Different Considerations in Mirror Photogrammetry

OKAMOTO, Atsushi; AKAMATU, Shin-Ichi

Different Considerations in Mirror Photogrammetry

By

Atsushi OKAMOTO and Shin-Ichi AKAMATU

(Received June 19, 1992)

Abstract

The self calibration of a non-metric camera without object space information requires an assumption that the interior orientation is unchanged between more than two photographs taken of the same three-dimensional object. However, this criterion may not be valid for many non-metric cameras. On the other hand, it is not very difficult to manufacture high-quality front-surfaced mirrors with practically no undulation. Also, by placing such a plane mirror close to an object so that a non-metric camera can record not only the object directly but also its mirror reflection, the above assumption can be satisfied rigorously between the normal and satellite pictures taken on the same negative. In addition, constraints are generated among the exterior orientation parameters of the stereopair of mirror photographs. Based on both these findings and the potential theory of overlapped photographs (Okamoto [1986]), this paper discusses the self calibration problem of non-metric cameras employing plane mirrors in detail. Further, the constraints regarding the exterior orientation of overlapped mirror photographs prove to be replaced by coplanarity equations and object space information redundantly used. Finally, corrections techniques for mirror distortions are presented.

INTRODUCTION

Mirrors have long been used mainly in close-range photogrammetry for the purpose of obtaining stereocoverage of hidden areas which cannot be registered in both photographs in a conventional manner. Further, spherical or round-surfaced objects can be reconstructed photogrammetrically by arranging a pair of cameras and two plane mirrors adequately (Kratky [1975], and Veress and Munjy [1983]). Little, however, has been done in the way of investigating the role of mirrors in central-projective geometry (Mikhail [1968]). Thus, the geometrical potential of mirrors has not been fully utilized in many close-range applications of mirror photographs.

In our previous research (Okamoto and Akamatu [1992]), geometrical properties of mirror photographs have been explored fundamentally. Also, an orientation method of a stereopair of normal and satellite pictures taken on the same film has been presented, which has the following characteristics that

(1) Parameters defining the mirror plane are employed as the orientation unknowns in

Division of Global Environment Engineering Kyoto University Yoshida-Honmachi, Sakyo-ku Kyoto 606–01 Japan

order to avoid the problem of expressing constraints among the exterior orientation elements of the stereopair in functional form, and

(2) The general law of reflection is introduced in the orientation calculation.

On the other hand, Kratky [1975] developed another orientation technique of such a stereopair of mirror photographs, which adopts the orientation elements of the reflected satellite picture (the mirror reflection of the satellite picture) as the orientation unknowns.

In this paper, the self calibration problem of non-metric cameras using plane mirrors is discussed under the assumption that the mirrors are free of distortion, the two orientation methods of overlapped mirror photographs are compared, when all the orientation unknowns are determined simultaneously, and correction techniques of mirror distortions are proposed, when they are not negligibly small.

SELF-CALIBRATION PROBLEM OF NON-METRIC CAMERAS USING PLANE MIRRORS

In this section, the self calibration problem of non-metric cameras using plane mirrors will be discussed based on the orientation theory of mirror photographs presented in our previous paper (Okamoto and Akamatu [1992]).

EXPLANATION OF THE PRINCIPLE

Let two plane mirrors be placed close to an object in such a way that a non-metric camera can register not only the object directly but also their mirror reflections, as is

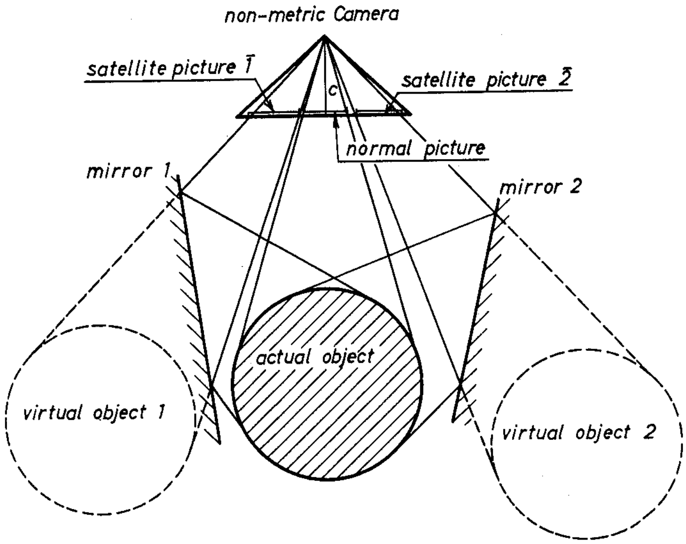

Fig. 1 Self calibration problem of one non-metric camera using two plane mirrors.

demonstrated in Fig. 1. Under the assumptions that the same interior orientation parameters are valid throughout the image plane and that the two mirrors are free of distortion, the interior geometry of the normal picture and the two satellite pictures $\bar{1}$ and $\bar{2}$ can be defined by the same central projective elements. Thus, the orientation problem of these three photographs can reduce to that for the case where the interior orientation is unchanged between three pictures taken of the same object. In the general case of photogrammetry, where a picture has 11 independent central projective parameters (the six exterior and five interior orientation elements), we have ten constraints among the interior orientation parameters of three such photographs. Further, these ten constraints can be classified into eight first-grouped and two second-grouped ones (Okamoto [1986]). It follows that all the interior orientation unknowns of the three photographs can be provided without object space information. When the linear part of the disturbing feature of non-metric cameras is negligibly small, as is the case in many close-range applications, the geometry of a picture can be determined by nine independent central-projective elements (the six exterior and three interior orientation elements). In this case, twelve constraints must be introduced for the interior orientation of the normal picture and the satellite pictures $\bar{1}$ and $\bar{2}$, namely, six constraints showing that we have non-linear systematic errors in measured image coordinates of the three pictures and six constraints describing that the three photographs have the same three conventional interior orientation parameters. In addition, these 12 constraints include eight first-grouped ones. From this fact it can be seen that the normal picture and the two satellite pictures $\bar{1}$ and $\bar{2}$ have the potential to provide all the interior orientation unknowns.

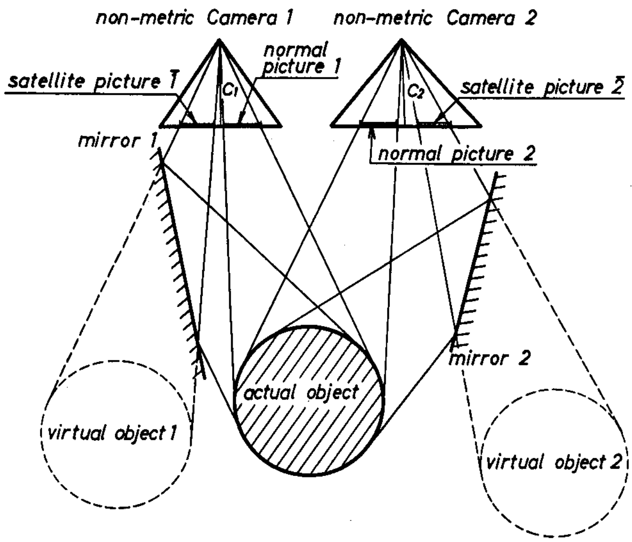

Fig. 2 Self calibration problem of two non-metric cameras using two plane mirrors.

Next, we will treat the case of employing two non-metric cameras and two plane mirrors (See Fig. 2). In this configuration, the normal picture 1 and the satellite picture $\bar{1}$ have the same interior orientation parameters. Also, the interior geometry of the satellite picture $\bar{2}$ is the same as that of the normal picture 2. Thus, in the general case of photogrammetry, this system can be specified by 10 constraints regarding the interior orientation of the four overlapped photographs. According to the potential theory of overlapped photographs (Okamoto [1986]), these 10 constraints can be divided into eight first-grouped and two second grouped ones. Consequently, the 10 interior orientation unknowns can be provided from the potential of the four pictures. In the usual case in close-range photogrammetry, 14 constraints are generated among the interior orientation elements of four such photographs. Eight among the 14 constraints describe the characteristics of the linear systematic errors and the remaining six pertain to the conventional interior orientation of the four pictures. In addition, the 14 constraints include eight first-grouped ones. This fact shows that the six interior orientation unknowns of the four photographs can be determined without object space information.

THE SELF CALIBRATION CALCULATION

This paragraph describes the self calibration calculation for the first example (Fig. 1) in the general case of photogrammetry. As for the five interior orientation parameters of

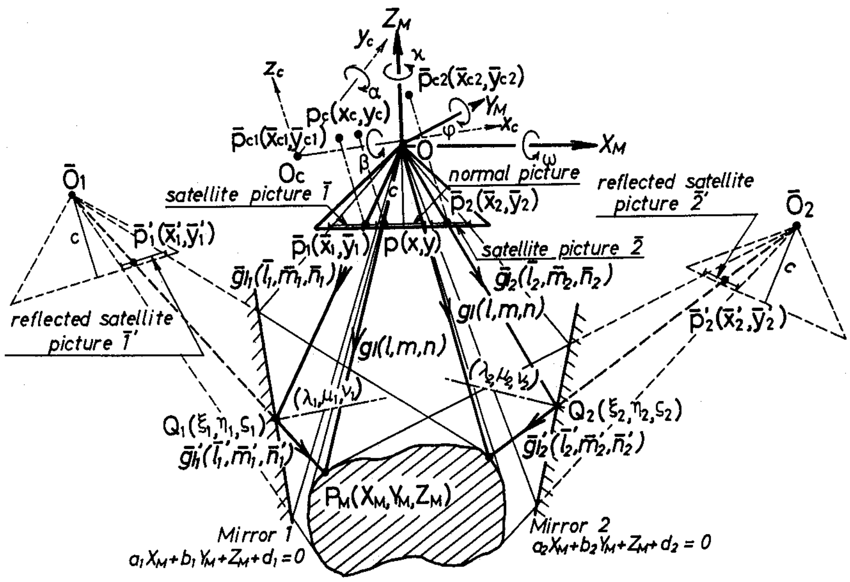

Fig. 3 Self calibration calculation for the first example. The planimetric coordinates of the projection center of the image plane are given as (x_{c0}, y_{c0}) with respect to the comparator coordinate system (x_c, y_c, z_c).

a photograph, we can, without loss of generality, select the following elements; the planimetric coordinates (x_{c0}, y_{c0}) of its projection center referred to the comparator coordinate system, its principal distance, c, and two rotation parameters (α, β) defining the inclination of the comparator coordinate axes with respect to the image plane (See Fig. 3). Further, the model space coordinate system (X_M, Y_M, Z_M) is taken as a right-handed, rectangular-Cartesian system with its origin at the projection center of the picture and with its $X_M - Y_M$ plane parallel to the picture plane. The collinearity condition between an image point and the corresponding model point will be given, respectively, for the normal picture and satellite photographs $\bar{1}$ and $\bar{2}$ as follows.

With regard to the normal picture, the transformed picture coordinates $({}_MX_p, {}_MY_p, {}_MZ_p)$ are described in the form

$$\begin{pmatrix} {}_MX_p \\ {}_MY_p \\ {}_MZ_p \end{pmatrix} = \begin{pmatrix} (x_c - x_{c0} + c \cdot \sin\alpha\cos\beta)\sec\alpha - (y_c - y_{c0} + c \cdot \sin\beta)\tan\alpha\tan\beta \\ (y_c - y_{c0} + c \cdot \sin\beta)\sec\beta \\ -c \end{pmatrix} \tag{1}$$

in which (x_c, y_c) denote measured plate coordinates of an image point $p(x, y)$ on the normal picture (Okamoto [1981, 1982]). Using Equation 1, direction cosines (l, m, n) of the imaging ray g can be obtained and the equation of g can be constructed in the model space coordinate system. Then, the collinearity condition relating the image point $p(x, y)$ and the corresponding model point $P_M(X_M, Y_M, Z_M)$ can be expressed as

$$\begin{aligned} X_M &= \frac{l}{n} Z_M = \frac{{}_MX_p}{{}_MZ_p} Z_M \\ Y_M &= \frac{m}{n} Z_M = \frac{{}_MY_p}{{}_MZ_p} Z_M \end{aligned} \tag{2}$$

Equation 2 contains only the five interior orientation unknowns $(x_{c0}, y_{c0}, c, \alpha, \beta)$.

An imaging ray for a satellite picture can be divided into a ray before reflection and a ray after reflection. Also, the equations of rays $\bar{g}_1$ and $\bar{g}_2$ before reflection for the satellite pictures $\bar{1}$ and $\bar{2}$ can be formed in the same manner as that of the g for the normal picture. Further, expressing the equations of the first and second mirror planes as

$$a_1 X_M + b_1 Y_M + Z_M + d_1 = 0 \tag{3}$$

and

$$a_2 X_M + b_2 Y_M + Z_M + d_2 = 0 \tag{4}$$

respectively, we can find the reflection points $Q_1(\xi_1, \eta_1, \zeta_1)$ and $Q_2(\xi_2, \eta_2, \zeta_2)$, and the direction cosines $(\lambda_1, \mu_1, \nu_1)$ and $(\lambda_2, \mu_2, \nu_2)$ of the normals to the first and second mirror planes (Okamoto and Akamatu [1992]). Then, the direction cosines $(\bar{l}_1', \bar{m}_1', \bar{n}_1')$ and $(\bar{l}_2', \bar{m}_2', \bar{n}_2')$ of the rays $\bar{g}_1'$ and $\bar{g}_2'$ after reflection can be obtained from the general law of reflection and the equations of $\bar{g}_1'$ and $\bar{g}_2$ can be constructed in the form

$$\bar{g}_1': \quad \frac{X_M - \xi_1}{\bar{l}_1'} = \frac{Y_M - \eta_1}{\bar{m}_1'} = \frac{Z_M - \zeta_1}{\bar{n}_1'} \tag{5}$$

$$\bar{g}_2': \quad \frac{X_M - \xi_2}{\bar{l}_2'} = \frac{Y_M - \eta_2}{\bar{m}_2'} = \frac{Z_M - \zeta_2}{\bar{n}_2'} \tag{6}$$

Finally, the collineariry conditions regarding the reflected satellite pictures $\bar{1}'$ and $\bar{2}'$ (the mirror reflections of the satellite pictures $\bar{1}$ and $\bar{2}$) can be described, respectively, as follows:

$$\begin{aligned} X_M &= \frac{\bar{l}_1'}{\bar{n}_1'}(Z_M - \zeta_1) + \xi_1 \\ Y_M &= \frac{\bar{m}_1'}{\bar{n}_1'}(Z_M - \zeta_1) + \eta_1 \end{aligned} \tag{7}$$

and

$$\begin{aligned} X_M &= \frac{\bar{l}_2'}{\bar{n}_2'}(Z_M - \zeta_2) + \xi_2 \\ Y_M &= \frac{\bar{m}_2'}{\bar{n}_2'}(Z_M - \zeta_2) + \eta_2 \end{aligned} \tag{8}$$

Equation 7 is functions of the five interior orientation parameters $(x_{c0}, y_{c0}, c, \alpha, \beta)$ of the picture and the three elements (a_1, b_1, d_1) of the first mirror plane. Also, Equation 8 contains the three parameters (a_2, b_2, d_2) defining the second mirror plane in addition to the five interior orientation elements.

The determination equations for the self calibration problem of a non-metric camera under consideration are constructed in the following manner. Equations 2 and 7 yield

$$\begin{aligned} X_M &= \frac{1}{n} Z_M, & Y_M &= \frac{m}{n} Z_M \\ X_M &= \frac{\bar{l}_1'}{\bar{n}_1'}(Z_M - \zeta_1) + \xi_1, & Y_M &= \frac{\bar{m}_1'}{\bar{n}_1'}(Z_M - \zeta_1) + \eta_1 \end{aligned} \tag{9}$$

which includes one equation equivalent to the coplanarity condition for the normal picture and the reflected satellite picture $\bar{1}'$. Also, the coplanarity condition provides four independent orientation unknowns (two mirror parameters and two interior orientation elements). Further, from Equations 2 and 8 we obtain

$$
\begin{aligned}
X_M &= \frac{1}{n} Z_M, & Y_M &= \frac{m}{n} Z_M \\
X_M &= \frac{\bar{l}_2'}{\bar{n}_2'}(Z_M - \zeta_2) + \xi_2, & Y_M &= \frac{\bar{m}_2'}{\bar{n}_2'}(Z_M - \zeta_2) + \eta_2
\end{aligned}
\tag{10}
$$

Equation 10 has the same geometrical characteristics as Equation 9. Regarding the overlapped part of the three photographs, we must apply the next equations obtained from Equations 2, 7, and 8;

$$
\begin{aligned}
X_M &= \frac{1}{n} Z_M, & Y_M &= \frac{m}{n} Z_M \\
X_M &= \frac{\bar{l}_1'}{\bar{n}_1'}(Z_M - \zeta_1) + \xi_1, & Y_M &= \frac{\bar{m}_1'}{\bar{n}_1'}(Z_M - \zeta_1) + \eta_1 \\
X_M &= \frac{\bar{l}_2'}{\bar{n}_2'}(Z_M - \zeta_2) + \xi_2, & Y_M &= \frac{\bar{m}_2'}{\bar{n}_2'}(Z_M - \zeta_2) + \eta_2
\end{aligned}
\tag{11}
$$

Equation 11 contains mathematically one equation equivalent to the coplanarity condition for the normal picture and the reflected satellite picture $\bar{1}'$, one equation which corresponds to the coplanarity condition for the normal picture and the reflected satellite picture $\bar{2}'$, and one equation required for the united model construction with the three overlapped photographs. Also, the model connection condition determines two independent orientation parameters (one mirror parameter (a scale factor) and one interior orientation element) (See Okamoto [1986]).

The 11 orientation unknowns (x_{c0}, y_{c0}, c, α, β, a_1, b_1, d_1, a_2, b_2, d_2) in this self calibration problem of constructing a model congruent to the object will be calculated as follows. We set up Equation 9 for two model points in the overlapped part between the first satellite picture $\bar{1}$ and the normal picture, Equation 10 for two model points in the overlapped part between the normal picture and the second satellite picture $\bar{2}$, and Equation 11 for two model points which have been imaged on the three pictures at the same time (See Fig. 4). The space coordinates of the six model points are treated as unknowns in this orientation calculation. Then, we have 28 independent equations with respect to 29 unknowns (the 11 orientation parameters plus 18 unknown coordinates of the six model points). This is an underdetermined system which is caused by the fact that two of the six mirror parameters pertain to a scale factor and one of these two unknowns cannot be thus determined in the process of the united model construction. Considering d_1 to be an arbitrary constant, we can, however, compute the five interior orientation elements of the non-metric camera by solving Equations 9, 10, and 11 with respect to 10 orientation parameters and the 18 unknown model coordinates. The united model formed in this process is similar to the object. In order to determine all the 11 orientation unknowns simultaneously, one length

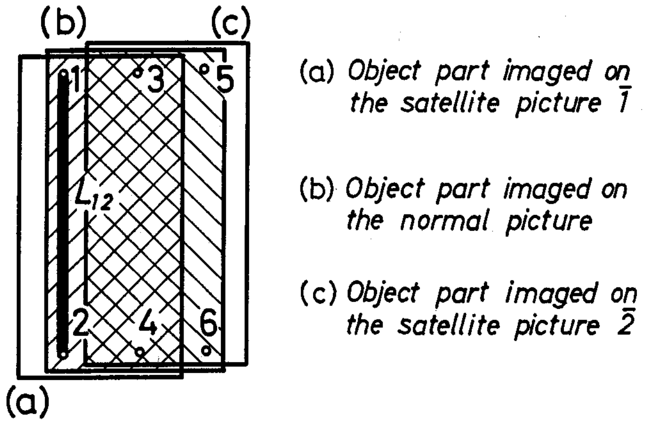

Fig. 4 Arrangement of orientation points and one length which is given as the object space control.

must be available as the object space control. Assume that a distance from point 1 to point 2 is known in the object space (See Fig. 4). The length L_{12} can be expressed with respect to the model space coordinate system (X_M, Y_M, Z_M) as

$$L_{12}=\sqrt{(X_{M1}-X_{M2})^2+(Y_{M1}-Y_{M2})^2+(Z_{M1}-Z_{M2})^2} \tag{12}$$

Adding Equation 12 to the determination equations (Equations 9, 10, and 11), we obtain 29 independent equations with respect to the 29 unknowns. Solving these 29 Equations, a united model congruent to the object can be constructed, which is very important in many close-range applications.

MATHEMATICAL MEANINGS OF SIMULTANEOUS DETERMINATION OF ALL ORIENTATION UNKNOWNS OF OVERLAPPED MIRROR PHOTOGRAPHS

We have three different solution approaches to the analytical orientation problem of photographs;

(1) Method to calculate orientation parameters of individual photographs based on the collinearity condition,
(2) Technique to divide the orientation procedure of a stereopair of photographs into the two main processes, relative and absolute orientation, and
(3) Method to determine all orientation unknowns of overlapped photographs simultaneously.

Among these three orientation techniques, the third approach using the collinearity equations as the determination equations may be applied most easily and rigorously in analytical photogrammetry. In this section, the geometrical characteristics of the simultaneous determination of all orientation unknowns of overlapped mirror pictures will be clarified in comparison with the first approach.

We will consider the orientation problem of a stereopair of a normal picture and a satellite picture which have been taken on the same film by placing a plane mirror close to an object. First, three parameters defining the mirror plane are adopted as the orientation unknowns. When a metric camera is employed for this purpose, we have nine orientation unknowns (the six exterior orientation elements plus the three mirror parameters). Two mirror parameters are provided during the phase of the relative orientation of the stereopair. The remaining seven elements can be determined in the process of the absolute orientation, if seven coordinates of three points are available as the object space controls. For the simultaneous determination of all the nine orientation unknowns, the collinearity equations for the stereopair are applied to the three control points (two points with the space coordinates given and a point with the height given). Then, we have 12 equations with respect to the 11 unknowns (the six exterior orientation parameters, the three mirror parameters, and two unknown planimetric coordinates of the point which has the known height). This is an overdetermined system which is caused by the fact that one coplanarity equation is used redundantly. However, a least-squares adjustment can overcome this problem without difficulties. In the usual case in close-range photogrammetry, a stereopair of normal and satellite pictures on the same negative have 12 independent orientation unknowns, namely, the six exterior orientation parameters and three interior ones of the normal picture, and three mirror parameters, because the geometry of the normal and satellite photographs can be defined by the same central-projective elements. The coplanarity condition in this case provides four independent orientation unknowns (two mirror parameters and two interior orientation elements). Also, the central-projective one-to-one correspondence between the model and object spaces can be expressed in terms of eight independent elements, which means that the unique determination of this transformation requires three control points (two points with the space coordinates given and one point with the planimetric coordinates known). Setting up the collinearity equations of the stereopair for four points including the three control points, we have 16 equations to obtain the 16 unknowns (the six exterior and three interior orientation parameters of the normal picture, the three mirror parameters, and four unknown coordinates of the four orientation points used) simultaneously. The discussion can readily be extended to the general case of mirror photogrammetry.

Next, we will analyze the normal and satellite pictures based on the orientation theory of a single photograph. However, unlike the previous case, orientation parameters of the reflected satellite picture (the mirror reflection of the satellite picture) are employed as the orientation unknowns. In metric photogrammetry, the orientation of the normal and reflected satellite pictures can be carried out separately, if we have three control points with the space coordinates given. For the simultaneous determination of all the 12 orientation parameters, the collinearity equations for the stereopair are applied to the three control points in order to obtain 12 (independent) equations. It will be noted that, in this orientation calculation, no constraints are introduced among the exterior orientation parameters of the reflected satellite picture, though we must consider mathematically three constraints due to the reflection of imaging rays for the satellite picture (Okamoto and Akamatu [1992]). From this fact we can see that the three constraints among the exterior orientation elements of the reflected satellite picture can be replaced by one coplanarity equation and two object space coordinates redundantly used. In other words, the 12 exterior orientation parameters of the normal and reflected satellite pictures can be regarded as independent, if we have more than two control points redundantly (See Kratky [1975]).

In the usual and general cases of mirror photogrammetry, the orientation problem of a stereopair of such photographs can be specified by two types of constraints; three constraints among the exterior orientation parameters of the stereopair and constraints describing that the interior orientation is unchanged between the normal and reflected satellite pictures. However, when the orientation theory of a single photograph is employed, these constraints can be replaced by coplanarity equations and object space information redundantly used.

CORRECTION FOR SYSTEMATIC ERRORS DUE TO MIRROR UNDULATION

In the preceding sections, mirror photogrammetry has been discussed under the assumption that mirrors are free of distortion. However, this criterion cannot be satisfied completely in practice. Therefore, in this section, geometrical properties of the mirror distortion will be explored in detail.

The general collinearity condition relating an object space (X, Y, Z) and the reflected satellite picture may be expressed in the form

$$\bar{x}_c' = \frac{\bar{A}_1' X + \bar{A}_2' Y + \bar{A}_3' Z + \bar{A}_4'}{\bar{A}_9' X + \bar{A}_{10}' Y + \bar{A}_{11}' Z + 1}$$
$$\bar{y}_c' = \frac{\bar{A}_5' X + \bar{A}_6' Y + \bar{A}_7' Z + \bar{A}_8'}{\bar{A}_9' X + \bar{A}_{10}' Y + \bar{A}_{11}' Z + 1} \qquad (13)$$

in which $(\bar{x}_c', \bar{y}_c')$ denote measured plate coordinates for the reflected satellite picture. Equation 13 can also be described in terms of six exterior orientation elements $(\bar{\omega}', \bar{\phi}', \bar{\kappa}', \bar{X}_0', \bar{Y}_0', \bar{Z}_0')$ and five interior orientation ones $(\bar{x}_{c0}', \bar{y}_{c0}', \bar{c}', \bar{\alpha}', \bar{\beta}')$ of the reflected satellite picture. Assume that the mirror distortion may be modeled in polynimial form, i.e.,

$$\begin{aligned} \Delta\bar{x}' &= e_0 + e_1\bar{x}' + e_2\bar{y}' + e_3(\bar{x}')^2 + e_4\bar{x}'\bar{y}' + e_5(\bar{y}')^2 + \cdots \\ \Delta\bar{y}' &= h_0 + h_1\bar{x}' + h_2\bar{y}' + h_3(\bar{x}')^2 + h_4\bar{x}'\bar{y}' + h_5(\bar{y}')^2 + \cdots \end{aligned} \quad (14)$$

where $(\bar{x}', \bar{y}')$ indicate ideal photo coordinates of the reflected satellite picture. The coefficients $(e_3, e_4, e_5, \ldots, h_3, h_4, h_5, \ldots)$ of the non-linear part of the mirror distortions cannot be absorbed by those $\bar{A}_i'$ $(i = 1, \ldots, 11)$ of Equation 13. On the other hand, the linear part of Equation 14

$$\begin{aligned} \Delta\bar{x}_1' &= e_0 + e_1\bar{x}' + e_2\bar{y}' \\ \Delta\bar{y}_1' &= h_0 + h_1\bar{x}' + h_2\bar{y}' \end{aligned} \quad (15)$$

corresponds to a two-dimensional affine transformation whose parameters are absorbed by the coefficients of the general collinearity equations. In other words, the elements $(e_0, e_1, e_2, h_0, h_1, h_2)$ defining the linear part of the mirror distortions give influences on the 11 photogrammetric orientation parameters $(\bar{\omega}', \bar{\Phi}', \bar{\kappa}', \bar{X}_0', \bar{Y}_0', \bar{Z}_0', \bar{x}_{c0}', \bar{y}_{c0}', \bar{c}', \bar{\alpha}', \bar{\beta}')$ of the reflected satellite picture.

From the discussions above, we can conclude as follows.

(1) In the general case of mirror photogrammetry
- (a) When a satellite picture (or the reflected satellite picture) is analyzed based on the orientation theory of a single photograph, the linear part of the mirror distortions need not be considered,
- (b) In the orientation problem of a stereopair of normal and satellite pictures on the same film, constraints among the orientation parameters of the stereopair are disturbed by the linear part of the mirror distortions. Thus, the general collinearity condition in algebraic form may be applied effectively to both photographs,
- (c) Parameters defining the non-linear part of the mirror distortions can be provided from the coplanarity condition of corresponding rays.

(2) With regard to the usual and metric cases of mirror photogrammetry
- (a) In the orientation problem of a single mirror photograph, the satellite picture (or the reflected satellite picture) has always eleven independent central projective unknowns, and
- (b) In the analysis of a stereopair of normal and satellite pictures on the same film, the 11 coefficients $\bar{A}_i'$ of the collinearity condition for the reflected satellite picture can be regarded as independent, if we have more control points than mathematically required.

In the coordinate measurement of normal and satellite pictures on the same film, a problem may arise in that we cannot observe them stereoscopically. However, printing two types of diapositives (a side-reversed diapositive and a diapositive without reversal of position) from the negative, we can overcome this problem. In addition, the general collinearity condition could be applied to both normal and reflected satellite pictures in order to eliminate all possible linear errors of the two diapositives. The non-linear errors due to lens distortion, effects of lack of film flatness, mirror undulation, and so on can be removed by the potential of the overlapped mirror photographs.

CONCLUSIONS

The self calibration problem of non-metric cameras using plane mirrors has been discussed under the assumption that the mirrors are free of distortion, and the following properties have been clarified that

(1) The use of two plane mirrors yields the potential to provide all interior orientation unknowns of three overlapped mirror photographs taken on the same film, and

(2) A united stereo model congruent to the object can be formed if one length is available as the object space control.

Next, two different types of orientation approaches of overlapped mirror photographs:

(a) a method of employing parameters defining the mirror planes as the orientation unknowns, and

(b) a technique of adopting orientation elements of the reflected satellite pictures (the mirror reflections of the satellite pictures at the exposure instant) as the orientation unknowns

have been compared, when all the orientation parameters are determined simultaneously. Through this investigation, it has been revealed that constraints regarding the exterior orientation of the overlapped mirror photographs can be replaced by coplanarity equations and object space information redundantly used. Finally, the orientation problem of mirror photographs has been analyzed for the case where the mirror distortions are not negligibly small. The results obtained are follows:

(1) Parameters describing the linear part of the mirror distortions are absorbed by the coefficients of the collinearity equations expressed in algebraic form, and

(2) Elements of the non-linear part of the mirror distortions can be determined from the coplanarity and model connection conditions of overlapped mirror photographs.

References

Kratky, V., 1975. Digital Modeling of Limbs in Orthopaedics. Photogrammetric Engineering and Remote Sensing, **41(6)**: 741–752.

Mikhail, E. M., 1968. Principles and Applications of Analytical Mirror Photogrammetry. Photogrammetric Engineering and Remote Sensing, **34(10)**: 1071–1078.

Okamoto, A., 1981. Orientation and Construction of Models, Part I: The orientation Problem in Close-Range Photogrammetry. Photogrammetric Engineering and Remote Sensing, **47(10)**: 1437–1454.

———, 1982a. Orientation and Construction of Models, Part IV: Further Considerations in Close-Range Photogrammetry. Photogrammetric Engineering and Remote Sensing, **48(8)**:

1353–1363.
———, 1986. Potential Theory in Photogrammetry. In Proceedings of the International Symposium of the ISPRS, Commission III, Rovaniemi, pp. 543–563.
Okamoto, A. and Akamatu, S., 1992. Fundamental Analytics of Mirror Photogrammetry. Paper submitted to the Editorial Office Memoirs of the Faculty of Engineering Kyoto University for possible publication.
Veress, S. A. and Munjy, R., 1983. Spatio-Temporal Position from Mirror Images. Photogrammetric Engineering and Remote Sensing, **49(2)**: 207–211.