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Fundamental Analytics of Mirror Photogrammetry

By

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Abstract

Mirror photogrammetry is discussed fundamentally in this paper. First, the relationship between an object and the satellite picture (a photograph taken of the virtual object) is investigated and an interesting property of the satellite picture is clarified that parameters defining the mirror surface in the object space coordinate system are absorbed only by the exterior orientation elements of the reflected satellite picture (the mirror reflection of the satellite picture). Based on this fact, the orientation problem of overlapped mirror photographs is solved for various configurations of cameras and mirrors. In addition, the orientation calculation using mirror parameters as the orientation unknowns is described by introducing the law of reflection in terms of direction cosines. The non-central projective parameters such as those defining lens distortion, effect of lack of film flatness, and mirror undulation are considered separately.

Introduction

In two-media photogrammetry, the refraction problem of imaging rays can be treated very easily by means of direction cosines (Rinner [1948, 1969]), Hoehle [1972]), Okamoto and Hoehle [1972]), Okamoto and Mori [1973]), and Okamoto [1982b, 1984]). The reflection problem of imaging rays in mirror photogrammetry may also be treated easily by using direction cosines, because the reflection and refraction are similar physical phenomena in optics. However, mirror photogrammetry is very different from two-media photogrammetry in that parameters describing refractive interfaces in the object space coordinate system pertain to the non-central projective ones and must thus be treated separately from the exterior and interior orientation elements of two-media photographs, while those of mirror surfaces belong to the central projective ones and are therefore absorbed by the coefficients of the collinearity condition relating an object point and its image point. Very little has been written that discusses mirror photogrammetry from such a point of view. Thus, the orientation problem of mirror photographs has not been fully solved, in particular, in the case of employing non-metric cameras.

In this paper, the relationship between mirror parameters and the exterior and interior orientation ones of a photograph taken by using the reflection of the mirror is explored

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precisely, the characteristics of the orientation problem of mirror photographs are explained for various configurations of plane mirrors and non-metric cameras, and the orientation calculation is formulated by means of the law of reflection expressed in terms of direction cosines.

Geometrical Characteristics of Satellite Pictures

The Collinearity Condition

We will assume that a plane mirror is placed close to an object in such a way that a camera registers not only the object but also its mirror reflection. The picture of the mirror reflection of the object is referred to as the satellite picture. The relationship between the object and the satellite picture will be considered as follows (See Fig. 1). When the camera records the object directly, the general collinearity condition is satisfied between an object point $P(X, Y, Z)$ and measured image coordinates (x_c, y_c) of its image point $p(x, y)$ as

$$x_c = \frac{A_1X + A_2Y + A_3Z + A_4}{A_9X + A_{10}Y + A_{11}Z + 1}$$
$$y_c = \frac{A_5X + A_6Y + A_7Z + A_8}{A_9X + A_{10}Y + A_{11}Z + 1} \qquad (1)$$

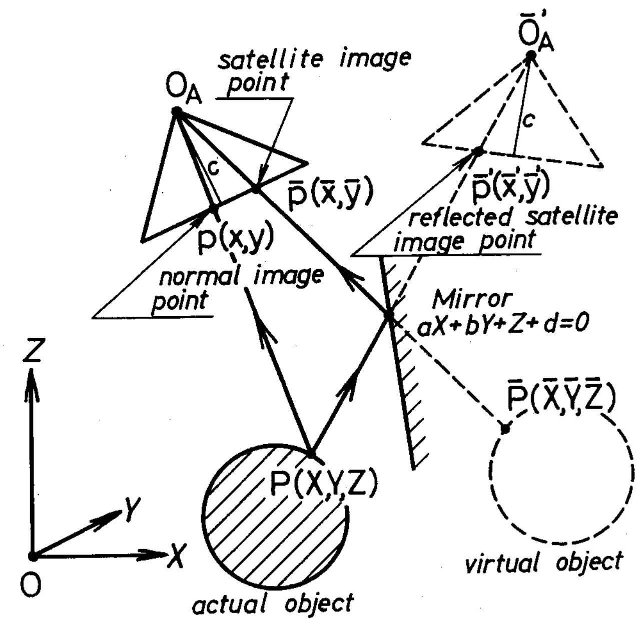

Fig. 1 Geometrical characteristics of a satellite picture for the case of using a camera and a plane mirror.

in which $A_i (i = 1,\ldots,11)$ are independent coefficients, if the measured image coordinates include linear systematic errors such as linear film deformation and linear measurement errors (Abdel-Aziz and Karara [1971], and Okamoto [1981, 1982a]). The same relationship is valid between the virtual object point $\bar{P}(\bar{X}, \bar{Y}, \bar{Z})$ (the corresponding point on the mirror reflection of the object) and measured coordinates $(\bar{x}_c, \bar{y}_c)$ of its satellite image point $\bar{p}(\bar{x}, \bar{y})$, i.e.,

$$\bar{x}_c = \frac{A_1\bar{X} + A_2\bar{Y} + A_3\bar{Z} + A_4}{A_9\bar{X} + A_{10}\bar{Y} + A_{11}\bar{Z} + 1} \qquad \bar{y}_c = \frac{A_5\bar{X} + A_6\bar{Y} + A_7\bar{Z} + A_8}{A_9\bar{X} + A_{10}\bar{Y} + A_{11}\bar{Z} + 1} \tag{2}$$

On the other hand, the relationship relating an actual object point $P(X, Y, Z)$ and its virtual object point $\bar{P}(\bar{X}, \bar{Y}, \bar{Z})$ is given by Veress and Munjy [1983] in the form

$$\begin{aligned} X &= u_1\bar{X} + u_2\bar{Y} + u_3\bar{Z} + u_4 \\ Y &= v_1\bar{X} + v_2\bar{Y} + v_3\bar{Z} + v_4 \\ Z &= w_1\bar{X} + w_2\bar{Y} + w_3\bar{Z} + w_4 \end{aligned} \tag{3}$$

which belong to the three-dimensional affine transformation. Also, the coefficients u_i, v_i, w_i $(i = 1,\ldots,4)$ are functions of three parameters (a, b, d) of the mirror plane, if it is expressed in the object space coordinate system (X, Y, Z) as

$$aX + bY + Z + d = 0 \tag{4}$$

The inverse transformation of Equation 3 yields

$$\begin{aligned} \bar{X} &= \bar{u}_1X + \bar{u}_2Y + \bar{u}_3Z + \bar{u}_4 \\ \bar{Y} &= \bar{v}_1X + \bar{v}_2Y + \bar{v}_3Z + \bar{v}_4 \\ \bar{Z} &= \bar{w}_1X + \bar{w}_2Y + \bar{w}_3Z + \bar{w}_4 \end{aligned} \tag{5}$$

We substitute Equation 5 into Equation 2 to obtain

$$\bar{x}_c = \frac{\bar{A}_1X + \bar{A}_2Y + \bar{A}_3Z + \bar{A}_4}{\bar{A}_9X + \bar{A}_{10}Y + \bar{A}_{11}Z + 1} \qquad \bar{y}_c = \frac{\bar{A}_5X + \bar{A}_6Y + \bar{A}_7Z + \bar{A}_8}{\bar{A}_9X + \bar{A}_{10}Y + \bar{A}_{11}Z + 1} \tag{6}$$

Coefficients $\bar{A}_i (i = 1,\ldots,11)$ of Equation 6 are obviously independent, because they include the independent coefficients $A_i (i = 1,\ldots,11)$ of Equation 2. From this fact it can be seen that the general collinearity condition is also valid between the actual object point $P(X, Y, Z)$ and the measured coordinates $(\bar{x}_c, \bar{y}_c)$ of its satellite image point $\bar{p}(\bar{x}, \bar{y})$. It follows that the orientation problem of a single satellite picture can be treated in the same manner as that of a conventional photograph. However, the position and attitude of the satellite picture at the exposure instant cannot be reconstructed, since Equations 2 and 6 have different coefficients, respectively.

Characteristics of Parameters Defining the Mirror Plane

In the preceding paragraph, we have seen that elements defining the mirror plane are absorbed by the orientation parameters of the satellite picture after the orientation. For the analysis of the orientation problem of the normal and satellite pictures which are taken on the same film at the same time, as is illustrated in Fig. 1, we need to explore more precisely the relationship between the three parameters of the mirror plane and the orientation elements of the satellite picture after the orientation (the photograph taken without the reflection of the mirror is termed here the normal picture.) For this purpose, we will employ the mirror reflection of the satellite picture at the exposure instant, which is referred to as the reflected satellite picture, because the collinearity condition relating an actual object point $P(X, Y, Z)$ and its image point $\bar{p}'(\bar{x}', \bar{y}')$ on the reflected satellite picture can be considered much more easily (See Fig. 1). First, the relationship between measured coordinates $(\bar{x}_c, \bar{y}_c)$ of a satellite image point $\bar{p}(\bar{x}, \bar{y})$ and the image coordinates $(\bar{x}', \bar{y}')$ of its corresponding point $\bar{p}'$ on the reflected satellite picture will be investigated. In the general case where the geometry of a picture is determined by 11 independent central projective parameters, the five interior orientation parameters can be selected as the planimetric coordinates (x_{c0}, y_{c0}) of its projection center referred to the comparator coordinate system, its principal distance c, and two rotation parameters (α, β) defining the relationship between the picture and comparator coordinate systems (See Okamoto [1981].). The last two rotation parameters are introduced for the correction of linear systematic errors included in the measured picture coordinates. In the case of using a plane mirror, we can consider the five interior orientation elements of the reflected satellite picture to be identical to those of the satellite picture at the exposure instant. Thus, the satellite picture point $\bar{p}(\bar{x}, \bar{y})$ and the corresponding point $\bar{p}'(\bar{x}', \bar{y}')$ on the reflected satellite picture may be assumed to be imaged on the same film, as is demonstrated in Fig. 2. Also, the satellite picture point $\bar{p}(\bar{x}, \bar{y})$ is expressed with respect to the comparator coordinate system (x_c, y_c, z_c) in the form

$$\begin{aligned} \bar{x}_c &= d_{11}\bar{x} + d_{12}\bar{y} + (x_{c0} - d_{13}c) \\ \bar{y}_c &= d_{21}\bar{x} + d_{22}\bar{y} + (y_{c0} - d_{23}c) \end{aligned} \qquad (7)$$

in which $d_{ij}(i = 1, 2; j = 1, 2, 3)$ are functions of the rotation parameters α and β about the comparator coordinate axes y_c and x_c (See Okamoto [1981].). On the other hand, the following relationship

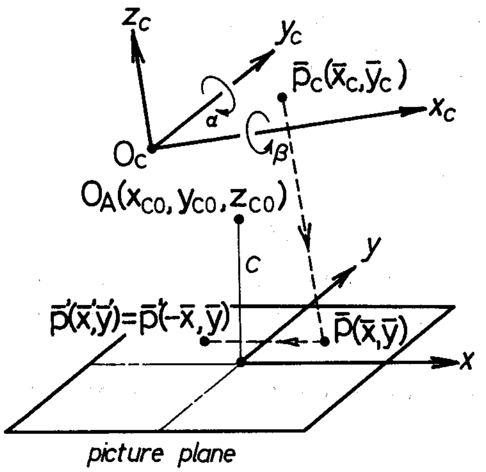

Fig. 2 The relationship between measured coordinates $(\bar{x}_c, \bar{y}_c)$ of a satellite image point $\bar{p}(\bar{x}, \bar{y})$ and image coordinates $(\bar{x}', \bar{y}')$ of the corresponding point $\bar{p}'$ on the reflected satellite picture.

$$\bar{x} = -\bar{x}', \qquad \bar{y} = \bar{y}' \tag{8}$$

is valid between $\bar{p}(\bar{x}, \bar{y})$ and $\bar{p}'(\bar{x}', \bar{y}')$. Then, Equation 7 can be rewritten as

$$\begin{aligned} \bar{x}_c &= -d_{11}\bar{x}' + d_{12}\bar{y}' + (x_{c0} - d_{13}c) \\ \bar{y}_c &= -d_{21}\bar{x}' + d_{22}\bar{y}' + (y_{c0} - d_{23}c) \end{aligned} \tag{9}$$

Equation 9 can be further modified as

$$\begin{pmatrix} \bar{x}' \\ \bar{y}' \end{pmatrix} = \begin{pmatrix} -d_{11} & d_{12} \\ -d_{21} & d_{22} \end{pmatrix}^{-1} \begin{pmatrix} \bar{x}_c & -(x_{c0} - d_{13}c) \\ \bar{y}_c & -(y_{c0} - d_{23}c) \end{pmatrix} = \begin{pmatrix} s_1\bar{x}_c + s_2\bar{y}_c + s_3 \\ s_4\bar{x}_c + s_5\bar{y}_c + s_6 \end{pmatrix} \tag{10}$$

in which $s_i (i = 1,\ldots,6)$ are functions of the five interior orientation parameters $(x_{c0}, y_{c0}, c, \alpha, \beta)$ which are common to the satellite and reflected satellite pictures.

From Equation 10, we can see that the image coordinates $(\bar{x}', \bar{y}')$ of point p' on the reflected satellite picture can be expressed in terms of only the five interior orientation parameters, which means that the relationship (Equation 10) does not contain the three elements (a, b, d) describing the mirror plane. Furthermore, with the image coordinates $(\bar{x}', \bar{y}')$ given, the orientation problem of the reflected satellite picture can be analyzed only with the exterior orientation parameters $(\bar{\phi}', \bar{\omega}', \bar{\kappa}', \bar{X}_0', \bar{Y}_0', \bar{Z}_0')$. Consequently, we can conclude that the three elements of the mirror plane are absorbed only by the six exterior orientation elements of the reflected satellite picture. In other words, the six exterior orientation parameters $(\bar{\phi}', \bar{\omega}', \bar{\kappa}', \bar{X}_0', \bar{Y}_0', \bar{Z}_0')$ are functions of the three parameters (a, b, d) of the mirror plane and the six exterior orientation parameters $(\phi, \omega, \kappa, X_0, Y_0, Z_0)$

of the satellite picture at the exposure instant, i.e.,

$$\bar{\tau}_j' = f_j(\tau_i(i = 1,\ldots,6),\ a,\ b,\ d) \qquad (j = 1,\ldots,6) \tag{11}$$

in which

$$\tau_i(i = 1,\ldots,6)\ :\ (\phi,\ \omega,\ \kappa,\ X_0,\ Y_0,\ Z_0)$$

$$\bar{\tau}_j'(j = 1,\ldots,6):\ (\bar{\phi}',\ \bar{\omega}',\ \bar{\kappa}',\ \bar{X}_0',\ \bar{Y}_0',\ \bar{Z}_0')$$

It follows that the mirror parameters (a, b, d) are absorbed by the six exterior orientation parameters $(\bar{\phi}, \bar{\omega}, \bar{\kappa}, \bar{X}_0, \bar{Y}_0, \bar{Z}_0)$ of the satellite picture after the orientation.

Orientation Problem of a Pair of Photographs with Plane Mirror

Basic Consideration

In this section, metric cameras are assumed to be used for the spatial determination of an object by means of a plane mirror. Also, the orientation problem of the mirror pictures will be discussed for the following two cases: the case where two different photographs are taken in such a way that the first camera registers the object directly and the second camera records only the virtual object (See Fig. 3) and the case where a metric camera registers not only the object but also its mirror reflection (See Fig. 4). In the former case, parameters defining the mirror plane in the object space coordinate system are absorbed by the exterior orientation elements of the reflected satellite picture $\bar{2}'$. Thus, employing the normal picture 1 and the reflected satellite picture $\bar{2}'$ enables one to perform

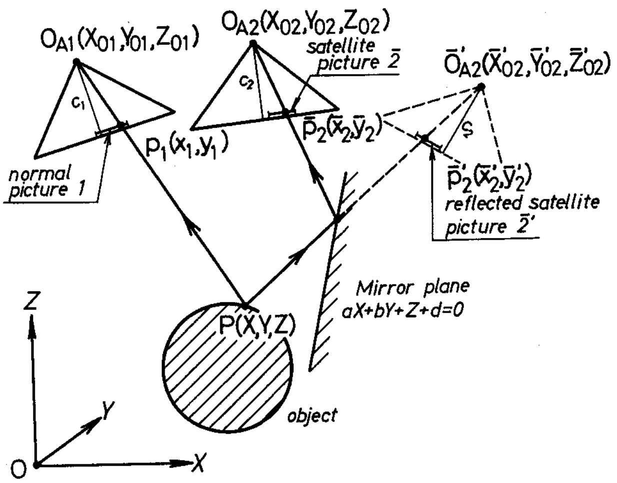

Fig. 3 The orientation problem of a stereopair of normal and satellite pictures 1 and $\bar{2}$ for the case of employing two cameras and one plane mirror.

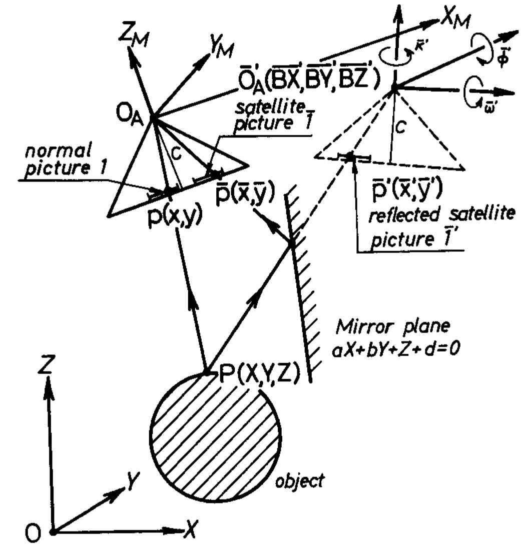

Fig. 4 The orientation problem of a stereopair of normal and satellite pictures 1 and $\bar{1}$ for the case of using a camera and a plane mirror. The six exterior orientation parameters ($\bar{\omega}'$, $\bar{\phi}'$, $\bar{\kappa}'$, $\overline{BX}'$, $\overline{BY}'$, $\overline{BZ}'$) of the reflected satellite picture $\bar{1}'$ are newly defined with respect to the model space coordinate system (X_M, Y_M, Z_M).

the orientation in a conventional manner. In addition, the mirror plane need not be determined in the object space coordinate system.

On the other hand, the orientation problem of the normal picture 1 and the reflected satellite photograph $\bar{1}'$ for the latter case must be analyzed in quite a different manner. Since the six exterior orientation parameters ($\bar{\phi}'$, $\bar{\omega}'$, $\bar{\kappa}'$, $\bar{X}_0'$, $\bar{Y}_0'$, $\bar{Z}_0'$) of the reflected satellite picture $\bar{1}'$ are functions of both the three parameters (a, b, d) of the mirror plane and the six exterior orientation elements (ϕ, ω, κ, X_0, Y_0, Z_0) of the normal picture 1 (the orientation parameters of the normal picture 1 are identical to those of the satellite picture $\bar{1}$ at the exposure instant), the reflected satellite picture $\bar{1}'$ has only three independent elements with respect to the normal picture 1. Under the assumption that the six exterior orientation elements of the normal picture 1 are equal to zero, those ($\bar{\phi}'$, $\bar{\omega}'$, $\bar{\kappa}'$, $\bar{B}_X'$, $\bar{B}_Y'$, $\bar{B}_Z'$) of the reflected satellite picture $\bar{1}'$ can be expressed in terms of only the three parameters (a, b, d) of the mirror plane as

$$\bar{\delta}_j' = f_j(a, b, d) \qquad (j = 1, \ldots, 6) \tag{12}$$

in which

$$\bar{\delta}_j'(j = 1, \ldots, 6): (\bar{\phi}', \bar{\omega}', \bar{\kappa}', \bar{B}_X', \bar{B}_Y', \bar{B}_Z')$$

Equation 12 means that we have three constraints among the six exterior orientation elements of the reflected satellite picture $\bar{1}'$, i.e.,

$$g_i(\bar{\phi}', \bar{\omega}', \bar{\kappa}', \bar{B}_X', \bar{B}_Y', \bar{B}_Z') = 0 \qquad (i = 1, 2, 3) \tag{13}$$

In addition, in the relative orientation process of a stereopair of photographs, the translation parameter along the X-axis of the model coordinate system can be taken arbitrarily. Hence, the five relative orientation elements of the reflected satellite picture $\bar{1}'$ and the three constraints among them are given, respectively, as follows:

$$\bar{\delta}_j' = f_j(a, b, d) \qquad (j = 1, \dots, 5) \tag{14}$$

and

$$g_i(\bar{\phi}', \bar{\omega}', \bar{\kappa}', \bar{B}_Y', \bar{B}_Z') = 0 \qquad (i = 1, 2, 3) \tag{15}$$

From Equations 14 and 15, we can see that one of the three parameters (a, b, d) describing the mirror plane in the model coordinate system can be taken arbitrarily in the relative orientation of the normal picture 1 and the reflected satellite picture $\bar{1}'$. It follows that the coplanarity condition of corresponding rays provides only two independent elements (Mikhail [1968] noted that the relative orientation of a stereopair of normal and satellite pictures can be determined by two independent elements, if the two photographs are taken on the same film.)

The Case of Using Non-Metric Cameras

Next, the orientation problem of mirror photographs will be discussed in the case of employing non-metric cameras instead of metric cameras. When we use two non-metric cameras and one plane mirror, as is shown in Fig. 3, the orientation theory in close-range photogrammetry can easily be applied to the analysis of the normal picture (the left picture) and the reflected satellite picture of the right photograph. On the other hand, when a non-metric camera registers both the object and its mirror reflection, the orientation problem of the normal and reflected satellite photographs must be considered under two kinds of constraints: the three constraints (Equation 13) among the exterior orientation parameters of the stereopair and constraints that the interior orientation parameters of the reflected satellite picture are identical to those of the normal picture. Also, these constraints have the following characteristics that

(i) The constraints due to the reflection of the mirror affect only the relative orientation of the stereopair. Thus, they are independent of the absolute orientation.
(ii) The constraints regarding the interior orientation of the stereopair are independent of the relative orientation of the stereopair and thus reduce the number of the absolute orientation parameters (Okamoto [1986]).

With the geometrical properties of the constraints to be introduced known, the orientation problem of the stereopair can readily be solved in the general case where a picture has 11 independent central projective elements and also in the usual case in close-range photogrammetry, where the geometry of a picture can be determined by nine independent central projective elements.

(1) The general case

In the general case of photogrammetry, a stereopair of photographs has 22 independent orientation unknowns. Seven among these 22 parameters are determined from the coplanarity condition of corresponding rays and these seven relative orientation elements can be classified into five exterior and two interior ones. Further, the general central projective transformation between the model and object spaces can be described in terms of the remaining 15 elements (seven exterior and eight interior orientation parameters). In mirror photogrammetry with a non-metric camera and a plane mirror, the normal and reflected satellite pictures have only 14 independent orientation unknowns because of the three constraints regarding the exterior orientation and the five constraints among the 10 interior orientation parameters of the stereopair. The three constraints due to the reflection of the mirror are absorbed by the coplanarity condition. Thus, the relative orientation of the stereopair can be determined by only four independent elements (two exterior and two interior orientation parameters). The remaining 10 unknowns are provided during the phase of the absolute orientation of the stereopair, because the five constraints regarding the interior orientation must be introduced in this process. From this fact we can see that the one-to-one correspondence between the model and object spaces can be determined uniquely with four control points (three points with the space coordinates known and a point with one of the space coordinates given).

(2) The usual case in close-range photogrammetry

In the usual case, a stereopair of conventional pictures have 18 independent orientation unknowns. However, the coplanarity condition can also provide seven independent elements. Therefore, the absolute orientation of the stereopair can be determined by the remaining 11 independent orientation parameters (seven exterior and four interior). In mirror photogrammetry under consideration, the number of independent orientation unknowns of the normal and reflected satellite pictures reduces to 12 owing to the three constraints due to the reflection of the mirror and the three constraints regarding the interior orientation of the stereopair. Four orientation elements (two exterior and two interior) can be provided from the coplanarity condition and the remaining eight are determined in the process of the absolute orientation of the stereopair. This is because the three constraints among the interior orientation parameters affect only the central projective one-to-one correspondence between the model and object spaces. The object space controls mathematically required are two points with the space coordinates known and one point with the planimetric coordinates given.

Use of Multiple Plane Mirrors

In the preceding section, the orientation problem of mirror photographs has been discussed in the case of using only one plane mirror. However, in many close-range

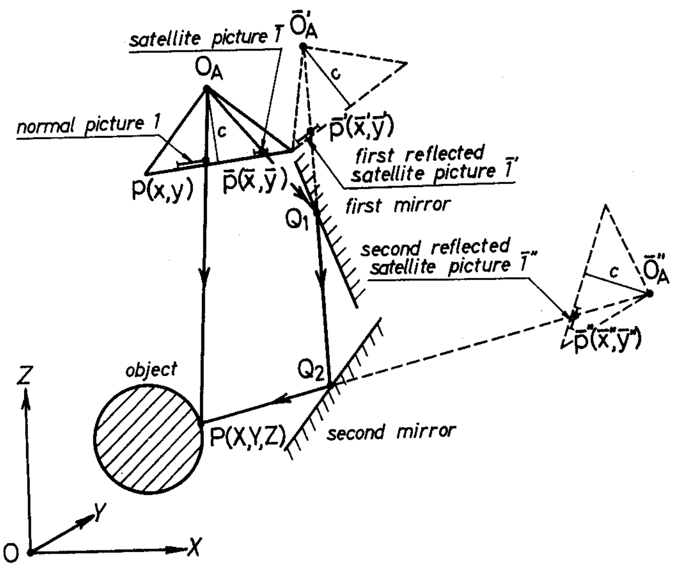

Fig. 5 The orientation problem of a stereopair of normal and satellite pictures 1 and $\bar{1}$ for the case of employing a camera and two plane mirrors.

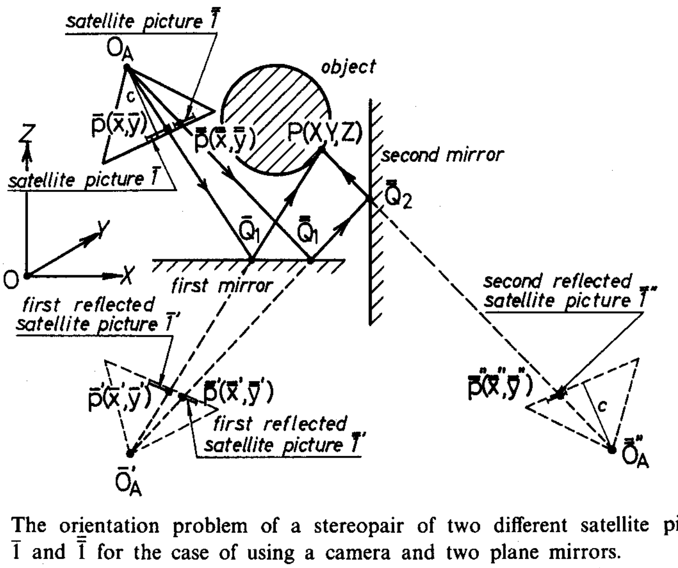

Fig. 6 The orientation problem of a stereopair of two different satellite pictures $\bar{1}$ and $\bar{\bar{1}}$ for the case of using a camera and two plane mirrors.

applications of mirror photogrammetry, the situation may often occur that we must employ multiple mirrors, as is demonstrated in Figs. 5 and 6. Considering that parameters defining the mirror planes in the object space coordinate system are absorbed by only exterior orientation elements of the reflected satellite pictures, it can be seen that these parameters affect only the relative orientation process of a stereopair of mirror photographs. In this section, the characteristics of the relative orientation of mirror photographs will be explained in the case where a camera records the object not only directly but also by using the reflection of multiple mirrors.

First, we will assume that two plane mirrors are placed in the object space in such a way that the normal and satellite pictures can be taken with the same camera at the same time (See Fig. 5). With respect to the normal picture 1, the reflected satellite picture $\bar{1}''$ regarding the second mirror has six independent orientation elements which are functions of the six parameters describing the two mirrors in the object space coordinate system. It will also be noted that interior orientation parameters of the reflected satellite picture $\bar{1}''$ are identical to those of the normal picture 1 and further that such constraints are independent of the relative orientation of the stereopair. Thus, the coplanarity condition of corresponding rays provides seven independent orientation elements (five exterior and two interior) in the general and usual cases of photogrammetry, and only five exterior orientation elements in the case of metric photogrammetry.

The discussion mentioned above can also be verified as follows (See Fig. 5). In general, the projection center 0_A of the picture taken, an object point P, and the reflection point Q_1 on the first mirror, and the reflection point Q_2 on the second mirror do not lie in the same plane. Thus, the parallel translation of the second mirror along the Z-axis of the object space coordinate system (X, Y, Z) prevents the corresponding rays $\overrightarrow{0_A P}$ and $\overrightarrow{Q_2 P}$ from intersecting. This means that three elements defining the second mirror plane with respect to the first mirror plane are necessary for constructing the stereo model. Further, the relative position and attitude of the first mirror plane with respect to the picture taken are given by two independent parameters. Consequently, in the case of metric photography, five independent orientation parameters are determined during the phase of the relative orientation of the normal picture 1 and the reflected satellite picture $\bar{1}''$.

Next, two plane mirrors are so arranged that a camera can register an object not only with the first mirror but also by using the reflection of two mirrors, as is illustrated in Fig. 6. In the orientation problem of a stereopair of pictures taken in such a mirror configuration, the first reflected satellite pictures $\bar{1}'$ and $\bar{\bar{1}}'$ may be regarded as the taken picture itself. Then, we can analyze the orientation problem of the two satellite pictures $\bar{1}$ and $\bar{\bar{1}}$ on the same film as that of the first reflected satellite picture $\bar{1}'$ and the second reflected satellite picture $\bar{\bar{1}}''$. Accordingly, this orientation problem can be solved in the same manner as for the normal and reflected satellite pictures 1 and $\bar{1}'$ in Fig. 4. In the case of metric photography, the stereo model can be constructed by determining two independent orientation parameters from the coplanarity condition of corresponding rays. The discussion on the case of using a non-metric camera in this mirror configuration is omitted here (See the preceding section.).

The explanation above can also be confirmed by exploring the geometrical properties of the corresponding rays $\overrightarrow{\bar{Q}_1P}$ and $\overrightarrow{\bar{\bar{Q}}_2P}$ in Fig. 6. Regarding the original photograph as the reference picture, the ray $\overrightarrow{\bar{Q}_1P}$ can be described with three parameters defining the first mirror plane and the corresponding ray $\overrightarrow{\bar{\bar{Q}}_2P}$ is a function of both the three elements of the first mirror and three for the second mirror plane. Thus, the ray $\overrightarrow{\bar{\bar{Q}}_2P}$ has only three independent elements with respect to $\overrightarrow{\bar{Q}_1P}$. In addition, a scale factor is involved in these parameters. Consequently, the intersection of the corresponding rays can be accomplished by providing two independent elements of the second mirror plane.

The case of using more than two plane mirrors can be treated in the same manner as above, because all parameters defining the mirror planes are absorbed by exterior orientation parameters of the reflected satellite pictures.

The Orientation Calculation

This section describes the orientation calculation of mirror photographs for the usual case in close-range photogrammetry, where a picture has nine independent central projective elements (the six exterior and three interior orientation ones). Also, in many close-range applications of mirror photogrammetry, absolute positioning with respect to the reference coordinate system is not a matter of major importance. Of greater importance is the relative positioning accuracy of points on the surface to be measured. The construction of a stereo model congruent to the object can meet this requirement perfectly. Thus, the object space control is established only in terms of distances. Furthermore, the treatment in this section has the following characteristics that

(i) The collinearity equations relating a model point and measured coordinates of its image point are employed as the determination equations, because the orientation calculation of solving the collinearity equations is much easier than using the coplanarity and similarity conditions.

(ii) The law of reflection is introduced, because constraints among the exterior orientation parameters of the reflected satellite picture are very difficult to express in functional form.

The Case of Using a Non-Metric Camera and a Plane Mirror

First, we will select a simple configuration of a non-metric camera and a plane mirror as is illustrated in Fig. 7. The model coordinate system (X_M, Y_M, Z_M) is taken as a right-handed, rectangular-Cartesian system with its origin at the projection center of the

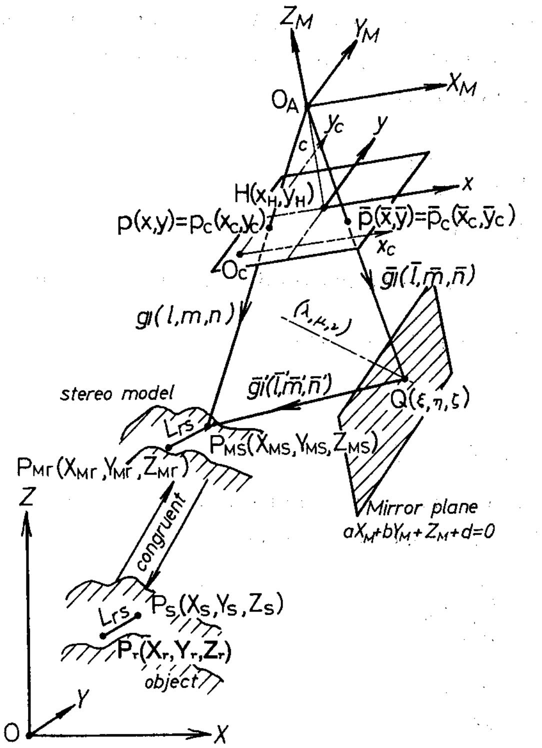

Fig. 7 Orientation calculation for constructing a stereo model congruent to the object using a stereopair of normal and satellite pictures taken by means of a non-metric camera and a plane mirror.

picture and further with its X_M-Y_M plane parallel to the picture plane. Thus, six exterior orientation parameters of the picture are equal to zero with respect to the model space coordinate system. Regarding the interior orientation parameters of the picture, the principal point coordinates are defined as (x_H, y_H) in the measured image coordinate system and the principal distance is designated by c. By expressing measured plate coordinates of a normal picture point p as (x_c, y_c) and those of the corresponding point $\bar{p}$ on the satellite picture as $(\bar{x}_c, \bar{y}_c)$, the collinearity condition between the model point $P_M(X_M, Y_M, Z_M)$ and the measured plate coordinates will be given for the normal and (reflected) satellite pictures, respectively, as follows.

With regard to the normal picture, an imaging ray $g(l, m, n)$ is described in the model space coordinate system as

$$g: \frac{X_M}{l} = \frac{Y_M}{m} = \frac{Z_M}{n} \tag{16}$$

in which (l, m, n) denote direction cosines of the ray g. In order to obtain the direction cosines, we will first find space coordinates $({}_MX_p, {}_MY_p, {}_MZ_p)$ of the normal picture point p in the model space coordinate system, which are termed the transformed picture coordinates. They are given in the form

$$\begin{pmatrix} {}_MX_p \\ {}_MY_p \\ {}_MZ_p \end{pmatrix} = \begin{pmatrix} x_c - x_H \\ y - y_H \\ -c \end{pmatrix} \tag{17}$$

The direction cosines (l, m, n) can be described by means of Equation 17 as

$$l = {}_MX_p/A, \quad m = {}_MY_p/A, \quad n = {}_MZ_p/A \tag{18}$$

in which

$$A = \sqrt{{}_MX_p^2 + {}_MY_p^2 + {}_MZ_p^2}$$

Using the condition that the imaging ray must pass through the corresponding model point $P_M(X_M, Y_M, Z_M)$ leads to the collinearity condition:

$$\begin{aligned} X_M &= \frac{1}{n} Z_M = \frac{{}_MX_p}{{}_MZ_p} Z_M \\ Y_M &= \frac{m}{n} Z_M = \frac{{}_MY_p}{{}_MZ_p} Z_M \end{aligned} \tag{19}$$

Also, Equation 19 includes only three unknown orientation parameters (x_H, y_H, c).

On the other hand, the imaging ray for the satellite picture is subject to the reflection of the mirror. The ray before reflection is designated by $\bar{g}(\bar{l}, \bar{m}, \bar{n})$ and can be given in the same manner as the imaging ray g for the normal picture, i.e.,

$$\bar{g}: \frac{X_M}{\bar{l}} = \frac{Y_M}{\bar{m}} = \frac{Z_M}{\bar{n}} = \bar{\rho} \tag{20}$$

in which $(\bar{l}, \bar{m}, \bar{n})$ are direction cosines of $\bar{g}$ and $\bar{\rho}$ is an auxiliary parameter. The transformed picture coordinates $({}_MX_{\bar{p}}, {}_MY_{\bar{p}}, {}_MZ_{\bar{p}})$ of the satellite picture point $\bar{p}$ and the direction cosines are described, respectively, as follows.

$$\begin{pmatrix} {}_MX_{\bar{p}} \\ {}_MY_{\bar{p}} \\ {}_MZ_{\bar{p}} \end{pmatrix} = \begin{pmatrix} \bar{x}_c - x_H \\ \bar{y}_c - y_H \\ -c \end{pmatrix} \tag{21}$$

and

$$\bar{l} = {}_MX_{\bar{p}}/\bar{A}, \quad \bar{m} = {}_MY_{\bar{p}}/\bar{A}, \quad \bar{n} = {}_MZ_{\bar{p}}/\bar{A} \tag{22}$$

in which

$$\bar{A} = \sqrt{{}_M X_{\bar{p}}^{\ 2} + {}_M Y_{\bar{p}}^{\ 2} + {}_M Z_{\bar{p}}^{\ 2}}$$

The ray $\bar{g}'(\bar{l}', \bar{m}', \bar{n}')$ after the reflection is found by using the law of reflection. The mirror plane is expressed in the model space coordinate system in the form

$$aX_M + bY_M + Z_M + d = 0 \tag{23}$$

By means of Equations 20 and 23, we can calculate space coordinates (ξ, η, ζ) of the reflection point Q at which the ray $\bar{g}$ before reflection intersects the mirror plane, i.,e.,

$$\xi = \bar{\rho}\bar{l}, \quad \eta = \bar{\rho}\bar{m}, \quad \zeta = \bar{\rho}\bar{n} \tag{24}$$

in which

$$\bar{\rho} = \frac{-d}{a\bar{l} + b\bar{m} + \bar{n}}$$

Further, direction cosines (λ, μ, ν) of the normal to the mirror plane become

$$(\lambda, \mu, \nu) = \left(\frac{a}{a^2 + b^2 + 1}, \ \frac{b}{a^2 + b^2 + 1}, \ \frac{1}{a^2 + b^2 + 1}\right) \tag{25}$$

which is constant for all points on the mirror plane. By introducing the direction cosines $(\bar{l}, \bar{m}, \bar{n})$ of the ray $\bar{g}$ before reflection and those (λ, μ, ν) of the normal to the mirror plane into the law of reflection, we obtain direction cosines $(\bar{l}', \bar{m}', \bar{n}')$ of the ray $\bar{g}'$ after reflection as

$$\begin{aligned} \bar{l}' &= \bar{l} - 2\lambda\cos i \\ \bar{m}' &= \bar{m} - 2\mu\cos i \\ \bar{n}' &= \bar{n} - 2\nu\cos i \end{aligned} \tag{26}$$

in which

$$\cos i = \bar{l}\lambda + \bar{m}\mu + \bar{n}\nu$$

The reflection point $Q(\xi, \eta, \zeta)$ and the direction cosines $(\bar{l}', \bar{m}', \bar{n}')$ of the ray $\bar{g}'$ after reflection having been given, we can construct the equation of $\bar{g}'$ in the form

$$\bar{g}' : \frac{X_M - \xi}{\bar{l}'} = \frac{Y_M - \eta}{\bar{m}'} = \frac{Z_M - \zeta}{\bar{n}'} \tag{27}$$

which is equivalent to the equation of the (fictitious) imaging ray for the reflected satellite picture. Then, the collinearity condition regarding the reflected satellite picture can be expressed as

$$X_M = \frac{\bar{l}'}{\bar{n}'}(Z_M - \zeta) + \xi$$
$$Y_M = \frac{\bar{m}'}{\bar{n}'}(Z_M - \zeta) + \eta \tag{28}$$

Also, Equation 28 includes six orientation unknowns (x_H, y_H, c, a, b, d).

Writing down together the collinearity euations for the normal and reflected satellite pictures, we obtain the determination equations in the construction problem of a stereo model congruent to the object, i.,e.,

$$X_M = \frac{l}{n} Z_M, \qquad Y_M = \frac{m}{n} Z_M$$
$$X_M = \frac{\bar{l}'}{\bar{n}'}(Z_M - \zeta) + \xi, \qquad Y_M = \frac{m'}{\bar{n}'}(Z_M - \zeta) + \eta \tag{29}$$

Equation 29 contains mathematically one equation equivalent to the coplanarity condition of corresponding rays. Also, the coplanarity condition provides four independent orientation parameters (two exterior and two interior). However, a stereo model constructed only by means of the coplanarity condition is not similar to the object. In order to make the stereo model congruent to the object, the following expression

$$L_{rs} = \sqrt{(X_{Mr} - X_{Ms})^2 + (Y_{Mr} - Y_{Ms})^2 + (Z_{Mr} - Z_{Ms})^2} \tag{30}$$

must be valid for two distances given as the object space control, because the absolute orientation of the stereopair requires three control points (two points with the space coordinate given and one point with the planimetric coordinates known) and the degrees of freedom of such three points is two. Also, in the above expression, L_{rs} denotes a distance from point r to point s in the object space and the right hand side is the corresponding length described in terms of the model space coordinate system (X_M, Y_M, Z_M).

The six orientation unknowns (x_H, y_H, c, a, b, d) can be computed as follows. We set up Equation 29 for four model points at both ends of the two distances and Equation 30 for two lengths given. The space coordinates of the four model points are treated as unknowns in this orientation calculation. Then, we obtain 18 independent equations with respect to 18 unknowns (the six orientation parameters plus 12 unknown coordinates of the four model points).

The Case of Using Two Non-Metric Cameras and Two Plane Mirrors

A photogrammetric model reconstruction of round-shaped objects must be based on the availability of more than one stereopair of photographs to allow viewing of the object in its entirety. For this purpose, Kratky [1975] devised such an arrangement of two cameras

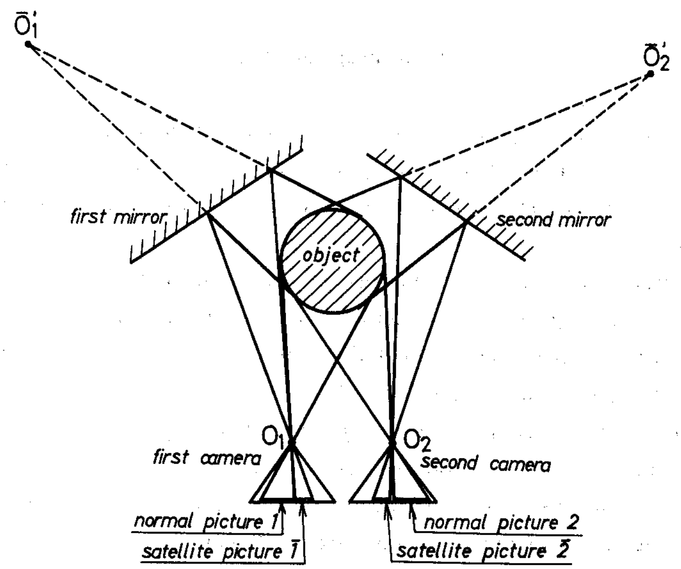

Fig 8 Orientation calculation for constructing a united stereo model congruent to the round-shaped object using two normal and two satellite pictures taken by means of two non-metric cameras and two plane mirrors.

and two plane mirrors as is shown in Fig. 8. Assuming that two different non-metric cameras are used in this measurement, we can discuss the problem of forming a united stereo model congruent to the object by using the two normal pictures and two reflected satellite pictures as follows. The parameters to be determined are six exterior orientation elements of the second camera, six interior orientation ones of the two non-metric cameras, and six coefficients describing the two plane mirrors, respectively, in the model space coordinate system, because six independent parameters of the two reflected satellite pictures can be replaced with the six coefficients of the two mirror planes by introducing the law of reflection. Further, we have six constraints regarding the interior orientation of the four overlapped pictures saying that the interior orientation elements of the reflected satellite photographs $\bar{1}'$ and $\bar{2}'$ are identical to those of the normal pictures 1 and 2, respectively. According to the potential theory (Okamoto [1986]), the introduction of such six constraints makes it possible to determine six unknown interior orientation parameters of the four pictures overlapped. Thus, a united model similar to the round-shaped object can be constructed without object space information. It follows that we can form the united stereo model congruent to the object, if only a distance is available as control. For further detailed discussion on this united model construction refer to the paper by Okamoto (1986)).

Concluding Discussions

Mirror photogrammetry has been discussed fundamentally in this paper. First, the relationship between an object and the satellite picture (a photograph taken of the mirror reflection of the object) has been explored and the following interesting facts have been revealed that:

(1) The collinearity condition is valid between the object and the satellite picture.

(2) Parameters defining the mirror surfaces in the object space coordinate system are absorbed only by the exterior orientation elements of the reflected satellite picture (the mirror reflection of the satellite picture).

(3) Consequently, the orientation of a single mirror photograph can be performed in the same manner as that for a conventional picture.

Based on the characteristics of the satellite picture found, we can solve the orientation problem of a stereopair of mirror photographs as follows.

(1) In the case of employing only one plane mirror

In this case a camera records not only the object directly but also its mirror reflection. The coplanarity condition of corresponding rays for the normal and satellite photographs provides only two exterior orientation parameters for the case of metric photography, two exterior and two interior orientation elements in the usual case in close-range photogrammetry, where the geometry of a picture is determined by nine independent central projective parameters. In the general case of photogrammetry, where a photograph has eleven central projective parameters, two exterior and two interior orientation elements can also be determined from the coplanarity condition. Accordingly, the one-to-one correpondence relating the model and object spaces can be described by seven exterior orientation parameters in the case of metric photography, by eight orientation parameters in the usual case in close-range photogrammetry, and by ten in the general case.

(2) In the case of adopting multiple plane mirrors

When a camera registers the object not only directly but also by means of more than one plane mirrors, no constraints are generated among the exterior orientation elements of the satellite picture. Hence, the orientation of the normal and satellite pictures can be carried out in the same manner as for a stereopair of conventional photographs. However, when we have two different satellite pictures on the same film, the stereopair must be analyzed by means of the method described in (1) or by the technique above. The selection of the orientation methods depends on the arrangement of plane mirrors located in the path of imaging rays for the two satellite pictures, respectively.

When we have constraints among the exterior orientation parameters of satellite pictures, the orientation calculation for overlapped mirror photographs can be formulated easily and generally by introducing the law of reflection expressed in terms of direction cosines, because we can select elements describing the mirror planes in the orientation coordinate system as the orientation unknowns in place of the dependent exterior orientation parameters of the satellite pictures.

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