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AUTHOR(S):

AKAI, Koichi; TAMURA, Takeshi

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# Seismic Response Analysis of Offshore Seabed with Depth-Proportional Shear Modulus

By

Koichi AKAI* and Takeshi TAMURA**

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**Abstract**

Referring to the results of the PS-logging performed at the boring site of the 150 m-deep seabed in Osaka Bay, it is found that there exists a relationship between the celerity of the transversal wave, $v_s$ (m/s), and the depth of soil layers, $z$ (m), as $v_s = 30z^{0.5}$. Other information obtained from the soil exploration also indicates that the seabed is almost normally consolidated at the site. These data show that the shear modulus increases proportionally with depth. In this paper, the characteristic function of such a ground is deduced by solving the fundamental differential equation, and the procedure of seismic response analysis is described. By the numerical calculation for a modeled seabed subjected to a simulated irregular seismic excitation at the base ground, it is known that, at the mudline, all responses reach their maximum values. In particular, the acceleration response attains as high as 4.7 times the input ground motion.

## 1. Introduction

The seabed of Osaka Bay consists of soil layers: an alluvial marine layer, a terrace deposit and a series of diluvial layers where marine deposits are numbered as Ma 12, Ma 11, ----, Ma 0 from the top to the bottom layer. However, the general aspect of the offshore seabed in Osaka Bay is characterized by its clay-rich constitution compared to the inland area. Sometimes, the designers of civil engineering structures face severe soil conditions of lacking sand/gravel layers upon which the foundations of heavy structures should be supported. In addition to such a feature, the seabed is almost normally consolidated at the site, which is predicted by a small overconsolidation ratio in the upper diluviums[1]. Some data of PS-logging[2] show that there exists a linear correlation between the celerity of the transversal wave and the square root of depth. This means that

* Professor, Department of Civil Engineering, Kyoto University, Kyoto 606, Japan
** Associate Professor, ditto

the dynamic shear modulus of the seabed ground increases proportionally with depth.

At present, we have several big civil engineering projects in and around Osaka Bay. In such projects are involved not only the land reclamation along the bay, such as Kobe Port Island (436 ha) and Osaka South Port (930 ha), but also the construction of the Kansai International Airport (to be opened in 1993). This is a marine airport constructed on a man-made island (511 ha) located about 5 km off the sea coast (Senshu-oki), around which the water depth is 16 m to 19 m. The island is to be connected with the main land by a 3.8 km-long double deck bridge for road and railway. It is inevitable for designers, therefore, to consider the seismic behavior of such large-scale offshore structures.

## 2. Interpretation of PS-logging Data

Fig. 1 shows an example of the results of the suspension-type PS-logging of the 150 m-deep seabed in Osaka Bay. The interpretation of this logging data is

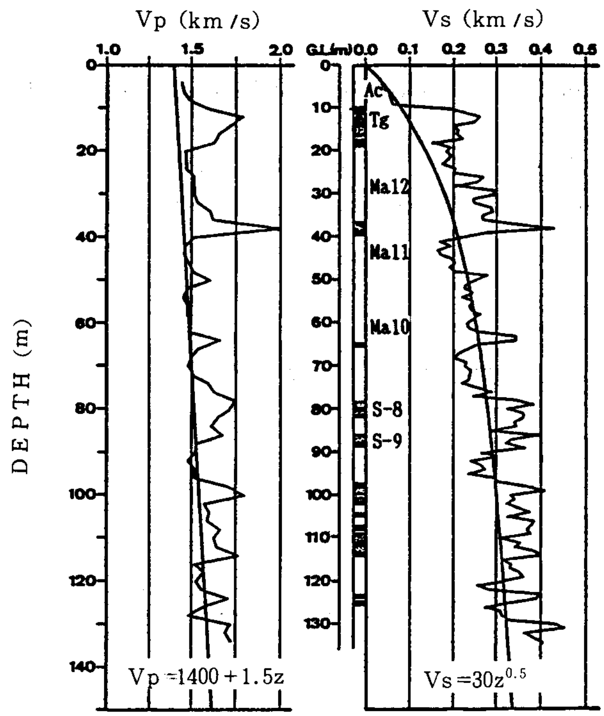


Fig. 1 Result of PS-logging at the boring site of Senshu-oki in Osaka Bay

as follows.

1) There exists a relationship between the celerity of the transversal wave, $v_s$ (m/s), and the depth, $z$ (m), as well as between the shear modulus of soil layers, $G$ (tf/m²) and the depth, $z$:

$$\left.\begin{aligned} &v_s=30z^{0.5}(i.e.,\ z=100\text{m};\ v_s=300\text{m/s}) \\ &G=900\rho z \end{aligned}\right\} \tag{1}$$

2) A similar correlation can be obtained between the celerity of the longitudinal wave, $v_p$ (m/s), and the depth, $z$. Namely, $v_p$= 1400 m/s at $z$= 0 m (mudline) and it increases proportionally with depth. At $z$= 100 m, $v_p$= 1550 m/s.

3) The only exceptional phenomenon can be seen in the so-called upper diluvial clay strata, Ma 12-Ma 10. The celerity observed, $v_s$= 150 - 300 m/s, is larger than the value of Eq. (1).

4) Generally speaking, $v_s$ in the sand/gravel layers is much larger than that in the clay layers. Therefore, Eq. (1) is not applicable to coarse grained soils. It can be said, however, that Eq. (1) is still appropriate for the seabed in Osaka Bay, because the clay layers are predominant in this offshore area as described earlier.

## 3. Chracteristics of Ground Vibration with Depth-Proportional Shear Modulus

The equation of motion for the vibration of a semi-infinite layer subjected to a horizontal seismic motion, $u_g$, at the base (see Fig. 2) is

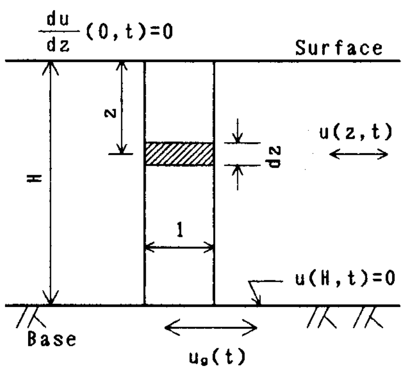


Fig. 2 Cross section and boundary conditions of a semi-infinite soil layer subjected to a horizontal seismic motion at its base

$$\rho(z)\frac{\partial^2 u}{\partial t^2}+c(z)\frac{\partial u}{\partial t}-\frac{\partial}{\partial z}\left[G(z)\frac{\partial u}{\partial z}\right]=-\rho(z)\frac{\partial^2 u_g}{\partial t^2} \tag{2}$$

where $\rho(z)$ denotes the mass density at a depth $z$, $c(z)$ the viscous damping coefficient, $G(z)$ the shear modulus and $u(z, t)$ the relative displacement at time, $t$. Assuming $\rho$ and $c$ are independent of depth, $z$, in the present case, we obtain from Eq. (1)

$$G=Kz \tag{1)'$$

Hence Eq. (2) becomes

$$\rho\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}-\frac{\partial}{\partial z}\left(Kz\frac{\partial u}{\partial z}\right)=-\rho\ddot{u}_g \tag{3}$$

Let us consider the homogeneous equation of Eq. (3) in order to investigate the dynamic characteristics of such a ground. Applying the variable separation method with the complex circular frequency $\omega$,

$$u(z, t)=v(z)\cdot w(t)=v(z)\cdot e^{i\omega t} \tag{4}$$

Substituting this expression into Eq. (3), we obtain the following ordinary differential equation for $v(z)$:

$$\frac{d^2 v}{dz^2}+\frac{1}{z}\frac{dv}{dz}+\frac{\rho\omega^2-ci\omega}{Kz}v=0 \tag{5}$$

Transforming $z$ in a new variable $\zeta=az^\sigma$ with $\sigma=1/2$, $a=2\sqrt{(\rho\omega^2-ci\omega)/K}$, leads to the Bessel differential equation of order zero. Thus, the solution satisfying the boundary conditions that $v$ is finite at the ground surface, $z=0$ ($\zeta=0$), and $v=0$ at the base, $z=H$, is expressed by the following Bessel function of the first kind of order zero.

$$v=J_0(\zeta)=J_0\left(2\sqrt{\frac{\rho\omega^2-ci\omega}{K}z}\right)=J_0(2\lambda\sqrt{z})=J_0\left(\sqrt{\frac{z}{H}}\kappa_0\right) \tag{6}$$

where $\lambda$ denotes the eigenvalue of vibration and the individual value of $\kappa_0$;

$$\kappa_{on}=2\sqrt{\frac{\rho\omega_n^2-ci\omega_n}{K}H} \tag{7}$$

is determined by the roots of $J_0(\zeta)=0$.

From the boundary condition at the base, $z=H$, the general solution of Eq. (2) is expressed as:

$$u(z, t)=e^{-at}\sum_{n=1}^{\infty}(a_n \sin\beta_n t+b_n \cos\beta_n t)\cdot J_0(2\lambda_n\sqrt{z}) \tag{8}$$

where $a=\dfrac{c}{2\rho}$, $\beta_n=\dfrac{\sqrt{-c^2+4\rho K\lambda_n}}{2\rho}$

It can be recognized from Eq. (8) that the time-dependent term consists of the product of two components; namely, a uniform damping $e^{-at}$ and an individual trigonometric function (a normal vibration) $\begin{Bmatrix}\sin\beta_n t\\ \cos\beta_n t\end{Bmatrix}$. The depth-dependent term (*i. e.*, the modal solution), on the other hand, is expressed by the Bessel function of the square root of depth, $z$. This is the characteristic function of ground vibration where the shear modulus increases proportionally with depth. The mode of such a ground is indicated in Fig. 3.

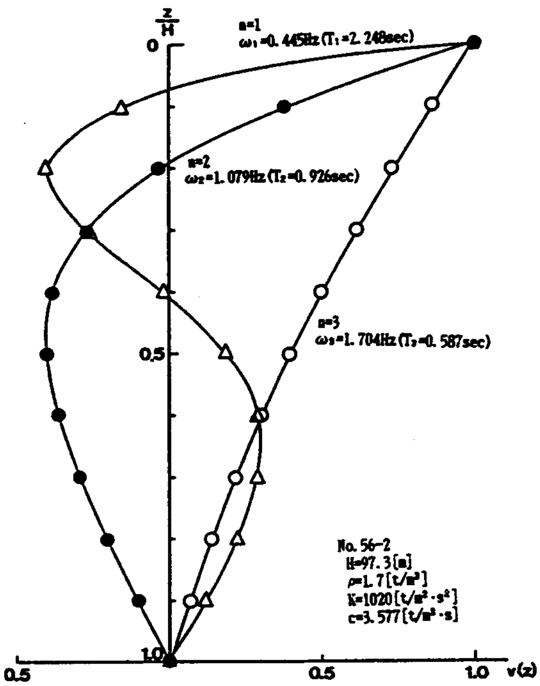


Fig. 3 Mode of ground vibration

## 4. Seismic Response of the Ground with Depth-Proportional Shear Modulus

Idriss and Seed[3] proposed some models to analyze the responses during earthquakes of soil layers with a depth-dependent shear modulus. They solved the following fundamental equation:

$$\rho\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}-\frac{\partial}{\partial z}\left(Kz^m\frac{\partial u}{\partial z}\right)=-\rho\ddot{u}_g \tag{9}$$

They noted however that the analytical method is effective for $m\leq 1/2$. It means that when $m>1/2$, a solution in terms of Bessel functions cannot be obtained. In their paper, therefore, comparisons for $m=0$ (constant elasticity) and $m=1/3$ are made between the closed-form solution and the numerical solution with a lumped-mass representation.

As described earlier, seabed grounds are almost normally consolidated at their site and, therefore, the shear modulus of soil layers, $G$, is approximately proportional to the depth, $z$. This means that the power $m$ in Eq. (9) equals unity (*i. e.*, $m>1/2$). Our present effort is oriented to obtain the closed-form solution of the seismic responses in such situations.

As a step to the solution of Eq. (3), we first analyze an equation:

$$\rho\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}-\frac{\partial}{\partial z}\left(Kz\frac{\partial u}{\partial z}\right)=-\rho\cdot\delta(t) \tag{10}$$

where $\delta(t)$ denotes the delta function. This corresponds to the case where an external force of unit pulse is applied to a vibratory system which is at rest at $t=0$.

Let it be assumed that an exernal force, $f$ (unit pulse), is applied for a very short time, $\varepsilon$, to a body with mass, $m$, at rest at $t=0$, as illustrated in Fig. 4.

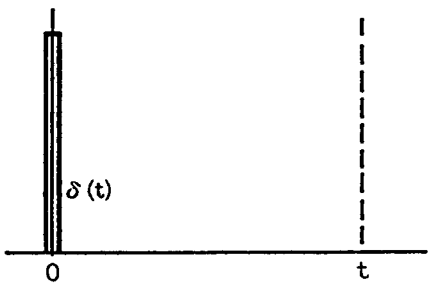


Fig. 4 Unit pulse

Namely,

$$m\ddot{x}=f=\frac{1}{\varepsilon} \qquad (0\leq t\leq\varepsilon)$$

Then,

$$m\dot{x}=\frac{t}{\varepsilon}\,;\ m\dot{x}=1 \text{ at } t=\varepsilon$$

$$mx=\frac{t^2}{2\varepsilon}\,;\ mx=\frac{\varepsilon}{2} \text{ at } t=\varepsilon$$

Hence, if $\varepsilon\rightarrow 0$, $mx\rightarrow 0$. It means that a unit pulse produces a change in the velocity of mass as $1/m$.

Thus, the problem of Eq. (10) can be reduced to solving the following equation:

$$\rho\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}-\frac{\partial}{\partial z}\left(Kz\frac{\partial u}{\partial z}\right)=0 \tag{10)'$$

under the initial condition,

$$u=0,\ \dot{u}=-1 \text{ at } t=0$$

Using the result of **3**. and considering the initial condition for Eq. (8); $u=0$ at $t=0$, then we get $b_n=0$. Therefore,

$$u(z,t)=e^{-\alpha t}\sum_{n=1}^{\infty}a_n\sin\beta_n t\cdot J_0(2\lambda_n\sqrt{z}) \tag{8)'$$

$$\therefore\frac{\partial u}{\partial t}=e^{-\alpha t}\sum_{n=1}^{\infty}\left\{-\alpha\sin\beta_n t+\beta_n\cos\beta_n t\right\}\cdot J_0(2\lambda_n\sqrt{z})$$

Another initial condition:

$$\frac{\partial u}{\partial t}=-1 \quad \text{at} \quad t=0$$

leads to

$$\sum_{n=1}^{\infty}a_n\beta_n J_0(2\lambda_n\sqrt{z})=-1$$

Putting $x\rightarrow 2\sqrt{z}$ and $a\rightarrow 2\sqrt{H}$ in the well-known formula,

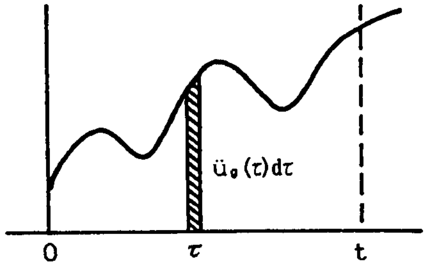

Fig. 5 Superposition of unit pulse

$$\sum_{n=1}^{\infty} \frac{2}{\lambda_n a J_1(\lambda_n a)} J_0(\lambda_n x) = 1 \qquad (J_0(\lambda_n a) = 0)$$

we then have

$$\sum_{n=1}^{\infty} \frac{1}{\lambda_n \sqrt{H} J_1(2\lambda_n \sqrt{H})} J_0(2\lambda_n \sqrt{z}) = 1$$

$$\therefore a_n = -\frac{1}{\beta_n \lambda_n \sqrt{H} J_1(2\lambda_n \sqrt{H})}$$

Thus, Eq. (8)′ is finally written as:

$$u(z,t) = -\frac{1}{\sqrt{H}} e^{at} \sum_{n=1}^{\infty} \frac{\sin \beta_n t}{\beta_n} \frac{J_0(2\lambda_n \sqrt{z})}{\lambda_n J_1(2\lambda_n \sqrt{H})} \tag{11}$$

Comparing the original problem (Eq. (3)) with Eq. (10), we have to replace $-\rho \cdot \delta(t)$ by $-\rho \ddot{u}_u(\tau) d\tau$. Therefore, referring to Fig. 5,

$$u(z,t) = -\frac{1}{\sqrt{H}} e^{-a(t-\tau)} \sum_{n=1}^{\infty} \frac{\sin \beta_n t}{\beta_n} \frac{J_0(2\lambda_n \sqrt{z})}{\lambda_n J_1(2\lambda_n \sqrt{H})} \ddot{u}_g(\tau) d\tau \tag{12}$$

By summing up the effect of vibration during the time $\tau = 0 \sim \tau$, the displacement response of ground under the input acceleration $\ddot{u}_g(t)$ is given by the following Duhamel integral.

$$u(z,t) = -\frac{1}{\sqrt{H}} \sum_{n=1}^{\infty} \frac{1}{\beta_n} \frac{J_0(2\lambda_n \sqrt{z})}{\lambda_n J_1(2\lambda_n \sqrt{H})} \int_0^t e^{-a(t-\tau)} \sin \beta_n (t-\tau) \ddot{u}_g(\tau) d\tau \tag{13}$$

Differentiating the above equation by the time, $t$, once or twice, we obtain the velocity response, $\dot{u}$, and the absolute acceleration response, $\ddot{u} + \ddot{u}_g$, respectively, as follows:

$$\dot{u}(z,t) = -\sqrt{\frac{K}{\rho H}} \sum_{n=1}^{\infty} \frac{J_0(2\lambda_n \sqrt{z})}{\beta_n J_1(2\lambda_n \sqrt{H})}$$

$$\times \int_0^t e^{-\alpha(t-\tau)} \cos \{\beta_n(t-\tau)+\gamma_n\} \ddot{u}_g(\tau)d\tau \tag{14}$$

$$\ddot{u}(z,t)+\ddot{u}_g(t)=\frac{K}{\rho}\frac{1}{\sqrt{H}}\sum_{n=1}^{\infty}\frac{\lambda_n J_0(2\lambda_n\sqrt{z})}{\beta_n J_1(2\lambda_n\sqrt{H})}$$

$$\times \int_0^t e^{-\alpha(t-\tau)} \sin \{\beta_n(t-\tau)+2\gamma_n\} \ddot{u}_g(\tau)d\tau \tag{15}$$

where, from Eqs. ( 7 ) and ( 8 ),

$$J_0(2\lambda_n\sqrt{H})=0,\ \alpha=\frac{c}{2\rho},\ \beta_n=\frac{\sqrt{4\rho K\lambda_n-c^2}}{2\rho},\ \gamma_n=\tan^{-1}\left(\frac{\alpha}{\beta_n}\right)$$

The shear strain of ground is expressed as the differentiation of Eq. (13) with respect to $z$, whereas the shear stress can be obtained by multiplying the strain by $G=Kz$.

## 5. Results of Numerical Calculation

Eqs. (13) ~ (15) are numerically calculated by decomposing the seismic data, $\ddot{u}_g$ $(t)$, into the Fourier series through the Fast Fourier Transformation.

### 5. 1 Mechanical soil parameters

( 1 ) Density $\rho$ (t/m³)

As the average value of soil densities for alluvial and diluvial clay strata in Osaka Bay, we take $\rho$= 1.70 t/m³.

( 2 ) Proportional constant $K$ (t/m²·s²)

As mentioned earlier, there exists a correlation expressed by Eq. ( 1 ) ; $G$= 900$\rho z$ (t/m·s²). Therefore, we should take $K$= 900 × 1.70 = 1530 t/m²·s². However, taking the strain-dependency of the dynamic shear modulus during the seismic vibration into account here, we multiply it by 2 / 3 to modify the value $K$. Thus, $K$= 2 / 3 × 1530 = 1020 t/m²·s².

( 3 ) Damping coefficient $c$ (t/s·m³)

Utilizing the field data of the PS-logging and referring to the laboratory result of the triaxial forced vibration test for alluvial clay specimen[4], we use $c$= 3.58 t/s·m³.

### 5. 2 Seismic data

( 1 ) Input seismic wave

The accelerogram of Tokachi-oki earthquake (May 16, 1968) observed at

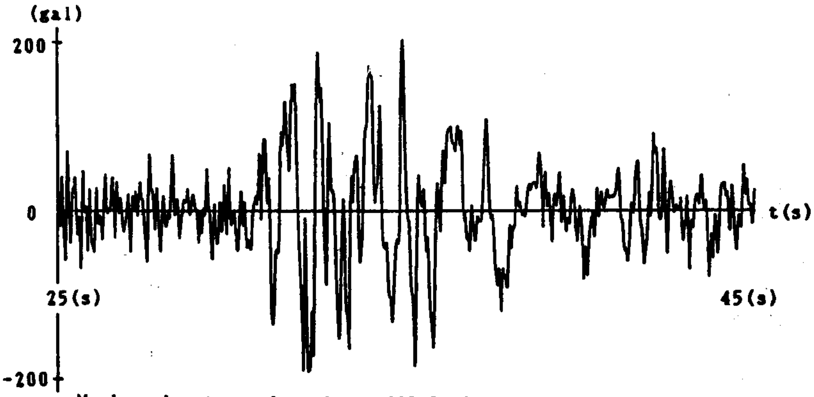


Fig. 6 Acceleration record used in analysis (E–W component, 1968 Tokachi–oki earthquake)

Hachinohe, Aomori Prefecture (E–W component with maximum acceleration 203.3 gal[5], see Fig. 6), is used as the input seismic wave.

(2) Input seismic base

According to the boring data obtained at the construction site of the Kansai International Airport in Osaka Bay, it was found that there existed a remarkable unconformity at the depth of $H=97.3$ m below mudline at the boring site, No. 56-2. Therefore, the sand/gravel layer overlying Ma 3 beneath the unconformity is chosen as the input seismic base where the dynamic impedance of ground varies extremely.

## 5.3 Analytical results

The analytical results for the above input data are shown in Fig. 7 (a) – (e), which correspond to the maximum absolute acceleration response $(\ddot{u}+\ddot{u}_g)_{max}$, the maximum velocity response $\dot{u}_{max}$, the maximum displacement response $u_{max}$, the maximum shear strain $\gamma_{max}$ and the maximum shear stress $\tau_{max}$, respectively. It is known from these figures that, at the mudline, $z=0$, all responses reach their maximum values along the vertical direction; namely, 16.6 cm in displacement, 78.4 kine in velocity and 959 gal in absolute acceleration. The last one is as high as 4.7 times the input ground motion ($\ddot{u}_g=203.3$ gal). Such a response amplification is very remarkable in the soil layer near the surface of the seabed.

Fig. 8 (a) – (c) illustrates the time-responses at the mudline of absolute acceleration, relative velocity and relative displacement, respectively. It can be understood that the peak values of responses have some time lags, when compared with the original input data shown in Fig. 6.

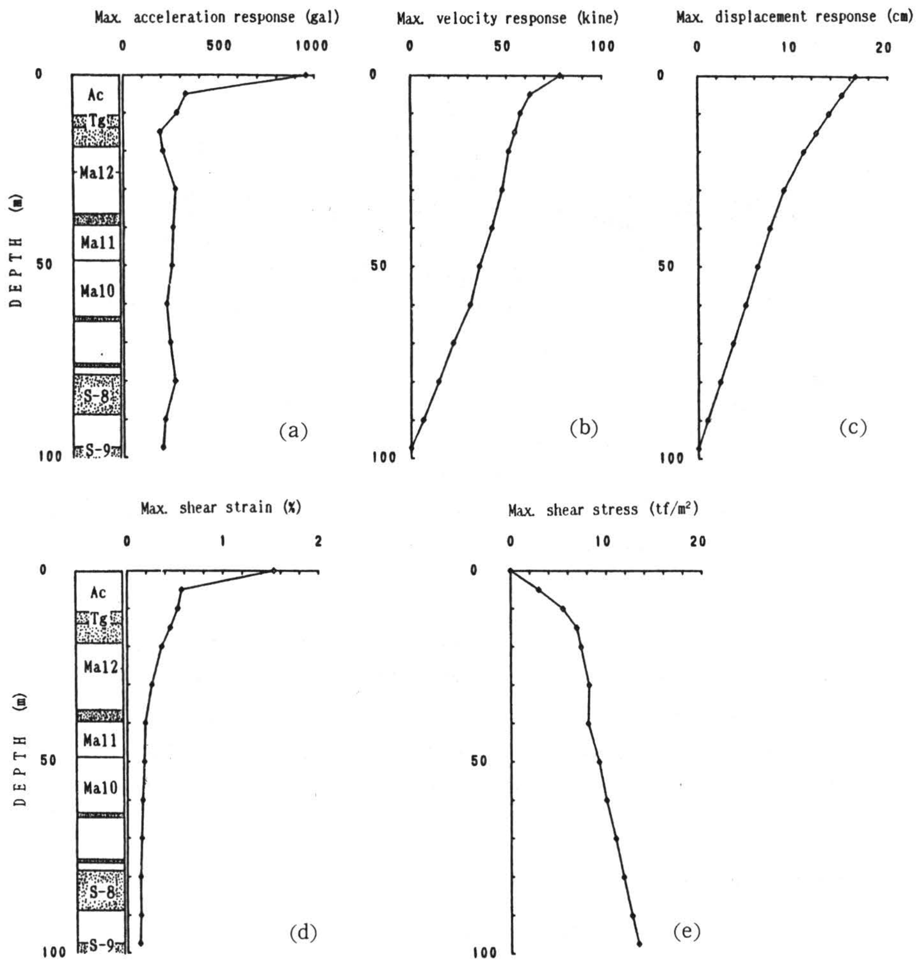


Fig. 7 Results of dynamic response analysis for a model of Senshu-oki seabed

## 6. Conclusions

In the present paper, an analytical method to obtain the seismic response of offshore seabed with a depth-proportional shear modulus, $G \propto z$, has been described in some detail.

The result of the PS-logging of the seabed in Osaka Bay leads to a fundamental equation of motion for the vibration of a semi-infinite layer subjected to a horizontal seismic motion at its base. It has been found out that the characteristics of motions are expressed as the product of three components; namely, a uniform damping, a normal vibration and a Bessel function of order zero in

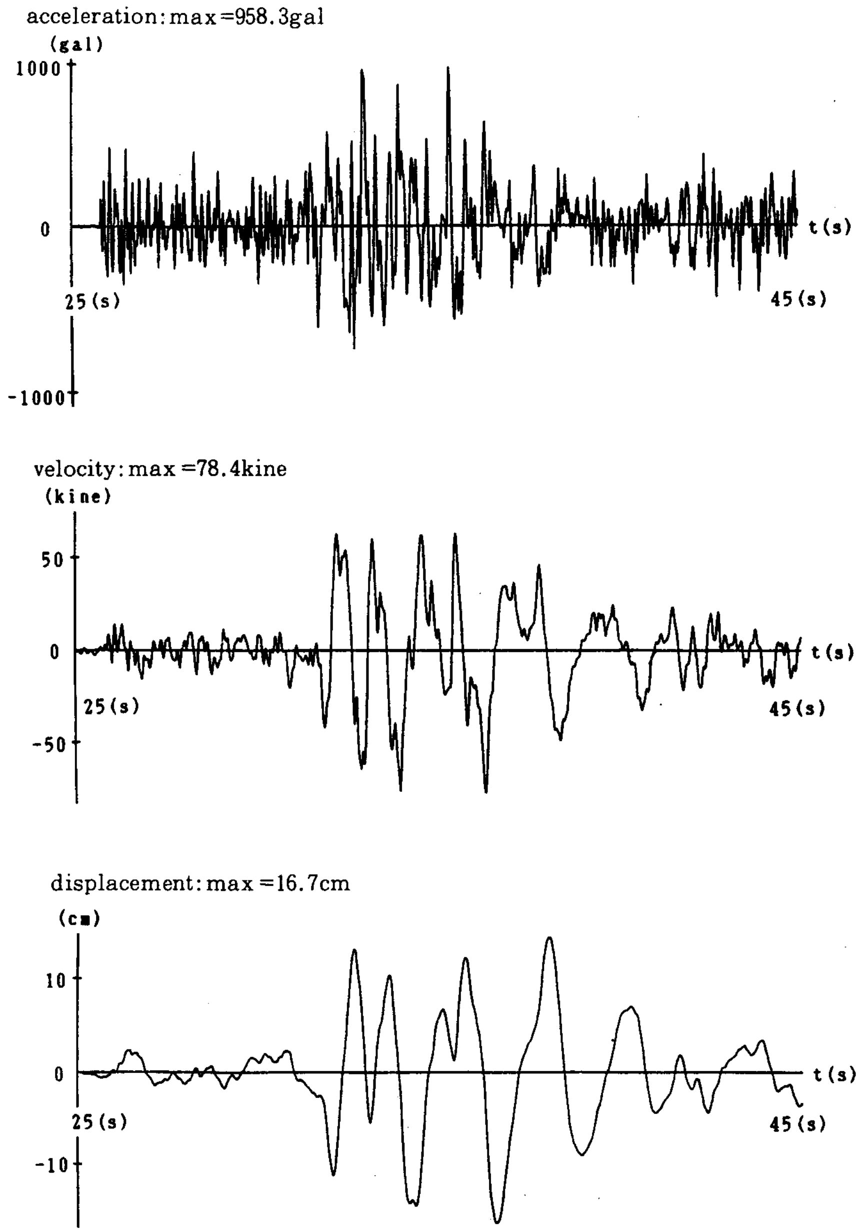


Fig. 8 Surface response of layer with depth-proportional shear modulus

terms of the square root of depth. Then, the closed-form solution of seismic responses in the form of the Duhamel integral has been deduced analytically for acceleration, velocity and displacement, as well as shear strain and shear stress in the ground.

By the numerical calculation for a modeled seabed subjected to a simulated irregular seismic excitation at the base ground, it is known that, at the mudline, all responses reach their maximum values. In particular, the acceleration response indicates as high as 4.7 times the input ground motion.

The analytical solution presented here is expected to give an appropriate result for actual seabed grounds, although the much more rigorous response analysis could be obtained by the numerical approach such as the lumped-mass solution or the finite element method.

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