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Steady State Analysis of Underground Electromagnetic Field Generated by Dipole Located over Ground

By

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Abstract

This paper presents the steady state analysis of an underground electromagnetic field induced by a sinusoidally excited dipole located over the ground. Theoretical derivation of the electromagnetic field is carried out in the complex frequency domain. The numerical method to calculate the obtained equation which contains complicated infinite integrals is presented.

Introduction

The lightning discharge effect is a radiation phenomenon of the electromagnetic field and there the electromagnetic field generated by discharge current propagates into space. This electromagnetic field propagates only into free space when the conductivity of the ground is assumed to be infinite, but if not the field propagates also into the ground.

If a transmission line such as a communication network is constructed near the lightning point, the electromagnetic field operates as a distributed source which induces surge voltage on the transmission line. When the peak value of the surge voltage exceeds the voltageproof of the communication equipment connected to the line, the equipment may brake down. Recently communication equipment has been made more compact, and voltage-resisting qualities have decreased, so effective prevention measures against surge voltage are demanded. For that reason a precise method of analysis of the electromagnetic field which generates the surge voltag is demanded.

To evaluate the electromagnetic field at the observation point, we calculate the fields generated by infinitely small electrical dipoles located on the lightning channel and integrate them along the channel[1].

When the observation point is located in free space, the method of analysing a

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steady state or transient state response of the electromagnetic field by lightning discharge has already been reported[(2),(3),(4)]. But under the ground, the propagation mode of the electromagnetic field is influenced by the conductivity of the ground and differs from that in free space.

For a current wave form with a very short duration time such as lightning discharge current, its frequency components range very widely. In this paper, we will consider the linear response of the electromagnetic field, and it may be enough to evaluate the propagation property for every considered frequency component. So, we will show a method to calculate the steady state response of the electromagnetic field in the case where the observation point is located under the ground. Since we are considering the steady state response, an electrical dipole is assumed to be excited by a sinusoidal current.

2. Underground electromagnetic field generated by dipole located over the ground

We consider the two domains separated by the boundary plane of $z=0$ as shown in Fig. 1. The wave source is a dipole of length dz and current I located on the z-axis at $z=z_0$. The electromagnetic field generated by this dipole has axial symmetry, so we analyse the field by cylindrical coordinates (r,ϕ,z). The Hertz vector generated by this dipole has only a z-component that is given by

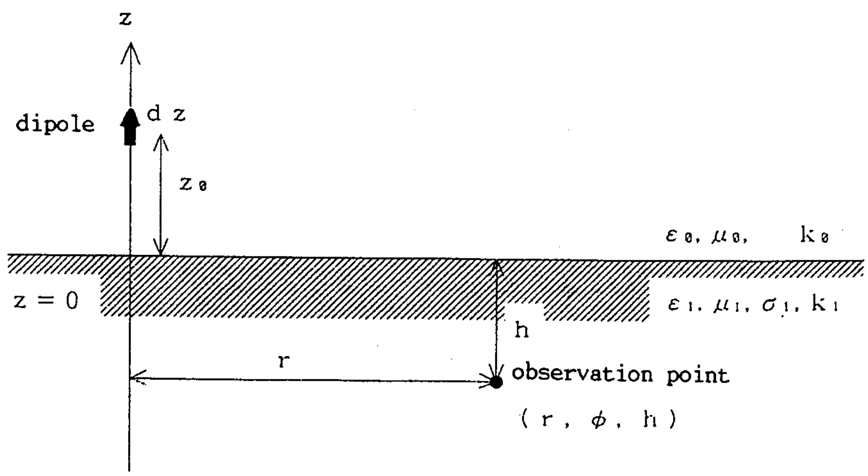

Fig. 1 Observation point and position of dipole

$$d\Pi_0 = -\frac{Idz}{4\pi\varepsilon_0 s}\int_0^\infty \left[\frac{u\exp(-|z-z_0|\xi_0)}{\xi_0} + R(u)\exp(-z\xi_0)\right] J_0(ur)du \quad z>0 \quad (1)$$

in free space and

$$d\Pi_1 = -\frac{Idz}{4\pi\varepsilon_0 s}\int_0^\infty T(u)\exp(z\xi_1)J_0(ur)du \quad z<0 \quad (2)$$

under the ground. Here s is the Laplace operator, J_0 is the 0th order Bessel function of the 1st kind, $R(u)$ and $T(u)$ are the reflection and refraction coefficients determined by the boundary conditions.

Other quantities are

$$\left.\begin{aligned} k_0^2 &= -\varepsilon_0\mu_0 s^2 \\ k_1^2 &= -\mu_1 s(\sigma_1+\varepsilon_1 s) \\ \xi_0 &= (u^2-k_0^2)^{1/2} \\ \xi_1 &= (u^2-k_1^2)^{1/2} \end{aligned}\right\} \quad (3)$$

where subindexes 0 and 1 denote the quantities in free space and underground. ε, μ, σ are permittivity, permiability and conductivity of the medium.

$d\Pi_0$ and $d\Pi_1$ are both independent on the ϕ-axis and the horizontal components of the electrical field and flux density are continuous at $z=0$, so the following relations hold true.

$$\frac{\partial(d\Pi_0)}{\partial z} = \frac{\partial(d\Pi_1)}{\partial z} \quad z=0 \quad (4)$$

$$k_0^2(d\Pi_0) = k_1^2(d\Pi_1) \quad z=0 \quad (5)$$

Using these relations, $R(u)$ and $T(u)$ become as follows:

$$R(u) = \frac{u}{\xi_0}\left[1 - \frac{2k_0^2\xi_1}{k_1^2\xi_0 + k_0^2\xi_1}\right]\exp(-z_0\xi_0) \quad (6)$$

$$T(u) = \frac{2k_0^2 u}{k_1^2\xi_0 + k_0^2\xi_1}\exp(-z_0\xi_0) \quad (7)$$

As shown in Fig. 1 the observation point is located under the ground, so we replace the axis of coordinate z by h. By these results the Hertz vector at (r,ϕ,h)

becomes as follows:

$$d\Pi_1 = -\frac{Idz}{4\pi\varepsilon_0 s}\int_0^{\infty}\frac{2k_0^2 u}{k_1^2\xi_0 + k_0^2\xi_1}\exp(-z_0\xi_0 + h\xi_1)J_0(ur)du \tag{8}$$

Applying vector analysis, the horizontal electrical field dE_r, the vertical electrical field dE_h, and the ϕ component of flux density dB_φ become as follows:

$$dE_r = -\frac{Idz}{4\pi\varepsilon_0 s}\int_0^{\infty}\frac{2k_0^2\xi_1 u^2}{k_1^2\xi_0 + k_0^2\xi_1}\exp(-z_0\xi_0 + h\xi_1)J_1(ur)du \tag{9}$$

$$dE_h = -\frac{Idz}{4\pi\varepsilon_0 s}\int_0^{\infty}\frac{2k_0^2 u^3}{k_1^2\xi_0 + k_0^2\xi_1}\exp(-z_0\xi_0 + h\xi_1)J_0(ur)du \tag{10}$$

$$dE_\varphi = -\frac{Idz}{4\pi\varepsilon_0 s}\int_0^{\infty}\frac{2k_0^2 k_1^2 u^2}{k_1^2\xi_0 + k_0^2\xi_1}\exp(-z_0\xi_0 + h\xi_1)J_1(ur)du \tag{11}$$

Here J_1 is the 1st order Bessel function of the 1st kind.

3. Steady state analysis of underground electromagnetic field

3.1 Current source and integrand

In analysing the lightning discharge effect, the electrical dipoles distribute continuously along the lightning channel and the frequency components of the discharge current exist over a wide range.

In this case we assume that a current source is given as follows:

$$\left.\begin{array}{l} I(z_0,\omega) = I_0\exp(i\omega t) \\ I_0\text{:peak value of current source} \\ \omega\text{:frequency of current source} \\ z_0\text{:location of dipole} \end{array}\right\} \tag{12}$$

Electromagnetic fields generated by this dipole have a common time factor $\exp(i\omega t)$, so we abbreviate this time factor and treat only phasor components. To do so, we substitute $s = i\omega = i2\pi f$ in equations (9), (10) and (11).

Since an analytical procedure to obtain the infinite integral contained in every equation is impossible, we use the numerical integration method.

The integrand contained in every infinite integral (we call it u integral below) of equations (9), (10) and (11) has a sharp peak caused by the term $1/(k_1^2\xi_0+k_0^2\xi_1)$ and heavy oscillations created by Bessel function and the integration mentioned above is very complicated. In order to evaluate the numerical integration we divide the region of integration into three subregions. The origin of the method is shown in reference (6).

In the following section, we will show a modified method to evaluate u integrals.

3.2 Evaluation of u integral

When evaluating the u integral contained in equations (9), (10) and (11), we use Simpson's numerical integration formula. This formula is given by

$$\int_{u_0}^{u_N} f(u)du \fallingdotseq \frac{\Delta}{3}\,(f_0+4f_1+2f_2+4f_3+\cdots+2f_{N-2}+4f_{N-1}+f_N) \tag{13}$$

where f_i's, $i=0,1,2,\cdots,N$ are values of integrand at the points

$$\left.\begin{array}{l} u_i=u_0+i\Delta \quad i=0,1,2,\cdots,N \\ \Delta=(u_N-u_0)/N \\ N\text{: even number} \end{array}\right\} \tag{14}$$

selected at equal distances along the u axis.

Variation of the integrand depends heavily on the position of dipole z_0, frequency of current source f, and observation point r,h, and it is important to determine a suitable division number N according to the subregions where numerical integration must be done. Determination of this number N and evaluation of the integral are done according to the following scheme.

① First, select $N=100$ and find out the integrated value and set it to Y.
② Take $N \leftarrow 2N$ and obtain integrated value Y'.
③ If $|Y-Y'|/|Y|<d$ (d: allowed error) or $N>N_0$ (N_0: maximum division number) holds true go to ④, otherwise take $Y \leftarrow Y'$ and go to ②.
④ Select $(16Y-Y')/15$ as the integrated value.

3.3 Calculation of Bessel function

In evaluating the u integral we must calculate the values of the integrand which contains the Bessel function at more than a thousand points selected by equation (14) or excution time in computation becomes too long. In order to make the calculation faster we try to use the vector processor. In this case the choice of the method to calculate the values of Bessel functions J_0 and J_1 arises. These functions are given as follows.

$$J_0(x)=\sum_{k=0}^{\infty}\frac{(-1)^k(x/2)^{2k}}{(k!)^2} \tag{15}$$

$$J_1(x)=\sum_{k=0}^{\infty}\frac{(-1)^k(x/2)^{2k+1}}{k!(k+1)!} \tag{16}$$

In general they are calculated using scientific subroutine library, but it is impossible to use this method when calculation is done by vector processor. So, we show how to develop a practical calculation method.

Convergence of the integrand with respect to u is due mainly to the exponential term $\exp(-z_0\xi_0+h\xi_1)$. In the considered region we can conclude $u\gg|k_0|,|k_1|$ and the exponential term is approximated by

$$f(u)=\exp[-(z_0+h)u]. \tag{17}$$

A value u_p is given for $f(u_p)$ which is sufficiently small so that it does not affect the integrated result and integration must be done in the interval $0\leqq u\leqq u_p$. For this value u_p we will select 40000 points at equal intervals in $0\leqq ur\leqq u_p r$. The values of functions $J_0(ur)$ and $J_1(ur)$ at these points are stored as a numerical table. The value of functions at arbitrary points is obtained by linear interpolation.

4. Numerical examples

Fixed values of parameters used in calculation are as follows:

constants of the earth

σ_1:0.01S/m

μ_1:$4\pi\times10^{-7}H/m$ $(=\mu_0)$

ε_r:10 $(=\varepsilon_1/\varepsilon_0)$

calculation of u integral

d:0.1

N_0:5000

dipole moment

Idz:1.0$A\cdot m$

Other values are listed at every example.

4.1 Examples obtained by changing position of dipole

In this calculation the observation point is fixed at $r=100$ m and $h=5$ m.

Fig. 2 shows the variations of horizontal components of the electrical field to the position of dipole z at frequencies 10 kHz and 100 kHz. From these results, the dependence of the field on frequency becomes strong as the frequency increases. The de-pendence on the position of the dipole appears the same at both frequencies. The effect appears the same for vertical electrical field or flux density.

4.2 Examples obtained by changing frequency of dipole

In this calculation the observation point is the same as in the previous case and the position of the dipole is fixed at $z_0=100$ m. Fig. 3 shows a variation of the horizontal electrical field to frequency f. Dependence on frequency becomes strong as frequency increases. The effect appears the same for a vertical electrical field or flux density.

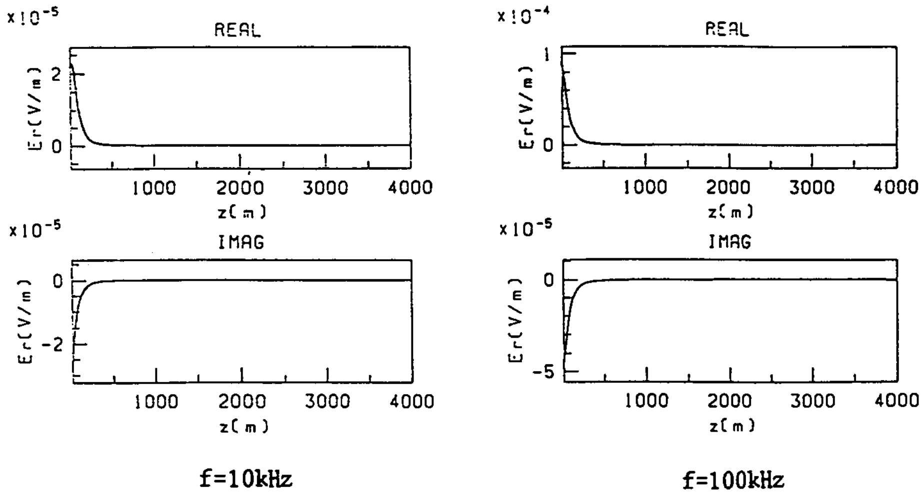

Fig. 2 Examples obtained by changing position of dipole

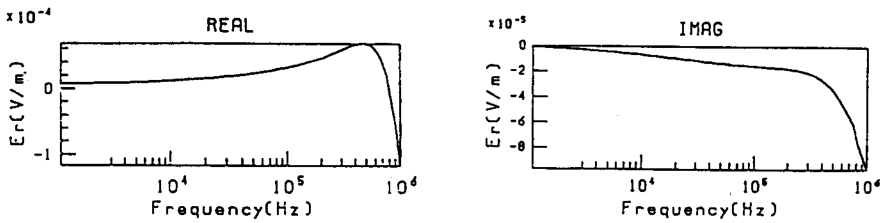

Fig. 3 Examples obtained by changing frequency of dipole

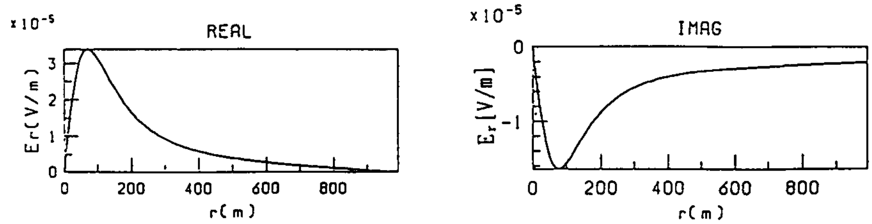

Fig. 4 Examples obtained by changing position of observation point

4.3 Examples obtained by changing position of observation point

Fig. 4 shows the variation of the horizontal electrical field to distance r where $z_0=100$ m, $h=5$ m and $f=100$ kHz. The effect is strongest nearby $r=40$ m and becomes weak as the distance increases. The effect appears the same for other fields.

5. Concluding remarks

In this paper we gave a method to analyse a steady state response of an underground electromagnetic field generated by a sinusoidally excited electrical dipole located over the ground. An equation to express every electromagnetic field presents a very complicated function with many parameters. The problem regarding the numerical integration method occupies an important position and the research of this paper aims at presenting a general method which can be used in any situation.

We gave our general consideration based on the analysis of numerical values. From the results obtained, the effect of the electrical dipole for every electromagnetic field is summarized as follows:

When the position of the dipole lies near the ground surface, the strength of the induced field decreases concentrically as the distance from the dipole increases, regardless of frequency. But as the position of the dipole becomes higher, the effect of frequency appears and attenuation by frequency becomes lower as frequency increases.

Further research topics will include analysis of transient electromagnetic fields using the results obtained.

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