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# Seismic Damage Assessment of RC Structures using Different Hysteretic Models

By

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and Satoru FUJISAWA\*

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## Abstract

Estimation of seismic damage of a structure varies depending on the assumed hysteretic rules and input excitations due to indices being calculated from earthquake response time histories. In this study, effects of the different hysteretic models on damage indices were studied. First, the response of RC bridge piers during earthquakes was calculated using different hysteretic models and input motions. Then, seismic damage was evaluated by 1) a damage index based on a linear combination of the maximum deformation ratio and the energy dissipation during cyclic loadings, and 2) damage spectra of damage index, ductility and absorbed hysteretic energy for structures with various natural periods. Results showed that the non-degrading maximum value directed model was accurate enough for seismic damage analysis while the bilinear model underestimated damage because of its linear response to the low intensity cyclic loadings. The maximum value directed model was also needed to predict the damage index from the maximum velocity or the spectral intensity of the input motions.

## 1. Introduction

Seismic damage analysis of existing reinforced concrete (RC) structures is important for the total lifeline system as well as for its own integrity. Many seismic safety indices were proposed for RC structures, most of which use the ductility factor, absorbed hysteretic energy, or a combination of these two. These indices are calculated using inelastic response analysis which requires an idealized hysteretic model of the structure. The more precise hysteretic model provides a more precise response at the expense of more calculation time. For this reason, a simple hysteretic model like the bilinear model is often used to model the structures.<sup>1),2)</sup> Different hysteretic models result in different earthquake responses,<sup>3)</sup> but how precise a hysteretic model is required for damage analysis is not clear. Furthermore, damage related spectra such as energy response spectra have also been precisely

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studied using a bilinear model.<sup>4)</sup> The effect of assumed hysteretic models on damage spectra is important to be evaluated.

In this paper, damage was evaluated using several hysteretic models, assuming that the most precise hysteretic model always gives the right damage assessment. The first part of this paper describes the effect of hysteretic models on the damage index, and the latter part discusses the effect of hysteretic models on the damage spectra.

## 2. Hysteretic Models for Force-Deformation Relation

Linear analysis is valid in cases of small response of the structure, and is widely used such as the response spectra method because of its convenience in calculation. However, nonlinear analysis is needed in designing important structures, to check the response during a large future earthquake. In the analysis of real structures, an idealization of their hysteretic behavior is essential, and many idealized hysteretic models have been proposed.

In this paper, a skeleton curve was assumed as piecewise linear as shown in Fig. 1. Cracks occur in concrete at point C; reinforcing bars yield at point Y; restoring force shows its maximum value at point M; and the structure collapses at point U. In the following, the 4 hysteretic models used in this analysis are described. A schematic diagram for each model is shown in Fig. 2.

(a) Linear model—Restoring force relates linearly to the deformation as shown in Fig. 2(a). Initial stiffness (line O-C in Fig. 1) was used for the constant stiffness

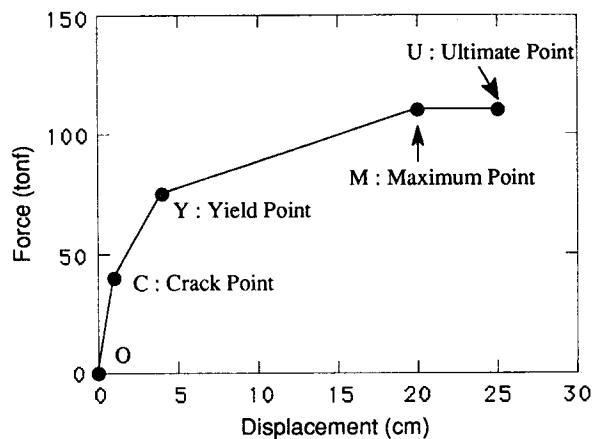


Fig. 1 Force-deformation curve for RC member.

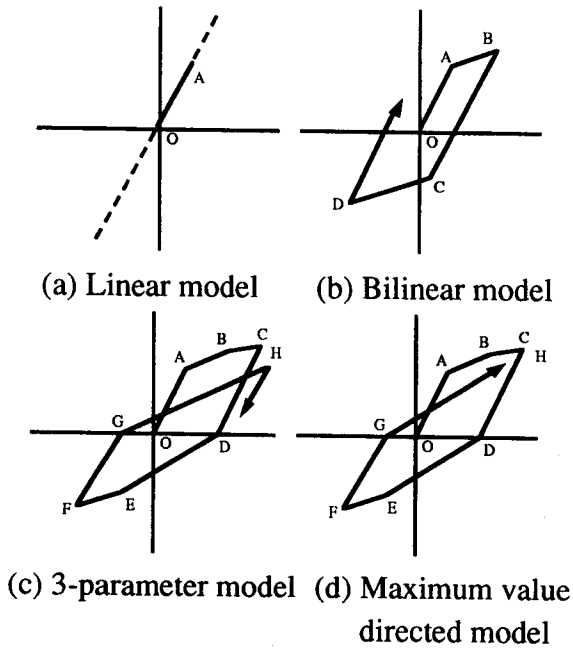


Fig. 2 Hysteretic models used in analysis.

in this simulation. Although a linear model is seldom used in damage analysis, a simulation was carried out in this study to compare its behavior to the other nonlinear models.

(b) Bilinear model—The bilinear model is the simplest nonlinear model that has two stiffnesses to represent elastic and inelastic behavior as shown in Fig. 2(b). Responses may change depending on the connecting point of two stiffness lines. In this study, line O-C and line Y-M in Fig. 1 represent two stiffnesses, and the crossing point of these two lines is defined as point A in Fig. 2(b).

(c) 3-parameter model—Park, Reinhorn and Kunnath<sup>5)</sup> proposed this degrading trilinear type hysteretic model as shown in Fig. 2(c) for RC structures. The three parameters represent stiffness deterioration, strength deterioration and pinching effect due to the shear cracks. There are many other degrading trilinear models, however, this model was selected in this study because of its strong relation to the damage index which is to be mentioned in the next section. Response based on this model was considered to be true in this study because of the assumption that the most precise hysteretic model always derives the most correct response. The three parameters were set to  $\alpha=3.0$ ,  $\beta=0.62$  and  $r=1.0$  according to reference 5.

(d) Maximum value directed model—An ordinal trilinear model shown in Fig. 2(d) was also discussed. The hysteretic response of this model directs its previous maximum or minimum point. Although the 3-parameter model shown above gives the precise response, the necessary parameters are difficult to determine exactly. The maximum value directed model neglects behavior that the 3-parameter model precisely describes; however, the stiffness deterioration during the loading process is a dominant parameter in determining the global shape of hysteretic loops.

### 3. Damage Indices using Different Hysteretic Models

Using the 4 hysteretic models described in the previous section, the response of an RC bridge pier was simulated. The model has dimensions of 2 m in diameter, 13 m in height, 100 tonf (0.98 MN) in weight and 250 tonf (2.45 MN) in reaction force of the beams. It was modeled as an SDOF system whose restoring force-deformation relation at its top during one direction loading is shown in Fig. 1, as described in the previous section. Its initial natural frequency was calculated to 2 Hz and the damping ratio of 5% was assumed.

#### 3.1 Damage index

Many indices have been proposed for seismic damage analysis; some of them use ductilities, some use only hysteretic energy and the others use a combination of ductility and energy. During an earthquake, RC structures may incur large deformations in one direction as well as undergo cyclic deformations. Park, Ang and Wen proposed the “damage index” in the form of a linear combination of a deformation term and a hysteretic energy term as follows:<sup>9)</sup>

$$D = \frac{\delta_m}{\delta_u} + \frac{\beta}{P_y \delta_u} \int dE \quad (1)$$

in which  $D$  is the damage index,  $\delta_m$  is the maximum deformation,  $\delta_u$  is the ultimate deformation,  $P_y$  is the yield strength,  $\int dE$  is the absorbed hysteretic energy and  $\beta$  is an empirical coefficient. A damage index of more than 1 represents severe damage or collapse. Among the many indices used to evaluate damage of the structures, the maximum deformation, absorbed hysteretic energy and the linear combination of these two are the basic indices.

#### 3.2 Input motions

As input motions affect the response of the structures, several sinusoidal wa-

ves and four earthquake records were used in this analysis. The earthquake records used are : the NS component of the El Centro record of the 1940 Imperial valley earthquake, the NS component of the Hachinohe record of the 1968 Tokachi-Oki earthquake, the NS component of the Akita record of the 1983 Nihonkai-Chubu earthquake and the EW component of the SCT record of the 1985 Mexico earthquake.

Fig. 3 shows their acceleration time histories and Fig. 4 shows their response spectra for 5% damping. The SCT record of Fig. 3(d) shows a sinusoidal wave

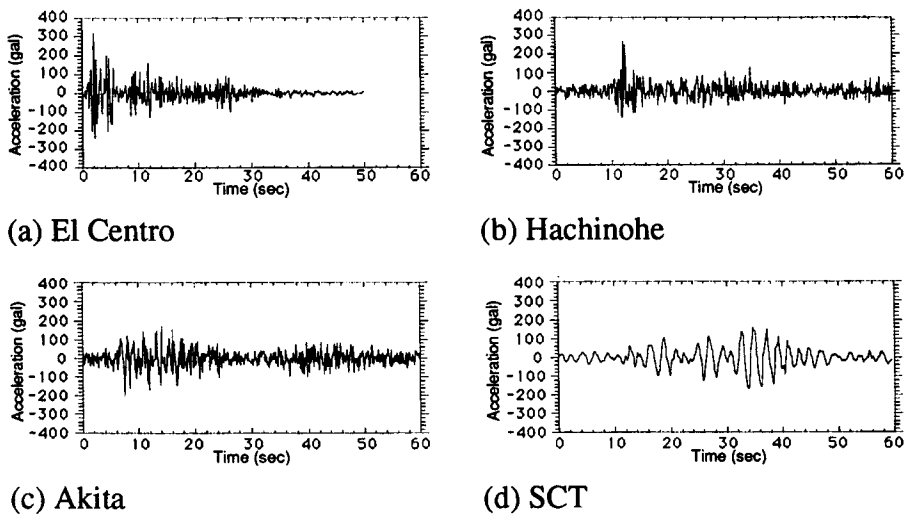


Fig. 3 Earthquake records used in this study.

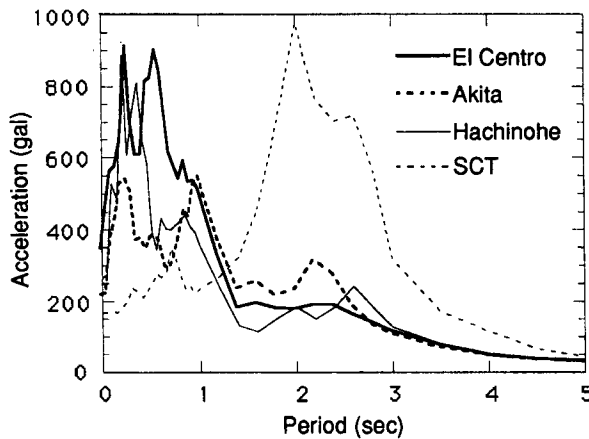


Fig. 4 Response spectra of input motions.

form which may induce a resonance response in a structure with a natural period of 2 seconds, while the other 3 records affect the structures in the shorter natural period range. The El Centro record has a relatively short duration of its main shock.

To see the effect of the strength of the input motions, amplification factors of 0.5, 1.0 and 1.5 were used for each input motion.

### 3.3 Numerical simulation on damage index

Fig. 5 shows examples of the hysteretic force-deformation response of each model for the case of the El Centro record. Damage indices for various earthquake records are shown in Fig. 6. They show different tendencies for different input motions. Fig. 7 shows the damage indices for the sine waves whose period is from 0.2 to 3.0 seconds. Lines for the linear and the bilinear models show lines similar to the resonance curve.

Further, Fig. 8 shows the ratio of the first and the second terms of the damage index, which represent damage due to large deformation and to hysteretic energy,

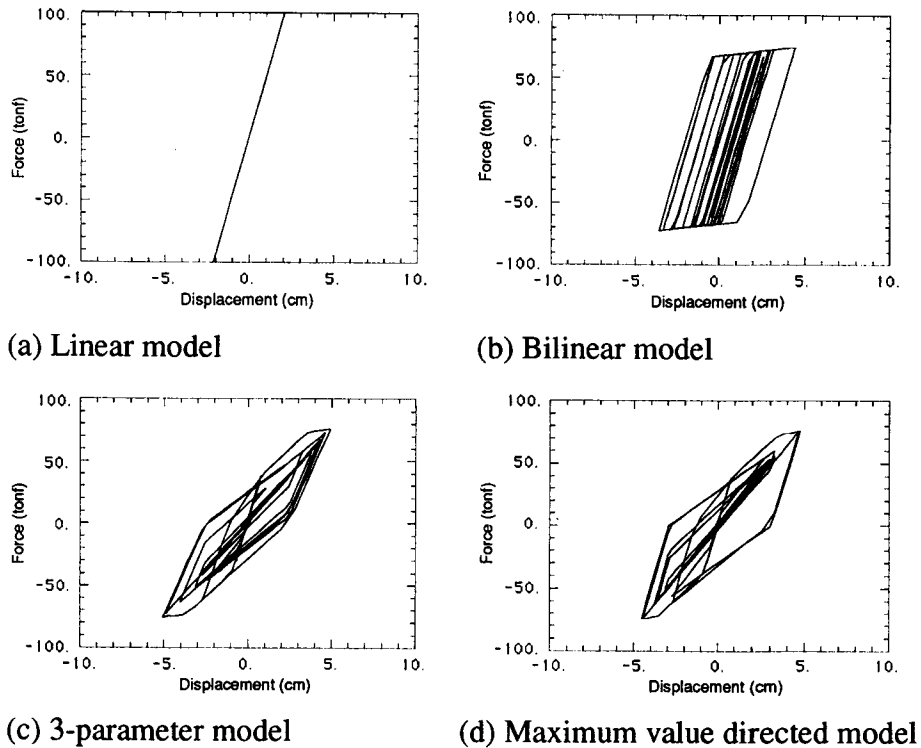
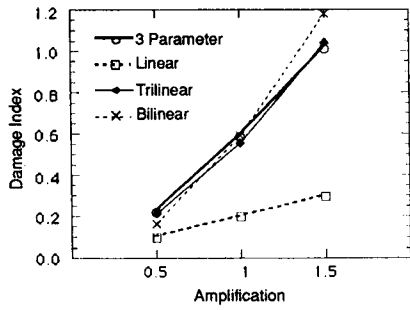
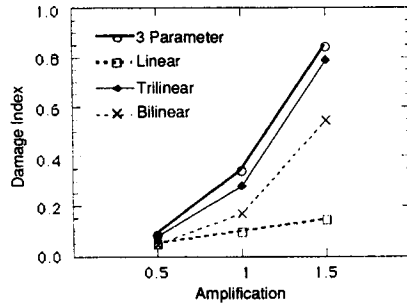


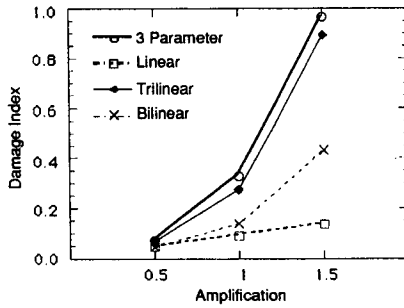
Fig. 5 Hysteretic responses for El Centro record.



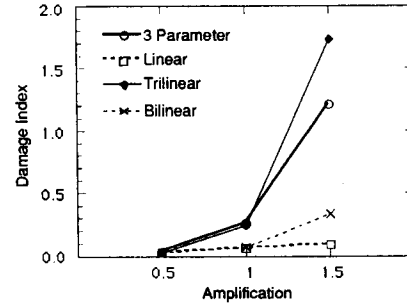
(a) El Centro



(b) Hachinohe



(c) Akita



(d) SCT

Fig. 6 Damage index for different hysteric models and input motions.

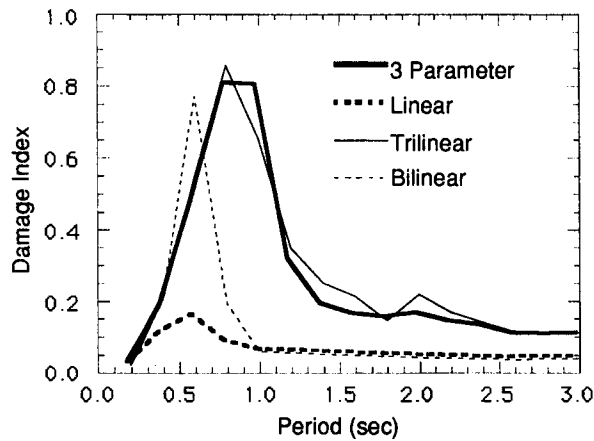


Fig. 7 Damage index for various periods of sinusoidal excitations.



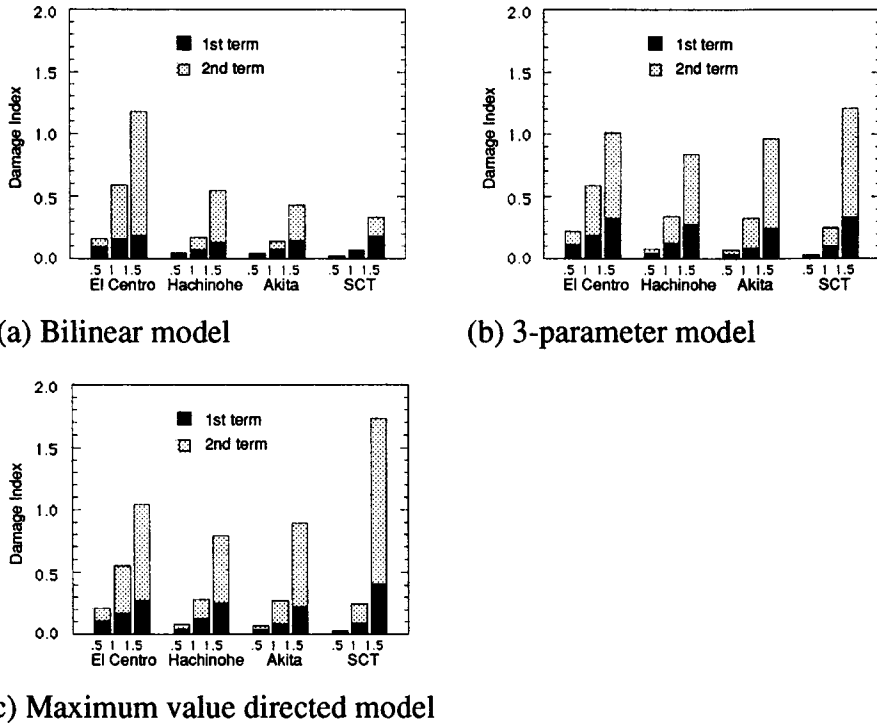


Fig. 8 Ratio of 1st and 2nd terms of damage index.

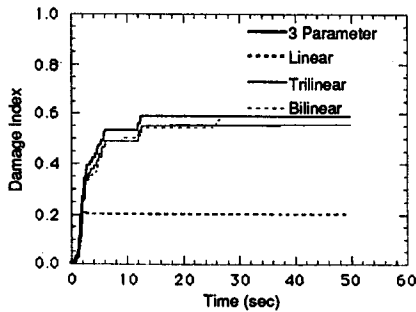
respectively. This figure does not include the linear model because its 2nd term makes no contribution to the damage index. The damage index-time histories are shown in Fig. 9.

(a) Linear model

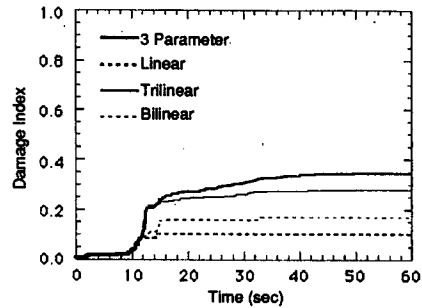
As the response of the linear model shows no hysteretic behavior and the deformation is also small compared with other nonlinear models, the damage index is too small in every case. The linear model should not be used in damage analysis.

(b) Bilinear model

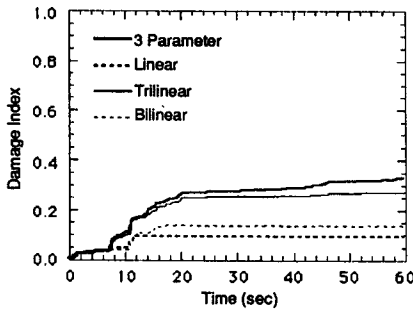
The damage index of the bilinear model shows quite different behavior if its response enters the second stiffness or not. If the response does not overcome its yield level as for the SCT record scaled to 0.5 or 1.0 times (Fig. 6-d), the damage index is the same as the linear model. For the sine wave input shown in Fig. 7, it shows the same values as the linear model for small response. However, once the resonance occurs, it shows a larger value than even the 3-parameter model because of its parallelogram shaped hysteretic loop shown in Fig. 5-b; i.e., it can absorb more hysteretic energy than the degrading type hysteresis loop of Fig. 5-c.



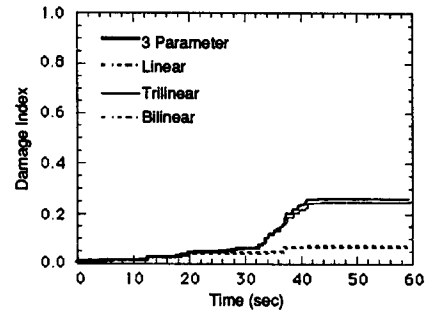
(a) El Centro



(b) Hachinohe



(c) Akita



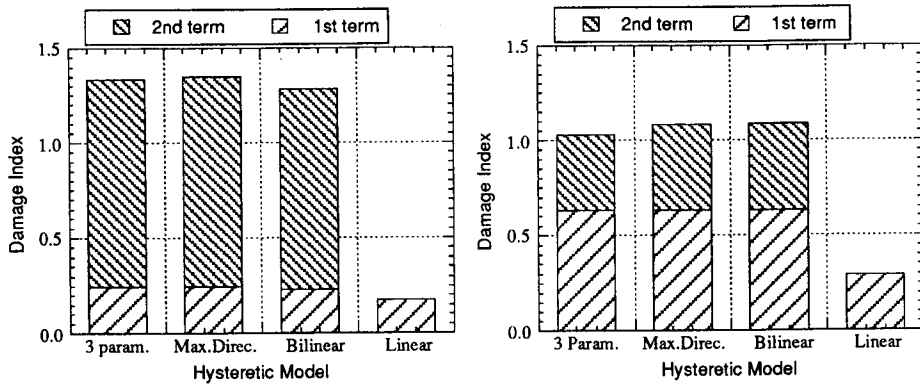
(d) SCT

Fig. 9 Damage index time histories of different hysteretic models.

As in Figs. 8-a and 8-b, the ratio between 1st and 2nd terms differs from that in the bilinear and 3-parameter models. The deformation of the bilinear model is smaller than of the other models in every case which is shown as a solid bar in Fig. 8.

They showed almost the same value for only the El Centro record. There are 2 possible reasons for this phenomena: one is that the dominant period of this record covers the natural period of the assumed model, and the other is its impulsive wave form. Fig. 10 (a) shows the damage index to a filtered white noise from 1 to 3 Hz which covers the natural period of the model (2 Hz), and Fig. 10 (b) shows the response to an impulse scaled to 5000 gal. The damage index of the bilinear model and of the 3-parameter model to the filtered white noise showed different values, and the index to the strong impulse showed almost the same values. Thus, this phenomena was caused by the impulsive input motion. The bilinear model estimates the damage with accuracy only for an impulsive input motion.

Furthermore, as the hysteretic loops remained open for weak input motion,



(a) Filtered white noise from  
1 to 3 Hz

(b) Impulse scaled to 5000 gal

Fig. 10 Ratio of 1st and 2nd terms of damage index for different input motions.

the damage due to small cyclic response after the main large response was neglected. This was observable in Fig. 9 which shows the damage index time histories. The damage index of the 3-parameter model abruptly increased when the input motion showed the maximum acceleration because of the large inelastic deformation. The damage index continued increasing after the large deformation with the result that damage even for the small cyclic response was evaluated. On the contrary, the damage index of the bilinear model remained at the same value after the large deformation.

As a result, the bilinear model usually underestimates damage when compared with the 3-parameter model.

(c) Maximum value directed model

As shown in Fig. 6, the difference between the 3-parameter model and the maximum value directed model was at most 3% except when the SCT record was scaled to 1.5 times larger. The judgment, from the damage index, of whether the structure will collapse or not was the same in every simulation. They also showed almost the same values for the sinusoidal waves shown in Fig. 7.

For the SCT record scaled to 1.5 times larger, the area of the hysteresis loops became small for the 3-parameter model compared with the maximum value directed model because of the stiffness degrading in the unloading process and the strength degradation due to absorbed hysteretic energy. This was also observed from the ratio of the 1st term to the 2nd term as shown in Fig. 8 (c). The damage index according to the maximum value directed model may differ from the 3-pa-

parameter model for the waves which cause numerous large deformations, however, the indices from both models exceed 1.0 for these strong input motions which result in the same judgment of the structure as totally collapsed. Thus, the maximum value directed model is accurate enough for seismic damage analysis provided that precise earthquake response is unessential.

### 3.4 Relation between the indices for earthquake motions and the damage index

Calculation of the damage index requires the time histories of the restoring force and the deformation which are usually estimated from inelastic response analysis. Therefore, a simple equation which approximates the damage index from another simple index of input motion is valid for utilizing the damage indices. In this section, the maximum velocity of the input motion  $v_{max}$  and the spectrum intensity  $SI$ , which are considered to have a high correlation with damage to the structure, are calculated in order to find a correlation with the damage index using different hysteretic models.

Fig. 11 shows the correlation between the damage index and the maximum velocity of the input motions  $v_{max}$ , and Fig. 12 shows the correlation between the damage index and the spectrum intensity  $SI$ .<sup>6)</sup> The  $x$ -axes of these figures represent  $v_{max}$  or  $SI$  of each earthquake record scaled to 0.5, 1.0 and 1.5 times larger than the original record, and they were sorted according to their values regardless of earthquakes. As the SCT record has somewhat special wave form from the other general earthquake records due to its special ground condition, Figs. 11 (b) and 12 (b) were plotted with the results for the SCT records excluded while Figs. 11 (a) and 12 (a) were plotted using all records. As the maximum velocity  $v_{max}$  or the spectrum intensity increases to a certain level, the damage index abruptly

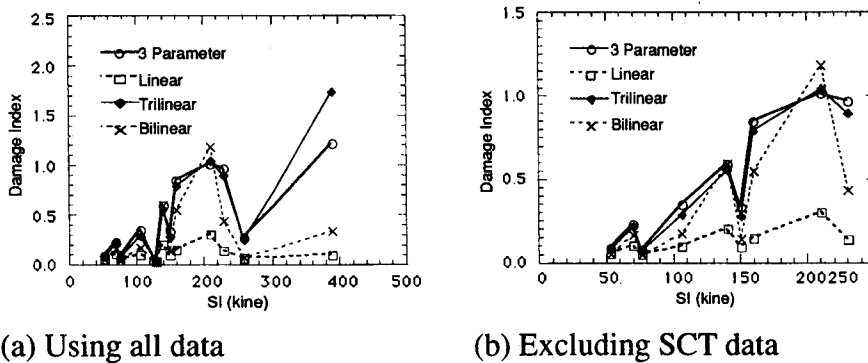


Fig. 11 Relation of damage index to the maximum velocity of input motions.

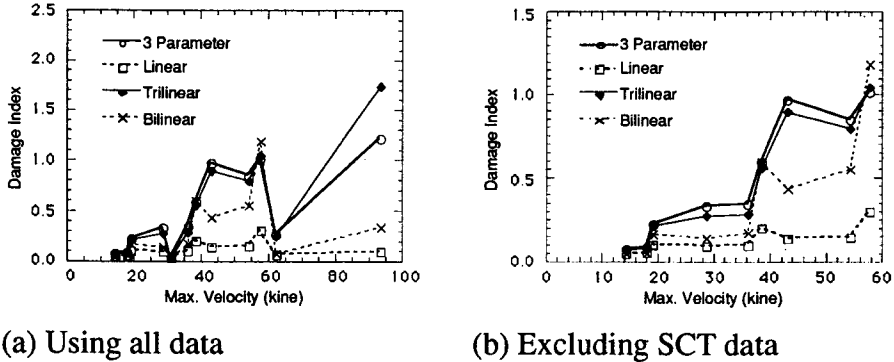


Fig. 12 Relation of damage index to spectral intensity; SI.

increases. Although the relationship is nonlinear, linear regression was used in this paper because the total number of data are not enough for nonlinear analysis.

The correlation coefficient using the 3-parameter model for the damage index and the maximum velocity  $v_{max}$  is 0.85 for the case using all records and 0.9 for the case excluding the SCT record, which is high enough to assume linear function between them. As the maximum velocity exceeds 40 kine, the damage index reached 1 in Fig. 11.

The correlation between the damage index and the spectrum intensity  $SI$  is also high to have a correlation coefficient of 0.8.  $SI$  is more suitable to be used for the assessment of general structures located in a wide area than of the individual structure as in this study, because  $SI$  is defined as the mean velocity response of the structures with natural periods of from 0.1 to 2.5 seconds. The relation of the maximum velocity or the spectrum intensity and the damage index estimated using the bilinear model showed less correlation than the 3-parameter model. Furthermore, the damage index evaluated from  $v_{max}$  or  $SI$  usually showed smaller values than the 3-parameter model, while the maximum value directed model could be used to estimate an index similar to the 3-parameter model. The bilinear model is inadequate to evaluate the relation between the damage index and an index of the input motion.

Because both the maximum velocity and the spectrum intensity are indices only for the input motion, analysis capable of considering information about the structure is needed especially for special wave forms such as the SCT record.

#### 4. Damage spectra using different hysteretic models

##### 4.1 Damage spectra

In this section, damage of the structures with various natural periods was expressed as the damage spectrum. The damage spectrum can be defined for any index related to damage such as ductility, absorbed hysteretic energy, or the damage index, as a function of the natural period of structure. This section studies the response of the sturcutre model which satisfies the design code subjected to the design earthquake motion. A skeleton curve assumed as a perfectly elasto-plastic model (Fig. 13) which satisfies the Japanese seismic code for highway bridges,<sup>7)</sup> was used because of its simplicity. First, the maximum restoring force  $P_Y = P_U$  was defined to satisfy the horizontal seismic load as follows:

$$\frac{P_Y}{m} = \frac{P_U}{m} = k_h \cdot g \quad (2)$$

in which,  $m$  is the mass of the structure,  $g$  is the gravitational acceleration,  $k_h$  is the design horizontal seismic coefficient determined by the following equation:

$$k_h = c_z \cdot c_G \cdot c_I \cdot c_T \cdot k_{h0} \quad (3)$$

in which,  $c_z$  is a modification factor for zone,  $c_G$  is a modification factor for ground condition,  $c_I$  is a modification factor for importance,  $c_T$  is a modification factor for structural response and  $k_{h0}$  is a standard design horizontal seismic coefficient which is set to 0.2.  $k_h$  should not be less than 0.1.

Using a circular natural frequency  $\omega$ , the yield deformation  $\delta_Y$  is written as follows:

$$\delta_Y = \frac{P_Y}{m \cdot \omega^2} \quad (4)$$

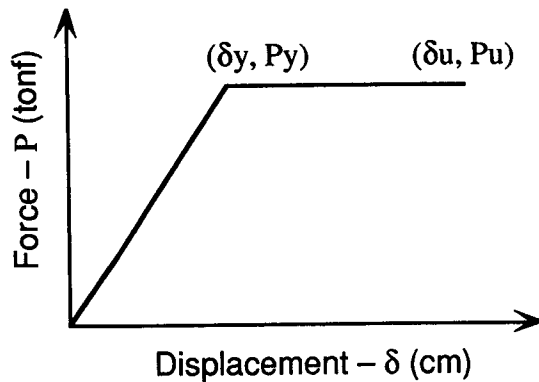


Fig. 13 Perfectly elasto-plastic model assumed for damage spectra.

The ultimate deformation is determined by checking the bearing capacity of RC piers for lateral force. The ultimate strength should satisfy the following equation:

$$\frac{P_u}{m} = k_{he} \cdot g \quad (5)$$

in which,  $k_{he}$  is an equivalent horizontal seismic coefficient to be checked for the bearing capacity of RC piers for lateral force, determined using the next equation:

$$k_{he} = \frac{k_{hc}}{\sqrt{2\mu - 1}} \quad (6)$$

in which,  $\mu$  is an allowable ductility factor,  $k_{hc}$  is the horizontal seismic coefficient for checking bearing capacity determined from the following equation:

$$k_{hc} = c_z \cdot c_I \cdot c_R \cdot k_{hc0} \quad (7)$$

in which,  $c_z$  and  $c_I$  are the same modification factors as Eq. (3),  $c_R$  is a modification factor for structural response and  $k_{hc0}$  is a standard horizontal seismic coefficient set to 1.0.

From Eqs. (2), (6) and (7), the required ductility factor is derived as follows:

$$\mu = \frac{1}{2} \left( \frac{k_{hc}}{k_h} \right)^2 + \frac{1}{2} \quad (8)$$

Let the structure be assumed to collapse in flexural failure, the ultimate deformation is derived as follows:

$$\delta_U = (1 + \mu \cdot \alpha - \alpha) \cdot \delta_Y \quad (9)$$

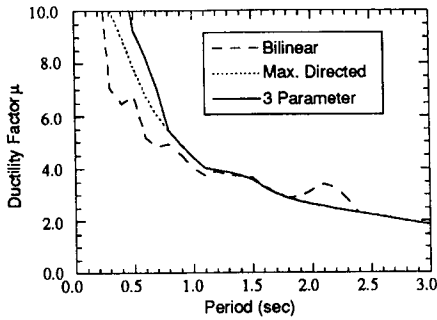
in which,  $\alpha$  is a safety factor set to 1.5.

Using this procedure, a skeleton curve for a structure with an assumed natural period is determined in the form of deformation-restoring force per unit mass. In this paper,  $c_z$  and  $c_I$  are set to 1.0, and the other factors are determined for each ground condition: stiff, medium and soft. A suitable input earthquake record is selected based on the seismic code for each soil condition.<sup>7)</sup> In this paper, the recommended earthquake record for stiff soil is called Type 1, medium soil is called Type 2 and soft soil is called Type 3. They were multiplied 3 times to check the ultimate behavior of structures.

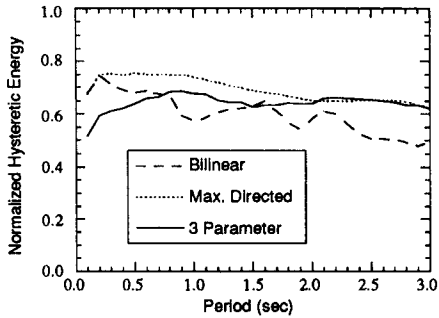
All parameters for the 3-parameter model were set to 1.0 to make the model undergo a heavy degrading process for the purpose of accentuating the differences among the hysteretic models.

**4.2 Numerical simulations on damage spectra**

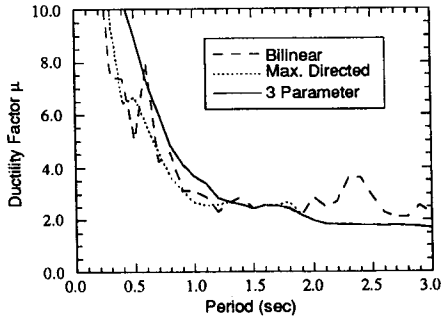
Fig. 14 shows the damage spectra for the ductility factor, Fig. 15 shows the damage spectra for the absorbed hysteretic energy normalized by the total input energy, and Fig. 16 shows the damage spectra for the damage index. In Fig. 16,



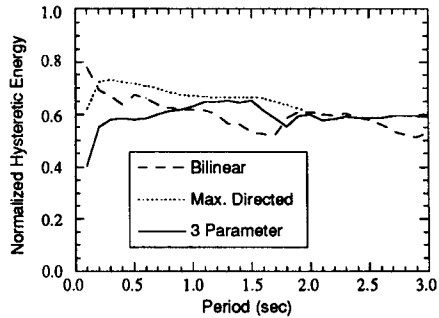
(a) Type 1 earthquake



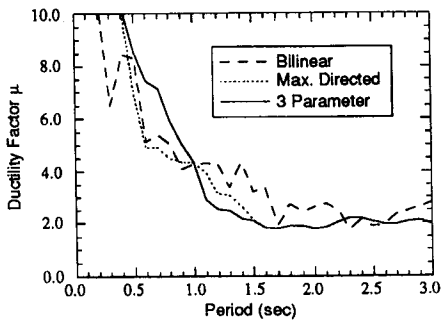
(a) Type 1 earthquake



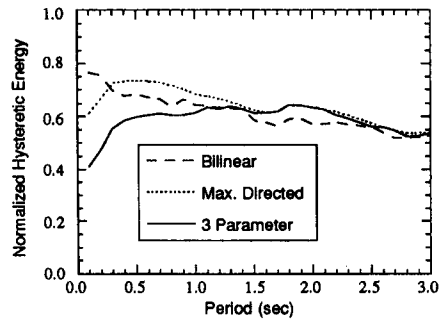
(b) Type 2 earthquake



(b) Type 2 earthquake



(c) Type 3 earthquake

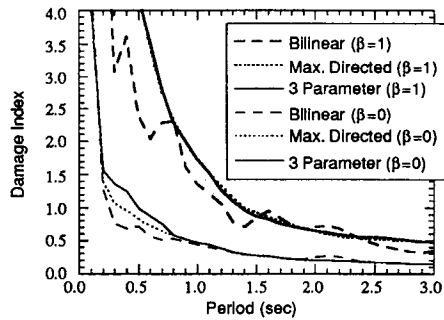


(c) Type 3 earthquake

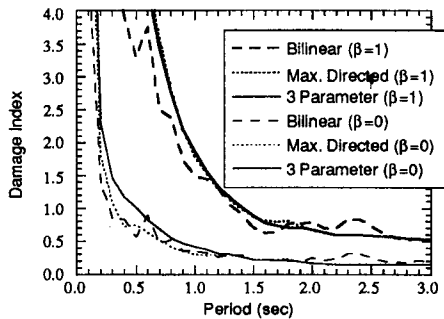
Fig. 14 Damage spectra for ductility factor.

Fig. 15 Damage spectra for absorbed hysteretic energy.

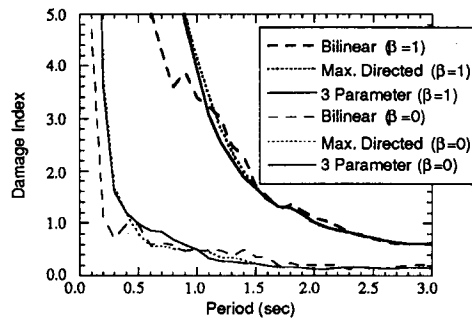




(a) Type 1 earthquake



(b) Type 2 earthquake



(c) Type 3 earthquake

Fig. 16 Damage spectra for damage index.

the lines for  $\beta=0.0$  are also plotted giving the contribution of only the 1st term of the damage index. As the 1st term and the 2nd term are added linearly, a damage index corresponding to any value of  $\beta$  can be interpolated from this figure.

(a) The maximum value directed model

The maximum value directed model underestimated ductility factors of less

than 10% for short period structures (Fig. 14). In addition, it showed the same estimates as the 3-parameter model for the structures whose natural period is longer than 1.0 second on stiff ground (Fig. 14-a), longer than 1.3 seconds on medium ground (Fig. 14-b), and longer than 1.5 seconds on soft ground (Fig. 14-c).

On the contrary, this model overestimated hysteretic energy in the short period range (Fig. 15). This difference was caused by the strength degradation process existing in the 3-parameter model due to the large deflection. For the structures whose natural period is longer than 2.0 seconds, this model showed the same estimates of the absorbed hysteretic energy as the 3-parameter model.

This model gave the same damage index as the 3-parameter model (Fig. 16). As the 1st term of the damage index is defined as the ratio of the maximum deformation to the ultimate deformation instead of to the yield deformation similar to the ductility factor, the difference in the ductility factor had little effect on the damage index. Furthermore, the 1st term of the damage index gave smaller values than the 3-parameter model to the short period structures and the 2nd term gave larger values, resulting in a smaller difference as a total. The difference between the maximum value directed model and the 3-parameter model was less than 5%.

(b) The bilinear model

The bilinear model underestimated the ductility factor by about 20% in the short period range, and overestimated it at most 20% for the long period range (Fig. 14). This model is the same as the linear model unless the deformation becomes larger than the yield point. It shows an abrupt change of its nature around the yield point.

The bilinear model underestimated the normalized ratio of the hysteretic energy to the total input energy by at most 20% in the range of periods longer than 1.0 second, and overestimated it in the shorter periods (Fig. 15).

The bilinear model also underestimated the damage index (Fig. 16). The judgment for vulnerability of whether the structure will collapse or not is almost the same as with the 3-parameter model. However, the index according to the bilinear model should be handled carefully because the index showed only half the value of the 3-parameter model for some periods.

## 5. Conclusions

The effects of the different hysteretic models on the damage index were studied. On the assumption that a precise inelastic hysteretic model called the 3-parameter model will show the real damage to the RC structures, more simple hystere-

tic models were examined for their ability to approximate the damage index and the damage spectra. The main conclusions are as follows:

1. The bilinear model usually underestimates the damage index compared with a precise hysteretic model such as the 3-parameter model. Damage assessment of RC structures requires at least the maximum value directed model.
2. The maximum value directed model approximates the damage index to within 5% accuracy except for an input motion with numerous cyclic waves of large amplitude such as the SCT record of the 1985 Mexico earthquake. However, the decision of whether the structure will collapse or not is the same as with the 3-parameter model, even for the SCT record. Degrading of the strength and unloading stiffness due to the cyclic loadings can be neglected for damage analysis unless a precise earthquake response is needed.
3. The damage index can be evaluated with high accuracy from the maximum velocity or the spectrum intensity of the input motion. However, the relation between the index of the input motion and the damage index using the bilinear model showed less correlation than the 3-parameter model, whereas the maximum value directed model showed the same high correlation as the 3-parameter model. The bilinear model is inadequate to evaluate the relation between the damage index and an index of the input motion.
4. The damage spectra for the damage index, ductility and the normalized hysteretic energy were calculated using the bilinear model, the maximum value directed model, and the 3-parameter model. The damage spectra using the maximum value directed model showed errors of at most 10% compared with the 3-parameter model, while the bilinear model underestimated or overestimated the damage spectra by at most 50%. The damage spectra for ductility and the hysteretic energy should be calculated using a precise hysteretic model, and at least the maximum value directed model is required for the damage index spectra.

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