4\beta^2}$*, and* $q_{Bi}^A = \frac{D_B}{3}$*, where both teams invest only in market A.*

The A-outcome is an equilibrium outcome if and only if $\frac{D_A}{D_B} \geq \theta_A$ *and the A-outcome is set as a continuation equilibrium in the two-merger subgame. Under the same conditions,* *both teams merge in equilibrium.*

(b) Define the B-outcome as the set of R&D efforts and outputs $x^B_{Ai} = 0$, $x^B_{Bi} = \frac{4D_B}{5}$, $q^B_{Ai} = \frac{D_A}{3}$, *and* $q^B_{Bi} = \frac{3D_B}{5}$, *both teams invest only in market B.*

The B-outcome is an equilibrium outcome if and only if $\frac{D_A}{D_B} \leq \hat{\theta}_B$. *Under the same conditions and with ER1, none of the teams merge in equilibrium.*

(c) Define the asymmetric outcome as the set of R&D efforts and outputs

$$(x^{\diamondsuit A}_A, x^{\diamondsuit B}_A, x^{\diamondsuit A}_B, x^{\diamondsuit B}_B) = \begin{cases} \left(\frac{4\beta D_A}{9-8\beta^2}, 0, 0, D_B\right) & \textit{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \left(\frac{D_A}{\beta}, 0, 0, D_B\right) & \textit{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases},$$

$$(q^{\diamondsuit A}_A, q^{\diamondsuit B}_A, q^{\diamondsuit A}_B, q^{\diamondsuit B}_B) = \begin{cases} \left(\frac{3D_A}{9-8\beta^2}, \frac{(3-4\beta^2)D_A}{9-8\beta^2}, 0, D_B\right) & \textit{if} \quad \beta < \frac{\sqrt{3}}{2} \\ (D_A, 0, 0, D_B) & \textit{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases},$$

where a team (denominated $\diamondsuit A$*) invests only in market A, and the other team (denominated* $\diamondsuit B$*) invests only in market B.*

The asymmetric outcome is an equilibrium outcome if and only if an asymmetric outcome is set as a continuation equilibrium in the two-merger subgame, $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$, *and* $\hat{\theta}_A \geq \frac{D_A}{D_B} \geq \hat{\theta}_B$, *or* $\beta \in \left[\frac{\sqrt{3}}{2}, 1\right)$ *and* $\frac{D_A}{D_B} \geq \hat{\theta}_B$. *Under the same conditions for* $\frac{D_A}{D_B} \neq \hat{\theta}_B$ *with ER1, only team* $\diamondsuit A$ *merges in equilibrium.*

The duopoly model has three types of equilibrium outcomes. This chapter denominates this group of equilibrium outcomes as *market outcomes*. In every market outcome, each team invests only in one market, a result of R&D cost assumptions. Any team that becomes a conglomerate in equilibrium invests only in market A. As the monopoly benchmark, the incentive to merge is to use the R&D lab in market A.

The symmetric A- and B-outcomes are equivalent to the outcomes in the monopoly

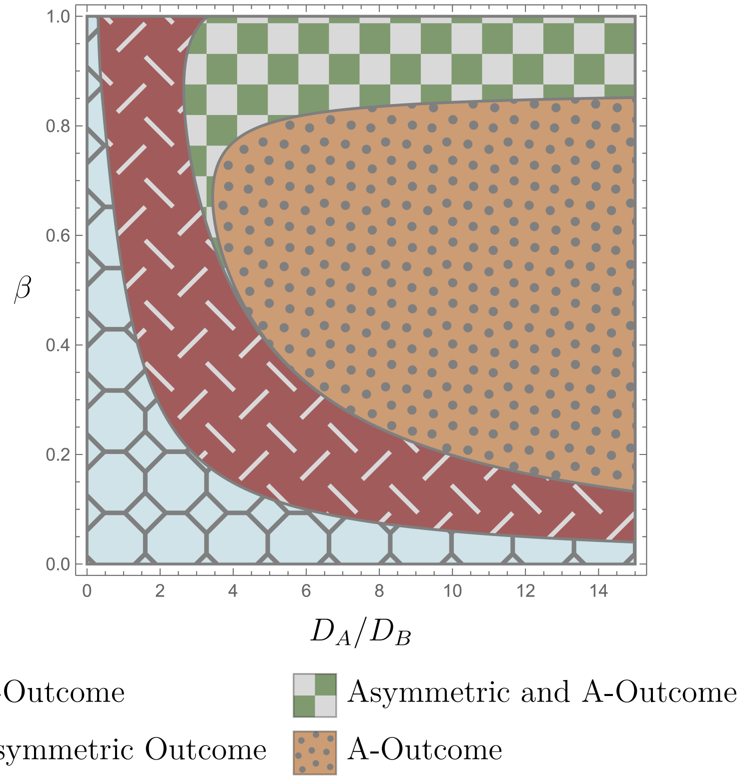

Figure 2.2: Duopoly's equilibria.

benchmark in that all the R&D labs are used only in the best and most profitable market. If market B is more profitable, none of the firms merge and both standalone firms in B invest in R&D only in their market. If market A is more profitable, two conglomerates are formed and both invest only in market A. Given that $\hat{\theta}_B < \theta_M < \theta_A$, for the B- and A-outcome to be an equilibrium outcome, the market ratio must be lower (higher) relative to the monopoly benchmark.

While both teams benefit from investing in the best market, they are hurt by the increased competition. There are two types of competition in the symmetric outcomes: the usual quantity competition and R&D competition. Both types increase the overall R&D effort and production in the symmetric outcome relative to the monopoly benchmark ($2x^k_{ki} > x^{Mk}_k$, $2q^A_{ki} > q^{MA}_k$ and $2q^B_{ki} > q^{MB}_k$). Furthermore, quantity competition is more intensified in a market with R&D investments than in the same market without them ($q^A_{Ai} > q^B_{Ai}$ and $q^B_{Bi} > q^A_{Bi}$).

The asymmetric outcome, which does not have an equivalent solution in the monopoly benchmark, exists in the duopoly model for intermediate values of the market ratio.[5] Here, only one team merges, becomes a conglomerate, and invests only in market A; the standalone firm in market B of the other team invests only in its market. As the profitability of the markets is not too different, the size of the markets is not the main motivation for investments. In the asymmetric outcome, there is always a team that invests in the worst market. Instead, the main incentive in the asymmetric outcome is to avoid competition. Indeed, when the asymmetric outcome is the equilibrium outcome the original framework of the duopoly is radically changed, as explained in Corollary 1, which follows directly from Proposition 2.

Corollary 1. *When the asymmetric outcome is an equilibrium outcome, the conglomerate produces nothing in market B; thus, the standalone firm is a monopoly in market B. If $\beta \geq \frac{\sqrt{3}}{2}$, the conglomerate is a monopoly in market A.*

In the asymmetric outcome equilibrium, the R&D effort and output of the standalone firm in market B are equivalent to the ones in the monopoly's MB outcome. Whether the conglomerate is a monopoly in market A depends on technological compatibility. When $\beta < \frac{\sqrt{3}}{2}$, R&D investments are overly expensive such that the conglomerate cannot afford the necessary R&D effort to seize all the demand. When $\beta \geq \frac{\sqrt{3}}{2}$, the conglomerate can invest in the necessary amount of R&D to capture all the demand, preventing the

[5]The asymmetric outcome also exists simultaneously with the A-outcome for high values of the market ratio, as in Figure 2.2. This region coincides with the one where both outcomes are in equilibrium in the two-merger subgame. The multiplicity originates in the two-merger subgame in the second stage. The equilibrium outcome is then determined by which continuation equilibrium is played in the two-merger subgame. When $\beta \geq \frac{\sqrt{3}}{2}$, the asymmetric outcome always exists as an equilibrium in the two-merger subgame no matter how profitable market A is relative to B because team $\Diamond A$ captures all the demand in market A when the technological compatibility is high; hence, it is not optimal for team $\Diamond B$ to deviate to A.

standalone firm from participating in the market A.

If the technological compatibility is larger than one half, the conglomerate invests more in R&D than in the monopoly's MA outcome ($x_A^{\Diamond A} \geq x_A^{MA}$ if and only if $\beta \geq \frac{1}{2}$). Even if the conglomerate is a monopolist, it must invest more because of the existing threat of a competitor. If $\beta < \frac{1}{2}$, R&D is overly expensive; hence, its effectiveness to gain a competitive advantage is severely reduced. Thus, in the asymmetric outcome, the conglomerate prefers not to invest as much as in the monopoly benchmark, where R&D does not have a role in competition and its only use is to reduce costs. In comparison to the A-outcome, the overall R&D in market A is (weakly) smaller in the asymmetric outcome ($x_A^{\Diamond A} \leq 2x_{Ai}^A$), being equal only when $\beta = \frac{\sqrt{3}}{2}$.

There exists a threshold $\bar{\theta} \in (\hat{\theta}_B, \theta_A)$, defined in Appendix 2.2, such that when the asymmetric outcome is the equilibrium outcome and the market ratio is below $\bar{\theta}$, the conglomerate is the team that invests in the least profitable market A, while the team of standalone firms invests in the most profitable market B. Hence, the team profits of the standalone firms are greater than the profits of the conglomerate. The merger is more beneficial to the non-merging firms than to the conglomerate. This phenomenon resembles the merger paradox of Salant et al. (1983).

The results explain the conglomerate discount, as stated in Proposition 3. Appendix 2.3 presents the proof.

Proposition 3. *When $\frac{D_A}{D_B} \geq \frac{9-4\beta^2}{5\beta^2}$, the profit of a conglomerate in the two-merger equilibrium is less than the sum of the profits of its standalone counterparts in the non-merger outcome.*

The conglomerate discount appears as a kind of prisoner dilemma in the model.[6]

[6]It follows that $\frac{9-4\beta^2}{5\beta^2} \geq \theta_A$ if $\beta \in \left(0, 4\sqrt{\frac{3}{101}}\right]$. Hence, the conglomerate discount does not always

R&D competition hurts the profits; thus, when the market ratio is adequately large, the teams would be better in the non-merger outcome as the R&D efforts are concentrated in the worst market B. However, given the rival's strategy, any team prefers to invest in the best market A, creating two conglomerates in equilibrium and increasing the level of competition in market A. Thus, excess R&D competition causes the conglomerate discount in the model.

2.5 Welfare and policy implications

The asymmetric outcome reduces the competition when it is an equilibrium outcome. However, unlike the symmetric outcomes, in the asymmetric outcome R&D is invested in both markets. Thus, the consumers are hurt by the decreased competition, but the consumers of one market benefit from the increased R&D. The overall effect is not clear; hence, the policy implications are not straightforward. Appendix 2.4 presents the welfare analysis for a better understanding. This chapter presents the results on the producer surplus, consumer surplus, and social welfare regarding the asymmetric outcome in Proposition 4. Further, Figure 2.3 contrasts the asymmetric equilibrium with the results on consumer surplus and social welfare.

Proposition 4. *Define the total producer surplus as TPS, total consumer surplus as TCS, and total social welfare as TW. Denote the A-, B-, and asymmetric outcome with the superscript A, B, and $\diamondsuit$, respectively. Define the thresholds* $\bar{\gamma} = \frac{(9-8\beta^2)\sqrt{5(9-4\beta^2)}}{6\beta\sqrt{2(9-23\beta^2+12\beta^4)}}$,

occur when two conglomerates are formed in equilibrium. The R&D cost function induces outcomes where firms focus on only one market. However, it is not enough to explain the conglomerate discount as it further depends on the market ratio and technological compatibility.

$\bar{\beta} = \frac{\sqrt{23-\sqrt{97}}}{2\sqrt{6}}$,

$$\gamma_A = \begin{cases} \frac{\sqrt{5}(9-8\beta^2)(9-4\beta^2)}{12\beta\sqrt{81-189\beta^2+120\beta^4-16\beta^6}} & if \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{\sqrt{5}(9-4\beta^2)}{3\sqrt{(4\beta^2-3)(15-4\beta^2)}} & if \quad \beta > \frac{\sqrt{3}}{2} \end{cases}, \quad \gamma_B = \begin{cases} \frac{3\sqrt{11}(9-8\beta^2)}{20\beta\sqrt{9-7\beta^2}} & if \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{11}}{5\sqrt{5}} & if \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}.$$

The thresholds satisfy $\hat{\theta}_B < \gamma_B$, $\gamma_B < \gamma_A$ *for* $\beta \neq \frac{\sqrt{3}}{2}$, $\gamma_A < \hat{\theta}_A$ *for any* $\beta < \frac{\sqrt{3}}{2}$, $\bar{\beta} < \frac{\sqrt{3}}{2}$, *and* $\bar{\gamma} > \hat{\theta}_A$ *for any* $\beta \in (0, \bar{\beta})$.

(a) $TPS^{\diamondsuit} > \max\{TPS^A, TPS^B\}$.

(b) $TCS^{\diamondsuit} \geq TCS^A$ *if* $\frac{D_A}{D_B} \leq \gamma_A$ *and* $\beta \neq \frac{\sqrt{3}}{2}$, *and* $TCS^{\diamondsuit} > TCS^A$ *if* $\beta = \frac{\sqrt{3}}{2}$. $TCS^{\diamondsuit} \geq TCS^B$ *if* $\frac{D_A}{D_B} \geq \gamma_B$.

Thus, $TCS^{\diamondsuit} > \max\{TCS^A, TCS^B\}$ *if* $\gamma_A \geq \frac{D_A}{D_B} \geq \gamma_B$ *and* $\beta \neq \frac{\sqrt{3}}{2}$, *or* $\frac{D_A}{D_B} \geq \gamma_B$ *and* $\beta = \frac{\sqrt{3}}{2}$. *Under the same conditions, the asymmetric outcome is an equilibrium outcome.*

(c) $TW^{\diamondsuit} > \max\{TW^A, TW^B\}$ *if* $\beta < \bar{\beta}$ *and* $\frac{D_A}{D_B} \leq \bar{\gamma}$, *or* $\beta \geq \bar{\beta}$. *Thus, whenever the asymmetric outcome is an equilibrium outcome, it is the social welfare dominating outcome.*

The statement in part (a) of Proposition 4 is not surprising because, in the asymmetric outcome, the firms gain from their R&D investments and there is no competition; in the symmetric outcome, the product and R&D competitions harm the firms. Thus, why can the symmetric outcomes be supported in equilibrium? By the nature of the asymmetric outcome, there is always a team investing in the worst market. For extreme values of the market ratio, such a team has incentives to change its merger decision to deviate to a more profitable symmetric outcome.

Under the conditions of part (b) of Proposition 4, the reduction of competition in the asymmetric outcome is compensated by the increase in the overall R&D effort. This situation mainly occurs when the quantities produced in the markets end up being similar.

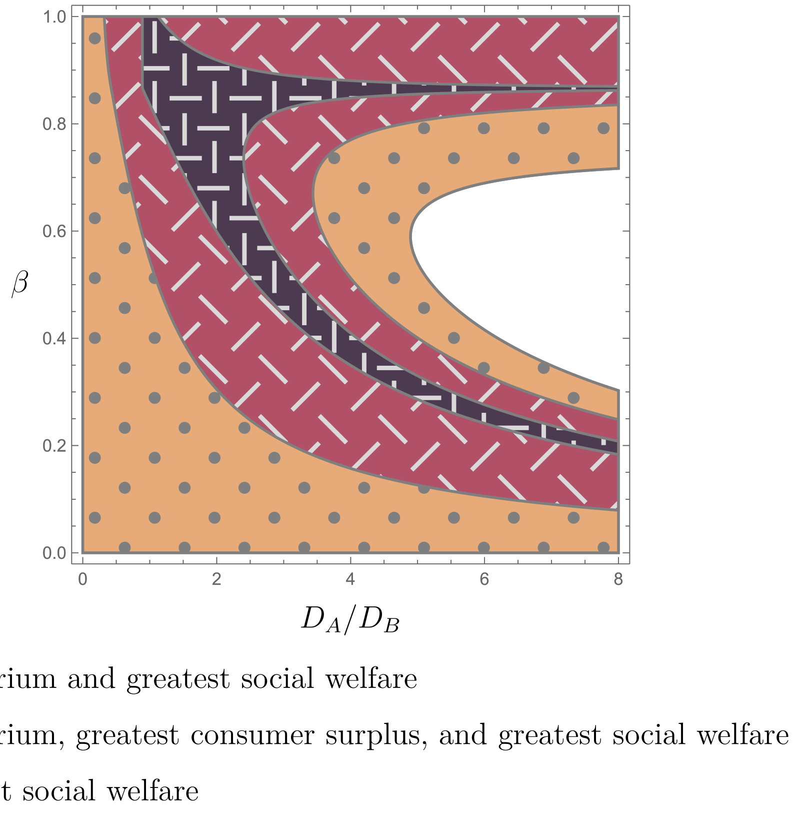

Figure 2.3: Asymmetric outcome: equilibrium, consumer surplus and social welfare.

For example, as in Figure 2.3, the asymmetric outcome is the best for the consumers when $D_A = D_B$ and β is adequately large (i.e., when the technological compatibility is close to being perfect). This same intuition applies to the dominance area of consumer surplus that extends to the lower right region of Figure 2.3. An increase in the market ratio must be compensated by a decrease in technological compatibility to maintain the similarity in produced quantities. The dominant area of consumer surplus also converges to $\beta = \frac{\sqrt{3}}{2}$. At $\beta = \frac{\sqrt{3}}{2}$, the R&D efforts in market A in the A-outcome and asymmetric outcome are equal. Thus, at $\beta = \frac{\sqrt{3}}{2}$, the asymmetric outcome dominates the A-outcome, as, in the former outcome, R&D is invested in market B and, thus, has more overall R&D effort.

If the conditions of part (b) do not hold but the asymmetric outcome is the equilibrium outcome, then the consumers are better-off in some symmetric outcome. If the size of market B is considerably greater than A, then an equilibrium where only one conglomerate merger occurs is harmful to the consumers and, thus, the competition authority should prevent that merger. When market A is considerably greater than B and $\beta \neq \frac{\sqrt{3}}{2}$, the policy is more intricate. Here, the consumers would be better-off in the A-outcome; hence, the policy implication is that the authorities should encourage a second conglomerate merger to reach the two-merger outcome. However, such a policy is outside the jurisdiction of competition authorities like the European Commission.

Part (c) of Proposition 4 states that the asymmetric outcome in equilibrium always achieves the greatest total welfare. Thus, implementing any kind of policy would go against the interest of the overall society. Nevertheless, this welfare result is attributed mainly to the producer surplus. Competition authorities generally prioritize consumers. Therefore, regulation of the asymmetric outcome remains valuable.

If the conditions in part (c) of Proposition 4 are not satisfied, the A-outcome is the social welfare dominant outcome.[7] Further, the A-outcome is also an equilibrium outcome. The high market ratio signifies that market A contributes the most to the total consumer surplus. Though the conglomerate discount might be observed, low technological compatibility implies that the competition in market A in the A-outcome is not exceedingly intense. Thus, the consumers in market A benefit from the competition without greatly hurting the profits of the firms.

2.6 Conclusion

This chapter examined how shifting R&D capabilities through a conglomerate merger affects competition in a Cournot duopoly framework with technologically related firms. Whether a conglomerate merger depresses competition depends on the R&D investment

[7] The B-outcome is never the social welfare dominating outcome. In this market outcome, the competition greatly reduces the profits, as there is perfect technological compatibility.

behavior. In symmetric equilibria, all firms invest only in the most profitable market. If market B is the most profitable, none of the firms merge, as there are no incentives to use the R&D lab in market A. If market A is the most profitable, two conglomerates are created in equilibrium, as there are incentives to use the R&D lab in market A. These kinds of equilibria preserve the original framework of the duopoly, because two firms produce in each market.

In asymmetric equilibria, one conglomerate is formed to invest in market A, while one standalone firm invests in market B. With this strategy, the firms avoid product and R&D competition, and at least one market is monopolized. Further, the consumers of all markets benefit from the R&D unlike in the symmetric outcomes. The positive effect of the R&D is offset by the effect of decreased competition when the market sizes are uneven. A conglomerate merger to allocate resources from a profitable market to a less profitable market is harmful to consumers; thus, competition authorities should prevent it. In the inverse scenario, a conglomerate merger to allocate resources from an unprofitable market to a more profitable market is not harmful to consumers, but a higher number of conglomerate mergers would be better. Hence, the policy implication is to enforce more mergers; however, competition authorities like the European Commission do not have the power to implement such a policy.

The welfare results might be overturned under a different R&D framework. The scope of the chapter was limited to shifting R&D capabilities in one direction by the assumption that only the firms in market B own an R&D lab. Given such asymmetry, the identity of the markets is an important factor on whether the policy implication is outside the jurisdiction of competition authorities. A path for future research is to consider that resources can be shifted in both directions. This extension of the model might be useful to understand which characteristics of the markets determine the type of policy.

This chapter constructs the model assuming a static setting with imperfect technology incompatibility. In the long term, the incompatibility may disappear, and R&D efficiencies or spillovers may arise. Under such conditions, it might be possible to achieve a more preferable outcome from the perspective of the consumer or social welfare, such as an outcome with a product and R&D competition in all markets. An avenue for future research is to extend the model with more general R&D and cost assumptions. Beyond more general or complex functional forms, a dynamic setting might be necessary to factor in the process of adoption of new technology. This line of research is worth pursuing, as formulating more appropriate policies might require a better understanding of the conglomerate effects of mergers on welfare beyond competitive issues.

Chapter 3

Agency Problems in a Competitive Conglomerate with Production Constraints

3.1 Introduction

Agency frameworks have been used extensively to explain the inner mechanism of conglomerates, firms that participate in several industries. The hypothesis that the conglomerate discount is caused by agency problems is of particular importance (Maksimovic & Phillips, 2007). Broadly speaking, the conglomerate discount theory claims that the conglomerate is less than the sum of the values of its individual parts (Berger & Ofek, 1995). One argument is that conflicts of interest inside the conglomerate creates inefficiencies in capital allocation (Busenbark et al., 2017), thus causing the conglomerate discount. While the capital allocation efficiency literature is substantial, it usually neglects the conglomerate's strategic interaction with the market. Indeed, the interactions of conglomerates, such as Amazon, are not limited to those that happen within the firm, Amazon also has to deal with competitors, such as E-bay in the online retail market or Netflix in the video streaming market.

If agency problems have an effect in a conglomerate, it is reasonable to assume that the same agency problems also have an effect on the markets where the conglomerate participates, and by extension, on social welfare. Despite this, explicit policies regarding conglomerate effects related to agency problems are lacking. Indeed, only some conglomerate effects are monitored and regulated. For instance, competition authorities' concern for conglomerates effects when these have the potential to lessen competition (Markovits, 2014). In the EU "Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings" (2008) , the only non-coordinated conglomerate effect listed is foreclosure. Specifically, bundling and tying practices.

This chapter explores the reciprocal effects between agency problems and market competition. We develop a tractable theoretical model with three defining characteristics. First, there is a conglomerate (or multi-market firm) facing oligopolistic competition. We assume that the conglomerate competes as the leader in two duopoly markets with a Stackelberg-Cournot framework with heterogeneous goods. The two demand functions are independent of each other.

Second, the conglomerate has production constraints. The conglomerate has a common pool of resources that functions as the input for both products. If the resources are low, there is an opportunity cost of producing in one market or the other.

Third, there is an agency problem inside the conglomerate. We consider an adverse selection model. We assume that a conglomerate consists of its headquarters and two division managers. The headquarters' profit depends on both divisions, while the managers' utility depends only on the performance of their own division. The headquarters decide how to allocate the resources across the divisions. The managers, using these resources, take production decisions in their respective divisions.

The headquarters do not know the value of the intercept of the demand in one of the markets, but the manager running that division does. That intercept of demand is a random variable that can take two values: high or low. Through a contract mechanism headquarters might obtain that information from the manager to allocate resources contingent on the manager's report.

The payoffs of the headquarters and the manager are different, so the manager might have incentives to lie. Depending on the state, there exist an ideal level of resources that maximizes the profit of a division. However, that ideal value might not be achieved if the resources are low enough. In this case, the motivation behind the manager's report is to obtain a level of resources as close as possible to the ideal value. Thus, the contract mechanism must offer allocations contingent on the state such that the manager receives the amount closest to the ideal value only if the truth is reported.

We first solve a benchmark with symmetric information. We find two equilibria. First, the unrestricted equilibrium, where the resources are plenty and the production constraint is not binding. Second, the restricted equilibrium, where the resources are not plenty, the production constraint is binding, and thus, there is an opportunity cost of production.

We find that the solutions of the model under asymmetric information and the benchmark are equivalent if the resources of the conglomerate are large enough, that is, the first-best contract can be achieved. Conversely, if the resources are low, the conglomerate can only induce truth-telling with the second-best contract. Depending on the level of resources, the second-best contract is separated or pooled.

A separating second-best contract is achieved if the resources are not too low. The mechanism enables headquarters to distinguish the true value of the demand and to allocate resources accordingly. With this contract, in comparison to the benchmark in any state, the conglomerate produces more in the market with asymmetric information and less in the market with the certain demand. The benchmark's high state allocation in the market with uncertain demand is too close to the low state's ideal value. Hence, the headquarters allocate more resources in the high state so that the resources are further away from the low state's ideal value. Simultaneously , more resources are also allocated in the low state to approach them to the low state's ideal value. Conversely, less resources are allocated in the market with certain demand in both states as there is an opportunity cost of producing in one market or the other.

If the resources are very low, a separating contract is too costly, and thus, the headquarters is content with a pooling contract. The headquarters know the value of the demand with the mechanism, but the resource allocation plan remains the same regardless of the information revealed. With this contract, the production is equivalent to the outcome of a model where none of the players inside the conglomerate knows the true value of the demand, so the information is useless with this scheme.

Furthermore, we analyze the ex-ante and post social welfare. We show that under certain conditions the social welfare improves in the model under asymmetric information in comparison to the symmetric information benchmark. Because there is an opportunity cost of production, the allocation distortion caused by the second-best contracts transfers surplus from one market to the other. Welfare might improve if surplus is transferred from the worst market to the best market. Welfare is more likely to increase if the goods are complements, as the surplus of the follower firms and consumers change in the same direction as the variations in the conglomerate's production.

Particular to the ex-ante welfare, we find that the separating contract sometimes improves it if the market with uncertain demand is large on average in comparison to the market with certain demand. In contrast, we find that the pooling contract never improves the ex-ante welfare.

Finally, we analyze how the degree of differentiation delimits the type of equilibrium. Under symmetric information, at higher levels of substitutability or complementarity the restricted equilibrium is more likely to occur. Under asymmetric information and when the resources are low but not too low, at higher levels of substitutability or complementarity the second-best contract is more likely to be implemented. Under asymmetric information, and when the resources are very low, at higher levels of substitutability or complementarity the pooling equilibrium is more likely to occur.

This chapter is organized as follows: Section 3.2 reviews the related literature. Section 3.3 specifies the model structure. Section 3.4 presents the symmetric information benchmark. Section 3.5 solves and analyzes the model. Section 3.6 analyzes social welfare. Section 3.7 analyzes the effect of the degree of differentiation on the delimitation of the type of equilibrium. Section 3.8 concludes.

3.2 Literature review

This chapter is relevant to the literature related to resource allocation and agency problems. Especially pertinent is the literature where production constraints are explicitly considered. The theoretical model of Harris, Kriebel, and Raviv (1982) shows that using a transfer pricing scheme as an allocation mechanism is cost minimizing and induces the divisions of a firm to tell the truth. A transfer pricing scheme is feasible even if the resource constraint is binding. Cachon and Lariviere (1999) consider a model where a supplier allocates a limited capacity to multiple downstream retailers. One of their main results is that the mechanism that maximizes the total profits of the retailers cannot induce truth-telling if the capacity is binding. In our model, the first-best contract can be achieved even when the production constraint is binding, if the resources are not too low. This implies that the managers tell the truth while maximizing the profit of their own divisions.

Another strand of literature about resource allocation focuses on financial resources within a conglomerate. Particularly, the corporate finance's capital allocation efficiency literature investigates whether the distribution of financial resources across divisions in a multi-market firm matches with their respective performance, that is, whether a high-prospects division receives more than a low-prospects division (Busenbark et al., 2017). Studies on efficient allocation ascertain that the firm prioritizes the most profitable endeavors over the less profitable ones (see for example Stein (1997) ,Maksimovic and Phillips (2002), Brusco and Panunzi (2005))

Opposing the theory of efficient allocation, there is literature proposing that agency problems cause inefficiency in capital allocation. In theoretical research, Rajan et al. (2000) predict that as the diversity increases, the transfers from better-opportunities divisions to worse-opportunities divisions increases. The reason is that allocating resources to the weak division improves the contribution of this division to the joint profit, increasing the strong division's incentives to invest efficiently. In Stein and Scharfstein (2000), the division managers of weak divisions engage in rent-seeking behavior, which is costly for the firm. To mitigate this behavior, the CEO can allocate capital inefficiently to the weak divisions. In Wulf (2009), the core division manager sends distorted information to the headquarters to influence the division of capital in favor of the core division and against the small division. In empirical contributions, Rajan et al. (2000) provide evidence supporting their theoretical hypothesis. In Arrfelt et al. (2013), a backward-looking logic leads to over-investment (under-investment) in low (high) expectations divisions.

We include agency problems in a conglomerate in the form of an adverse selection problem between headquarters and managers. However, the adverse selection not always cause inefficient resource allocation. Both the first and second-best contract can be achieved. The former can be interpreted as efficient and the second as inefficient. Thus, our model reconciles the theories of the efficient and inefficient allocation literature.

Literature focusing on the allocation of resources in competitive conglomerates is scarce. One of them is the study by Levinthal and Wu (2010). In their model, the authors assume two multi-market firms competing in two markets. These firms have the ability to relocate a fixed amount of resource across markets. Because the resource is finite, there is an opportunity cost in transferring the resource from one market to the other one. In equilibrium, there are more incentives to allocate resources to one market as the size of that market increases. Similarly, in our model, the headquarters allocate more resources to the greatest market under the first-best contract. However, under the second-contract, the headquarters might prioritize the worst market.

There are some other theoretical studies about competitive conglomerates, but they mainly incorporate foreclosure as the conglomerate effect. In Granier and Podesta (2010), a gas and electrical firms can price discriminate only after a merger, by selling their products in a bundle. In Tan and Yuan (2003), they study divestitures by assuming two competing conglomerates, each one supplying a group of goods. Within the conglomerate, the goods are complements, while across the conglomerates the goods are substitutes.

3.3 The model

A conglomerate firm participates in two markets that are not related horizontally or vertically, denoted by $k \in \{C, N\}$. The division in market N is a newly acquired division, while the division market C is the core business (or original business) of the firm. The conglomerate is run by a risk-neutral headquarters, which we assume is the owner of the firm, and two managers, each one in charge of one division.

We assume that the conglomerate is competing with one standalone firm in each one of the markets. The firms are denoted by $i \in \{1, 2\}$, where the conglomerate is 1 and the standalone firm is 2. We consider a sequential quantity competition (Stackelberg-Cournot) with heterogeneous goods, where the conglomerate is the leader in both markets and standalone firms are the followers.

We assume that a representative consumer in market k has a quasi-linear utility function with the form $U_k(q_{k0}, q_{k1}, q_{k2}) = q_{k0} + v_k(q_{k1}, q_{k2})$, where q_{k0} is the quantity of the numeraire good, q_{ki} is the output of firm i, and $v_k(q_{k1}, q_{k2})$ is given by:

$$v_k(q_{k1}, q_{k2}) = D_k(q_{k1} + q_{k2}) - \frac{1}{2}\left(q_{k1}^2 + 2\alpha q_{k1}q_{k2} + q_{k2}^2\right)$$

where $\alpha \in [-1, 1]$ is a constant measuring the degree of differentiation of the good and D_k is the intercept of the demand in market k. It stands that the goods are substitutes when $\alpha > 0$ and are complements when $\alpha < 0$. We assume a common α in both markets so that the only variable differentiating the markets is the intercept of the demand.

The utility function generates the following inverse demand function faced by firm i in market k:

$$P_{ki}\left(q_{ki}, q_{kj}\right) = D_k - q_{ki} - \alpha q_{kj}$$

where $j \in \{1, 2\}$ for $j \neq i$ and P_{ki} is the price of firm i in market k. In the core market, all the players know the value of the intercept, D_C, which is a positive constant. In the new market, the standalone firm and the division manager know the value of the intercept but the conglomerate's headquarters do not. However, the headquarters know that the intercept is a random variable that can take two values: high value D_N^H with probability $p_H \equiv p$ and low value D_N^L with probability $p_L \equiv 1 - p$, where $p \in (0, 1)$ is a constant. These priors are common knowledge. We assume that $D_N^H > D_N^L > 0$ and $D_C > 0$.

Although the demand functions are independent of each other, we assume that the products of both markets use a common input in their production process. For simplicity, we assume that the products are produced only with this common input. The conglomerate is endowed with a positive exogenous amount of input X, which is allocated between the divisions for them to produce their respective products. We refer to this endowment as the resources of the conglomerate. The production function for the conglomerate's product k is $q_{k1} = x_k$, where x_k is the amount of input. The total amount of input assigned to both divisions must satisfy that $x_N + x_C \leq X$. We can write this restriction in terms of quantities as $q_{N1} + q_{C1} \leq X$. We refer to the last inequality as the production–possibility constraint of the conglomerate. As for the standalone firms, we assume that in any scenario they have enough resources to operate without constraints in each one of their markets. Therefore, we ignore the production-possibility constraints of the standalone firms.

A corner solution for the conglomerate would entail producing nothing in one of the markets. As we are interested in the scenario where the conglomerate participates in both markets, we make two assumptions to guarantee interior solutions. First, for any s

with $s \in \{H, L\}$:

$$X \geq \frac{(2-\alpha)}{2(2-\alpha^2)} |D_C - D_N^s| \tag{3.1}$$

This implies that the firm has enough resources to operate in both markets in any state.

Second:

$$D_N^H - D_N^L < \frac{2D_C}{3} \tag{3.2}$$

This assumes that the difference between the intercepts of the demand of the new market in both states is small relative to the intercept of the core market.

The headquarters are in charge of the allocation of resources across the divisions. Each manager is in charge of producing and supplying the good to the market in their respective division. The headquarters' payoff is the sum of the profits of both divisions minus an exogenous fixed compensation for both managers. Each manager's utility come from the fixed compensation and the profit of their own division. The latter component is explained by a preference in empire building, which in this case is interpreted as the desire to manage a profitable division.[1] Without loss of generality, we normalize the fixed compensation of both managers to zero, so the headquarters' payoff is simply the profit of the overall firm, while each manager's utility is equivalent to the profit of their own division. The reservation utility of both managers is set to be zero. If a manager does not receive at least their reservation utility, they quit.

It is possible for the headquarters to ask the manager in division N for the value of D_N. However, the maximization of the overall profit does not necessarily imply the maximization of the profit of division N, so the manager might have incentives to not

[1]Empire building is mentioned in Stein and Scharfstein (2000) as an explanation of why managers profit from their own divisions while the principal profits from all divisions, thus creating the agency problem. In Wulf (2009) and Bernardo, Cai, and Luo (2001) managers gain utility as their allocation of capital increases. Empire building is given as a reason for this in Bernardo et al. (2001). In our model, managers desire to maximize the profits of their own division, but they do not necessarily desire a larger allocation of resources, as the profits are decreasing for a large enough production.

report truthfully. To induce truth-telling, the headquarters establish a contract obliging the manager to produce a specific amount of output contingent on the announced value of the demand in market N.[2]

We assume that the headquarters commit to allocate enough resources to the manager to produce the agreed quantities. Headquarters do not have incentive to give to the manager in division N extra resources as they would be wasted. Headquarters might have incentives to allocate less than the necessary resources, but this strategy would preclude the manager to fulfill the contract.

The sequence of events is as follows:

1. The headquarters offer a contract to the manager of division N. The contract establishes that the manager has to produce $q_N\left(D_N^s\right)$ if the reported state is s.

2. The manager of division N reports the value of the demand in N.

3. The headquarters allocate resources simultaneously to both divisions depending on the report of the manager.

4. Both managers set the output in their respective markets.

5. The standalone firms set their output in both markets after observing the quantities produced by the conglomerate.

We solve the game in the following sections. Furthermore, we concentrate on pure strategies.

[2] Managerial compensation contracts might also include an endogenous fixed payment and a profit share rule (see for example Bernardo et al. (2001)). Here, we are not interested in optimal contracts but rather in the effect of agency problems in the markets. Thus, we simplify the problem by considering output as the only the component of the contract, as its allocation is what generates the agency problem.

3.4 Symmetric information benchmark

Here, it is assumed that the headquarters know the true value of the demand in market N. Given that that conglomerate has already selected $\check{q}_{k1}$, the standalone firm in market C solves the following problem:

$$\max_{q_{C2} \geq 0} \quad \left(D_C - q_{C2} - \alpha \check{q}_{C1}\right) q_{C2}$$

While the standalone firm in market N and state s solves the following problem:

$$\max_{q^s_{N2} \geq 0} \quad \left(D^s_N - q^s_{N2} - \alpha \check{q}_{N1}\right) q^s_{N2}$$

The best response functions of the standalone firms in market C and N in state s are respectively as follows:

$$q^B_{C2}\left(\check{q}_{C1}\right) = \frac{D_C - \alpha \check{q}_{C1}}{2} \text{ and } q^{sB}_{N2}\left(\check{q}_{N1}\right) = \frac{D^s_N - \alpha \check{q}_{N1}}{2} \tag{3.3}$$

The headquarters' maximization problem in state s is:

$$\begin{aligned}
&\max_{\forall k, q^s_{k1} \geq 0} \quad \left(D_C - q^s_{C1} - \alpha q^B_{C2}\left(q^s_{C1}\right)\right) q^s_{C1} + \left(D^s_N - q^s_{N1} - \alpha q^{sB}_{N2}\left(q^s_{N1}\right)\right) q^s_{N1} = \\
&\max_{\forall k, q^s_{k1} \geq 0} \quad \frac{1}{2}\left(\left((2-\alpha)D_C - (2-\alpha^2)q^s_{C1}\right) q^s_{C1} + \left((2-\alpha)D^s_N - (2-\alpha^2)q^s_{N1}\right) q^s_{N1}\right) \qquad (3.4) \\
&\text{s.t.} \qquad q^s_{N1} + q^s_{C1} \leq X
\end{aligned}$$

where after the equal sign we substitute (3.3) into the headquarters' objective function. Whether the restriction is binding depends on the parameters of the problem. First, we assume that the resources are plenty, and hence that the restriction is not binding. We denote this solution with U. The optimal outputs of the conglomerate in market C and N in state s are as follows:

$$q_{C1}^{U} = \frac{2-\alpha}{2(2-\alpha^2)} D_C \text{ and } q_{N1}^{sU} = \frac{2-\alpha}{2(2-\alpha^2)} D_N^s$$

and the optimal outputs of the standalone firms in market C and N in state s are as follows:

$$q_{C2}^{U} = \frac{4-\alpha^2-2\alpha}{4(2-\alpha^2)} D_C \text{ and } q_{N2}^{sU} = \frac{4-\alpha^2-2\alpha}{4(2-\alpha^2)} D_N^s$$

U is an equilibrium in state s if the resources are plenty, specifically:

$$X \geq \frac{2-\alpha}{2(2-\alpha^2)} \left(D_C + D_N^s\right) = \Omega^s$$

Second, we assume that the resources are not plenty in state s, that is $X < \Omega^s$. The restriction in (3.4) is binding and the problem in state s can be rewritten as follows in terms of q_{N1}^s:

$$\max_{q_{N1}^s \geq 0} \quad \frac{2-\alpha^2}{2} \left(\left(\frac{(2-\alpha)D_C}{2-\alpha^2} - X + q_{N1}^s \right) (X - q_{N1}^s) + \left(\frac{(2-\alpha)D_N^s}{2-\alpha^2} - q_{N1}^s \right) q_{N1}^s \right)$$

We denote this solution with R. From the first order condition (FOC) the optimal outputs of the conglomerate in state s are:

$$q_{C1}^{sR} = q_{C1}^{U} - \frac{\theta^{sR}}{2} \text{ and } q_{N1}^{sR} = q_{N1}^{sU} - \frac{\theta^{sR}}{2}$$

and the optimal outputs of the standalone firms in state s are:

$$q_{C2}^{sR} = q_{C2}^{U} + \alpha\frac{\theta^{sR}}{4} \text{ and } q_{N2}^{sR} = q_{N2}^{sU} + \alpha\frac{\theta^{sR}}{4}$$

where $\theta^{sR} = q_{N1}^{sU} + q_{C1}^{U} - X$ is the conglomerate's deficit in the resources needed to achieve U in state s. Thus, in R each one of the conglomerate's divisions faces a reduction in production equal to half the production deficit. Consequently, an increase in the resources increments the production in both divisions in half the increase of X ($\frac{\partial q_{k1}^{sR}}{\partial X} = \frac{1}{2}$). Given (3.1), it follows that $q_{k1}^{sR} \geq 0$ and $q_{k2}^{sR} \geq 0$ for all s. We summarize the results of this section in Proposition 5.

Proposition 5. *The equilibria in the symmetric information benchmark are characterized as follows:*

a) When $X \geq \Omega^H$, U is an equilibrium in both states.

b) When $\Omega^H > X \geq \Omega^L$, R is an equilibrium in the high state and U is an equilibrium in the low state.

b) When $X < \Omega^L$, R is an equilibrium in both states.

When the equilibrium is U, the conglomerate has plenty of resources, and thus, there is not an opportunity cost to produce in one market or the other. Each division of the conglomerate functions as a standalone firm, without considering the other market when taking decisions. As expected, in market N the conglomerate and standalone firm produce more in the high state than in the low state ($q_{Ni}^{HU} > q_{Ni}^{LU}$). Contrastingly, the outputs of both firms in market C are independent of the state.

When the equilibrium is R, the conglomerate faces an opportunity cost to produce in one market or the other as the resources are not enough to produce the optimal output in both markets. In comparison to U, with the production restriction in R, the conglomerate produces less in both markets in any state ($q_{C1}^{U} > q_{C1}^{sR}$ and $q_{N1}^{sU} > q_{N1}^{sR}$). Similar to U, in R the conglomerate produces more in market N if the state is high ($q_{N1}^{HR} > q_{N1}^{LR}$). In this case, the output in market C depends of the state. Hence, given the trade-off between market C and N, the conglomerate produces less in market C if the state is high ($q_{C1}^{LR} > q_{C1}^{HR}$).

The effect of the conglomerate's production constraint on standalone firms depends on whether the goods are substitutes or complements. If the goods are substitutes, in any state both standalone firms are better off in R than in U because their production increases ($q_{C2}^{U} < q_{C2}^{sR}$ and $q_{N2}^{sU} < q_{N2}^{sR}$). Conversely, if the goods are complements, in any state both standalone firms are worse off because their production decreases ($q_{C2}^{U} > q_{C2}^{sR}$ and $q_{N2}^{sU} > q_{N2}^{sR}$). Similar to the conglomerate, the standalone firm in market N produces more in the high state ($q_{N2}^{HR} > q_{N2}^{LR}$). This same comparison in market C depends on α. If the goods are substitutes, the standalone firm in C produces more in the high state ($q_{C2}^{HR} > q_{C2}^{LR}$). Conversely, if the goods are complements, it produces more in the low state ($q_{C2}^{HR} < q_{C2}^{LR}$). This is consistent with the observed behavior of the conglomerate in market C ($q_{C1}^{LR} > q_{C1}^{HR}$).

3.5 Information revelation

By the revelation principle, we can restrict our attention to only the truth-telling situations. The problem of the headquarters is:

$$\max_{\forall k, \forall s, q_{k1}^s \geq 0} \quad \sum_{s \in \{H,L\}} p_s \left(P_{C1}\left(q_{C1}^s, q_{C2}^B\left(q_{C1}^s\right)\right) q_{C1}^s + P_{N1}^s\left(q_{N1}^s, q_{N2}^{sB}\left(q_{N1}^s\right)\right) q_{N1}^s \right) \tag{3.5}$$

$$\text{s.t.} \quad \forall s, q_{N1}^s + q_{C1}^s \leq X$$

$$P_{N1}^H\left(q_{N1}^H, q_{N2}^{HB}\left(q_{N1}^H\right)\right) q_{N1}^H \geq P_{N1}^H\left(q_{N1}^L, q_{N2}^{HB}\left(q_{N1}^L\right)\right) q_{N1}^L \tag{3.6}$$

$$P_{N1}^L\left(q_{N1}^L, q_{N2}^{LB}\left(q_{N1}^L\right)\right) q_{N1}^L \geq P_{N1}^L\left(q_{N1}^H, q_{N2}^{LB}\left(q_{N1}^H\right)\right) q_{N1}^H \tag{3.7}$$

$$\forall s, P_{C1}\left(q_{C1}^s, q_{C2}^B\left(q_{C1}^s\right)\right) q_{C1}^s \geq 0 \tag{3.8}$$

$$\forall s, P_{N1}^s\left(q_{N1}^s, q_{N2}^{sB}\left(q_{N1}^s\right)\right) q_{N1}^s \geq 0 \tag{3.9}$$

Where the restrictions (3.6) and (3.7) are the high and low incentive compatibility constraints (IC), respectively. The restrictions (3.8) and (3.9) are the individual rationality constraints (IR) of the manager of division C and N, respectively. The IC constraints ensure that a truthful report is optimal for the manager. The IR constraints impose that the managers receive at least their reservation utility.[3]

The problem is simplified by substituting (3.3) into (3.5)-(3.9). To further simplify, we substitute q_{C1}^U and q_{N1}^{sU} into (3.5), q_{N1}^{HU} into (3.6), q_{N1}^{LU} into (3.7), q_{C1}^U into (3.8), and q_{N1}^{sU} (3.9). The simplified problem is as follows:

[3] We assumed that any manager quits if their gain is not at least their reservation utility. Thus, we include the IR constraint of the manager of the division C even though there is no adverse-selection between the headquarters and that manager.

$$\max_{\forall k, \forall s, q_{k1}^s \geq 0} \quad \frac{2-\alpha^2}{2} \sum_{s \in \{H,L\}} p_s \left(\left(2q_{C1}^U - q_{C1}^s\right) q_{C1}^s + \left(2q_{N1}^{sU} - q_{N1}^s\right) q_{N1}^s \right) \tag{3.10}$$

$$\text{s.t.} \quad \forall s, q_{N1}^s + q_{C1}^s \leq X \tag{3.11}$$

$$\left(q_{N1}^H - q_{N1}^L\right)\left(2q_{N1}^{HU} - q_{N1}^H - q_{N1}^L\right) \geq 0 \tag{3.12}$$

$$\left(q_{N1}^H - q_{N1}^L\right)\left(q_{N1}^H + q_{N1}^L - 2q_{N1}^{LU}\right) \geq 0 \tag{3.13}$$

$$\forall s, 2q_{C1}^U \geq q_{C1}^s \tag{3.14}$$

$$\forall s, 2q_{N1}^{sU} \geq q_{N1}^s \tag{3.15}$$

3.5.1 First-best contract

Here, we derive the conditions on the parameters that allow to achieve the solution in the symmetric information benchmark with the contract mechanism. First, when $X \geq \Omega^H$, the allocation of resources is such that the division N in state s can produce q_{N1}^{sU}, achieving the maximum unconstrained profit, and thus, the ideal level of output for the manager in division N and the headquarters. Hence, the manager does not have incentives to lie, the IC constraints are satisfied and the first-best is achievable.

Second, when $\Omega^H > X \geq \Omega^L$, the allocation of resources is ideal in the low state but not in the high state. In this case the resources are not plenty enough in the high state, so the maximum unconstrained level of profit is not achieved in neither of the divisions. Because the profit of any division is quadratic and concave in the output, if the ideal output is not achievable, the preferred alternative is a level of output as close as possible to the ideal output. In this scenario, it follows that $q_{N1}^{HU} > q_{N1}^{HR} > q_{N1}^{LU}$. Here, the resources are not too low, so the deficit caused by the production constraint is not too severe to distort the high state output below the unrestricted low state output. Thus, the high state manager of division N prefers the output of the high state, so there are no incentives to lie. If the state is low, the ideal output is achieved in division N, so the manager of division N does not have incentives to lie in this state either. Therefore, the first-best is achievable in this case.

Third, when $X < \Omega^L$, the allocation of resources in any state does not allow to reach the ideal level of output in neither of the divisions. The high state manager of division N does not have incentives to lie as $q_{N1}^{HU} > q_{N1}^{HR} > q_{N1}^{LR}$. However, if the state is low, there might be incentives to lie. If q_{N1}^{HR} is closer to q_{N1}^{LU} than q_{N1}^{LR}, it is profitable for the manager to lie. In the converse case, q_{N1}^{HR} is so high that q_{N1}^{LR} is preferred. Intuitively, the manager might not lie if that results in an unprofitable overproduction.

The low state IC constraint (3.13) holds if $q_{N1}^{HR} + q_{N1}^{LR} \geq 2q_{N1}^{LU}$. Thus, the first-best contract is achieved if and only if:

$$X \geq \frac{2-\alpha}{4(2-\alpha^2)}\left(2D_C + 3D_N^L - D_N^H\right) = \hat{\Omega} \tag{3.16}$$

With (3.2), it follows that $\hat{\Omega} > 0$. Moreover, given that $\hat{\Omega} < \Omega^L$, (3.16) is not guaranteed to hold. Without (3.2), if the right side of the inequality in (3.16) is non-positive, (3.16) will hold as $X > 0$. Intuitively, without the assumption, the difference $D_N^H - D_N^L$ might be so large that lying when the state is low always results in overproduction. We state the main result of the first-best contract in Proposition 6.

Proposition 6. *A contract achieves the first-best outcome if and only if $X \geq \hat{\Omega}$. The equilibria in the first-best outcome are characterized as follows:*

a) When $X \geq \Omega^H$, U is an equilibrium in both states.

b) When $\Omega^H > X \geq \Omega^L$, R is an equilibrium in the high state and U is an equilibrium in the low state.

b) When $\Omega^L > X \geq \hat{\Omega}$, R is an equilibrium in both states.

3.5.2 Second-best contract

Suppose that $X < \hat{\Omega}$ so that the first-best contract is not achieved. Here, (3.13) is binding, so it follows that either $q_{N1}^H = q_{N1}^L$ or $q_{N1}^H + q_{N1}^L = 2q_{N1}^{LU}$.[4] With $q_{N1}^H = q_{N1}^L$, we obtain a pooling equilibrium candidate. With $q_{N1}^H + q_{N1}^L = 2q_{N1}^{LU}$, we get a separating equilibrium candidate. We compute the headquarters' ex-ante expected profit for each candidate and compare them to establish the existence of equilibria. We state formally these equilibria in Proposition 7.

Proposition 7. *(a) A separating equilibrium S, exists if and only if $\hat{\Omega} > X \geq \check{\Omega}$, where:*

$$\check{\Omega} = \frac{(2-\alpha)}{2(2-\alpha^2)}\left(D_C + 2D_N^L - D_N^H\right)$$

In this equilibrium, the outputs of the conglomerate are:

$$q_{N1}^{HS} = q_{N1}^{HR} + (1-p)\theta^S, \quad q_{N1}^{LS} = q_{N1}^{LR} + p\theta^S,$$
$$q_{C1}^{HS} = q_{C1}^{HR} - (1-p)\theta^S, \quad q_{C1}^{LS} = q_{C1}^{LR} - p\theta^S$$

where $q_{N1}^{HS} > q_{N1}^{LS}$ always holds. The outputs of the standalone firms are:

$$q_{N2}^{HS} = q_{N2}^{HR} - \frac{\alpha(1-p)\theta^S}{2}, \quad q_{N2}^{LS} = q_{N2}^{LR} - \frac{\alpha p\theta^S}{2},$$
$$q_{C2}^{HS} = q_{C2}^{HR} + \frac{\alpha(1-p)\theta^S}{2}, \quad q_{C2}^{LS} = q_{C2}^{HR} + \frac{\alpha p\theta^S}{2}$$

[4]For all the proofs related to the second-best contract, see Appendix 3.

where[5]*:*

$$\theta^S = \frac{2-\alpha}{4(2-\alpha^2)}(2D_C + 3D_N^L - D_N^H) - X$$

(b) A pooling equilibrium P, exists if and only if $X \leq \check{\Omega}$. In this equilibrium, the output of the conglomerate for any k is:

$$\bar{q}_{k1}^P = pq_{k1}^{HR} + (1-p)q_{k1}^{LR}$$

Furthermore, the outputs of the standalone firms are:

$$q_{N2}^{HP} = q_{N2}^{HR} + \frac{\alpha(1-p)\theta^P}{2}, \quad q_{N2}^{LP} = q_{N2}^{LR} - \frac{\alpha p \theta^P}{2}, \quad \bar{q}_{C2}^P = pq_{C2}^{HR} + (1-p)q_{C2}^{LR}$$

where:

$$\theta^P = \frac{(2-\alpha)}{4(2-\alpha^2)}(D_N^H - D_N^L)$$

We illustrate all the equilibria found in the symmetric and asymmetric information cases for each state in Figure 3.1.

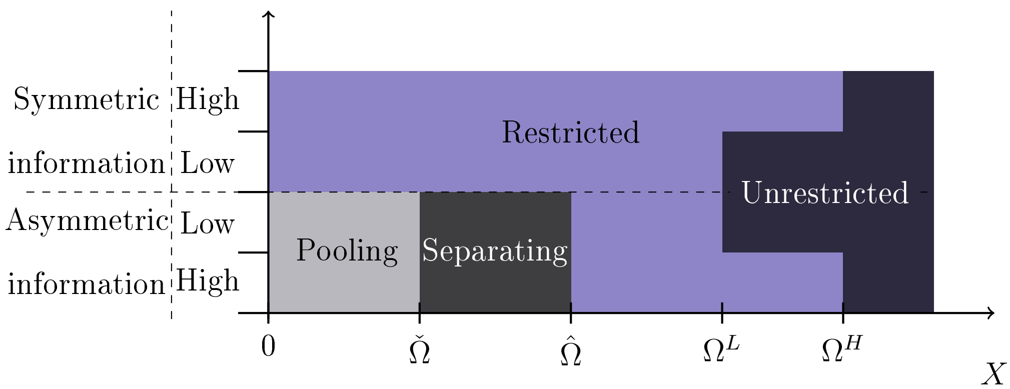

Figure 3.1: Existence of Equilibria.

In equilibrium $\hat{E}$, for $\hat{E} = \{P, S\}$, the restricted output of the conglomerate in state s is distorted in $\theta^{\hat{E}}$, weighted by the probability of state s not occurring. The distortion

[5]Notice that $\theta^S > 0$ as $X < \hat{\Omega}$.

θ^S is the deficit in resources needed to achieve the first-best contract, while θ^P is the gap in the conglomerate's restricted first-best production in market N between states ($\theta^P = q_{N1}^{HR} - q_{N1}^{LR}$). The conglomerate maximizes its expected payoff in the equilibrium with the smallest distortion. Thus, the equilibrium P's condition of existence $X \leq \check{\Omega}$ is equivalent to $\theta^P \leq \theta^S$.

At $X = \check{\Omega}$, the headquarters are indifferent between the separating and pooling equilibria. Here, it holds $q_{N1}^{LU} = q_{N1}^{HR}$. Furthermore, to satisfy (3.13), q_{N1}^{HS} and q_{N1}^{LS} must be equally distanced from q_{N1}^{LU}. Thus, the low state output in S coincides with the output in P ($q_{N1}^{LS} = \overline{q}_{N1}^{P}$). The low state manager produces the same in either S or P. However, the headquarters are indifferent with the high state manager producing either q_{N1}^{HS} or $\overline{q}_{N1}^{P}$, because at this point both options distorts q_{N1}^{HR} in the same magnitude, but in different directions.

The mechanism in equilibrium S is such that the conglomerate produces more in market N in the high state than in the low state ($q_{N1}^{HS} > q_{N1}^{LS}$). This is desirable for the conglomerate in the sense that market N produces more (less) when the state is high (low).

In market N in any state, the conglomerate produces more in equilibrium S than in R but less than in U ($q_{N1}^{sU} > q_{N1}^{sS} > q_{N1}^{sR}$). Due to the restricted production, the increased production in market N results in a decreased production in market C ($q_{C1}^{sS} < q_{C1}^{sR}$). The headquarters prefer the outcome R, so the output in the second-best contract is as close as possible to the ones in R. In state s, the headquarters transfer θ^S weighted by the probability of state s not occurring from market C to N. The mechanism incentivizes the low state manager to tell the truth in two ways. First, it increases the manager's utility in comparison to outcome R. Second, lying when the true state is low results in overproduction, hurting the profit in market N.

Regarding the standalone firms, if the goods are substitutes, in state s the firm in market N is worse off in S in comparison to R ($q_{N2}^{sS} < q_{N2}^{sR}$) and the firm in market C is better off ($q_{C2}^{sS} > q_{C2}^{sR}$). If the goods are complements, in state s the firm in market N is better off ($q_{N2}^{sS} > q_{N2}^{sR}$) and the firm in market C is worse off ($q_{C2}^{sS} < q_{C2}^{sR}$).

When the equilibrium is P, the resources are so low that headquarters cannot implement an effective mechanism to differentiate the states in market N. Even though the mechanism induces truth-telling, P is equivalent to the outcome of a model where none of the players inside the conglomerate knows the true value of the demand, and hence, the conglomerate operates under uncertainty. Indeed, a solution where the same output $\overline{q}_{N1}$ is produced in any state can also be obtained by setting the intercept of the demand in market N equal to its expected value $pD_N^H + (1-p)\, D_N^L$.

In comparison to the outcome R, in equilibrium P the conglomerate produces less in market N if the state is high and produces more if the state is low ($q_{N1}^{HR} > \overline{q}_{N1}^{P} > q_{N1}^{LR}$), while in market C it produces more if the state is high and produces less if the state is low ($q_{C1}^{HR} < \overline{q}_{C1}^{P} < q_{C1}^{LR}$).

Comparing the standalone firms' production outputs between the outcome R and equilibrium P, it follows that if the goods are substitutes, the production in P in market N is higher if the state is high and lower if the state is low ($q_{N2}^{HP} > q_{N2}^{HR} > q_{N2}^{LR} > q_{N2}^{LP}$), while in market C the production is lower if the state is high and higher if the state is low ($q_{C2}^{HR} > \overline{q}_{C2}^{P} > q_{C2}^{LR}$). If the goods are complements, the production in P in market N is lower if the state is high and higher if the state is low ($q_{N2}^{HR} > q_{N2}^{HP} > q_{N2}^{LP} > q_{N2}^{LR}$), while in market C the production is higher if the state is high and lower if the state is low ($q_{C2}^{HR} < \overline{q}_{C2}^{P} < q_{C2}^{LR}$).

Whether the equilibrium is separating or pooling, a reduction of p approaches q_{k1}^{LS} or $\overline{q}_{k1}^{P}$ to q_{k1}^{LR}, while an increase of p approaches q_{k1}^{HS} or $\overline{q}_{k1}^{P}$ to q_{k1}^{HR}. This trade-off between the low and high states occurs because (3.13) is binding. As the probability of state s increases, the second-best quantities of state s become closer to the first-best quantities. Conversely, as the probability of state s decreases, the gap between the second and first-best quantities increases. Thus, as the probability of state s increases, the ex-post profit of the conglomerate improves in state s, while the ex-post profit in the other state decreases. A change in p affects the equilibrium output of the standalone firms in the same direction as the ones of the conglomerate. This is simply because of the role as followers of the standalone firms.

The resource allocation in the second-best contract can be interpreted as inefficient as the allocation in the first-best contract can improve the expected profit of the conglomerate. This meaning of inefficiency differs from the definition generally used in the literature of inefficient allocation. In those studies, allocating extra resources to weaker divisions is seen as inefficient. However, in our model the interpretation of inefficient does not depend on the relative profitability of the markets. For example, $D_N^H > D_N^L > D_C$, so that market N is undoubtedly better than market C. In equilibrium S, even though the headquarters assign more resources to the best market and less to the worst market, it is still considered inefficient in our model.

3.6 Social welfare

We compare the ex-ante and post total social welfare between the second-best contracts and their counterpart in symmetric information benchmark: equilibrium R. We first discuss the ex-post total social welfare. This is of interest when the policy maker knows the true value of the demand in market N. The ex-post social welfare is the sum of the ex-post producer and consumer surplus, which we define hereunder.

The ex-post total producer surplus in state s and equilibrium E for $E = \{R, P, S\}$ is the sum of the profits of both standalone firms, and the ex-post profit of the conglomerate

In equilibrium E, market k and state s the standalone firm's profit is $\pi_{k2}^{sE} = (q_{k2}^{sE})^2$, while the conglomerate's profit is $\pi_{k1}^{sE} = P_{k1}^s\left(q_{k1}^{sE}, q_{k2}^{sE}\right) q_{k1}^{sE}$. Hence, the ex-post total producer surplus in state s and equilibrium E is $\Pi^{sE} = \pi_{C1}^{sE} + \pi_{N1}^{sE} + \pi_{C2}^{sE} + \pi_{N2}^{sE}$.

The ex-post consumer surplus in market k, state s, and equilibrium E can be computed by:

$$CS_k^{sE} = v_k^s(q_{k1}^{sE}, q_{k2}^{sE}) - P_{k1}^s\left(q_{k1}^{sE}, q_{k2}^{sE}\right) q_{k1}^{sE} - P_{k2}^s\left(q_{k1}^{sE}, q_{k2}^{sE}\right) q_{k2}^{sE}$$

Therefore, the ex-post total social welfare in state s and equilibrium E is:

$$W^{sE} = CS_N^{sE} + CS_C^{sE} + \Pi^{sE} = v_N^s(q_{N1}^{sE}, q_{N2}^{sE}) + v_C^s(q_{C1}^{sE}, q_{C2}^{sE})$$

The results of the ex-post total social welfare are in Proposition 8.

Proposition 8. *a) $W^{sP} \geq W^{sR}$ for any s if and only if* [6]*:*

$$1 - p_s \leq \frac{2((1-\alpha)(7-\alpha-3\alpha^2)+1)\hat{\theta}^s}{(2-\alpha)(4-3\alpha^2)(D_N^H - D_N^L)} \tag{3.17}$$

where $\hat{\theta}^L = D_N^L - D_C$ and $\hat{\theta}^H = D_C - D_N^H$.

[6] Condition (3.17) is non-empty. Consider $D_C = 2$, $D_N^H = 4$, $D_N^L = 3$, $p = 0.9$, $\alpha = 0.2$ and $X = 1$. This satisfies (3.1), (3.2), $X < \check{\Omega}$, and the low state (3.17). Consider $D_C = 8$, $D_N^H = 6$, $D_N^L = 3$, $p = 0.1$, $\alpha = 0.2$, and $X = 3$. This satisfies (3.1), (3.2), $X < \check{\Omega}$, and the high state (3.17).

b) $W^{sS} \geq W^{sR}$ *for any* s *if and only if* [7]*:*

$$1 - p_s \leq \frac{((1-\alpha)(7-\alpha-3\alpha^2)+1)(D_N^s - D_C)}{2(4-3\alpha^2)(2-\alpha^2)\theta^S} \tag{3.18}$$

The contract reallocates surplus from one market to the other. The losses caused by the contract to the agents in one market is profitable for their counterparts in the other market. Thus, the asymmetry of information will improve the social welfare if the earnings in one market exceed the losses of the other.

For the consumers, they will be better-off in a specific market in the second-best contract than in R if the conglomerate overproduces in that market. Conversely, the consumers of the other market will be worse-off due to the reduction of the conglomerate's production in that market. That is, there is a group of winning consumers and a group of losing consumers.

As for the standalone firms, in any state there is always one firm that is better-off and one that is worse-off in the second-best contract equilibria in comparison to R. Thus, just like the consumers, there is one winning standalone firm and one losing firm. The winning (losing) firm will operate in the same market as the winning (losing) consumers only if the goods are complements.

Finally, while the total profit of the conglomerate in state s $(\pi_{C1}^{sE} + \pi_{N1}^{sE})$ is better in R than in the second-best contract equilibria, at the individual market level the conglomerate is always worse off in the market with underproduction but is better-off in the market with overproduction.

The overall loss of profit of the conglomerate is concordant with the conglomerate

[7]Condition (3.18) is non-empty. Consider $D_C = 2$, $D_N^H = 4$, $D_N^L = 3$, $p = 0.9$, $\alpha = 0.2$, $X = 2$. This satisfies (3.1), (3.2), $\hat{\Omega} > X \geq \check{\Omega}$, and the low state (3.18). Consider $D_C = 4.51$, $D_N^H = 6$, $D_N^L = 3$, $p = 0.1$, $\alpha = 0.2$, and $X = 2.7$. This satisfies (3.1), (3.2), $\hat{\Omega} > X \geq \check{\Omega}$, and the high state (3.18).

discount theory. The ex-post conglomerate discount in state s and in equilibrium $\hat{E}$, that is, the conglomerate's loss caused by the asymmetric information is $(2-\alpha^2)(1-p_s)^2(\theta^{\hat{E}})^2$. The conglomerate discount increments exponentially at increments of the distortion in production and in the probability of state s not occurring.

The denominators of the right side of (3.17) and (3.18) are positive, but depending on the parameters, the numerators can be negative, positive or zero. Any inequalities cannot hold if its respective numerator is non-positive, hence there are necessary conditions. The necessary condition for (3.17) in the low state is $D_N^L > D_C$, for (3.17) in the high state is $D_C > D_N^H$, and for (3.18) is $D_N^s > D_C$. The logic of the conditions is that the best market has to be the one where the conglomerate overproduces due to the information revelation scheme.

If the necessary condition holds and the right side of (3.17) or (3.18) is less than 1, then there exists a p that achieves $W^{s\hat{E}} \geq W^{sR}$. The role of the probability here is to avoid a bad scenario for the conglomerate in the second-best contract equilibria. As discussed earlier, the conglomerate is better-off as the probability approaches 0 or 1.

Now, we proceed to analyze the ex-ante total social welfare. This is relevant when the policy maker does not know the true value of the demand. We compute the ex-ante total social welfare as follows:

$$EW^E = pW^{HE} + (1-p)W^{LE}$$

The results of the ex-post total social welfare are in Proposition 9.

Proposition 9. *a)* $EW^P < EW^R$ *always holds.*

b) $EW^S \geq EW^R$ *if and only if:*

$$X \geq \hat{\Omega} + \frac{(1-\alpha)(7-\alpha-3\alpha^2)+1}{4(2-\alpha^2)(4-3\alpha^2)} \left(2D_C - D_N^L - D_N^H\right) = \Omega^W \tag{3.19}$$

Part (a) of Proposition 9 states that the ex-ante total welfare is always worse with asymmetric information in the pooling equilibrium. The pooling contract transfers surplus from market N to market C in the high state. Conversely, it transfers surplus from C to N in the low state. The overall effect of the transfers is a reduction in the expected welfare. Intuitively, the cause of the loss of surplus is that headquarters do not use the information gained with the pooling contract.

Part (b) of Proposition 9 states that the separating second-best contract sometimes improves the ex-ante welfare. The separating equilibrium exists only if $\hat{\Omega} > X \geq \check{\Omega}$. Thus, an X exists such that (3.19) is satisfied and S is an equilibrium if $\hat{\Omega} > \Omega^W$. This last inequality holds if and only if $D_N^L + D_N^H > 2D_C$. In essence, if the mean of the demand of market N is greater than the demand of market C, there exists a high enough X such that the ex-ante total welfare is better in equilibrium S than in R.

A high X is required as it implies higher total production. The intuition behind $D_N^L + D_N^H > 2D_C$ is similar to the one in the ex-post analysis. In equilibrium S in any state the conglomerate overproduces in market N and underproduces in market C. Thus, the market size condition implies that the market where the conglomerate overproduces must be the best (in average).

Further, equilibrium S always welfare dominates R if $\check{\Omega} \geq \Omega^W$. That condition holds if and only if the goods are complements ($\alpha < 0$) and if $D_N^L + D_N^H > \theta^W D_C$, where:

$$\theta^W = \frac{(1-\alpha)(7-\alpha-3\alpha^2)+1}{2\alpha(\alpha-1)} \tag{3.20}$$

When the goods are complements, it follows $\theta^W > 2$. Thus, (3.20) implies that market N must be better than C in more than the average. The complementary condition is required so that the overproduction in market N benefits both the consumers and standalone firms in that market.

In the ex-ante case, the conglomerate is worse off with asymmetric information. The ex-ante conglomerate discount in equilibrium $\hat{E}$ is $(2-\alpha^2)(1-p)p(\theta^{\hat{E}})^2$. As the ex-post case, the conglomerate discount increments exponentially at increments of the distortion in production. The conglomerate discount disappears as the probability goes towards 0 or 1 because the value of D_N becomes more certain. Contrastingly, the conglomerate discount is at its highest at $p = 1/2$, when each state is equally probable.

3.7 Effects of the degree of differentiation on the delimitation of equilibrium

Now, we analyze the effect of the degree of differentiation on the thresholds that delimit the type of equilibrium. These thresholds are Ω^L, Ω^H, $\hat{\Omega}$, and $\check{\Omega}$ (as shown in Figure 3.1). All these thresholds can be written and hence be interpreted in terms of q^U_{C1}, q^{LU}_{N1} and q^{HU}_{N1}. Thus, α affects these thresholds and the unrestricted outputs in a similar manner. As the analysis of all these variables is similar, we only explicitly check q^U_{C1}. The derivative of q^U_{C1} with respect to α is:

$$\frac{\partial q^U_{C1}}{\partial \alpha} = \frac{(4\alpha - \alpha^2 - 2)D_C}{2(2-\alpha^2)^2} \tag{3.21}$$

As D_C and the denominator of (3.21) are always positive, (3.21) has the same sign as $(4\alpha - \alpha^2 - 2)$. It follows that $(4\alpha - \alpha^2 - 2)$ is equal to zero when $\bar{\alpha} = 2 - \sqrt{2}$. Thus, the factor $(4\alpha - \alpha^2 - 2)$ is positive when $\alpha > \bar{\alpha}$ and negative when $\alpha < \bar{\alpha}$.

The derivatives of all Ω^L, Ω^H, $\hat{\Omega}$, $\check{\Omega}$, q_{N1}^{LU}, and q_{N1}^{HU} with respect to α also have the same sign as $(4\alpha - \alpha^2 - 2)$. We state this result in Proposition 10.

Proposition 10. *a) On the range $[-1, \bar{\alpha}]$, all q_{C1}^{U}, q_{N1}^{LU}, q_{N1}^{HU}, Ω^L, Ω^H, $\hat{\Omega}$, and $\hat{\Omega}$ are decreasing in α.*

b) On the range $[\bar{\alpha}, 1]$, all q_{C1}^{U}, q_{N1}^{LU}, q_{N1}^{HU}, Ω^L, Ω^H, $\hat{\Omega}$, and $\hat{\Omega}$ are increasing in α.

Proposition 10 implies that the type of equilibrium under symmetric information depends on α. Fix all the parameters other than X and α. There is X such that the unrestricted outcome is an equilibrium in both states for values of α close to $2 - \sqrt{2}$. The unrestricted outcome is an equilibrium in the low state and the restricted outcome is in the high state for values of α not too close but not too far from $2 - \sqrt{2}$. The restricted outcome is an equilibrium in both states for values of α quite far from $2 - \sqrt{2}$. Thus, higher levels of complementarity and substitutability increase the likelihood of the production-possibility constraint being binding.

Under asymmetric information, there is X such that the first best contract is achieved for values of α close to $2 - \sqrt{2}$. The second-best contract is most likely to coincide with the first-best one at an intermediate level of substitutability, and any departure from that level toward either substitutability or complementarity makes the attainment of the first-best outcome less likely. Further, the type of the second-best contract depends on α. The pooling equilibrium is more likely at higher levels of complementarity or substitutability.

3.8 Conclusion

We developed an adverse selection model of a conglomerate with restricted production participating as the leader in two duopoly markets with a Stackelberg-Cournot framework with heterogeneous goods. We derive two equilibria in the symmetric information benchmark. If the resources of the conglomerate are plenty, the production-possibility constraint is not binding and the resulting equilibrium is U. If the resources are scarce, the equilibrium is R.

The first-best contract is achievable if the resources are high enough when the information is asymmetric. Thus, U and R also exist as equilibria in this case. If the resources are low enough, only the second-best contract is possible. Here, we derive two more equilibria. If the resources are not too low, a separating equilibrium S exists. If the resources are too low, a pooling equilibrium P exists. In the S equilibrium, headquarters can allocate the resources accordingly to the manager's report. Specifically, headquarters allocate more resources to market N in the high state than in the low state. In P the production plan of the conglomerate is the same regardless of the state.

We proved that the social welfare might improve with asymmetric information depending on the parameters. The implementation of the contract mechanism leads to a reallocation of the production plan of the conglomerate, transferring surplus from one market to another. If the increase of surplus in one market exceeds the decrease of surplus in the other market, the social welfare will be better in the second-best contract. One requirement for this is that the best market must be the one with the increased surplus. Furthermore, welfare is more likely to improve if the goods are complements, because the surplus of the standalone firms and consumers will move in the same direction as the variations in the conglomerate's production.

We find that the separating contract sometimes improves the ex-ante welfare. The separating contract transfers surplus from market C to market N in any state, thus a requirement to improve the welfare is that the market N must be better in average than market C. If market N is larger than C in more than the average and the goods are complements, the separating contract always improves the ex-ante welfare. In contrast, the pooling contract never improves welfare. This contract also transfers surplus across markets, but because the information acquired is not used in a meaningful way, the overall effect is a reduction in the expected welfare.

Ideally, the policy authority should interfere when the asymmetric information hurts the social welfare. Measuring asymmetric information might be difficult in practice. However, our model predicts effects of the asymmetric information might be problematic in terms of the resources and the sizes of the markets, which are variables more easily measurable. First, agency problems are more likely to be problematic when the resources of the conglomerate are low. Second, in most cases the welfare is likely to fall if the conglomerate diversifies in markets that are smaller than its core business.

While it is unlikely that the policy authority will be able to directly regulate agency problems, it can restrict the expansion of the conglomerate to new markets, preventing the creation of scenarios with agency problems. Thus, researching this kind of conglomerate effect and its policy implications is worth it even though the nature of such effect is abstract in practice. Thus, a possible extension of the model is to study a different agency framework, such as moral hazard. We leave this for future research.

We analyzed how changes in the degree of differentiation α determines the type of equilibrium. We find that the adverse selection is more likely to cause a distortion with higher levels of substitutability or complementarity. This result suggest that variables related to competition are relevant to understand phenomena within conglomerates, such as the conglomerate discount. We only examined a Stackelberg-Cournot framework, so a possible line of research is to study the inner dynamics of a conglomerate in other competitive scenarios, assuming simultaneous competition or price competition.

Chapter 4

Conglomerate Merger and Divestment Dynamics

4.1 Introduction

Empirical evidence suggests that in the firm's life cycle the corporate structure might change through diversification and divestitures (Matsusaka (2001), Fluck and Lynch (1999)). We define diversification as the business operation that expands a firm, which is done with mergers or acquisitions. Contrastingly, we refer to divestitures as the business operation that shrinks a firm, for example, by selling a division of the firm. In this chapter, we develop a discrete-time, infinite horizon theoretical model where a monopolist can take diversifying and divesting decisions. Specifically, the monopolist can buy and sell a firm from another market. We utilize numerical methods to obtain approximate results.

We consider mergers of the conglomerate-type. Namely, mergers where the merged parties come from markets that are not related horizontally or vertically. In our model, we assume that a monopolist can buy or sell another monopolist firm from a not related horizontally or vertically market. In the literature, the firm's incentives to diversify/divest are generally associated with conglomerate mergers. We concentrate in two types of incentives.

First, the firms might diversify/divest to search for profitable markets. We assume that the demand of one of the markets follows a Markov process. Thus, the most profitable market might change through time, and hence, the monopolist might prioritize a different market in each period. This is consistent with Matsusaka (2001). In their model, a firm enters and exits different industries searching for markets that are a good match for its organizational capabilities.

Second, in the model of Fluck and Lynch (1999), the incentive to form a conglomerate is to get financing in periods of distress, and once the financing requirements have been satisfied, the conglomerate divests as the merger is no longer needed. In a similar manner, in our model we assume that the firm can obtain a cost reducing variable through a merger. In this chapter we refer to that variable as "capital", however it can also be considered as "resources". Additionally, we assume that capital is lost when a divestiture occurs.

The technical aspects of our model are based on the horizontal merger dynamic models with capacity constraint of Gowrisankaran (1999) and Chen (2009). In these models, firms must invest to obtain production capacity in each period as such capacity depreciates through time. Capacity is a relevant variable to define the mergers. Assumedly, the capacity of a new merged firm equals the sum of the capacities of the original firms members of the merger. In that regard, the "capital" in our model fulfills the same function as "capacity", as it can be obtained (lost) with an acquisition (divestiture).

The main difference with our model and the models of Gowrisankaran (1999) and Chen (2009) is the type of merger. Unlike horizontal mergers, conglomerates mergers do not change the number of firms competing in a particular market. Hence, it is easier to characterize a divestment with a conglomerate framework than with a horizontal one.

After a horizontal merger, the identity of the original firms is lost as they become a homogeneous entity. In the case of conglomerate mergers, the original firms become divisions of the conglomerate. Thus, after a divestment the divisions simply return to be standalone firms.

Similar to Gowrisankaran (1999) and Chen (2009), we add an external capital market as an alternative way to obtain capital in the model. In a conglomerate framework, this dual way of obtaining capital within the firm or from an external source is a relevant topic in the internal capital market efficiency literature. This literature analyses the efficiency of the internal capital markets relative to the external ones (Busenbark et al., 2017).

This chapter is organized as follows: Section 4.2 describes the model structure. Section 4.3 specifies the details of the computation of the model. Section 4.4 presents the results.

4.2 The model

A monopolist in an infinite-horizon discrete-time industry takes production and capital related decisions to maximize its long term profit given its discount factor $\delta \in (0, 1)$. We denote time with t, however we use it only when there is a need to differentiate between periods. The monopolist's capital stock K can be use to reduce the cost of production. New capital can be acquired through two channels: by buying it in the external capital market or by acquiring a different firm with its own capital stock. The capital stock diminishes in two ways. First, it depreciates at a constant rate $\phi \in (0, 1)$ in each period. Second, if the monopolist has acquired a different firm, it can be sold to get a one-period reward. The firm can be sold together with some capital to increase the one-period reward.

There are two markets that are not related horizontally or vertically, denoted by

$f \in \{C, N\}$. We assume that those markets share some similarities in their production process, and thus, they can share capital. Market C is the core or original market of a monopolist that can expand its business by acquiring another monopolistic firm in the new market N. We refer to the firm in market C as the core firm, the firm in market N as the new firm, and the firm born from the merger of the core firm and the new firm as the conglomerate. When in a conglomerate, we assume that only the new firm can be sold. The only decision maker in our model is the core firm. Thus, when the core firm is not merged, it only takes decisions in market C; and when it is merged, it takes decisions in both markets C and N. Conversely, when the new firm is not merged, its actions are assumed to be exogenous. In market f, the firm faces the following inverse demand function:

$$P_f(q_f) = D_f - q_f$$

where P_f and q_f are the price and the output in market f, respectively. In the core market, D_C is a positive constant. However, in the new market, D_N is an exogenous continuous-valued Markov process. The marginal cost in market f is given by:

$$MC_f(\theta_f) = \frac{\beta_{f1}}{\beta_{f2} + \theta_f}$$

where β_{f1} and β_{f2} are positive constants, and θ_f is the capital allocated to market f.

Because we assumed only two markets, there are only two merging-separating states: the firms are merged or they are not. The core firm can acquire the new firm only if the current state is "separated", and can sell the new firm only if the current state is "merged". When the firms are separated, the new firm can be acquired at a take-or-leave-it positive constant price $AP = \alpha_0 + \tau_I \kappa$, where α_0 is the base acquisition price, κ is the current capital stock of the new firm, and τ_I is the price per unit of capital exchanged through merging/separating. When the firms are merged, the new firm can be sold at a price $SP = \alpha_1 + \tau_I \gamma(1-\phi)K$, where α_1 is the base selling price and γ is the proportion of capital after depreciation that is sold together with the new firm.

Now we describe the state space. There is one discrete state variable, $M \in \{0, 1\}$, which equals 1 if the firm is merged and equals 0 otherwise. There are two continuous state variables: the capital stock $K \in [0, \bar{K}]$ and the value of the intercept of the demand of the new market $D_N \in [\underline{D}_N, \bar{D}_N]$. We assume that $\bar{K}$, $\underline{D}_N$ and $\bar{D}_N$ are nonnegative constants.

Each period of the model has three stages. In the first stage, there are three discrete action variables. First, the merging-separating decision $m \in \{0, 1\}$, which equals 1 if the core firm chooses to merge and equals 0 otherwise. Second, the amount of capital bought in the capital market $k \in \{0, 1, 2, ..., \bar{k}\}$, which we assume can be acquired only in integer values at a constant price per unit τ_E. We assume that $\bar{k}$ is a positive integer. Third, the proportion of capital after depreciation that is sold together with the new firm $\gamma \in \{0, 0.1, 0.2, ..., 0.9, 1\}$, which is selected each time the core firm chooses to separate.

In the second stage the core firm chooses how much capital to allocate to the firms it owns, selecting $\theta_f \in [0, K]$ in market f. In the third stage the core firm chooses the output on the firms it owns, producing $q_f \in [0, \infty]$ in market f. We assume that the actions θ_f and q_f are continuous variables.

Now we formalize the state transition functions. For clarity, we utilize the subscript

t. The new demand state transition function is given by[1]:

$$D_{Nt+1} = \min\{\max\{D_{Nt} + \epsilon_{t+1}, \underline{D}_N\}, \bar{D}_N\}$$

where ϵ_t is an i.i.d normal $\mathcal{N}(\mu, \sigma^2)$ shock. The merging-separating state updates according to the merging-separating decision, thus $M_{t+1} = m_t$. The capital stock state transition function is given by[2]:

$$K_{t+1} = M_t m_t (1-\phi) K_t + (1-M_t)(1-\phi) K_t + k_t + (1-M_t) m_t \kappa + M_t (1-m_t)(1-\gamma)(1-\phi) K_t$$

where the first term is the stock of capital left from the previous period after depreciation when both the state and action are "merged'. The second term is the stock of capital left from the previous period after depreciation when the state is "separated". The third term is the capital bought in the capital market. The fourth term is the capital gained through a merger. The fifth term is the capital left from the previous period after depreciation and for selling the new firm.

In the second and third stages, θ_f and q_f do not affect any of the states; that is, the behavior in the second and the third stages in each period does not affect future actions. Hence, each stage can be analyzed separately. Further, the second and third stages can be solved as a single-period problem. We solve the model in a backward manner. We explain the problem and our solution method of each one of the stages hereunder.

[1]The numerical method requires boundaries in the state space. The optimal state path must be inside those boundaries to obtain an accurate approximation of the solution. Considering this, a large enough state space interval must be set. However, the new demand does not depend on any action and hence its path is exogenous. Thus, the boundaries of the new demand are included explicitly in its state transition function, otherwise its path would go beyond the boundaries regardless of the size of the interval.

[2]With this type of state transition function, K_t is always nonnegative given any nonnegative initial value K_0.

4.2.1 Third stage: Production

In any period t, for any $D_N \in [\underline{D}_N, \bar{D}_N]$ and given that m and θ_f have been already chosen, the profit from market f is given by:

$$\pi_{3f}(q_f) = \left(D_f - q_f - \frac{\beta_{f1}}{\beta_{f2} + \theta_f} \right) q_f \tag{4.1}$$

If $m = 0$, the problem is to select $q_C \geq 0$ to maximize $\pi_{3C}(q_C)$. If $m = 1$, the problem is to select $q_C \geq 0$ and $q_N \geq 0$ to maximize $\pi_{3C}(q_C) + \pi_{3N}(q_N)$. In either case, we can compute an analytical solution, which for any f is the usual monopoly solution:

$$q_f^* = \frac{D_f}{2} - \frac{\beta_{f1}}{2(\beta_{f2} + \theta_f)} \tag{4.2}$$

To guarantee an interior solution independent of θ_f, we require that $D_f > \frac{\beta_{f1}}{\beta_{f2}}$.

4.2.2 Second stage: Capital allocation

In any period t, for any $(D_N, K) \in [\underline{D}_N, \bar{D}_N] \times [0, \bar{K}]$ and given that m has been already chosen, by substituting (4.2) into (4.1) it follows that:

$$\pi_{2f}(\theta_f) = \left(\frac{D_f}{2} - \frac{\beta_{f1}}{2(\beta_{f2} + \theta_f)} \right)^2$$

If $m = 0$, the problem is to select $\theta_C \in [0, K]$ to maximize $\pi_{2C}(\theta_C)$. The solution here is simply $\theta_C^{**} = K$. If $m = 1$, the problem is to select $(\theta_C, \theta_N) \in [0, K] \times [0, K]$ to maximize $\pi_{2C}(\theta_C) + \pi_{2N}(\theta_N)$ subject to $\theta_C + \theta_N \leq K$. Given that the restriction is binding, the problem of this stage when $m = 1$ can be simplified as follows:

$$\max_{\theta_C \in [0,K]} \quad \pi_{2CN}(\theta_C) = \left(\frac{D_C}{2} - \frac{\beta_{C1}}{2(\beta_{C2} + \theta_C)}\right)^2 + \left(\frac{D_N}{2} - \frac{\beta_{N1}}{2(\beta_{N2} + K - \theta_C)}\right)^2 \qquad (4.3)$$

Because (4.3) is a continuous function on a close interval, a solution is guaranteed to exist. However, computing an analytical interior solution is quite difficult as is requires to solve a polynomial of degree 4. Thus, we solve (4.3) numerically. We utilize the `fmincon` solver provided in MATLAB's Optimization Toolbox to solve (4.3) in every state[3] $(D_N, K) \in [\underline{D}_N, \bar{D}_N] \times [0, \bar{K}]$ to obtain the solution $\theta_C^*(D_N, K)$.

4.2.3 First stage: Decisions on the stock of capital

In the first stage, in a single period the reward function is given by:

$$R(M, K, D_N, m, k, \gamma) =$$

$$(1-m)\pi_{2C}(K) + m\pi_{2CN}(\theta_C^*(D_N, K)) - \tau_E k - (1-M)m(\alpha_0 + \tau_I \kappa) + M(1-m)(\alpha_1 + \tau_I \gamma(1-\phi)K)$$

where the first term is the profit for market operations when the firms are separated, the second term is the profit for market operations when the firms are merged, the third term is the cost for buying in the capital market, the fourth term is the merger's cost, and the fifth term is the income for selling a firm. The Bellman's equation associated with this problem is (we use the subscript t):

$$V(M_t, K_t, D_{Nt}) = \max_{m_t, k_t, \gamma_t} [R(M_t, K_t, D_{Nt}, m_t, k_t, \gamma_t) + \delta E_\epsilon V(M_{t+1}, K_{t+1}, D_{Nt+1})]$$

[3] Even though D_N and K were assumed to be continuous, the numerical method requires to fully discretize the state space.

where E_ϵ is the expected value operator. We utilize `dpsolver` from the CompEcon Toolbox to solve this dynamic optimization problem. The solver computes an approximate solution of the Bellman equation utilizing the collocation method. A thorough explanation of how to use this solver can be found in Miranda and Fackler (2004).

4.3 Computation on the model

For comparison, we run different configurations of the model in section 3. Such configurations are benchmark versions of the model and versions of the full model with different parameters. For every configuration of the model we set the following parameters, unless otherwise stated: $D_C = 20$, $\beta_{C1} = 10$, $\beta_{C2} = 1$, $\beta_{N1} = 10$, $\beta_{N2} = 1$, $\bar{K} = 200$, $\phi = 0.3$, $\underline{D}_N = 10$, $\bar{D}_N = 30$, $\kappa = 55$, $\alpha_0 = 200$, $\alpha_1 = 300$, $\delta = 0.9$, $\mu = 0$, $\sigma^2 = 0.25$, $\bar{k} = 30$, $\tau_I = 3$ and $\tau_E = 4$.

The parameters were set considering some factors. Given that D_N is a Markov process, there is a dynamic where the most profitable market might change through time depending on the value of D_C. Thus, we set D_C in the middle of the state space of D_N to facilitate this dynamic. For our result in the production stage to be valid, D_f needs to be greater than the marginal cost in market f. Thus, we set $D_C > \frac{\beta_{C1}}{\beta_{C2}}$ and $\underline{D}_N > \frac{\beta_{N1}}{\beta_{N2}}$.

We set $\bar{k} < \kappa$, so that higher volumes of capital are transacted through mergers than through the external capital market. Nevertheless, in none of the results $\bar{k}$ or close values are an optimal action, implying that our selection of $\bar{k}$ does not restrict the action of the core firm. We also set a large $\bar{K}$ relative to $\bar{k}$ and κ to avoid restricting the optimal path.

Given the boundaries of D_N, we choose a low value for the variance of ϵ so that D_N takes different values inside its state space interval. A very high value of the variance might result in D_N taking frequently the value of the upper or lower bound. Moreover, we set the mean of ϵ to zero to obtain a fair number of negative and positive shocks. Furthermore, it is reasonable to assume that the base price of a firm is quite expensive, thus we choose a high value for α_0 and α_1.

Because the full model is stochastic, we present the optimal path with a Monte Carlo simulation. To compute the simulations we use `dpsimul` from the CompEcon Toolbox. The usual method to analyze the optimal path is to run several simulations and thereafter to compute the average of the simulated paths in each period. However, such method might result insubstantial in our case as the merging-separating decision only takes two values. Thus, we graph the path of the relevant state and action variables in a 3-D surface plot. We set in the horizontal axis the time period, while the vertical axis points out each one of the performed simulations. Specifically, each integer in the vertical axis corresponds with one simulation. For every configuration of the model, we perform 10 simulations and set the time to 100 periods. The initial states are $K_0 = 50$, $D_{N0} = 20$ and $M_0 = 0$. For comparison, we set the same exogenous path of the new demand in every configuration of the model.

For the first set of benchmarks, we assume that the core firm cannot take merging-separating decisions. Thus, there are two scenarios. In the first benchmark, termed as $B1$, the core firm is a standalone firm. In the second benchmark, termed as $B2$, the core firm is a conglomerate. In both benchmarks, the only action variable is the capital bought in the external market. Further, the state variable of the capital stock is present. However, only $B2$ has the state variable of the new market demand. Thus, $B1$ is deterministic while $B2$ is stochastic.

In the third benchmark, termed as $B3$, we eliminate the action of the proportion of capital sold in a separation, and assume that γ is an exogenous constant, chosen by the core firm before the beginning of the industry dynamics. Here we set $\gamma = 0.5$.

We run two configurations of the full model. In the first configuration, termed as $F1$, the parameters are as assumed initially. In the second configuration, termed as $F2$, we assume $\tau_I > \tau_E$; specifically, $\tau_I = 4$ and $\tau_E = 3$. In $F1$, the price per unit of capital when buying/selling a firm is lower than the price in the external capital market. Thus, we are implying that there is a discount for acquiring a large volume of capital through a merger. In $F2$, the capital bought in the external capital market is cheaper even though this channel deals with a lower volume of capital. We set $\bar{k}$ in $F1$ and $F2$ equal to 15 and 20, respectively. We reduce the value of $\bar{k}$ in comparison to the benchmarks for computational efficiency. We set this value higher in $F2$ given that the external capital market is more appealing there and a lower value might restrict the actions of the core firm.

The technical aspects of the implementation of the collocation method for each configuration of the model are in Appendix 4.1. The MATLAB code for $F2$ is in Appendix 4.2.

4.4 Results

We show the common new demand path for every configuration of the model in Figure 4.1. For the following analysis we use the demand of the new market as the point of reference.

We present the optimal merging-separating decision for $B3$, $F1$ and $F2$ in Figures 4.2, 4.3 and 4.4, respectively. Because the merging-separating state updates according to the merging-separating decision, the results on that state are redundant, and thus we omit them. We observe two merging-separating patterns in $B3$, $F1$ and $F2$. Each pattern is seemingly associated with particulars values of the demand of the new market. We state formally the dynamic of these patterns in the following observation.

Observation 1. *a) In intervals of time when D_N is high, the core firm's decision is to merge and to stay merged.*

b) In intervals of time when D_N is low, the core firm merges and separates intermittently.

The intuition for Proposition 1 is the same for any configuration of the model. In part a) the core firm has incentives to remain in a conglomerate form because the new market is quite profitable. As long as D_N remains high through time, there are incentives to be a conglomerate in the long run. In part b) the new market is not so profitable, thus the core firm in a conglomerate prefers to sell the new firm and obtain a high one-period payment. However, in the following period after the separation, the standalone core firm will decide to merge again. Here the incentive is mainly to obtain a high amount of capital with the new firm. Therefore, in this case there are not incentives to remain in the merged or separated state in the long run.

The difference among $B3$, $F1$ and $F2$ is the frequency of occurrence of the patterns. Thus, how low or high the new demand must be to lead to one pattern or another depends on the configuration of the model. In $F2$ the decision to separate is taken with the least frequency, followed by $B3$. In $F2$ the most expensive channel is a merger, thus this channel is used less in comparison to $B3$ and $F1$. In $B3$ the frequency of separations is lower than in $F1$ because in the latter the core firm has more control on the separation conditions, and thus there are more incentives to separate.

We present the optimal capital bought in the external market capital market for $B1$, $B2$, $B3$, $F1$ and $F2$ in Figures 4.5, 4.6, 4.7, 4.8 and 4.9, respectively. In $B1$, because

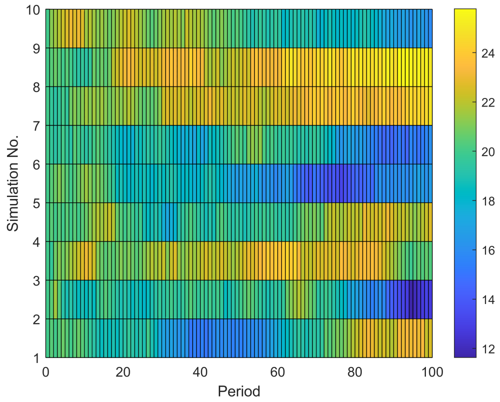

Figure 4.1: Demand of the new market-Path

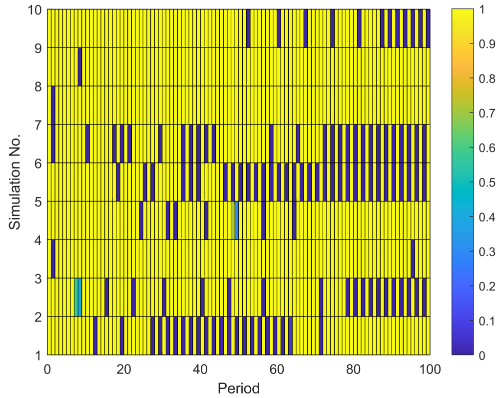

Figure 4.2: Merging/separating-Optimal decision-$B3$

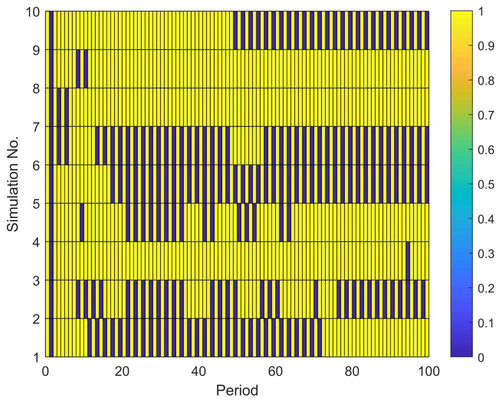

Figure 4.3: Merging/separating-Optimal decision-$F1$

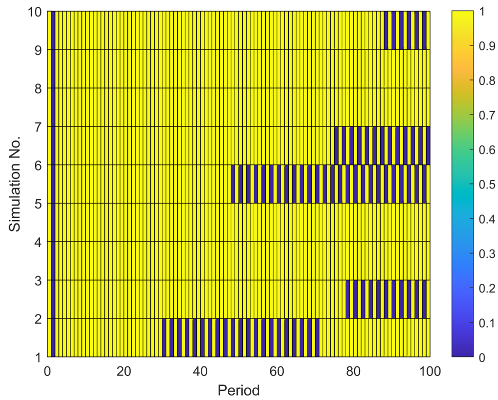

Figure 4.4: Merging/separating-Optimal decision-$F2$

its deterministic nature, the capital bought externally converges to an steady state. In every simulation, in every period 2 units of capital are bought once the initial capital is depleted. In $B2$ usually 4 units of capital are bought per period, twice as $B1$ because the monopolist participates in two markets in $B2$. In $B2$ the demand of the new market affects the capital bought externally. If D_N is very high, the monopolist in rare occasions might buy 5 units of capital in one period. If D_N is low, the monopolist intermittently buys between 3 and 4 units of capital each period.

In $B3$ we observe two predominant patterns. First, in intervals of time where the core firm remains merged, usually 4 units of capital are bought. In conglomerate form the only mean to obtain new capital is through the capital market, thus we observe a behavior similar to $B1$ and $B2$. Second, in intervals of time where the core firm separates intermittently, usually no capital is bought. As capital can be obtained constantly through mergers, the external capital market is ignored, as is the worst way to obtain capital. In both patterns there are some disturbances where the monopolist might buy up to 7 units of capital in one period. $F1$ is similar to $B3$, but now the disturbances are much less frequent in periods where no capital is bought. Because capital is obtained more frequently through a merger in $F1$, the core firm relies less in the external capital market.

In $F2$ we observe a similar pattern as $B3$ and $F1$ except for the periods of separation. Here the core firm buys a large volume of capital (around 13 units) in each period. By the differential in the capital's prices, the core firm gains a profit by buying cheaper capital in the external market to sell it at a higher price with a separation.

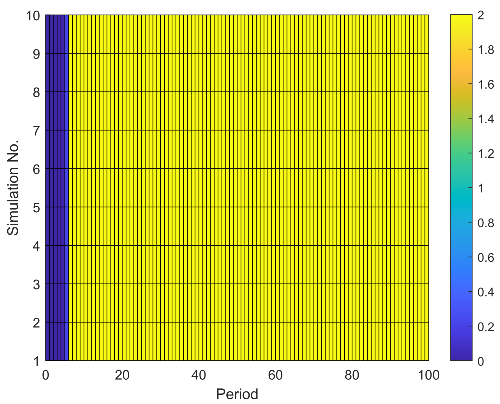

Figure 4.5: Capital bought externally-Optimal decision-$B1$

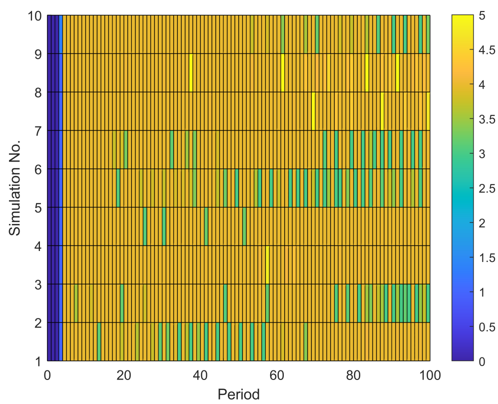

Figure 4.6: Capital bought externally-Optimal decision-$B2$

We present the optimal proportion of capital sold in a separation for $F1$ and $F2$ in Figures 4.10 and 4.11, respectively. When the core firm separates in $F1$, the proportion of capital sold after depreciation is quite high, ranging between 70% and 80%. In $F2$ as selling capital through a separation is quite profitable, the core firm sells all its capital.

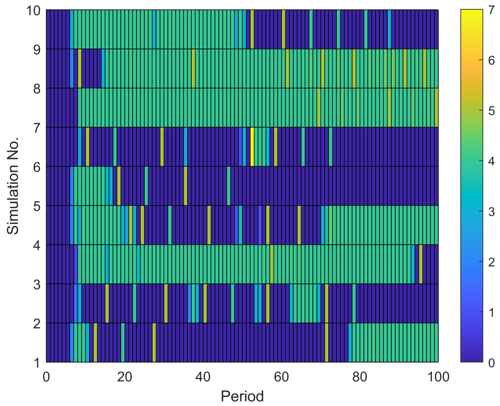

Figure 4.7: Capital bought externally-Optimal decision-$B3$

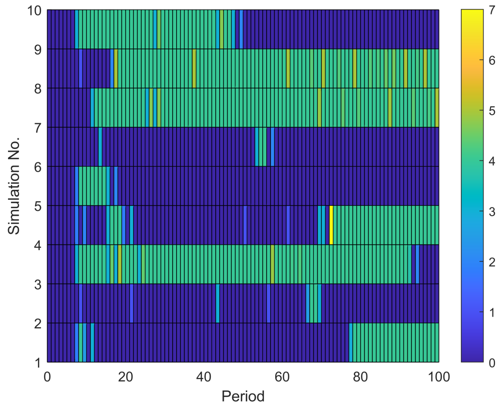

Figure 4.8: Capital bought externally-Optimal decision-$F1$

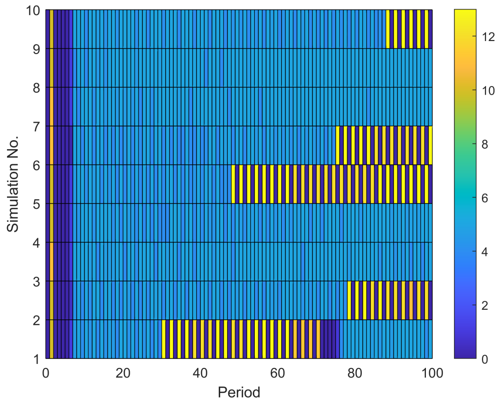

Figure 4.9: Capital bought externally-Optimal decision-$F2$

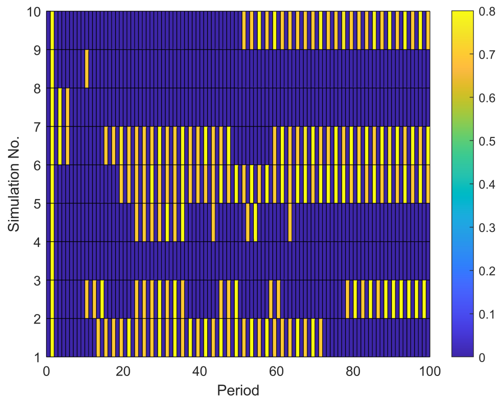

Figure 4.10: Capital sold-Optimal decision-$F1$

We display the optimal path of the capital stock in Figures for $B1$, $B2$, $B3$, $F1$ and $F2$ in Figures 4.12, 4.13, 4.14, 4.15 and 4.16, respectively. In $B1$ and $B2$, the capital stock follows the same logic as the capital bought externally because the external capital market is the only channel available to acquire capital in those benchmarks.

In $B3$, $F1$ and $F2$, in periods where the core firm remains merged, at the beginning the capital stock is high, but it diminishes rapidly after some periods. After that the level of capital stock remains low, as the only new capital acquired in these periods is through the external capital market. In periods where the core firm separates intermittently, the firm gains capital with a merger but loses it with a separation and thus the capital stock path behaves intermittently as the merging-separating decision. The level of the capital stock in periods of separation is different among the three configurations as the proportion of capital sold in separation also differs.

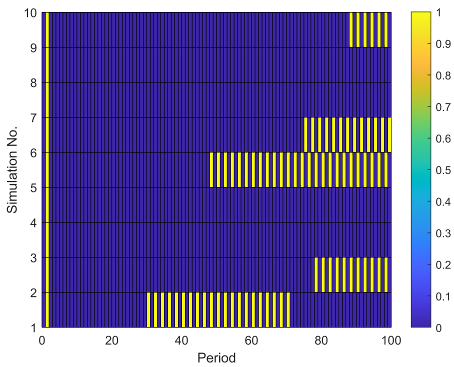

Figure 4.11: Capital sold-Optimal decision-$F2$

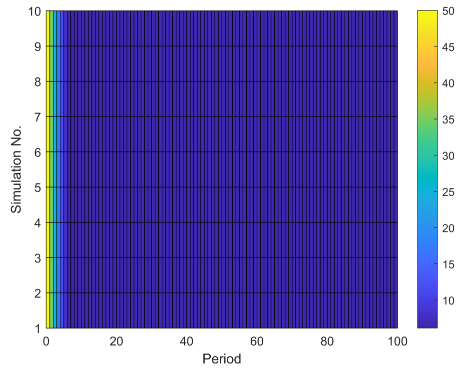

Figure 4.12: Capital stock-Path-$B1$

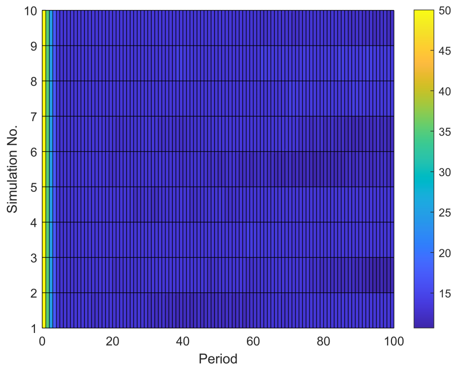

Figure 4.13: Capital stock-Path-$B2$

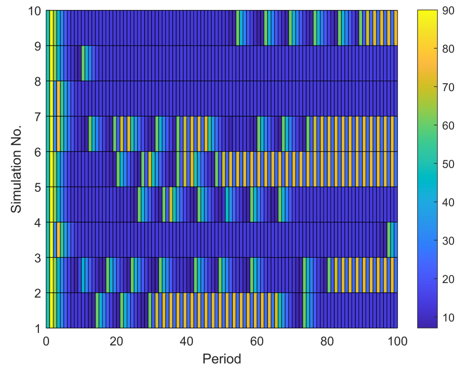

Figure 4.14: Capital stock-Path-$B3$

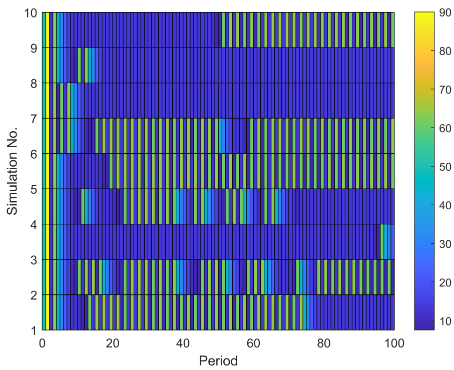

Figure 4.15: Capital stock-Path-$F1$

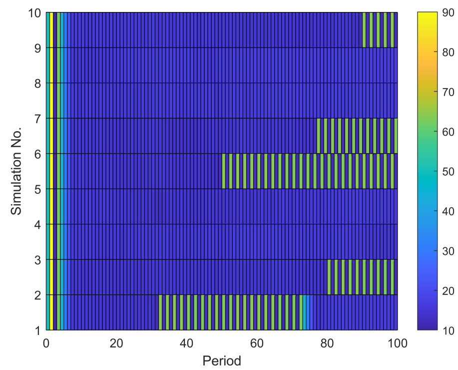

Figure 4.16: Capital stock-Path-$F2$

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Appendices

Appendix 2.1: Monopoly benchmark

Assume that each market is a monopoly. In the first stage, the single team decides whether to merge. In the second stage, if the team chose to merge, the conglomerate sets its R&D effort. If the team chose not to merge, only the standalone firm in market B sets it R&D effort. In the third stage, the output is set in each market.

Assume that the team of monopoly firms merges in the first stage. The joint-profit maximization problem in the third stage is

$$\max_{q_A, q_B \geq 0} \left(D_A - q_A + \beta x_A\right) q_A + \left(D_B - q_B + x_B\right) q_B - \frac{1}{2}\left(x_A + x_B\right)^2. \tag{A.1}$$

The optimal output as a function of the R&D effort is

$$q_A(x_A) = \frac{D_A + \beta\, x_A}{2}, \quad q_B(x_B) = \frac{D_B + x_B}{2}. \tag{A.2}$$

In the second stage, by substituting (A.2) into (A.1), the conglomerate's problem is

$$\max_{x_A, x_B \geq 0} \left(\frac{D_A + \beta\, x_A}{2}\right)^2 + \left(\frac{D_B + x_B}{2}\right)^2 - \frac{1}{2}\left(x_A + x_B\right)^2. \tag{A.3}$$

When the value of either x_A or x_B is adequately high, the objective function in (A.3) is decreasing in that variable for any non-negative value of the other variable. Hence, the objective function must be bounded above in the non-negative region. Given that the problem is constrained by $x_A \geq 0$ and $x_B \geq 0$, a solution is guaranteed to exist.

The determinant of the associated Hessian matrix is given by $D = \frac{1}{4}(-\beta^2 - 2) < 0$; thus, the objective function is not concave. Then, this problem does not yield an interior solution. Nevertheless, because a solution exists, the optimal R&D effort must be a corner solution. There are two candidates for the optimal solution: the firm invests only in market A ($x_B = 0$), or the firm invests only in market B ($x_A = 0$). Those outcomes are denoted as MA and MB, respectively.

First, in the MA outcome, from the first order condition (FOC), the R&D effort, the output, and the team profit are given by

$$x_A^{MA} = \frac{\beta D_A}{2-\beta^2}, \quad x_B^{MA} = 0, \quad q_A^{MA} = \frac{D_A}{2-\beta^2}, \quad q_B^{MA} = \frac{D_B}{2}, \quad \pi_{AB}^{MA} = \frac{{D_A}^2}{2(2-\beta^2)} + \frac{{D_B}^2}{4}.$$

Second, in the MB outcome, the objective function in (A.3) is concave with respect to x_B when $x_A = 0$. From the FOC, the R&D effort, the output, and the team profit π are given by

$$x_A^{MB} = 0, \quad x_B^{MB} = D_B, \quad q_A^{MB} = \frac{D_A}{2}, \quad q_B^{MB} = D_B, \quad \pi_{AB}^{MB} = \frac{{D_A}^2}{4} + \frac{{D_B}^2}{2}.$$

The solution in the second stage depends on which profit is greater. It holds that $\pi_{AB}^{MA} \geq \pi_{AB}^{MB}$ if and only if $\frac{D_A}{D_B} \geq \theta_M = \frac{\sqrt{2-\beta^2}}{\beta}$. Thus, MA is the solution when $\frac{D_A}{D_B} \geq \theta_M$, and MB is the solution when $\frac{D_A}{D_B} \leq \theta_M$.

Now assume that the monopoly firms do not merge in the first stage. In the solution, the R&D effort, the output, and the team profit are the same as MB for any value of $\frac{D_A}{D_B}$.

In the first stage, when $\frac{D_A}{D_B} > \theta_M$, the team of monopoly firms merge to achieve the outcome MA. When $\frac{D_A}{D_B} \leq \theta_M$, the team of monopoly firms is indifferent on whether to merge. Thus, by ER1, the team chooses not to merge and achieve the outcome MB.

Appendix 2.2: Duopoly

Second and third stages

Two-merger subgame

Each one of the firms in market A merges with one of the firms in B. In the third stage, the merged firm chooses non-negative quantities q_{Ai} and q_{Bi}, given that x_{Ai} and x_{Bi} have already been selected in the second stage. The payoff function in the third stage of the conglomerate is

$$(D_A - q_{Ai} - q_{Aj} + \beta x_{Ai})\, q_{Ai} + (D_B - q_{Bi} - q_{Bj} + x_{Bi})\, q_{Bi} - \frac{1}{2}(x_{Ai} + x_{Bi})^2 . \qquad \text{(A.4)}$$

The equilibrium output as a function of the R&D effort is

$$q_{Ai}(x_{Ai}, x_{Aj}) = \begin{cases} \frac{D_A + 2\beta x_{Ai} - \beta x_{Aj}}{3} & \text{if } \frac{D_A}{\beta} + 2x_{Ai} \geq x_{Aj} \text{ and } \frac{D_A}{\beta} + 2x_{Aj} \geq x_{Ai} \\ 0 & \text{if } \frac{D_A}{\beta} + 2x_{Ai} < x_{Aj} \\ \frac{D_A + \beta x_{Ai}}{2} & \text{if } \frac{D_A}{\beta} + 2x_{Aj} < x_{Ai} \end{cases}, \qquad \text{(A.5)}$$

$$q_{Bi}(x_{Bi}, x_{Bj}) = \begin{cases} \frac{D_B + 2x_{Bi} - x_{Bj}}{3} & \text{if } D_B + 2x_{Bi} \geq x_{Bj} \text{ and } D_B + 2x_{Bj} \geq x_{Bi} \\ 0 & \text{if } D_B + 2x_{Bi} < x_{Bj} \\ \frac{D_B + x_{Bi}}{2} & \text{if } D_B + 2x_{Bj} < x_{Bi} \end{cases}. \qquad \text{(A.6)}$$

In the second stage, conglomerate i chooses non-negative x_{Ai} and x_{Bi}. By substituting (A.5) and (A.6) into (A.4), the payoff function in the second stage is given by

$$\pi_{Ai}(x_{Ai}, x_{Aj}) + \pi_{Bi}(x_{Bi}, x_{Bj}) - \frac{1}{2}\left(x_{Ai} + x_{Bi}\right)^2, \tag{A.7}$$

where

$$\pi_{Ai}(x_{Ai}, x_{Aj}) = \begin{cases} \frac{(D_A + 2\beta x_{Ai} - \beta x_{Aj})^2}{9} & \text{if} \quad \frac{D_A}{\beta} + 2x_{Ai} \geq x_{Aj} \\ & \quad \text{and} \quad \frac{D_A}{\beta} + 2x_{Aj} \geq x_{Ai} \\ 0 & \text{if} \quad \frac{D_A}{\beta} + 2x_{Ai} < x_{Aj} \\ \frac{(D_A + \beta x_{Ai})^2}{4} & \text{if} \quad \frac{D_A}{\beta} + 2x_{Aj} < x_{Ai} \end{cases},$$

$$\pi_{Bi}(x_{Bi}, x_{Bj}) = \begin{cases} \frac{(D_B + 2x_{Bi} - x_{Bj})^2}{9} & \text{if} \quad D_B + 2\,x_{Bi} \geq x_{Bj} \\ & \quad \text{and} \quad D_B + 2\,x_{Bj} \geq x_{Bi} \\ 0 & \text{if} \quad D_B + 2\,x_{Bi} < x_{Bj} \\ \frac{(D_B + x_{Bi})^2}{4} & \text{if} \quad D_B + 2\,x_{Bj} < x_{Bi} \end{cases}.$$

At $x_{Ai} > \frac{D_A}{\beta} + 2x_{Aj}$ the derivative with respect to x_{Ai} of the payoff function (A.7) is

$$\frac{\beta(D_A + \beta x_{Ai})}{2} - x_{Ai} - x_{Bi},$$

which is always negative. At $x_{Bi} > D_B + 2\,x_{Bj}$, the derivative with respect to x_{Bi} of the payoff function (A.7) is

$$\frac{D_B + x_{Bi}}{2} - x_{Ai} - x_{Bi},$$

which is always negative. It is then suboptimal for conglomerate i to play any $x_{Ai} > \frac{D_A}{\beta} + 2x_{Aj}$ or $x_{Bi} > D_B + 2\,x_{Bj}$. By symmetry, it is also suboptimal for conglomerate j

to play any $x_{Aj} > \frac{D_A}{\beta} + 2x_{Ai}$ or $x_{Bj} > D_B + 2\,x_{Bi}$. Thus, per (A.7), all the equilibria of the two-merger subgame can be found by considering the following payoff function for conglomerate i

$$\frac{(D_A + 2\beta x_{Ai} - \beta x_{Aj})^2}{9} + \frac{(D_B + 2x_{Bi} - x_{Bj})^2}{9} - \frac{1}{2}\left(x_{Ai} + x_{Bi}\right)^2 \tag{A.8}$$

on the region $\left[0, \frac{D_A}{\beta} + 2x_{Aj}\right] \times [0, D_B + 2\,x_{Bj}]$. The determinant of the associated Hessian matrix of (A.8) is $D = \frac{8}{9}(-\frac{1}{9}\beta^2 - 1) < 0$, which implies that the function is not concave on $\left[0, \frac{D_A}{\beta} + 2x_{Aj}\right] \times [0, D_B + 2\,x_{Bj}]$. Thus, no interior solution exists. However, given that (A.8) is a continuous function on a compact rectangle, a maximum is guaranteed to exist. Therefore, the equilibrium must be a corner solution.

With ER2, there are only three equilibrium candidates: a symmetric candidate where both invest only in A ($x^*_{Ai} = x^*_{Aj} > 0$ and $x^*_{Bi} = x^*_{Bj} = 0$), termed as the A-outcome; a symmetric candidate where both teams invest only in market B ($x_{Ai} = x_{Aj} = 0$ and $x_{Bi} = x^*_{Bj} > 0$), termed as the B-outcome; and an asymmetric candidate with the form $x^*_{Aj} = x^*_{Bi} = 0$, $x^*_{Ai} > 0$ and $x^*_{Bj} > 0$, termed as the asymmetric outcome.

A-outcome Let $x_{Bi} = x_{Bj} = 0$. The function in (A.8) when $x_{Bi} = 0$ is concave with respect to x_{Ai}. From the FOC, the R&D effort, output, and team profit are given by

$$x^A_{Ai} = \frac{4\beta D_A}{9 - 4\,\beta^2}, \quad x^A_{Bi} = 0, \quad q^A_{Ai} = \frac{3D_A}{9 - 4\,\beta^2}, \quad q^A_{Bi} = \frac{D_B}{3}, \tag{A.9}$$
$$\pi^A_i = \frac{(9 - 8\beta^2){D_A}^2}{(9 - 4\beta^2)^2} + \frac{{D_B}^2}{9}.$$

This appendix now examines when the A-outcome is an equilibrium. Suppose the rival plays $x_{Aj} = x^A_{Ai}$ and $x_{Bj} = 0$; thus, the objective function of conglomerate i is given by

$$\frac{1}{9}\left(\frac{(9-8\beta^2)D_A}{9-4\beta^2}+2\beta x_{Ai}\right)^2+\frac{(D_B+2x_{Bi})^2}{9}-\frac{1}{2}\left(x_{Ai}+x_{Bi}\right)^2 \tag{A.10}$$

on the region $\left[0, \frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}\right] \times [0, D_B]$.

Suppose conglomerate i plays the upper bound of x_{Ai}; that is, $x_{Ai} = \frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}$. The derivative of (A.10) with respect to x_{Ai} evaluated at $x_{Ai} = \frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}$ is $\frac{(8\beta^2-9)D_A}{(9-4\beta^2)\beta} - x_{Bi}$, which is always negative. Therefore, any $\left(\frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}, x_{Bi}\right)$ is not a solution to (A.10).

The remainder solution candidates to (A.10) are $(0, x_{Bi})$ and (x_{Ai}, D_B), where the first candidate x_{Bi} is an interior solution given by the FOC. This appendix starts with the first candidate. The function in (A.10) when $x_{Ai} = 0$ is concave with respect to x_{Bi}. Thus, the solution given by the FOC is $x_{Bi} = 4D_B$, which is outside the feasible region. Therefore, this strategy is not a solution of (A.10). For the second solution candidate, there are two cases to consider. First, assume the strategy where $x_{Bi} = D_B$ and x_{Ai} is an interior solution given by the FOC. From the FOC of (A.10), with respect to x_{Ai} at $x_{Bi} = D_B$, it follows that

$$x_{Ai} = \frac{4\beta D_A}{9-4\beta^2} - \frac{9D_B}{9-8\beta^2}.$$

Further, to satisfy the assumption of $x_{Ai} > 0$, $\frac{D_A}{D_B} > \frac{9(9-4\beta^2)}{4\beta(9-8\beta^2)}$ must hold. The profit in this case, denoted by π_i^{D4}, is

$$\pi_i^{D4} = \frac{(9-8\beta^2){D_A}^2}{(9-4\beta^2)^2} + \frac{(9-4\beta^2){D_B}^2}{9-8\beta^2} - \frac{4\beta D_A D_B}{9-4\beta^2}.$$

Given $\frac{D_A}{D_B} > \frac{9(9-4\beta^2)}{4\beta(9-8\beta^2)}$, $\pi_i^A > \pi_i^{D4}$ always holds. Thus, conglomerate i does not deviate in this case.

Second, assume that conglomerate i plays $x_{Bi} = D_B$ and $x_{Ai} = 0$. The profit in this case, denoted by π_i^{D5}, is

$$\pi_i^{D5} = \frac{(9-8\beta^2)^2 {D_A}^2}{9(9-4\beta^2)^2} + \frac{{D_B}^2}{2}.$$

When $\frac{D_A}{D_B} < \frac{(9-4\beta^2)\sqrt{7}}{4\beta\sqrt{9-8\beta^2}}$, it follows that $\pi_i^A < \pi_i^{D5}$. Thus, conglomerate i deviates unilaterally in this case. Therefore, the A-outcome can be sustained as an equilibrium if and only if $\frac{D_A}{D_B} \geq \theta_A = \frac{(9-4\beta^2)\sqrt{7}}{4\beta\sqrt{9-8\beta^2}}$.

B-outcome Let $x_{Ai} = x_{Aj} = 0$. The function in (A.8) when $x_{Ai} = 0$ is concave with respect to x_{Bi}. From the FOC, the R&D effort, output, and team profit are given by

$$x_{Ai}^B = 0, \quad x_{Bi}^B = \frac{4D_B}{5}, \quad q_{Ai}^B = \frac{D_A}{3}, \quad q_{Bi}^B = \frac{3D_B}{5}, \qquad \text{(A.11)}$$
$$\pi_i^B = \frac{{D_A}^2}{9} + \frac{{D_B}^2}{25}.$$

This appendix now examines when the B-outcome is an equilibrium. Suppose the rival plays $x_{Aj} = 0$ and $x_{Bj} = x_{Bi}^B$; thus, the objective function of conglomerate i is given by

$$\frac{(D_A + 2\beta x_{Ai})^2}{9} + \frac{(\frac{D_B}{5} + 2x_{Bi})^2}{9} - \frac{1}{2}\left(x_{Ai} + x_{Bi}\right)^2 \qquad \text{(A.12)}$$

on the region $\left[0, \frac{D_A}{\beta}\right] \times \left[0, \frac{13D_B}{5}\right]$.

Suppose conglomerate i plays the upper bound of x_{Bi}; that is, $x_{Bi} = \frac{13D_B}{5}$. The derivative of (A.12) with respect to x_{Bi} evaluated at $x_{Bi} = \frac{13D_B}{5}$ is $-\frac{D_B}{5} - x_{Ai}$, which is always negative. Therefore, any $\left(x_{Ai}, \frac{13D_B}{5}\right)$ is not a solution to (A.12).

The remainder solution candidates to (A.12) are $(x_{Ai}, 0)$ and $\left(\frac{D_A}{\beta}, x_{Bi}\right)$, where the first candidate x_{Ai} is an interior solution given by the FOC. This appendix starts with the first candidate. The function in (A.12) when $x_{Bi} = 0$ is concave with respect to x_{Ai}.

Thus, the solution given by the FOC is

$$x_{Ai} = \frac{4\beta D_A}{9 - 8\,\beta^2}. \tag{A.13}$$

For (A.13) to be an interior solution, $\frac{4\beta D_A}{9-8\,\beta^2} < \frac{D_A}{\beta}$ must be satisfied. This expression holds if and only if $\beta < \frac{\sqrt{3}}{2}$. Therefore, a necessary condition for (A.13) to be a solution is $\beta < \frac{\sqrt{3}}{2}$. The profit of conglomerate i in this case, denoted by π_i^{D1}, is

$$\pi_i^{D1} = \frac{{D_A}^2}{9 - 8\beta^2} + \frac{{D_B}^2}{225}.$$

When $\frac{D_A}{D_B} > \frac{\sqrt{9-8\beta^2}}{5\beta}$, it follows that $\pi_i^B < \pi_i^{D1}$; therefore, conglomerate i deviates unilaterally in this case.

For the second candidate, there are two cases to consider. First, assume the strategy where $x_{Ai} = \frac{D_A}{\beta}$, and x_{Bi} is an interior solution given by the FOC. From the FOC of (A.12), with respect to x_{Bi} at $x_{Ai} = \frac{D_A}{\beta}$, it follows that

$$x_{Bi} = \frac{4D_B}{5} - \frac{9D_A}{\beta}.$$

Further, to satisfy the assumption of $x_{Bi} > 0$, $\frac{D_A}{D_B} < \frac{4\beta}{45}$ must hold. The profit in this case, denoted by π_i^{D2}, is

$$\pi_i^{D2} = {D_A}^2 + \frac{{D_B}^2}{25} - \frac{4D_A D_B}{5\beta} + \frac{4{D_A}^2}{\beta^2}.$$

Given $\frac{D_A}{D_B} < \frac{4\beta}{45}$, $\pi_i^B > \pi_i^{D2}$ always holds. Thus, conglomerate i does not deviate in this case.

Second, assume that conglomerate i plays $x_{Ai} = \frac{D_A}{\beta}$ and $x_{Bi} = 0$. The profit in this case, denoted by π_i^{D3}, is

$$\pi_i^{D3} = \frac{(2\beta^2 - 1){D_A}^2}{2\beta^2} + \frac{{D_B}^2}{225}.$$

When $\frac{D_A}{D_B} > \frac{4\beta}{5\sqrt{16\beta^2-9}}$ and $\beta > 3/4$, it follows that $\pi_i^B < \pi_i^{D3}$. Conglomerate i then deviates unilaterally in this case.

Notice that when $\beta \in \left(\frac{3}{4}, \frac{\sqrt{3}}{2}\right)$, depending on the market ratio, conglomerate i can deviate to π_i^{D1} or π_i^{D3}. Given that $\frac{\sqrt{9-8\beta^2}}{5\beta} < \frac{4\beta}{5\sqrt{16\beta^2-9}}$ for any $\beta \in \left(\frac{3}{4}, \frac{\sqrt{3}}{2}\right)$, to sustain the B-outcome as an equilibrium the market ratio must only satisfy $\frac{D_A}{D_B} \leq \frac{\sqrt{9-8\beta^2}}{5\beta}$.[4]

In conclusion, the B-outcome can be sustained as an equilibrium if and only if $\frac{D_A}{D_B} \leq \theta_B$, where

$$\theta_B = \begin{cases} \frac{\sqrt{9-8\beta^2}}{5\beta} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{4\beta}{5\sqrt{16\beta^2-9}} & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}.$$

Asymmetric outcome Let $x_{Aj} = 0$ and $x_{Bi} = 0$. First, suppose an equilibrium candidate where x_{Bj} is given by the FOC. From the FOC of (A.8) with respect to x_{Bj} at $x_{Aj} = x_{Bi} = 0$, it follows that $x_{Bj} = 4D_B$, which is outside the feasible region. Hence, this candidate is not a solution. Second, suppose x_{Bj} is equal to the upper bound; that is, $x_{Bj} = D_B$. Thus, the objective function of conglomerate i is given by

$$\frac{(D_A + 2\beta x_{Ai})^2}{9} - \frac{{x_{Bi}}^2}{18} - \frac{{x_{Ai}}^2}{2} - x_{Bi}x_{Ai} \tag{A.14}$$

on the region $\left[0, \frac{D_A}{\beta}\right] \times [0, 3D_B]$.

It is easy to see that (A.14) is strictly decreasing in $x_{Bi} \geq 0$. Thus, (A.14) must be maximized at $x_{Bi} = 0$. Given that (A.14) is concave in x_{Ai} at $x_{Bi} = 0$, the optimal x_{Ai}

[4]If $\frac{D_A}{D_B} \in \left(\frac{\sqrt{9-8\beta^2}}{5\beta}, \frac{4\beta}{5\sqrt{16\beta^2-9}}\right]$, conglomerate i will deviate to π_i^{D1}. When $\frac{D_A}{D_B} > \frac{4\beta}{5\sqrt{16\beta^2-9}}$, conglomerate i can deviate to π_i^{D1} or π_i^{D3}.

must be an interior solution if it is inside the feasible region. From the FOC, $x_{Ai} = \frac{4\beta D_A}{9-8\beta^2}$. The result is inside the feasible region if and only if $\frac{4\beta D_A}{9-8\beta^2} < \frac{D_A}{\beta}$. This inequality holds if and only if $\beta < \frac{\sqrt{3}}{2}$.

This appendix denotes the conglomerate that invests only in market A (B) as $\diamondsuit A$ ($\diamondsuit B$). When (A.14) is maximized, the R&D effort, the output, and the team profit are given by

$$(x_A^{\diamondsuit A}, x_A^{\diamondsuit B}, x_B^{\diamondsuit A}, x_B^{\diamondsuit B}) = \begin{cases} \left(\frac{4\beta D_A}{9-8\beta^2}, 0, 0, D_B\right) & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \left(\frac{D_A}{\beta}, 0, 0, D_B\right) & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}, \tag{A.15}$$

$$(q_A^{\diamondsuit A}, q_A^{\diamondsuit B}, q_B^{\diamondsuit A}, q_B^{\diamondsuit B}) = \begin{cases} \left(\frac{3D_A}{9-8\beta^2}, \frac{(3-4\beta^2)D_A}{9-8\beta^2}, 0, D_B\right) & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ (D_A, 0, 0, D_B) & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases},$$

$$(\pi^{\diamondsuit A}, \pi^{\diamondsuit B}) = \begin{cases} \left(\frac{{D_A}^2}{9-8\beta^2}, \frac{(3-4\beta^2)^2 {D_A}^2}{(9-8\beta^2)^2} + \frac{{D_B}^2}{2}\right) & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \left(\frac{(2\beta^2-1){D_A}^2}{2\beta^2}, \frac{{D_B}^2}{2}\right) & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}.$$

This appendix considers two cases that depend on β.

Case 1: Let $\beta < \frac{\sqrt{3}}{2}$. It suffices to verify the strategy of conglomerate j to examine when the asymmetric outcome is an equilibrium. When conglomerate i plays $x_{Ai} = x_A^{\diamondsuit A}$ and $x_{Bi} = 0$, the objective function of conglomerate j is given by

$$\frac{1}{9}\left(\frac{3(3-4\beta^2)D_A}{9-8\beta^2} + 2\beta x_{Aj}\right)^2 + \frac{(D_B + 2x_{Bj})^2}{9} - \frac{1}{2}(x_{Aj} + x_{Bj})^2 \tag{A.16}$$

on the region $\left[0, \frac{9D_A}{(9-8\beta^2)\beta}\right] \times [0, D_B]$.

Suppose conglomerate j plays the upper bound of x_{Aj}; that is, $x_{Aj} = \frac{9D_A}{(9-8\beta^2)\beta}$. The derivative of (A.16) with respect to x_{Aj} evaluated at $\frac{9D_A}{(9-8\beta^2)\beta}$ is $\frac{\left[4\beta^2(9-4\beta^2)-27\right]D_A}{3(9-8\beta^2)\beta} - x_{Bj}$, which is always negative. Therefore, any $\left(\frac{9D_A}{(9-8\beta^2)\beta}, x_{Bj}\right)$ is not a solution to (A.16). Moreover, ABj playing $x_{Aj} = 0$ and x_{Bj} being given by the FOC is not a solution to (A.16), given that $x_{Bj} = 4D_B$, which is outside the feasible region.

The remainder solution candidates to (A.16) are $(x_{Aj}, 0)$ and (x_{Aj}, D_B), where x_{Aj} is an interior solution given by the FOC in both cases. For the first candidate, from the FOC of (A.16) with respect to x_{Aj} at $x_{Bj} = 0$, it follows that

$$x_{Aj} = \frac{12\beta(3-4\,\beta^2)D_A}{(9-8\,\beta^2)^2}.$$

Conglomerate j gains a profit of

$$\pi_{ABj}^{D6} = \frac{9(3-4\,\beta^2)^2{D_A}^2}{(9-8\,\beta^2)^3} + \frac{{D_B}^2}{9}.$$

When $\frac{D_A}{D_B} > \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$ and $\beta < \frac{\sqrt{3}}{2}$, it follows that $\pi^{\Diamond B} < \pi_{ABj}^{D6}$; therefore, conglomerate j deviates unilaterally in this case.

For the second candidate, from the FOC of (A.16) with respect to x_{Aj} at $x_{Bj} = D_B$, it follows that

$$x_{Aj} = \frac{12\beta(3-4\,\beta^2)D_A - 9(9-8\,\beta^2)D_B}{(9-8\,\beta^2)^2}.$$

Further, to satisfy the assumption of $x_{Aj} > 0$, $\frac{D_A}{D_B} > \frac{9(9-8\,\beta^2)}{12\beta(3-4\,\beta^2)}$ must hold. Given that $\frac{9(9-8\,\beta^2)}{12\beta(3-4\,\beta^2)} > \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$, and the asymmetric outcome is not an equilibrium when $\frac{D_A}{D_B} > \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$, this case does not provide new information.

Thus, when $\beta < \frac{\sqrt{3}}{2}$, the asymmetric outcome can be sustained as an equilibrium if and only if $\frac{D_A}{D_B} \leq \hat{\theta}_A = \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$.

Case 2: Let $\beta \geq \frac{\sqrt{3}}{2}$. Again, it suffices to verify the strategy of conglomerate j. When conglomerate i plays $x_{Ai} = x_A^{\diamond A}$ and $x_{Bi} = 0$, the problem of conglomerate j is given by

$$\frac{(D_B + 2x_{Bj})^2}{9} - \frac{(9 - 8\beta^2){x_{Aj}}^2}{18} - \frac{{x_{Bj}}^2}{2} - x_{Bj}x_{Aj} \tag{A.17}$$

on the region $\left[0, \frac{3D_A}{\beta}\right] \times [0, D_B]$.

The function in (A.17) is strictly decreasing in $x_{Aj} \geq 0$. Thus, (A.17) must be maximized at $x_{Aj} = 0$. Moreover, conglomerate j playing $x_{Aj} = 0$ and x_{Bj} being given by the FOC is not a solution to (A.17), given that $x_{Bj} = 4D_B$, which is outside the feasible region. Hence, the only solution of (A.17) is $x_{Aj} = 0$ and $x_{Bj} = D_B$. Therefore, when $\beta \geq \frac{\sqrt{3}}{2}$, the asymmetric outcome always exists as an equilibrium.

Summary of the two-merger subgame The thresholds satisfy $\theta_B < \theta_A$ and $\theta_A < \hat{\theta}_A$ for any $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$. Thus, an equilibrium always exists in the two-merger subgame. This appendix summarizes the equilibrium results of the two-merged subgame in Proposition A.1, illustrated in Figure A.1.

Proposition A. 1. *An equilibrium always exists in the two-merged subgame. With ER2, all the equilibria are characterized as follows:*

(a) The A-outcome is an equilibrium if and only if $\frac{D_A}{D_B} \geq \theta_A$.

(b) The B-outcome is an equilibrium if and only if $\frac{D_A}{D_B} \leq \theta_B$,

(c) The asymmetric outcome is an equilibrium if and only if $\beta < \frac{\sqrt{3}}{2}$ *and* $\frac{D_A}{D_B} \leq \hat{\theta}_A$, *or* $\beta \geq \frac{\sqrt{3}}{2}$.

Asymmetric equilibria are two-fold depending on the roles of the teams. Figure A.1 displays four regions. First, the B-outcome and two asymmetric outcomes are equilibria in the lower left region. Second, the unique equilibrium in the center-right region is the A-outcome. Third, the A and two asymmetric outcomes are equilibria in the upper right

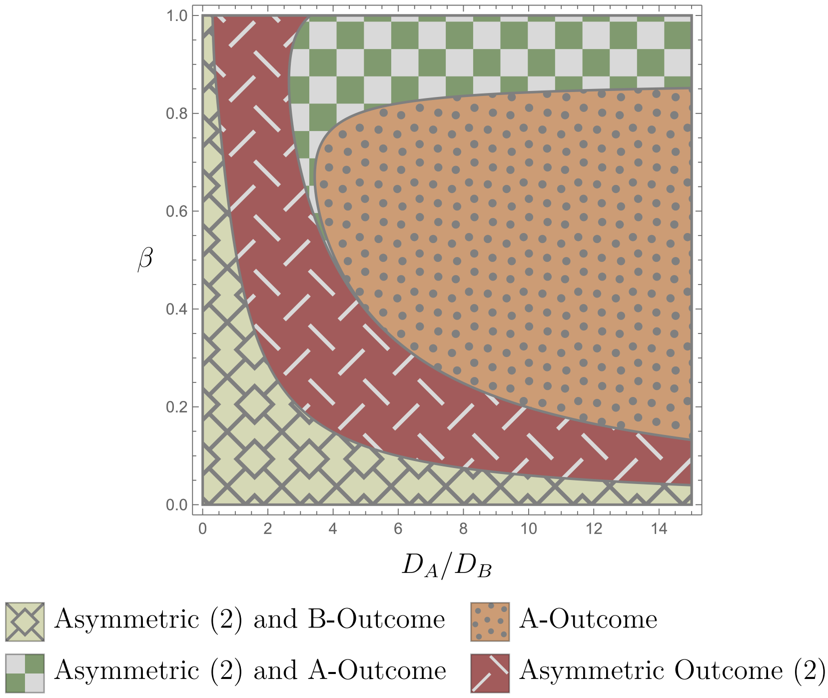

Figure A.1: Existence of equilibria in the two-merger subgame.

region. Fourth, two asymmetric outcomes are equilibria in the middle region.

One-merger subgame

Only one firm in A merges with one firm in B. The non-merged team can only invest in market B; the conglomerate can invest in any market. With ER2, the only equilibria are the B-outcome, as described by (A.11), and the asymmetric outcome, as described by (A.15).

The B-outcome is an equilibrium in the one-merger subgame under the same condition as the two-merger subgame. However, the asymmetric outcome is always an equilibrium in the one-merged subgame. Team $\diamondsuit B$ is always the non-merged team when the asymmetric outcome is the equilibrium of the one-merge subgame. A condition involving the market size is not necessary, as team $\diamondsuit B$ cannot deviate to market A.

This appendix summarizes the equilibrium results of the one-merged subgame in Proposition A.2, illustrates in Figure A.2.

Proposition A. 2. *With ER2, all the equilibria in the one-merger subgame are characterized as follows:*

(a) The B-outcome is an equilibrium if and only if $\frac{D_A}{D_B} \leq \theta_B$.

(b) The asymmetric outcome is always an equilibrium.

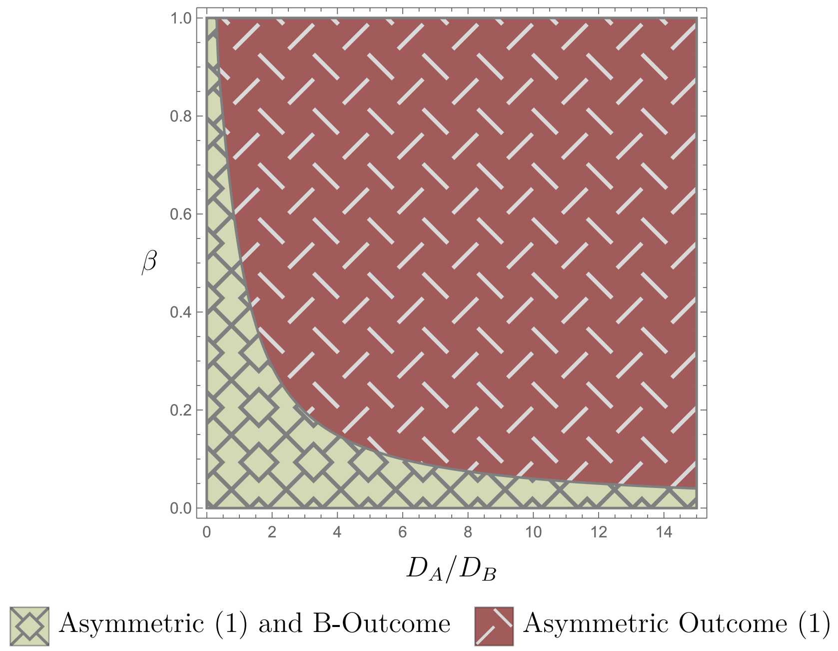

Figure A.2: Existence of equilibria in the one-merger subgame.

Figure A.2 displays two regions. First, the B-outcome and one asymmetric outcome are equilibria in the lower left region. In the second region, one asymmetric outcome is an equilibrium.

Zero-merger subgame

In the zero-merger subgame, none of the teams are merged. With ER2, the only equilibrium is the B-outcome as described by (A.11). The B-outcome always exists as an equilibrium, as none of the teams can deviate to market A. This appendix summarizes the equilibrium results of the zero-merger subgame in Proposition A.3.

Proposition A. 3. *With ER2, the B-outcome always exists as the unique equilibrium of the zero-merger subgame.*

First stage

The team payoffs in the first stage are the profits in (A.9), (A.11), and (A.15). This appendix defines four main scenarios for the first stage, each scenario corresponding to one region in Figure A.1. There are multiple equilibria in the second stage for each scenario; thus, this appendix considers every possible combination of equilibria and establishes sub-scenarios. Table A.3 presents the normal form of the game.

Table A.3: Normal form of the first stage

		A2,B2	
		Do not merge	Merge
A1,B1	Do not merge	π^B,π^B	$\pi_1^{DM,M}$,$\pi_2^{DM,M}$
	Merge	$\pi_1^{M,DM}$,$\pi_2^{M,DM}$	$\pi_1^{M,M}$,$\pi_2^{M,M}$

Where –contingent on the parameters– $\left(\pi_1^{M,M}, \pi_2^{M,M}\right) \in \left\{ \left(\pi^{\Diamond B}, \pi^{\Diamond A}\right), \left(\pi^{\Diamond A}, \pi^{\Diamond B}\right), \left(\pi^{A}, \pi^{A}\right), \left(\pi^{B}, \pi^{B}\right) \right\}$, $\left(\pi_1^{M,DM}, \pi_2^{M,DM}\right) \in \left\{ \left(\pi^{\Diamond A}, \pi^{\Diamond B}\right), \left(\pi^{B}, \pi^{B}\right) \right\}$ and $\left(\pi_1^{DM,M}, \pi_2^{DM,M}\right) \in \left\{ \left(\pi^{\Diamond B}, \pi^{\Diamond A}\right), \left(\pi^{B}, \pi^{B}\right) \right\}$.

The asymmetric equilibria are two-fold in the two-merger subgame but are not in the one-merger subgame. Thus, there are two possible ways to allocate the payoffs from the asymmetric outcome in the profile (M, M); however, there is only one way in the profiles (DM, M) and (M, DM): the team that chooses to merge always invests only in market A.

This appendix restates a set of thresholds from the second and third stages and defines new ones to define the scenarios and sub-scenarios. First, for the previously defined thresholds, the appendix sets $\theta_1 \equiv \hat{\theta}_A$, $\theta_2 \equiv \theta_A$, and $\theta_6 \equiv \theta_B$. The new thresholds come from comparing the payoffs of the first stage. It follows that $\pi^B \leq \pi^{\Diamond A}$ if and only if $\frac{D_A}{D_B} \geq \theta_5$, $\pi^{\Diamond B} \leq \pi^A$ if and only if $\frac{D_A}{D_B} \geq \theta_3$, $\pi^{\Diamond A} \geq \pi^{\Diamond B}$ if and only if $\frac{D_A}{D_B} \geq \theta_4$, and

$\pi^B \geq \pi^{\diamond B}$ if and only if $\frac{D_A}{D_B} \geq \theta_7$, where:

$$\theta_3 = \begin{cases} \frac{\sqrt{7}(9-8\beta^2)(9-4\beta^2)}{12\beta\sqrt{81-180\beta^2+128\beta^4-32\beta^6}} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{\sqrt{7}(9-4\beta^2)}{3\sqrt{2(9-8\beta^2)}} & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}, \quad \theta_4 \equiv \bar{\theta} = \begin{cases} \frac{(9-8\beta^2)}{4\sqrt{2}\beta\sqrt{1-\beta^2}} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{\beta}{\sqrt{2\beta^2-1}} & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}$$

$$\theta_5 \equiv \hat{\theta}_B = \begin{cases} \frac{3\sqrt{9-8\beta^2}}{10\sqrt{2}\beta} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{2}\beta}{5\sqrt{16\beta^2-9}} & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}, \quad \theta_7 = \begin{cases} \frac{3\sqrt{23}(9-8\beta^2)}{20\beta\sqrt{9-10\beta^2}} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{23}}{5\sqrt{2}} & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}.$$

It follows that $\theta_1 > \theta_2$ for any $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$, $\theta_2 > \theta_3 > \theta_4 > \theta_5 > \theta_6$ for any β and $\theta_7 > \theta_5$ for any β.

Scenario 1: $\frac{D_A}{D_B} > \theta_1$ and $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$. The equilibrium in the two-merger subgame is the A-outcome; in the one-merger subgame, it is the asymmetric outcome. Here, it holds that $\pi^B < \pi^{\diamond A}$ and $\pi^{\diamond B} < \pi^A$. If player 2 plays (DM), the best strategy for player 1 is to play (M). If player 2 plays (M), the best strategy for player 1 is to play (M). The dominant strategy for player 1 is then (M). Given that the payoffs are symmetric, the dominant strategy for player 2 is also (M). Thus, the equilibrium in the first stage is the profile (M, M).

Therefore, when $\frac{D_A}{D_B} > \hat{\theta}_A$ and $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$, the equilibrium of the first stage corresponds to the A-outcome. Moreover, in that equilibrium, two conglomerates are formed.

Scenario 2: $\theta_1 \geq \frac{D_A}{D_B} \geq \theta_2$ and $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$ or $\frac{D_A}{D_B} \geq \theta_2$ and $\beta \in \left[\frac{\sqrt{3}}{2}, 1\right)$. For the two-merger subgame, the A-outcome and asymmetric outcome exist as an equilibrium. For the one-merger subgame, the asymmetric outcome is the equilibrium. It holds that $\pi^B < \pi^{\diamond A}$, $\pi^{\diamond B} < \pi^A$, and $\pi^{\diamond B} < \pi^{\diamond A}$.

Scenario 2.1: The A-outcome is set in the profile (M, M). This scenario is analogous to Scenario 1. Hence, the equilibrium in the first stage is the profile (M, M).

Scenario 2.2: The asymmetric outcome is set in the profile (M, M), and player 1's payoff in the profile (M, M) is $\pi^{\diamond A}$. If any player plays (DM), the best strategy for the other player is to play (M). If player 2 plays (M), the best strategy for player 1 is to play (M). When player 1 plays (M), player 2 is indifferent to playing either (DM) or (M). Thus, the equilibria in the first stage are the profiles (M, DM) and (M, M). With ER1, only the profile (M, DM) is an equilibrium.

Scenario 2.3: The asymmetric outcome is set in the profile (M, M), and player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. By symmetry with Scenario 2.2, the equilibria in the first stage are the profiles (DM, M) and (M, M). With ER1, only the profile (DM, M) is an equilibrium.

Therefore, when $\hat{\theta}_A \geq \frac{D_A}{D_B} \geq \theta_A$ and $\beta \in \left(0, \frac{\sqrt{3}}{2}\right)$, or $\frac{D_A}{D_B} \geq \theta_A$ and $\beta \in \left[\frac{\sqrt{3}}{2}, 1\right)$, if the equilibrium played in the two-merger subgame is the A-outcome, the equilibrium of the first stage corresponds to the A-outcome. Moreover, in that equilibrium two conglomerates are formed.

If the equilibrium played in the two-merger subgame is the asymmetric outcome, then any equilibrium of the first stage corresponds to the asymmetric outcome. Adding ER1, in any equilibrium of the first stage, the outcome with one conglomerate always occurs.

Scenario 3: $\theta_2 > \frac{D_A}{D_B} > \theta_6$. The asymmetric outcome is the equilibrium in the two- and one-merger subgames.

Scenario 3.1: $\theta_2 > \frac{D_A}{D_B} > \theta_5$; thus, it holds that $\pi^B < \pi^{\Diamond A}$. Hence, (DM, DM) is never an equilibrium, as both players have incentives to deviate. Accordingly, at least one of the other profiles is an equilibrium, hence the asymmetric outcome is always an equilibrium.

Scenario 3.1.1: $\theta_2 > \frac{D_A}{D_B} > \theta_4$; thus, it holds that $\pi^{\Diamond B} < \pi^{\Diamond A}$. Player 1's payoff in the profile (M, M) is $\pi^{\Diamond A}$. This scenario is analogous to Scenario 2.2. Hence, the equilibria in the first stage are the profiles (M, DM) and (M, M). With ER1, only the profile (M, DM) is an equilibrium.

Scenario 3.1.2: $\theta_2 > \frac{D_A}{D_B} > \theta_4$; thus, it holds that $\pi^{\Diamond B} < \pi^{\Diamond A}$. Player 1's payoff in the profile (M, M) is $\pi^{\Diamond B}$. This scenario is analogous to Scenario 2.3. Hence, the equilibria in the first stage are the profiles (DM, M) and (M, M). With ER1, only the profile (DM, M) is an equilibrium.

Scenario 3.1.3: $\frac{D_A}{D_B} = \theta_4$; thus, it holds that $\pi^{\Diamond B} = \pi^{\Diamond A}$. Hence, the payoffs in the profile (M, M) are symmetric. If any player plays (DM), the best strategy for the other player is to play (M). The profiles (M, M), (M, DM), and (DM, M) have the same symmetric payoffs. Therefore, (M, M), (M, DM), and (DM, M) are the equilibria of the first stage. With ER1, only the profiles (M, DM) and (DM, M) are equilibria.

Scenario 3.1.4: $\theta_4 > \frac{D_A}{D_B} > \theta_5$; thus, it holds that $\pi^{\Diamond B} > \pi^{\Diamond A}$. Player 1's payoff in the profile (M, M) is $\pi^{\Diamond A}$. If any player plays (DM), the best strategy for the other player is to play (M). If player 2 plays (M), the best strategy for player 1 is to play (DM). When player 1 plays (M), player 2 is indifferent to playing either (DM) or (M). Therefore, there are two equilibria in the first stage: the profiles (M, DM) and (DM, M).

Scenario 3.1.5: $\theta_4 > \frac{D_A}{D_B} > \theta_5$; thus, it holds that $\pi^{\diamond B} > \pi^{\diamond A}$. Player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. By symmetry with Scenario 3.1.4, the equilibria in the first stage are the profiles (M, DM) and (DM, M).

Therefore, when $\theta_A > \frac{D_A}{D_B} > \hat{\theta}_B$, any equilibrium of the first stage corresponds to the asymmetric outcome. Adding ER1, in any equilibrium, one team merges.

Scenario 3.2: $\frac{D_A}{D_B} = \theta_5$; thus, it holds that $\pi^{\diamond B} > \pi^{\diamond A} = \pi^B$.

Scenario 3.2.1: Player 1's payoff in the profile (M, M) is $\pi^{\diamond A}$. If player 2 plays (M), the best strategy for player 1 is to play (DM). If player 1 plays (M), player 2 is indifferent to playing either (DM) or (M). If any player plays (DM), the other player is indifferent to playing either (DM) or (M). Thus, the equilibria of the first stage are the profiles (DM, DM), (M, DM), and (DM, M). With ER1, the only equilibrium is the profile (DM, DM).

Scenario 3.2.2: Player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. By symmetry with Scenario 3.2.1, the equilibria of the first stage are the profiles (DM, DM), (M, DM), and (DM, M). With ER1, only the profile (DM, DM) is an equilibrium.

Therefore, when $\frac{D_A}{D_B} = \hat{\theta}_B$, one of the equilibria is the profile (DM, DM), which corresponds to the B-outcome. Any other equilibrium corresponds to the asymmetric outcome. Adding ER1, none of the teams merge in equilibrium.

Scenario 3.3: $\theta_5 > \frac{D_A}{D_B} > \theta_6$; thus, it holds that $\pi^{\diamond B} > \pi^B > \pi^{\diamond A}$.

Scenario 3.3.1: Player 1's payoff in the profile (M, M) is $\pi^{\diamond A}$. If player 2 plays (M), the best strategy for player 1 is to play (DM). If any player plays (DM), the best strategy for the other player is to play (DM). Therefore, the equilibrium in the first stage is the profile (DM, DM).

Scenario 3.3.2: Player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. By symmetry with Scenario 3.3.1, the equilibrium of the first stage is the profile (DM, DM).

Scenario 4: $\frac{D_A}{D_B} \leq \theta_6$. The B and asymmetric outcomes are equilibria in the two- and one-merger subgames. It holds that $\pi^{\diamond B} > \pi^B > \pi^{\diamond A}$. Thus, the profile (DM, DM) is always an equilibrium. Furthermore, the asymmetric outcome is never an equilibrium as the player with the payoff of $\pi^{\diamond A}$ has incentives to deviate to obtain either π^B or $\pi^{\diamond B}$.

Scenario 4.1: The asymmetric outcome is set in the profiles (DM, M), (M, DM), and (M, M). Player 1's payoff in the profile (M, M) is $\pi^{\diamond A}$. This scenario is analogous to Scenario 3.3.1. Hence, the equilibrium of the first stage is the profile (DM, DM).

Scenario 4.2: The asymmetric outcome is set in the profiles (DM, M), (M, DM), and (M, M). Player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. This scenario is analogous to Scenario 3.3.2. Hence, the equilibrium of the first stage is the profile (DM, DM).

Scenario 4.3: The B-outcome is set in the profiles (DM, M), (M, DM), and (M, M). Both players are indifferent to playing either (DM) and (M) regardless of the other player's strategy. Thus, all the profiles are equilibria. With ER1, only the profile (DM, DM) is an equilibrium.

Scenario 4.4: The asymmetric outcome is set in the profiles (DM, M) and (M, DM). The B-outcome is set in the profile (M, M). If player 2 plays (DM), the best strategy for player 1 is to play (DM). If player 2 plays (M), the best strategy for player 1 is to play (DM). The dominant strategy for player 1 is then (DM). Given that the payoffs are symmetric, the dominant strategy for player 2 is also (DM). Thus, the equilibrium in the first stage is the profile (DM, DM).

Scenario 4.5: The asymmetric outcome is set in the profile (M, M). The B-outcome is set in the profiles (DM, M) and (M, DM). Player 1's payoff in the profile (M, M) is $\pi^{\diamond A}$. If player 2 plays (M), the best strategy for player 1 is to play (DM). If player 1 plays (M), the best strategy for player 2 is to play (M). If any player plays (DM), the other player is indifferent to playing either (DM) or (M). Hence, the profiles (DM, M) and (DM, DM) are equilibria of the first stage. With ER1, only the profile (DM, DM) is an equilibrium.

Scenario 4.6: The asymmetric outcome is set in the profile (M, M). The B-outcome is set in the profiles (DM, M) and (M, DM). Player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. By symmetry with Scenario 4.5, the profiles (M, DM) and (DM, DM) are equilibria of the first stage. With ER1, only the profile (DM, DM) is an equilibrium.

Scenario 4.7: The asymmetric outcome is set in the profile (M, DM). The B-outcome is set in the profiles (DM, M) and (M, M). If player 1 plays (M), the best strategy for player 2 is to play (DM). If player 2 plays (DM), the best strategy for player 1 is to play (DM). Finally, given that the profiles (DM, DM), (DM, M), and (M, M) have the same symmetric payoffs, the equilibria of the first stage are the profiles (DM, M) and (DM, DM). With ER1, only the profile (DM, DM) is an equilibrium.

Scenario 4.8: The asymmetric outcome is set in the profile (DM, M). The B-outcome is set in the profiles (M, DM) and (M, M). By symmetry with Scenario 4.7, the profiles (M, DM) and (DM, DM) are equilibria of the first stage. With ER1, only the profile (DM, DM) is an equilibrium.

Scenario 4.9: The asymmetric outcome is set in the profiles (M, DM) and (M, M). The B-outcome is set in the profile (DM, M). Player 1's payoff in the profile (M, M) is $\pi^{\diamond A}$. If player 1 plays (DM), player 2 is indifferent to playing either (DM) or (M). The dominant strategy for player 1 is (DM). Thus, the equilibria of the first stage are the profiles (DM, M) and (DM, DM). With ER1, only the profile (DM, DM) is an equilibrium.

Scenario 4.10: The asymmetric outcome is set in the profiles (DM, M) and (M, M). The B-outcome is set in the profile (M, DM). Player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. By symmetry with Scenario 4.9, the profiles (M, DM) and (DM, DM) are equilibria of the first stage. With ER1, only the profile (DM, DM) is an equilibrium.

Scenario 4.11: The asymmetric outcome is set in the profiles (DM, M) and (M, M). The B-outcome is set in the profile (M, DM). Player 1's payoff in the profile (M, M) is $\pi^{\diamond A}$. If player 1 plays (DM), the best strategy for player 2 is to play (DM). If player 1 plays (M), the best strategy for player 2 is to play (M). If player 2 plays (M), the best strategy for player 1 is to play (DM). Finally, when player 2 plays (DM), player 1 is indifferent to playing either (DM) or (M). Thus, the only equilibrium of the first stage is the profile (DM, DM).

Scenario 4.12: The asymmetric outcome is set in the profiles (M, DM) and (M, M). The B-outcome is set in the profile (DM, M). Player 1's payoff in the profile (M, M) is $\pi^{\diamond B}$. By symmetry with Scenario 4.11, the only equilibrium of the first stage is the profile (DM, DM).

Therefore, when $\frac{D_A}{D_B} \leq \hat{\theta}_B$, any equilibrium of the first stage corresponds to the B-outcome. Adding ER1, neither team merges in equilibrium.

Appendix 2.3: Conglomerate discount

It follows that $\pi^A \leq \pi^B$ if and only if $\frac{D_A}{D_B} \geq \frac{9-4\beta^2}{5\beta^2}$.

Appendix 2.4: Social welfare

This appendix defines the total producer surplus in the A-outcome, B-outcome, and asymmetric outcome as $TPS^A = 2\pi^A$, $TPS^B = 2\pi^B$, and $TPS^{\diamond} = \pi^{\diamond A} + \pi^{\diamond B}$, respectively. Comparing the total producer surplus of the asymmetric outcomes with the symmetric ones, it follows that $TPS^{\diamond} > TPS^B$ and $TPS^{\diamond} > TPS^A$.

The consumer surplus in market k can be computed by

$$CS_k = v_k(q^*_{k1}, q^*_{k2}) - p_{k1}(q^*_{k1}, q^*_{k2}) \cdot q^*_{k1} - p_{k2}(q^*_{k1}, q^*_{k2}) \cdot q^*_{k2} = \frac{1}{2}(q^*_{k1} + q^*_{k2})^2,$$

where q^*_{ki} is the equilibrium quantity of firm i in market k. The second equality follows from the fact that, in equilibrium, $p_{k1}(q^*_{k1}, q^*_{k2}) = p_{k2}(q^*_{k1}, q^*_{k2})$ holds.

This appendix defines the total consumer surplus in the A-outcome, B-outcome, and asymmetric outcome as $TCS^A = 2(q^A_{Ai})^2 + 2(q^A_{Bi})^2$, $TCS^B = 2(q^B_{Ai})^2 + 2(q^B_{Bi})^2$, and

$TCS^{\diamondsuit} = \frac{1}{2}(q_A^{\diamondsuit A} + q_A^{\diamondsuit B})^2 + \frac{1}{2}(q_B^{\diamondsuit A} + q_B^{\diamondsuit B})^2$, respectively. It follows that $TCS^{\diamondsuit} \geq TCS^B$ if $\frac{D_A}{D_B} \geq \gamma_B$, $TCS^{\diamondsuit} \geq TCS^A$ if $\frac{D_A}{D_B} \leq \gamma_A$ and $\beta \neq \frac{\sqrt{3}}{2}$, and $TCS^{\diamondsuit} > TCS^A$ if $\beta = \frac{\sqrt{3}}{2}$, where

$$\gamma_A = \begin{cases} \frac{\sqrt{5}(9-8\beta^2)(9-4\beta^2)}{12\beta\sqrt{81-189\beta^2+120\beta^4-16\beta^6}} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{\sqrt{5}(9-4\beta^2)}{3\sqrt{(4\beta^2-3)(15-4\beta^2)}} & \text{if} \quad \beta > \frac{\sqrt{3}}{2} \end{cases}, \quad \gamma_B = \begin{cases} \frac{3\sqrt{11}(9-8\beta^2)}{20\beta\sqrt{9-7\beta^2}} & \text{if} \quad \beta < \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{11}}{5\sqrt{5}} & \text{if} \quad \beta \geq \frac{\sqrt{3}}{2} \end{cases}.$$

The thresholds satisfy $\gamma_B < \gamma_A$ for $\beta \neq \frac{\sqrt{3}}{2}$. Thus, for any β, there exists a $\frac{D_A}{D_B}$ such that the asymmetric outcome is the market outcome with the greatest total consumer surplus. Moreover, it follows that $\hat{\theta}_B < \gamma_B$ and $\gamma_A < \hat{\theta}_A$ for any $\beta < \frac{\sqrt{3}}{2}$. Hence, when the asymmetric outcome is the market outcome with the greatest total consumer surplus, it is also an equilibrium outcome.

Finally, this appendix defines the total social welfare in the A-outcome, B-outcome, and asymmetric outcome as $TW^A = TPS^A + TCS^A$, $TW^B = TPS^B + TCS^B$ and $TW^{\diamondsuit} = TPS^{\diamondsuit} + TCS^{\diamondsuit}$. It follows that $TW^{\diamondsuit} > TW^B$. Further, $TW^{\diamondsuit} \geq TW^A$ if $\beta < \bar{\beta}$ and $\frac{D_A}{D_B} \leq \bar{\gamma}$ or $\beta \geq \bar{\beta}$, where

$$\bar{\gamma} = \frac{(9-8\beta^2)\sqrt{5(9-4\beta^2)}}{6\beta\sqrt{2(9-23\beta^2+12\beta^4)}}, \quad \bar{\beta} = \frac{\sqrt{23-\sqrt{97}}}{2\sqrt{6}}.$$

The thresholds satisfy $\bar{\gamma} > \hat{\theta}_A$ for any $\beta \in \left(0, \bar{\beta}\right)$. Thus, whenever the asymmetric outcome is an equilibrium outcome, it is the social welfare dominating outcome.

Appendix 3: Second-best contract

Taking the sum of (3.12) and (3.13) yields $2\left(q_{N1}^H - q_{N1}^L\right)\left(q_{N1}^{HU} - q_{N1}^{LU}\right) \geq 0$. Because $q_{N1}^{HU} > q_{N1}^{LU}$, satisfying both (3.12) and (3.13) requires $q_{N1}^H > q_{N1}^L$.

In the second-best contract at least one IC constraint is binding. When $q_{N1}^{H} = q_{N1}^{L}$ both (3.12) and (3.13) are binding. From this we get a pooling equilibrium candidate. When $q_{N1}^{H} > q_{N1}^{L}$, we prove that the optimal contract only binds (3.13) ($q_{N1}^{H} + q_{N1}^{L} = 2q_{N1}^{LU}$). Suppose otherwise, so that optimally $q_{N1}^{H} + q_{N1}^{L} > 2q_{N1}^{LU}$. Under $X < \hat{\Omega}$ it holds $q_{N1}^{HR} + q_{N1}^{LR} < 2q_{N1}^{LU}$. Then, $q_{N1}^{s} > q_{N1}^{sR}$ for some s. Hence, (3.11) in state s is binding, so it holds $q_{C1}^{s} < q_{C1}^{sR}$. Reducing q_{N1}^{s} and increasing q_{C1}^{s} increases the profit in state s, a contradiction. Thus, there is a separating equilibrium candidate where $q_{N1}^{H} > q_{N1}^{L}$ and $q_{N1}^{H}+q_{N1}^{L} = 2q_{N1}^{LU}$. We explore the pooling and separating equilibria candidates hereunder.

Pooling equilibrium candidate P

Assume $q_{N1}^{H} = q_{N1}^{L} = \overline{q}_{N}$. Clearly, this implies that $q_{C1}^{H} = q_{C1}^{L} = \overline{q}_{C}$. Hence, (3.11) is the same in any state. Suppose that (3.11) is not binding. Solving the problem yields $\overline{q}_{C} = q_{C1}^{U}$ and $\overline{q}_{N} = pq_{N1}^{HU} + (1-p)q_{N1}^{LU} > q_{N1}^{LU}$, so (3.11) does not hold, and thus, it must be binding.

By substituting $q_{N1}^{H} = q_{N1}^{L} = \overline{q}_{N}$ and (3.11) into (3.10), the simplified problem in terms of $\overline{q}_{N}$ is as follows:

$$\max_{\overline{q}_N \geq 0} \frac{2-\alpha^2}{2} \sum_{s\in\{H,L\}} p_s \left(\left(2q_{C1}^{U} - X + \overline{q}_N\right)\left(X - \overline{q}_N\right) + \left(2q_{N1}^{sU} - \overline{q}_N\right)\overline{q}_N \right) \tag{A.18}$$

From the FOC of (A.18), the solution candidate for market k is $\overline{q}_{k1}^{P}$. The outputs of the standalone firms in state s are q_{N2}^{sP} and $\overline{q}_{C2}^{P}$. The non-negativity conditions of q_{N2}^{sP} and $\overline{q}_{C2}^{P}$ will hold if (3.14) and (3.15) are satisfied.

Now, we verify if $\overline{q}_{N1}^{P}$ and $\overline{q}_{C1}^{P}$ satisfy the remaining restrictions of the problem. First, given (3.1), it holds that $\overline{q}_{N1}^{P} \geq 0$ and $\overline{q}_{C1}^{P} \geq 0$. Second, because $q_{C1}^{U} > q_{C1}^{LR} > q_{C1}^{HR}$, (3.14) is satisfied as $2q_{C1}^{U} \geq pq_{C1}^{HR} + (1-p)q_{C1}^{LR}$ holds. Third, when $X \leq \check{\Omega}$, it holds $q_{N1}^{LU} \geq q_{N1}^{HR} > q_{N1}^{LR}$. Thus, (3.15) in the low state is satisfied as $2q_{N1}^{LU} \geq pq_{N1}^{HR} + (1-p)q_{N1}^{LR}$ holds. Fourth, (3.15) in the high state also holds as $q_{N1}^{HU} > q_{N1}^{LU}$.

Separating equilibrium candidate S

Assume $q_{N1}^{H} > q_{N1}^{L}$ and $q_{N1}^{H} + q_{N1}^{L} = 2q_{N1}^{LU}$. Suppose that (3.11) in the high state is not binding, thus $q_{C1}^{H} = q_{C1}^{U}$. Moreover, $q_{N1}^{H} > q_{N1}^{L}$ and $q_{N1}^{H} + q_{N1}^{L} = 2q_{N1}^{LU}$ imply that $q_{N1}^{H} > q_{N1}^{LU}$. Therefore, (3.11) in the high state does not hold, and hence, it must be binding.

Suppose that (3.11) in the low state is not binding. We substitute $q_{N1}^{H} + q_{N1}^{L} = 2q_{N1}^{LU}$ and (3.11) in the high state into (3.10) to obtain a problem only in terms of q_{N1}^{L} and q_{C1}^{L}. The simplified problem yields the following:

$$q_{N1}^{L} = \frac{(2-\alpha)(p\left(D_C + 3D_N^L - D_N^H\right) + D_N^L) - 2p(2-\alpha^2)X}{2(2-\alpha^2)(p+1)}, \quad q_{C1}^{L} = q_{C1}^{U}$$

If (3.11) in the low state is not binding, $q_{N1}^{L} + q_{C1}^{L} \leq X$ must hold. That condition is equivalent to:

$$X \geq \frac{(2-\alpha)((2p+1)D_C + (3p+1)D_N^L - pD_N^H)}{2(2-\alpha^2)(2p+1)}$$

which never holds when $X < \hat{\Omega}$. Therefore, (3.11) in the low state must be binding.

Substituting (3.11) in both states and $q_{N1}^{H} + q_{N1}^{L} = 2q_{N1}^{LU}$ into (3.10), the simplified problem in terms of q_{N1}^{H} is as follows:

$$\max_{q_{N1}^{H}\geq 0} \quad \frac{2-\alpha^2}{2}\Big[p\left(\left(2q_{C1}^{U}-X+q_{N1}^{H}\right)\left(X-q_{N1}^{H}\right)+\left(2q_{N1}^{HU}-q_{N1}^{H}\right)q_{N1}^{H}\right)\ldots$$
$$+(1-p)\left(\left(2q_{C1}^{U}-X+2q_{N1}^{LU}-q_{N1}^{H}\right)\left(X-2q_{N1}^{LU}+q_{N1}^{H}\right)+\left(2q_{N1}^{LU}-q_{N1}^{H}\right)q_{N1}^{H}\right)\Big] \tag{A.19}$$

From the FOC of (A.19), it follows that the solution candidate in market k and state s is q_{k1}^{sS}. The output of the standalone firms in market k and state s is q_{k2}^{sS}. Again, $q_{k2}^{sS}\geq 0$ will hold if (3.14) and (3.15) are satisfied.

Now, we verify if this candidate for the solution satisfies the remaining restrictions of the problem. We start verifying (3.12), which is equivalent to verify $q_{N1}^{HS}>q_{N1}^{LS}$. The previous inequality holds if and only if:

$$p<\frac{(2-\alpha)(D_N^L+D_C)-2(2-\alpha^2)X}{4(2-\alpha^2)\theta^S} \tag{A.20}$$

When $X\geq\check{\Omega}$, the right side of (A.20) is greater or equal than 1, so it always holds. Now we verify the non-negativity constraints. Given that $q_{N1}^{HS}>q_{N1}^{LS}$, then $q_{C1}^{HS}<q_{C1}^{LS}$ because (3.11) is binding in any state. Thus, proving that $q_{N1}^{LS}\geq 0$ and $q_{C1}^{HS}\geq 0$ suffices to verify the non-negativity constraints. Given (3.1), it follows that $q_{N1}^{LS}\geq 0$. Furthermore, $q_{C1}^{HS}\geq 0$ holds if and only if:

$$p\geq\frac{(2-\alpha)(3D_N^L+D_C)-6(2-\alpha^2)X}{4(2-\alpha^2)\theta^S} \tag{A.21}$$

When $X\geq\check{\Omega}$ and with (3.2), the right side of (A.21) is lower or equal than 0, so it always holds. Now, we verify (3.15). Because $q_{N1}^{HS}>q_{N1}^{LS}>0$ and $q_{N1}^{HS}+q_{N1}^{LS}=2q_{N1}^{LU}$, it follows $2q_{N1}^{LU}>q_{N1}^{HS}>q_{N1}^{LS}$. Thus, (3.15) is satisfied in any state. Finally, we verify (3.14). In the low state, (3.14) holds if, and only if:

$$p \geq \frac{2(2-\alpha^2)X - (2-\alpha)(D_N^L + 3D_C)}{4(2-\alpha^2)\theta^S} \tag{A.22}$$

As $X < \hat{\Omega}$, the right side of (A.22) is negative, so it always holds. Given that (3.14) in the low state is satisfied, (3.14) in the high state also holds as $q_{C1}^{LS} > q_{C1}^{HS}$.

The solution

The ex-ante expected profit of the conglomerate in equilibrium P is:

$$E\pi^P = \frac{2-\alpha^2}{2} \sum_{s\in\{H,L\}} p_s \left(\left(2q_{C1}^U - \overline{q}_{C1}^P\right) \overline{q}_{C1}^P + \left(2q_{N1}^{sU} - \overline{q}_{N1}^P\right) \overline{q}_{N1}^P \right)$$

The ex-ante expected profit of the conglomerate in equilibrium S is:

$$E\pi^S = \frac{2-\alpha^2}{2} \sum_{s\in\{H,L\}} p_s \left(\left(2q_{C1}^U - q_{C1}^{sS}\right) q_{C1}^{sS} + \left(2q_{N1}^{sU} - q_{N1}^{sS}\right) q_{N1}^{sS} \right)$$

It follows that $E\pi^S \geq E\pi^P$ if and only if:

$$X \geq \frac{(2-\alpha)}{2(2-\alpha^2)} \left(D_C + 2D_N^L - D_N^H\right) = \check{\Omega}$$

Given (3.2), it follows $\check{\Omega} > 0$. Moreover, because $\check{\Omega} < \hat{\Omega}$, then there exists an X such that $E\pi^S \geq E\pi^P$. Thus, in the second-best contract the equilibrium is S when $\hat{\Omega} > X \geq \check{\Omega}$, and it is P when $X \leq \check{\Omega}$.

Appendix 4.1: Implementation of the collocation method

We utilize a cubic spline approximation for every configuration of the model. Spline approximations are usually preferred to polynomial approximations when the function is non-smooth. In our model, the value function is non-smooth due to our discrete-action scheme. In particular, the discrete nature of merging-separating dynamic creates two regions in the state space where the value function behaves differently. We present the number of univariate basis functions and collocation nodes for each configuration of the model in Table A.4.

Configuration / State	$B1$	$B2$	$B3$	$F1$	$F2$
Capital stock	90	90	90	90	90
Demand of the new market	-	45	45	45	35

Table A.4: Number of basis functions and collocation nodes

To verify whether our approximation is accurate to an acceptable level we check the graph of the Bellman residuals. The residual is a function of the states variables. The residual functions of $B1$ and $B2$ are univariate and bivariate, respectively, thus they can be plotted normally. The residual functions of $B3$, $F1$ and $F2$ are trivariate. We present those residuals using two graphs, one for the merger state and other for the separate state.

We show the residuals of $B1$ and $B2$ in Figures A.5 and A.6, respectively. We display the residuals for the separate state of $B3$, $F1$ and $F2$ in Figures A.7, A.9 and A.11, respectively. We present the residuals for the merger state of $B3$, $F1$ and $F2$ in Figures A.8, A.10 and A.12, respectively.

For every configuration of the model, the residuals are very close to zero in the interior region. However, there are disturbances in the boundaries of the state transition

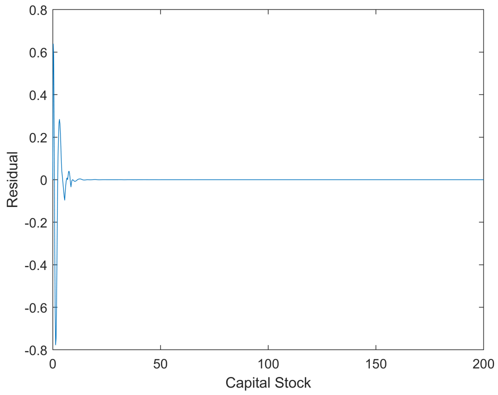

Figure A.5: Approximation Residual-$B1$

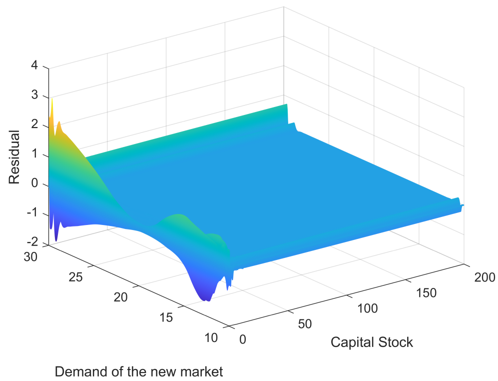

Figure A.6: Approximation Residual-$B2$

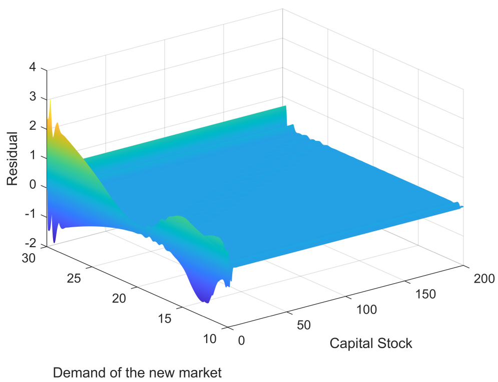

Figure A.7: Approximation Residual-Separate-$B3$

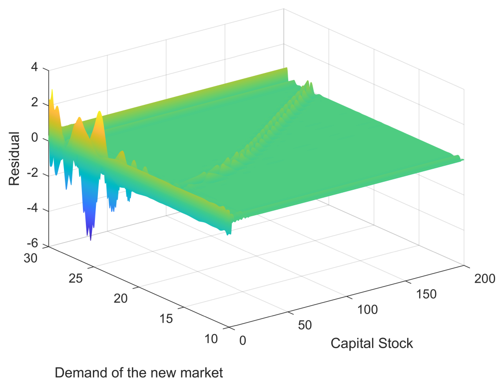

Figure A.8: Approximation Residual-Merger-$B3$

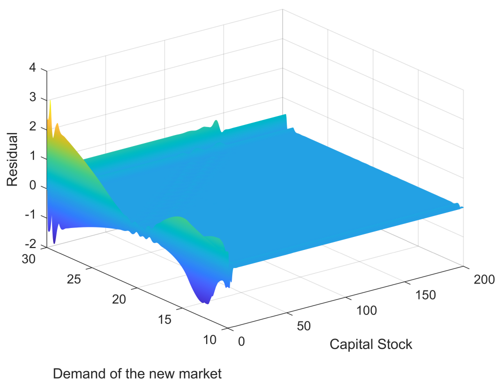

Figure A.9: Approximation Residual-Separate-$F1$

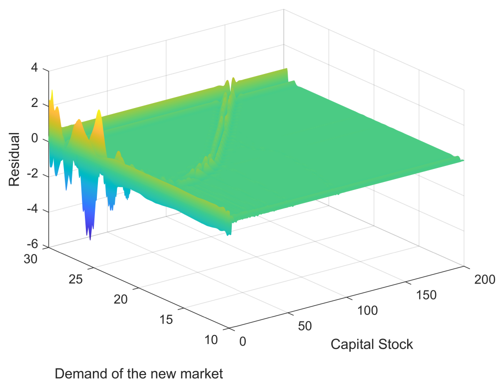

Figure A.10: Approximation Residual-Merger-$F1$

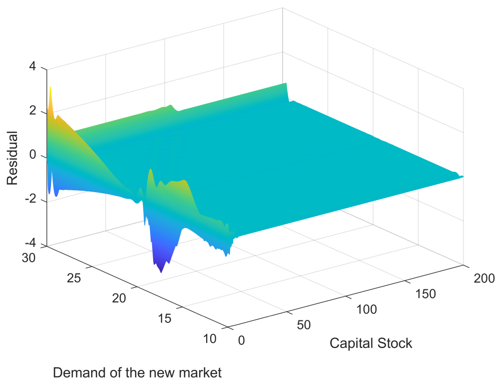

Figure A.11: Approximation Residual-Separate-$F2$

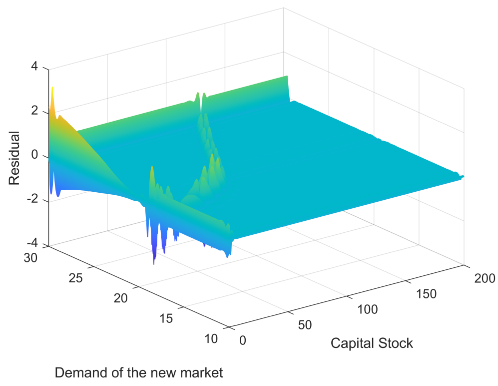

Figure A.12: Approximation Residual-Merger-$F2$

functions: the lower bound of the capital stock, and the upper and lower bound of the demand of the new market. Those disturbances do not surpass 5 in absolute value. The residuals might be lowered if the number of basis functions and collocation nodes is increased. Unfortunately, doing that would greatly increase the computational effort to run the model.

Appendix 4.2: MATLAB code (Configuration $F2$)

Main code (`Model.m`):

```
% PARAMETERS

dc=20;          %Demand core market
buc=10;         %Upper cost core market
bdc=1;          %Down cost core market
bun=10;         %Upper cost new market
bdn=1;          %Down cost new market
Kmin=0;         %Minimum capital stock
Kmax=200;       %Maximum capital stock
Ko=55;          %Capital stock standalone external firm
Pm=4;           %Price per unit of capital (Merger)
Ps=4;           %Price per unit of capital (Separation)
Pk=3;           %Price por unit of capital (Capital market)
KSmin=0;        %Minimum Proportion capital sold (separation)
KSmax=10;       %Maximum Proportion capital sold (separation)
KBmin=0;        %Minimum capital bought (Capital market)
KBmax=20;       %Maximum capital bought (Capital market)
psi= 0.3;       %Depreciation rate
delta= 0.9;     %Discount factor
Dmin= 10 ;      %Minimum demand external market
```

```
Dmax=30 ;       %Maximum demand external market
FB=200;         %Constant merging
FS=300;         %Constant separating


% DEFINE APPROXIMATION SPACE

  n      = [90 35];                                      % Degree of approximation
  smin   = [Kmin Dmin];                                  % Minimum state
  smax   = [Kmax Dmax];                                  % Maximum state
  fspace = fundefn('spli',n,smin,smax,[],[0;1]);         % Approximation space
  scoord = funnode(fspace);                                % State collocation grid coordinates
  snodes = gridmake(scoord);                             % Grid state

% CONSTRUCT ACTION SPACE

  x = {[0;1] [KBmin:KBmax]' 0.1*[KSmin:KSmax]'};      % Action space
  xgrid= gridmake(x) ;                                % Grid action

  % COMPUTE SHOCK DISTRIBUTION

  sigma= 0.25;                          % Covariance matrix
  mn=  0 ;                              % Mean
  sh =   4;                             % Number of shocks
  [e,w] = qnwnorm(sh,mn,sigma);         % Normal nodes and probabilities

% PACK MODEL STRUCTURE

  clear model
  model.func = 'Function';                                             % Model functions
  model.discount = delta;                                              % Discount factor
  model.e = e;                                                         % Shocks
  model.w = w;                                                         % Probabilities
  model.actions = xgrid;                                               % Model actions
  model.discretestates = 3;                                            % Index of discrete state
  model.params = {dc,buc,bdc,bun,bdn,Pk,psi,Pm,Ps,Kmax,Dmax,FB,FS,Ko,Dmin}; % Other parameters

% CALL SOLVER
  [c,s,v,x,resid] = dpsolve(model,fspace,snodes);   % Solve Bellman equation

% OPTIMAL ACTIONS

 x1=x(:,:,:,1); % Merging-separating decision
 x2=x(:,:,:,2); % Capital  bought decision
 x3=x(:,:,:,3); % Capital  sold decision

% RESIDUALS

 resid1=resid(:,:,1);  % Residuals (Separate state)
 resid2=resid(:,:,2);  % Residuals (Merger state)
```

```
% SIMULATION (ONE)

nyrs = 100;                                  % Number of periods
ss=[50 20 0];                                % Initial states
[ssim,xsim] = dpsimul(model,ss,nyrs,s,x);    % Simulation
s1path = squeeze(ssim(:,1,:));               % Simulated capital stock state
s2path = squeeze(ssim(:,2,:));               % Simulated demand external market state
s3path = squeeze(ssim(:,3,:));               % Simulated merging-separating state
x1path = squeeze(xsim(:,1,:));               % Simulated optimal merging-separating decision
x2path = squeeze(xsim(:,2,:));               % Simulated optimal capital bought decision
x3path = squeeze(xsim(:,3,:));               % Simulated optimal capital sold decision

% PLOT SIMULATION

 figure(1);
 plot(0:nyrs,s1path);
 title('Capital Stock');
 xlabel('Period');
 ylabel('Capital Stock');
 figure(2);
 plot(0:nyrs,s2path);
 title('Demand of the external market');
 xlabel('Period');
 ylabel('Demand of the external market');
 figure(3);
 plot(0:nyrs,s3path);
 title('Merging-Separating state');
 xlabel('Period');
 ylabel('Merging-Separating state');
 figure(4);
 plot(0:nyrs,x1path);
 title('Merging-Separating decision');
 xlabel('Period');
 ylabel('Merging-Separating decision');
 figure(5);
 plot(0:nyrs,x2path);
 title('Capital bought externally');
 xlabel('Period');
 ylabel('Capital bought externally');

 % SIMULATION (TEN)

rept=10;                                       % Number of simulations
ssm=ss.*ones(rept,3);                          % Initial states
rng('default')
god = rng
[ssimm,xsimm] = dpsimul(model,ssm,nyrs,s,x);   % Simulation
s1pathm = squeeze(ssimm(:,1,:));               % Simulated capital stock state
s2pathm = squeeze(ssimm(:,2,:));               % Simulated demand external market state
```

```
s3pathm = squeeze(ssimm(:,3,:));              % Simulated merging−separating state
x1pathm = squeeze(xsimm(:,1,:));              % Simulated optimal merging−separating decision
x2pathm = squeeze(xsimm(:,2,:));              % Simulated optimal capital bought decision
x3pathm = squeeze(xsimm(:,3,:));              % Simulated optimal capital sold decision

% PLOT EXPECTED STATE PATH

 figure(6);
 surface(0:nyrs,1:rept,s1pathm);
 %title('Capital Stock');
 xlabel('Period');
 ylabel('Simulation No.');
 h = colorbar;
 f1 = gcf;
 exportgraphics(f1,'Capital StockGA.png','Resolution',600)
 figure(7);
 surface(0:nyrs,1:rept,s2pathm);
 %title('Demand of the external market');
 xlabel('Period');
 ylabel('Simulation No.');
 h = colorbar;
 f2 = gcf;
 exportgraphics(f2,'Demand of the external marketGA.png','Resolution',600)
 figure(8);
 surface(0:nyrs,1:rept,s3pathm);
 %title('Merging−Separating state');
 xlabel('Period');
 ylabel('Simulation No.');
 h = colorbar;
 f3 = gcf;
 exportgraphics(f3,'Merging−Separating stateGA.png','Resolution',600)
 figure(9);
 surface(0:nyrs,1:rept,x1pathm);
 xlabel('Period')
 ylabel('Simulation No.')
 %title('Merging−Separating decision')
 h = colorbar;
 h.Limits = [0 1];
 f4 = gcf;
 exportgraphics(f4,'Merging−Separating decisionGA.png','Resolution',600)
 figure(10);
 surface(0:nyrs,1:rept,x2pathm);
 %title('Capital bought externally');
 xlabel('Period');
 ylabel('Simulation No.');
 h = colorbar;
 f5 = gcf;
 exportgraphics(f5,'Capital bought externallyGA.png','Resolution',600)
 figure(11);
 surface(0:nyrs,1:rept,x3pathm);
```

```
% title('Capital sold');
xlabel('Period');
ylabel('Simulation No.');
h = colorbar;
f6 = gcf;
exportgraphics(f6,'Capital soldGA.png','Resolution',600)
% PLOT RESIDUAL
figure(12)
hh=surf(s{1},s{2},resid1(:,:,1)');
% title('Approximation Residual: No merger');
xlabel('Capital Stock'); ylabel('Demand of the new market');
zlabel('Residual');
set(hh,'FaceColor','interp','EdgeColor','interp')
f7 = gcf;
exportgraphics(f7,'Approximation Residual No mergerGA.png','Resolution',600)
figure(13)
hh=surf(s{1},s{2},resid2(:,:,1)');
% title('Approximation Residual: Merger');
xlabel('Capital Stock'); ylabel('Demand of the new market');
zlabel('Residual');
set(hh,'FaceColor','interp','EdgeColor','interp')
f8 = gcf;
exportgraphics(f8,'Approximation Residual MergerGA.png','Resolution',600)
```

Auxiliary code (`Function.m`):

```
function out = Function(flag,s,x,e,dc,buc,bdc,bun,bdn,Pk,psi,Pm,Ps,Kmax,Dmax,FB,FS,Ko,Dmin)
switch flag
case 'f'; % REWARD FUNCTION
  %Second stage
 F=find(s(:,1)== max(s(:,1)) & s(:,2)==max(s(:,2)));
for i = 1:1:F(1,1)
fun = @(y) -(dc./2-buc./(2.*(bdc+y))).^2 - (s(i,2)./2-bun./(2.*(bdn+s(i,1)-y))).^2;
opts = optimoptions(@fmincon,'Algorithm','sqp');
problem = createOptimProblem('fmincon','objective',fun,'x0',0,'lb',0,'ub',s(i,1),'options',opts);
gs = GlobalSearch('Display','off');
y = run(gs,problem);
I(i,1)=y;
end
F1=size(s,1)/F(1,1)-1;
A=I;
for i = 1:F1
I=[I;A];
  end
  out =(1-x(:,1)).*((dc./2-buc./(2.*(bdc+s(:,1)))).^2) ...                %
     No merger
      +x(:,1).*((dc./2-buc./(2.*(bdc+I))).^2 + (s(:,2)./2-bun./(2.*(bdn+s(:,1)-I))).^2) ...     %
          Merger market
      -Pk*x(:,2)...                                                            %
          Cost of buying capital
```

```
        -(1-s(:,3)).*x(:,1).*(FB+Ko.*Pm) ...                                    % Cost when merging with market
        +s(:,3).*(1-x(:,1)).*(FS+x(:,3).*(1-psi).*s(:,1).*Ps) ;                % Income for selling market
case 'g'; % STATE TRANSITION FUNCTION
    out(:,1) = s(:,3).*x(:,1).*(1-psi).*s(:,1)+(1-s(:,3)).*(1-psi).*s(:,1) ...
        +x(:,2)+(1-s(:,3)).*x(:,1).*Ko+s(:,3).*(1-x(:,1)).*(1-x(:,3)).*(1-psi).*s(:,1);   % Capital stock
    out(:,2)=[ min([[max([s(:,2)+e Dmin*ones(size(s(:,2)))]')]' Dmax*ones(size(s(:,2)))]')]' ;   % Demand new market state
    out(:,3) = x(:,1);                                                         % Merging-separating state
end
```