Solving Dynamical Models Qualitatively : Causality Built in Dynamical Models : Topic : Common Sense Reasoning (simulation)

ISHIDA, Yoshiteru

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by

Yoshiteru ISHIDA

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Abstract

We define the "causality" which requires "time reference". With this causality, causal reasoning is carried out by verifying that any change is made by the causality. The verification is done by the comsumption of dt = + for each step. In order to make such causal reasoning possible, we carefully choose the base model, i.e. the dynamical model, which describes causality from what makes the change to what is changed. We also developed the qualitative similation algorithm using the causality built in the dynamical model. The power of the causality and simulation algorithm is demonstrated on two examples of the pressure regulator and the mass-spring system.

1 Introduction

Qualitative reasoning [1, 2] and causal reasoning [3] have been studied recently to give a causal account for the behavior of processes and devices without using conventional physics. They pointed out that using logical proof for making causal accounts has some undesirable features. They also proposed *mythical causality*, which summarizes physical action at a lower level. Iwasaki and Simon [3] used a method of causal ordering in a static model to determine the direction from cause to effect. Variables in a set of equations are successively determined by a method of Gaussian elimination starting from given values of variables imposed as exogenous conditions. Thereby the variables are ordered in solving the equations. De Kleer and Bobrow use higher order derivatives in qualitative reasoning of change [4]. Kuipers [5] also developed a simulation algorithm taking higher order derivatives into account. We studied a causality characterized by "the time reference" other than event dependency for the discussion of physical causality. Physical causality (or equivalently "change" through physical time) is

Division of Applied Systems Science
Kyoto University, Kyoto 606 Japan

intrinsically embedded in a dynamical model which states the causal relation between what is changed and what makes the change.

Our way of qualitative reasoning is different from theirs in the following two points:

(1) In reasoning; we defined another causality which refers to time strictly. Reasoning is done by tracking the behavior along the causality.

(2) In modeling; since we use the causality built in the dynamical model, we skip the qualitative modeling process. That is, we use a dynamical model as a qualitative model and solve it qualitatively.

Section 2 discussed the causality on the dynamical model. That causality is defined in terms of physical time. A cause-effect sequence is obtained by propagating signs along the time on the dynamical models. Section 3 discusses dynamical models as qualitative models. Section 4 presents a qualitative simulation algorithm based on the causality defined.

2 Causality built in Dynamical Models

2.1 Causality referring to time

Causality has been discussed without a formal definition. This has raised much confusion as to the difference between causality and logical inducibility. We call it "the causality" to distinguish it from the conventional one. The causality has the following two requirements, which seem intuitively sound for a causality for the discussion of dynamical change. When we say "the event A caused the event B", we must admit

(1) Time Reference: The even A occurred "before" the event B,

(2) Event Dependency: The occurrence of event B must be "dependent on" the occurrence of event A.

The "time reference" plays a crucial role in making a clear distinction between "the causality" and logical inducibility or algebraic derivability. In the original dynamical model of the form:

Measure of "change" = Power to make that "change"

contains the "built-in causal" direction from the right hand side to the left hand side. We restrict ourselves to interpretation of the form $dY/dt = X$ as follows: $X > 0$ caused $(dt > 0)$ or is capable of causing the event of Y increase $(dY > 0)$ and not in the opposite way as to the causality. The consumption of time dt should be claimed to verify the "built-in causality".

2.2. Causality in dynamical models

We formalize the "causality" by the propagation of signs in the dynamical model. In the propagation, time reference is included, since $dt = +$ is always needed to be consumed to conclude the causation.

Example 2.1. We use the same example of a pressure regulator as in that of [1]. In some level of abstraction, we have the variables expressing physical quantities as in Fig. 1. We can identify the causality in the feedback path. The flow also is caused by a driving force and by the available area for the flow. Further, the pressure at a point is caused by the flow through the point.

$$dX_s/dt = -a \cdot Po$$

$$dQ/dt = b \cdot (DP - c \cdot Q^2/Xs)$$

$$dPo/dt = e \cdot (Q^2 - f \cdot Po)$$

$$DP = Pi - Po$$

where a, b, c, e, and f are appropriately chosen positive constants. Note that this model is valid only in a restricted situation. For example, the flow must not be reversed, the exit should not be completely blocked and so forth.

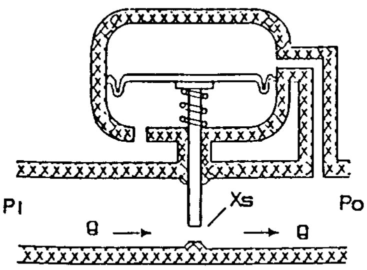

Fig. 1 Pressure Regulator [1]

Xs : availabel area for the flow through the valve

Po : pressure at outlet

Pi : pressure at inlet

DP : pressure drop across the value

Q : flow to the value

We can induce a cause-effect sequence by this model. Suppose Pi is disturbed ($\delta Pi = +$)[1] when the system is in a stationary state (all the time derivatives are zeros) then the initial sign vector is (δPi, δPo, δDP, δQ, δXs) = (+, 0, +, 0, 0). This in turn, has the effect of increasing Q. At the expense of dt = +, this will cause (+, 0, +, +, 0) at time t0 + dt (t0 is the initial time). Propagating the sign in the same manner, and by the consumption of dt, $\delta Xs = -$ successfully results as show in Table 1. So far we haven't needed indirect proof. The necessity of indirect proof such as "reductio ad absurdum" (R.A.A.) in [1] may be due to the incomplete

[1] δx denotes the variance from the equilibrium point of x.

Table 1. Qualitative state transition of pressure regulator exampe

q-time	δPi	dPi/dt	δPo	dPo/dt	δQ	dQ/dt	δXs	dXs/dt
0	+	0	0	0	0	0	0	0
0	+	0	0	0	0	+	0	0
1	+	0	0	0	+	+	0	0
1	+	0	0	+	+	?	0	0
2	+	0	+	+	?	?	0	0
2	+	0	+	?	?	?	0	—
3	+	0	?	?	?	?	—	—
3	+	0	?	?	?	?	—	?
4	+	0	?	?	?	?	?	?

δx indicates the variance from the initial stationary state $x=x0$ A qualitative simulation showing the consequence of $\delta Pi=+$

setting of initial condition and ad hoc modeling.

As another case, we will see the result of $\delta Po = +$ and no other disturbance initially. Then this will cause $\delta Xs = -$, $\delta Qi = -$. Po and hence DP become ambiguous. In the next step, all the change except $\delta Pi = 0$ become ambiguous. In the context this model describes, Pi is not affected by other variables. We cannot make any further propagation because of ambiguity. Interestingly, we can know why $dPo = +$ is obtained by propagating signs conversely along time. For the assignment (0, +, −, 0, 0), we do not find the assignment which consistently causes this assignment. Thus we conclude that this assignment is impossible in the context with which this model is concerned. However, the assignment (+, +, ?, ?, −) can be produced as the causal sequence of (0, 0, 0, +, 0). We will present an algorithm for qualitative simulation based on the built-in causality in the next section.

Many models can be built for the same target system. However, in order to discuss the built-in causality, dynamical models are needed. We cannot use static equations to discuss the causality. For example, the definitional equation of DP = Pi − Po has nothing to do with time (and hence causality). Although we can make a dynamical model by simply taking derivatives of both sides, it does not give any "causal" account. Further, when DP = p1 and Po = p0 are given, although we may say Pi = p2(= p1 + p0) is derived, we cannot say that Pi = p2 is "caused".

As for another example indicating the dynamical model is necessary, we cannot use a steady state equation: $Q^2 = f \cdot Po$ which is obtained by assuming $dPo/dt = 0$ in $dPo/dt = e \cdot (Q^2 - f \cdot Po)$. In fact, this equation only holds at the snapshot of the equilibrium point of dPo/dt = 0, thus the equation cannot be used to discuss the change using dPo/dt as done in [1]. In order to discuss the change of Po we must start from a dynamical model.

3 Dynamical Model as a Qualitative Model

Kuipers' theory [5] starts from abstracting the mathematical model preserving qualitative information in the model. The original dynamical model in which the causality is built-in is constructed by observing the phyical system in terms of (dynamical) effects among the physical entities (as opposed to observing the physical system as constraint among the physical entities). As already discussed, the original dynamical model has desirable "causality" built-in the model itself.

The original dynamical model is modeled whenever the causal direction that A = + together with time consumption $dt > 0$ will make (or have the effect) dB = +, is identified then it is described as: dB/dt = A + ...

Thus, we use the original dynamical model as our qualitative model, and solve it qualitatively in a similar manner to the numerical methods such as Newton-Raphson method for solving dynamical models.

We have shown that the cause-effect sequences based on the built-in causality are obtained by propagating signs of the states as well as dt = +. Although causal direction from a state to the other states can be obtained in the causal world, the constraints which come from the quantitative world are needed to see which variable reaches zero (or other landmark values) first among variables approaching zero. Constraints are used for ranking which variable reaches the specific point first. We will use whatever dynamical models, which can be equivalently derived from the original dynamical model, for such constraints. However, we use the following two heuristics to select the other dynamical models. (a) When x is approaching α and y is approaching β, and it is unknown which reaches first from the original model only, then another model must be derived by introducing a new variable $R = (x - \alpha)/(y - \beta)$. (b) When the convergence of the variable x *to* x_0 is of interest, then another model must be derived by introducing a new variable $k_1(x - x_0)^2 + k_2(d(x - x_0)/dt)^2 + \cdots$.

We used the triple value separated by the point as in [6] for the qualitative values of a variable and its higher order derivatives. The variable and its derivatives may have many points as their landmark values. For example, R has the important points λ_1, λ_2 in the example 4.1.

4 Qualitative Simulation

In order to clearly separate propagation through constraints and change which requires the consumption of dt = +, the algorithm is divided into two phases; static propagation and dynamic change. The former is the propagation of signs among variables which does not consume dt = +, while the latter is the change from a state to the other state.

4.1 The algorithm

The transient state (as opposed to the stationary state) of a variable is defined as the state in which some level of the state will be changed because of the non-zero of the higher derivatives. For example, $Q = -$, $dQ/dt = +$, $d^2Q/dt^2 = +$ (or 0) is the transient state, since Q will become zero in some lag time. The variable whose qualitative value must become zero earlier (this occurs by the heuristic (3) below) than the other transient state is also the transient state.

In order to determine which variable reaches the specific point first for variables in transient state, we use another constraint which is obtained from the original model by the model selection heuristic (a).

Heuristics used to specify which variable in a transient state changes first, are as follows:

(1) $(d^{n-1}x/dt^{n-1}, d^nx/dt^n) = (0, +)$ or $(0, -)$ then it must change to $(+, +)$ or $(-, -)$. This is the instant change rule [4].

(2) $(d^{n-1}x/dt^{n-1}, d^nx/dt^n, d^{n+1}x/dt^{n+1}) = (x, y, z)$ where $x = +$ or $-$, $y = -x$ and $z = 0$ or y (transient state), then it must change to $(0, y, z)$ under the condition that z (and hence y) does not change before that change. This heuristics comes from the value continuity rule stated in [4].

(3) Gien y = F(x), both x and y heading towards zero at time t_0, and $sgn(F(0)) = -sgn(y(t_0))$[2] then y must reach zero before x, since it must change sign before x reaches zero.

(4) If there is a relation $dx/dt = kx$ where $k \neq 0$ then any of x, $dx/dt, \ldots, d^nx/dt^n$ cannot be zero unless all of them are zero.

The heuristics of (1) must be applied before (2) and (3). (1) must be applied whenever it becomes applicable after applying (2) or (3). If there are many variables which are approaching zero (or other specific points), then they are ranked by (3) considering which reaches 0 (the specific points) earlier. When both X and Y are approaching zero, then another constraint is used by introducing a new variable R = X/Y.

Rule (3) corresponds to the confiict avoidance rule and value continuity rule [4].

Example 4.1. Let us consider the mass-spring system with friction [4] whose model is of the form:

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[2] $sgn(x)$ denotes the sign of x.

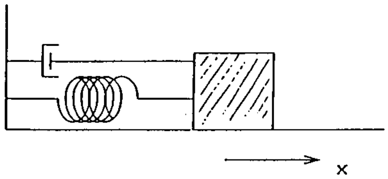

Fig. 2 Mass-spring system with a dashpot

(4–1) $dx/dt = v$

(4–2) $dv/dt = -kx - fv$ where k and f are positive constants.

(4–2) is the original form containing the built-in causality whereas (4–1) is the definition of v. The qualitative simulation for all possible initial sign assignment produces the four qualitatively different modes, which are all and the only possible modes for this system. In order to specify which variable reaches zero first among variables approaching zero, another constraint must be used by the model selection heuristic (a):

(4–3) $dR/dt = -R^2 - fR - k$ where $R = v/x$

The right hand side of (4–3) can be written as $-(R - \lambda_1)(R - \lambda_2)$ when $f^2 - 4k \geq 0 >$ where $0 > \lambda_1 \geq \lambda_2$. This relation is equivalent to the equations (4–1) and (4–2) through the relation $R = v/x$. It should be noted that the equation (4–3) and equations (4–1) and (4–2) provide qualitatively different constraints although they are equal in the quantitative world. The introduction of the constraint (4–3) with the landmark values λ_1 and λ_2 has the smae effect in terms of filtering spurious states as that of using the topological condition that the trajectory in the phase space (x, v) does not cross the two lines; $v = \lambda_1 x$ and $v = \lambda_2 x$ (as used in [8, 9]).

Tables 2-I-2-III show the simulations starting from all the possible initial patterns. These four patterns are all and the only qualitatively different patterns that are possible on the dynamical model (4–1) and (4–2). In the right most column, heuristics which are used to derive the change are indicated. Case I shows a deceasing oscillation (It is oscillation, because the final sign pattern is opposite to the initial sign pattern.)

As for the sign pattern (+ - + -) (case III), all of x, v, dv/dt and d^2v/dt^2 are approaching zero. Among them, $x = 0$ must precede other variables being zero by (3). Since x is not known to be a transient state, two cases, i.e. $x = 0$ (case III-1) and $x \neq 0$ (case III-2) are split. In case II-1*, x cannot be zero, since $R - \lambda_1 = 0$ must precede x = 0 (3) where $R - \lambda_1$ cannot be zero by (4), although x might be zero only by the qualitative information of (4–1) and (4–2). In case III-1**, again x cannot be zero by rules (3) and (4) although the sign pattern is the opposite pattern of the initial patterns where x becomes zero in the next step.

Table 2. The qualitative simulation for the mass-spring system

Case I
initial sign pattern $(x, v, dv/dt)=(+\ -\ -)$

x	v	dv/dt	d^2v/dt^2	
+	−	−	+	
+	−	0	+	(3) (2)
+	−	+	+	(1)
0	−	+	+	(3) (2)
−	−	+	+	(1)
−	−	+	0	(3) (2)
−	−	+	−	(1)
−	0	+	−	(3) (2)
−	+	+	−	(1)

Case II–2
initial sign pattern $(x, v, dv/dt)=(+\ +\ -)$

x	v	dv/dt	d^2v/dt^2	
+	+	−	−	
+	+	−	0	(3) (2)
+	+	−	+	(1)

The same patterns as that of case II–1 follow.

Case II–1
initial sign pattern $(x, v, dv/dt)=(+\ +\ -)$

x	v	dv/dt	d^2v/dt^2		$R-\lambda_1$	dR/dt	
+	+	−	+		+	−	
+	0	−	+	(3) (2)	+	−	
+	−	−	+	(1)	+	−	
+	−	0	+	(3) (2)	+	−	
+	−	+	+	(1)	+	−	
+	−	+	0	(3) (2)	+	−	
+	−	+	−		+	−	(3) (4) *

Case III–1
initial sign pattern $(x, v, dv/dt)=(+\ -\ +)$ case split (x becomes 0)

x	v	dv/dt	d^2v/dt^2		$R-\lambda_1$	$R-\lambda_2$	dR/dt	
+	−	+	−		−	−	−	
0	−	+	−	(3)	−	−	−	
−	−	+	−	(1)	+	+	−	
−	0	+	−	(3) (2)	+	+	−	
−	+	+	−	(1)	+	+	−	
−	+	0	−	(3) (2)	+	+	−	
−	+	−	−	(1)	+	+	−	
−	+	−	0	(3) (2)	+	+	−	
−	+	−	+		+	+	−	(3) (4) **

Case III–2
initial sign pattern $(x, v, dv/dt)=(+\ -\ +)$ case split (x does not become 0)

x	v	dv/dt	d^2v/dt^2		$R-\lambda_1$	$R-\lambda_2$	dR/dt
+	−	+	−		−	+	−
+	−	+	−	(3)	−	+	−

In order to investigate another qualitative aspect of whether or not the oscillation will converge, diverge or stay just periodical, we must cut the relations (4–1), (4–2) through the other hyperplane by the model selecting heuristic (b), i.e. $E = x^2 + (1/k)v^2$ thne $dE/dt = -fv^2$ follows. This relation indicates that E and hence x will eventually become zero as long as $f > 0$. However, the construction of such variables as E is ad hoc. We have shown that the qualitative stability can be checked only by the sign structure of the qualitative model [7].

5 Conclusion

We defined causality in physical systems by making time explicit. Since the causality is built in the dynamical model, a qualitative modeling process is skipped. Two other dynamical models are introduced; one for specifying which reaches a specific point first, and the other for investigating the convergence of variables.

It is shown that the method proposed here can simulate five qualitatively different modes for a mass-spring system with friction.

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