Simplified Formula for Axial Strains of Buried Pipes Induced by Propagating Seismic Waves

By

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Abstract

Pipe strains developed in buried straight pipes by horizontally propagating seismic waves are analyzed. Extensive discussion is made for the general slippage conditions between soils and pipes, as well as for the arbitrary angle of incidence of the longitudinal and transverse waves relative to the pipe axis. After the pipe strain solutions and their upper and lower bounds are obtained for the given values of the angle of incidence, solutions for the maximum pipe strains with unknown angles of incidence are discussed. In particular, simple approximate closed-form solutions for the maximum pipe strains developed herein should be useful for practical applications.

1. Introduction

Among major causes of structural damage to underground lifeline pipes during earthquakes, the effects of propagating seismic waves have been recognized and studied by many authors[1),2),5),6),8),9),10)]. Their results generally agree in the following: (1) Pipe failures are dominated by the ground strain. (2) Axial strain is of primary importance in comparison with the bending strain. (3) The mass effect of the pipe is negligile; i. e., a quasi-static analysis can be applied in finding the pipe strain imparted from the ground. (4) Slippage between the pipe and the surrounding soil makes the pipe strain smaller than the free field strain of the ground.

Such structural behaviors of buried pipes have been verified analytically, as well as from field observations. Some works have dealt with the stress concentration in curved pipes or junctions[1),8),9)]. The effects of the angle of incidence, namely the angle between the direction of the pipe axis and that of the wave propagation, have also been analyzed for some particular cases[5)].

In those earlier studies, certain limitied assumptions have been employed as to the type of seismic wave, angle of incidence, or the slippape conditions. This study

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deals with the response behavior of straight buried pipes subjected to a horizontal seismic wave propagation under more general loading conditions.

The analysis is made by using two pipe-soil interaction models. One uses the standard equivalent spring constant, and the other assumes a certain analytical function for soil displacement along the direction perpendicular to the pipe. A horizontal propagation of sinusoidal seismic waves at an arbitrary angle of incidence is assumed. Both the longitudinal and transverse waves are considered. Along with the exact solution under partial slippage between the soil and the pipe, approximate closed-form solutions are obtained.

After characterizing the input ground motions in Chapter 2, Chapter 3 deals with pipe strains for a given angle of incidence of the input seismic wave. It is a generalization of the pioneering works by Sakurai and Takahashi[8] and of those by Miyajima and Miyauchi[5] to general cases with longitudinal and transverse seismic waves, comprehensive representation of partial slippage conditions, and incorporating a closer lower bound solution. In Chapter 4, a maximum pipe strain with an unknown angle of incidence is discussed. After demonstrating its general behavior based on exact solutions, simple upper bound solutions are developed. It is shown under pipe-slippage that the maximum pipe strain for the longitudinal waves will be proportional to the cubic root of the free field normal strain. The maximum pipe strain for the transverse waves will be proportional to the square root of the free field shear strain. In Appendix A, the evalution of the spring constant and slippage conditions in pipe-soil interaction is discussed on the basis of the experimental works by Kuribayashi, Iwasaki, Kawashima and Miyata[4], from which some preliminary formulas are proposed.

2. Input Ground Motion

It is assumed that sinusoidal seismic waves propagate horizontally with an angle of incidence θ to the buried pipe, as shown in Fig. 1. Axial pipe strains caused by these seismic waves are dealt with. Longitudinal and transverse waves are considered, and discussion is made with a general understanding that they represent the Rayleigh waves and the Love waves, respectively. Recent developments in the strong motion

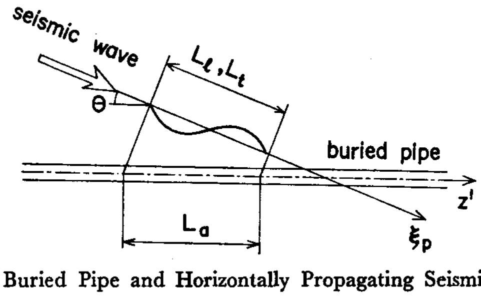

Fig. 1 Buried Pipe and Horizontally Propagating Seismic Wave.

seismology[3),11),13)] indicate that the major part of the energy of seismic motions, in frequency ranges affecting the ground strain, is carried by surface waves. Besides, when the effect of body waves on the pipe strains is concerned, formulation can be made in a similar manner.

Let the ground displacement caused by the seismic waves propagating along the ξ_p axis be represented by

$$u_l(t,\ \xi_p)=\frac{L_l}{2\pi}\varepsilon_l\sin\left(\omega t-\frac{2\pi}{L_l}\xi_t\right) \qquad (1)$$

and

$$u_t(t,\ \xi_p)=\frac{L_t}{2\pi}\gamma_t\sin\left(\omega t-\frac{2\pi}{L_t}\xi_p\right) \qquad (2)$$

Eq. (1) represents the effect of the longitudinal wave, in which u_l= the longitudinal displacement, L_l= the wave length, and ε_l= the normal strain.
Eq. (2) is for the transverse wave, in which u_t= the transverse displacement, L_t= the wave length, and γ_t= the shear strain. In these equations, ω= the circular frequency. The effective input displacement to the pipe for obtaining the axial pipe strain is represented by the apparent displacement $u_a(t,\ z')$, and the apparent wave length L_a along the pipe axis ; i. e.,

$$u_a(t,\ z')=u_G\sin\left(\omega t-\frac{2\pi}{L_a}z'\right)=\frac{L_a}{2\pi}\varepsilon_G\sin\left(\omega t-\frac{2\pi}{L_a}z'\right) \qquad (3)$$

where

$$L_a=\begin{cases} L_l/\cos\theta & \text{; longitudinal wave} \quad (4)\\ L_t/\cos\theta & \text{; transverse wave} \quad (4')\end{cases}$$

and

$$u_G=\begin{cases} \dfrac{L_l}{2\pi}\varepsilon_l\cos\theta & \text{; longitudinal wave} \quad (5)\\ \dfrac{L_t}{2\pi}\gamma_t\sin\theta & \text{; transverse wave} \quad (5')\end{cases}$$

The strain amplitude $\varepsilon_G=2\pi u_G/L_a$ represents the apparent normal free field strain along the pipe axis, and is given by

$$\varepsilon_G=\begin{cases} \varepsilon_l\cos^2\theta & \text{; longitudinal wave} \quad (6)\\ \gamma_t\sin\theta\cos\theta & \text{; transverse wave} \quad (6')\end{cases}$$

The relation between the apparent free field strain ε_G and the angle of incidence θ is shown in Fig. 2.

If the position along the pipe is represented by

$$z=\frac{L_a}{2}-\frac{L_a}{2\pi}\left(\omega t-\frac{2\pi}{L_a}z'\right) \qquad (7)$$

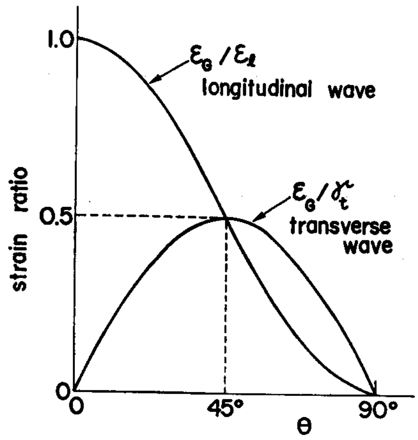

Fig. 2 Free Field Strain and Apparent Free Field Strain.

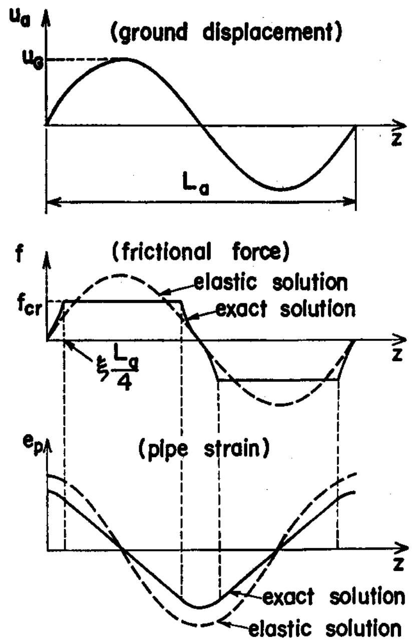

Fig. 3 Ground Displacement, Frictional Force, and Pipe Strain.

then Eq. (3) is rewritten as

$$u_a(z) = u_G \sin\frac{2\pi}{L_a}z = \frac{L_a}{2\pi}\varepsilon_G \sin\frac{2\pi}{L_a}z \quad \cdots\cdots (8)$$

as shown in Figs. 2 and 3.

3. Pipe Strains for a Given Direction of Incident Wave

3.1 Pipe Model

A standard method of calculating seismic loads imparted to buried pipes is to use a soil-pipe interaction model with an equivalent spring constant. Since the inertia force of the pipe can be neglected, the displacement is obtained from the following quasi-static equation of equilibrium.

$$\frac{d^2u_p}{dz^2} + \frac{f}{Ed} = 0 \quad \cdots\cdots (9)$$

where $u_p = u_p(z) =$ the pipe displacement, $E =$ the modulus of elasticity of the pipe material, $d =$ the wall thickness of the pipe, $f = f(z) =$ the seismic load acting per unit area of the pipe surface. The seismic load is determined from

$$f(z) = K(u_a - u_p) \quad ; \quad |u_a - u_p| \leq \Delta u_{cr} \quad \cdots\cdots (10)$$

or

$$f(z)=K\Delta u_{cr} \quad ; \quad |u_a-u_p|>\Delta u_{cr} \qquad (10')$$

where $K=$ the equivalent spring constant of the soil-pipe interaction per unit area, and $\Delta u_{cr}=$ the critical relative displacement for the initiation of slippage between the soil and the pipe. Eq. (10) applies to the elastic interaction forces, and Eq. (10′) holds for the portion of the pipe where the ground displacement relative to the pipe exceeds a frictional limit Δu_{cr}, and slippage takes place.

3.2 Pipe Strain Solutions Using Equivalent Spring Constant

(1) Exact solution

The exact solution for the type of interaction forces of Eqs. (10) and (10′) has been obtained by Miyajima and Miyauchi[5], though their discussion has been confined to the case of transverse waves. Herein, their solution is rewritten as Eqs. (11)~(13) which are more general in that they also include the case of the longitudinal waves.

$$u_p(z)=\begin{cases} u_G\left(\alpha_1 \sin\frac{2\pi}{L_a}z-\alpha_2 \sinh\lambda z\right) & ; \text{ elastic region} \qquad (11)\\ u_G\left[\alpha_3+\alpha_4\left\{\frac{z}{L_a}-2\left(\frac{z}{L_a}\right)^2\right\}\right] & ; \text{ plastic region} \qquad (11')\end{cases}$$

where

$$\left.\begin{aligned} &\alpha_1=1/\{(2\pi/\lambda L_a)^2+1\}\\ &\alpha_2=\frac{1}{\sin\lambda\xi\frac{L_a}{4}}\left\{\frac{\Delta u_{cr}}{u_G}-\frac{\sinh\xi\frac{\pi}{2}}{1+\left(\frac{\lambda L_a}{2\pi}\right)^2}\right\}\\ &\alpha_3=\sin\xi\frac{\pi}{2}-\frac{\Delta u_{cr}}{u_G}\left\{1+\frac{(2\xi-\xi^2)\lambda^2L_a^2}{32}\right\}\\ &\alpha_4=\frac{\lambda^2L_a^2}{4}\frac{\Delta u_{cr}}{u_G}, \qquad \lambda=\sqrt{\frac{K}{Ed}} \end{aligned}\right\} \qquad (12)$$

The parameter ξ defines the boundary of the elastic regions. It is determined from the following relation:

$$\frac{\Delta u_{cr}}{u_G}=\frac{(1-\alpha_1)\left(\frac{\lambda L_a}{2\pi}\cos\xi\frac{\pi}{2}\tanh\lambda\xi\frac{L_a}{4}+\sin\xi\frac{\pi}{2}\right)}{(1-\xi)\frac{\lambda L_a}{4}\tanh\lambda\xi\frac{L_a}{4}+1} \qquad (13)$$

Clearly, ξ varies in the range $0<\xi\leq 1$.

The axial pipe strain is represented by

$$e(z)=\frac{du_p}{dz}=\begin{cases}\varepsilon_G\left(\alpha_1\cos\frac{2\pi}{L_a}z-\alpha_2\lambda\frac{L_a}{2\pi}\cosh\lambda z\right) & ;\ \text{elastic region} \cdots\cdots\cdots\cdots (14)\\ \varepsilon_G\frac{\alpha_4}{2\pi}\left(1-4\frac{z}{L_a}\right) & ;\ \text{plastic region} \cdots\cdots\cdots\cdots (14')\end{cases}$$

From Eqs. (8), (10), (11), and (11′) the seismic load f is represented by

$$f(z)=\begin{cases}K_{u_G}\left\{(1-\alpha_1)\sin\frac{2\pi}{L_a}z+\alpha_2\sinh\lambda z\right\} & ;\ \text{elastic region} \cdots\cdots\cdots\cdots (15)\\ K\,\Delta u_{cr} & ;\ \text{plastic region} \cdots\cdots\cdots (15')\end{cases}$$

Setting $\xi=1$ in Eqs. (10) and (13) yields

$$\frac{\Delta u_{cr}}{u_G}=\frac{1}{1+(\lambda L_a/2\pi)^2}=1-\alpha_1$$

Solving this equation for u_G gives the values of u_G and ε_G on the intiation of slippage in the following form.

$$u_{G_s}=\Delta u_{cr}/(1-\alpha_1) \cdots\cdots (16)$$

$$\varepsilon_{G_s}=(2\pi/L_a)\,\Delta u_{cr}/(1-\alpha_1) \cdots\cdots (16')$$

This condition gives $\alpha_2=0$ which reduces Eq. (10) to the following elastic solution where no slippage occurs.

$$u_p(z)=u_G\alpha_1\sin\frac{2\pi}{L_a}z\ ;\ \text{elastic response} \cdots\cdots (11'')$$

In the same manner, setting $\alpha_2=0$ in Eqs. (14) and (15) gives the result for elastic response:

$$e(z)=\varepsilon_G\alpha_1\cos\frac{2\pi}{L_a}z \cdots\cdots (14'')$$

$$f(z)=Ku_G(1-\alpha_1)\sin\frac{2\pi}{L_a}z \cdots\cdots (15'')$$

Fig. 3 illustrates the relation between the elastic solution and the solution under partial slippage. It demonstrates that slippage between the soil and the pipe reduces the amplitude of the seismic load f and the pipe strain e_p, as pointed out earlier[5),8),9)].

The pipe strain amplitude, which is the value of $e(0)$, is obtained from Eq. (14) as

$$e_S=\begin{cases}\left(\alpha_1-\alpha_2\lambda\frac{L_a}{2\pi}\right)\varepsilon_G & ;\ \text{plastic solution} \cdots\cdots (17)\\ \alpha_1\varepsilon_G & ;\ \text{elastic solution} \cdots\cdots (17')\end{cases}$$

It may be pointed out that the second term on the right-hand side of Eq. (17) represents the decrement of the pipe strain due to slippage.

(2) Upper and lower bounds

When slippage between the soil and the pipe occurs, Eq. (13) requires iterative computations to determine ξ. Therefore closed-form approximate solutions will be useful. Herein, the upper and lower bounds for the pipe strain are discussed. Approximations can be made by assuming simplified distributions of $f(z)$ along the pipe, such as those shown in Fig. 4.

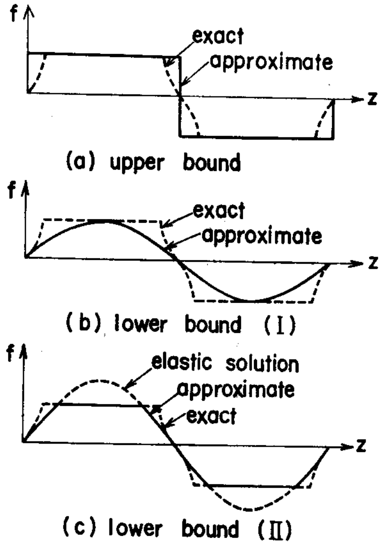

Fig. 4 Distribution of Frictional Forces for Approximate Pipe Strain.

The approximation in Fig. 4(a) assumes that slippage takes place everywhere along the pipe. This assumption has been used by Sakurai and Takahashi[8], Miyajima and Miyauchi[5], and Shinozuka and Koike[9]. It provides an upper bound on the pipe strain. The solution is obtained by taking the limit of $\xi\rightarrow 0$ in the foregoing exact solution. Then, the upper bound e_{S_u} on the pipe strain amplitude is obtained as

$$e_{S_u}=\frac{L_a\,K\,\Delta u_{cr}}{4Ed} \quad \cdots\cdots (18)$$

It should be noted that Eq. (18) gives an upper bound independent of the apparent free field strain. Indeed, e_{S_u} is the limited value to which e_S in Eq. (17) is asymptotic as ε_G tends to infinity.

The lower bound illustrated in Fig. 4(b) has been used by Shinozuka and Koike[9]. The distribution of $f(z)$ employed here is the elastic solution given in Eq. (15″) at the initiation of slippage, that is, for $u_G=u_{G_s}$. Therefore, by using Eq. (14″), a lower bound solution $e_{Sl}^{(1)}$ for the pipe strain amplitude is obtained as

$$e_{Sl}^{(1)}=\frac{2\pi}{L_a}u_{G_s}\alpha_1=\frac{2}{\pi}\,\frac{L_a\,K\,\Delta u_{cr}}{4Ed} \quad \cdots\cdots (19)$$

which is again independent of the apparent free field strain ε_G.

As Fig. 4(b) implies, the lower bound $e_{Sl}^{(1)}$ would not be close enough to the exact solution for some cases. Therefore, a closer lower bound solution is developed by using the approximation in Fig. 4(c). Here, the distribution of $f(z)$ is assumed to coincide with the elastic solution when its value is no more than $K\Delta u_{cr}$ and to take on a constant value of $K\Delta u_{cr}$ in the portion of the pipe where the elastic solution exceeds $K\Delta u_{cr}$. Then, the position on the z axis, defined by ξ' in Fig. 4(c), represents an approximate boundary of the elastic and plastic regions. Then, letting $f(z)=K\Delta u_{cr}$ in Eq. (15''), solving for z, and using it for $\xi' L/4$, we obtain

$$\xi'=\frac{2}{\pi}\sin^{-1}\left[\left\{1+\left(\frac{\lambda L_a}{2\pi}\right)^2\right\}\frac{\Delta u_{cr}}{u_G}\right]=\frac{2}{\pi}\sin^{-1}\frac{u_{Gs}}{u_G} \quad \cdots\cdots (20)$$

Then, using the boundary conditions that the slope of $\varepsilon_p(z)$ vanish at $z=0$, that the value and the slope of $\varepsilon_p(z)$ be continuous at $z=\xi' L/4$, and that $\varepsilon_p(z)$ vanish at $z=L/4$, a lower bound solution $e_l^{(2)}(z)$ is obtained as

$$e_l^{(2)}(z)=\begin{cases}\dfrac{L_a K\Delta u_{cr}}{4Ed}\left[1+\dfrac{2}{\pi}\left\{\dfrac{u_G}{u_{Gs}}\cos\dfrac{2\pi}{L_a}z-\sqrt{\left(\dfrac{u_G}{u_{Gs}}\right)^2-1}-\sin^{-1}\dfrac{u_{Gs}}{u_G}\right\}\right]; & 0\leq z\leq\xi'\dfrac{L_a}{4} \quad \cdots\cdots (21)\\[2ex] \dfrac{L_a K\Delta u_{cr}}{4Ed}\left(1-4\dfrac{z}{L_a}\right); & \xi'\dfrac{L_a}{4}<z\leq\dfrac{L_a}{4} \quad \cdots\cdots (21')\end{cases}$$

The corresponding lower bound for the pipe strain amplitude is obtained as

$$e_{Sl}^{(2)}=\frac{L_a K\Delta u_{cr}}{4Ed}\left\{1+\frac{2}{\pi}\left(\frac{u_G}{u_{Gs}}-\sqrt{\left(\frac{u_G}{u_{Gs}}\right)^2-1}-\sin^{-1}\frac{u_{Gs}}{u_G}\right)\right\} \quad \cdots\cdots (22)$$

It may be noted that $e_{Sl}^{(2)}$ tends to e_{Su} in Eq. (18) as $u_G\to\infty$, and it coincides with $e_{Sl}^{(1)}$ in Eq. (19) when $u_G=u_{Gs}$. In Fig. 5, the upper and lower bounds obtained above are compared with the exact solution. Observe that the lower bound $e_{Sl}^{(2)}$ gives a much closer approximation than the other bounds for all values of ε_G.

The estimation of the spring constant K and the critical relative displacement Δu_{cr} governing slippage conditions is an important subject that requires extensive research efforts. A preliminary discussion is made regarding this in Appendix A.

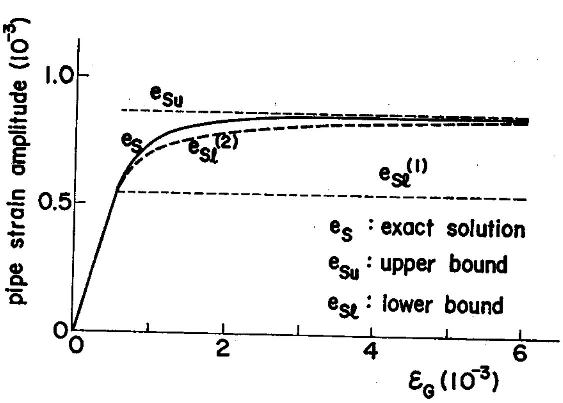

Fig. 5 Pipe Strain Amplitude. ($L/D=10^3$, $h/D=15$, $D/d=100$, $G/E=2.5\times10^{-4}$, $\gamma_{cr}=1.4\times10^{-4}$)

3.3 Pipe Strain Solution Using Displacement Profile Function

As discussed later in Appendix A, the spring constant and the critical relative displacement for the slippage condition will be indepedent of the depth of the pipe when the pipes are buried below a certain depth. In such cases, it may be more convenient to represent the rigidity and the slippage conditions of the soil in terms of physical constants pertinent to the soil. In this sense Shinozuka and Koike[9] proposed using a displacement profile function and to use the shear modulus to represent the soil rigidity, and the critical shear strain to represent the slippage condition. In this section, the pipe strains are analyzed using these concepts.

(1) Elastic solution and relation between the displacement profile function and the spring constant

It is assumed that the soil displacement around the pipe takes a certain functional form that coincides with the pipe displacement on the pipe surface, and converges to the apparent free field displacement u_a as the distance from the pipe increases. As proposed by Shinozuka and Koike[9], the displacement profile function $U(x)$ characterizes the soil deformation in the vicinity of the pipe, as shown in Fig. 6. The soil displacement around the pipe may then be represented by

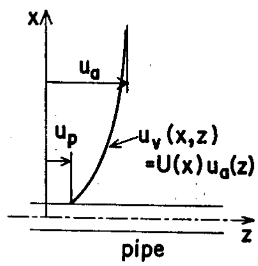

Fig. 6 Displacement Profile Function.

$$u_v(x, z) = U(x)\,u_a(z) \quad \cdots\cdots (23)$$

in which $U(x)=$ the displacement profile function.

The function $U(x)$ can be related to the spring constant K in the following manner. Given the functional form of $U(x)$, the shearing force $f(z)$ acting on the pipe surface without slippage takes the form

$$f(z) = G\phi_0 \frac{L_a}{2\pi}\varepsilon_G \sin\frac{2\pi}{L_a}z \quad \cdots\cdots (24)$$

where

$$\phi_0 = [dU/dx]_{x=D/2} \quad \cdots\cdots (25)$$

Substituting Eq. (24) into Eq. (9) leads to

$$u_p(z) = \left(\frac{L_a}{2\pi}\right)^3 \frac{G}{Ed}\phi_0\varepsilon_G \sin\frac{2\pi}{L_a}z \text{ ; elastic solution} \quad \cdots\cdots (26)$$

From the condition that the seismic load $K(u_a-u_p)$, determined by using the spring constant, be equal to $f(z)$ in Eq. (24), it follows that

$$G\psi_0 = K\left\{1-\left(\frac{L_a}{2\pi}\right)^2 \frac{G}{Ed}\psi_0\right\} \quad \cdots\cdots (27)$$

Hence,

$$K=\frac{G\psi_0}{1-\left(\frac{L_a}{2\pi}\right)^2 \frac{G}{Ed}\psi_0} \quad \cdots\cdots (28)$$

For the functional form of $U(x)$, Shinozuka and Koike[9] have proposed to use

$$U(x)=\zeta\frac{2\pi}{L_a}\, exp\left(-\zeta\frac{2\pi}{L_a}\frac{D}{2}\right) \quad \cdots\cdots (29)$$

where

$$\zeta=\frac{2\pi}{L_a}\frac{Ed}{G}$$

in which case Eq. (27) reduces to

$$\psi_0=\zeta\frac{2\pi}{L_a}\exp\left(-\zeta\frac{2\pi}{L_a}\frac{D}{2}\right) \quad \cdots\cdots (30)$$

Then Eq. (30) is rewritten as

$$K=\frac{\left(\frac{2\pi}{L_a}\right)^2 Ed}{\exp\left\{\left(\frac{2\pi}{L_a}\right)^2\frac{Ed}{G}\frac{D}{2}\right\}-1} \quad \cdots\cdots (31)$$

As the argument of the exponential function in Eq. (31) is much smaller than unity, K can be approximated by

$$K\cong\frac{2G}{D} \quad \cdots\cdots (32)$$

Eq. (32) gives very large values of K, compared to Eq. (A. 2) in Appendix A, based on an experimental result. This difference directly affects the elastic solution for the pipe strain. However, it is the critical force $K\Delta u_{cr}$, not K itself, that has primary effects on the pipe strains under partial slippage conditions. As will be seen later, the form of $U(x)$ will have influence only on determining the location of the bounday of the elastic and plastic regions on the pipe, and not so much on the estimated pipe strain.

(2) Upper and lower bounds on pipe strains under partial slippage conditions

When the displacement profile function $U(x)$ is used for estimating the pipe strains under slippage conditions, the upper and lower bound pipe strains corresponding to Eqs. (18), (19) and (22) are obtained in the same manner as in Fig. 4. The slippage condition in this case is given in terms of the critical shear strain γ_{cr} of the soil at interface. The shearing stress acting on the pipe surface is represented by $G\,\gamma_{cr}$. Under slippage it is also clear that

$$G\,\gamma_{cr}=K\,\Delta u_{cr} \quad \cdots\cdots (33)$$

Hence, from Eq. (18) we obtain the upper bound ϵ'_{S_u} of the pipe strain amplitude in the following form:

$$\epsilon'_{S_u}=\frac{L_a G\gamma_{cr}}{4Ed} \quad \cdots\cdots (34)$$

Likewise, the lower bound corresponding to Eq. (19) is obtained as

$$\epsilon'^{(1)}_{Sl}=\frac{2}{\pi}\frac{L_a G\gamma_{cr}}{4Ed} \quad \cdots\cdots (35)$$

The lower bound closer to the exact solution corresponding to Eq. (22) is obtained as follows. Again observing Fig. 4(c), and noting that the elastic solution for $f(z)$ is given by Eq. (24), the location ξ' of the boundary of the elastic region is represented by

$$\xi'=\frac{2}{\pi}\sin^{-1}\frac{2\pi\gamma_{cr}}{\psi_0 L_a \epsilon_G}=\frac{2}{\pi}\sin^{-1}\frac{\epsilon'_{Gs}}{\epsilon_G} \quad \cdots\cdots (36)$$

where

$$\epsilon'_{Gs}=\frac{2\pi\gamma_{cr}}{\psi_0 L_a} \quad \cdots\cdots (37)$$

in which ϵ'_{Gs} represents the apparent free field strain on the initiation of slippage. For a particular case where the displacement profile function is given by Eq. (29), Eq. (37) is written as

$$\epsilon'_{Gs}=\frac{1}{\zeta}\exp\left(\zeta\frac{2\pi}{L_a}\frac{D}{2}\right) \quad \cdots\cdots (38)$$

Then, analogous to Eq. (21), the lower bound $\epsilon'^{(2)}_l(z)$ for the pipe strain is obtained as

$$\epsilon'^{(2)}_l(z)=\begin{cases}\dfrac{L_a G\gamma_{cr}}{4Ed}\left[1+\dfrac{2}{\pi}\left\{\dfrac{\epsilon_G}{\epsilon'_{Gs}}\cos\dfrac{2\pi}{L_a}z-\sqrt{\left(\dfrac{\epsilon_G}{\epsilon'_{Gs}}\right)^2-1}-\sin^{-1}\dfrac{\epsilon'_{Gs}}{\epsilon_G}\right\}\right];\ 0\leq z\leq\xi'\dfrac{L_a}{4} & \cdots\cdots (39)\\[2ex] \dfrac{L_a G\gamma_{cr}}{4Ed}\left(1-4\dfrac{z}{L_a}\right);\quad \xi'\dfrac{L_a}{4}<z\leq\dfrac{L_a}{4} & \cdots\cdots (39')\end{cases}$$

The lower bound for the pipe strain amplitude is then represented by

$$\epsilon'^{(2)}_{Sl}=\frac{L_a G\gamma_{cr}}{4Ed}\left\{1+\frac{2}{\pi}\left(\frac{\epsilon_G}{\epsilon'_{Gs}}-\sqrt{\left(\frac{\epsilon_G}{\epsilon'_{Gs}}\right)^2-1}-\sin^{-1}\frac{\epsilon'_{Gs}}{\epsilon_G}\right)\right\} \quad \cdots\cdots (40)$$

4. Maximum Pipe Strains with Unknown Directions of Incident Waves

Since one can not predetermine the angle of incidence θ of the seismic wave relative to the pipe, it is reasonable to use the largest value of the pipe strains for the various values of θ. In this chapter, solutions are developed for the maximum pipe strains with unknown values of θ.

4.1 Elastic Solution

When no slippage occurs between the pipes and surrounding soils, the elastic solution applies, and a closed-form solution for the maximum pipe strain with the unknown θ can be readily derived. By substituting Eq. (4), (4'), (6) and (6') into Eq. (17), and referring to the first part of Eq. (12), the elastic solution for the pipe strain amplitude e_S is represented as a function of the input ground strain and the angle of incidence in the following form:

$$e_S = \begin{cases} \dfrac{\varepsilon_l \cos^2\theta}{1+\beta_l \cos^2\theta} \ ; & \text{longitudinal wave} \quad \cdots\cdots (41) \\[2ex] \dfrac{\gamma_t \sin\theta\cos\theta}{1+\beta_t \cos^2\theta} \ ; & \text{transverse wave} \quad \cdots\cdots (41') \end{cases}$$

in which

$$\beta_l = \left(\frac{2\pi}{\lambda L_l}\right)^2, \qquad \beta_t = \left(\frac{2\pi}{\lambda L_t}\right)^2 \quad \cdots\cdots (42)$$

The value of θ that maximizes e_S in the above expression is obtained as follows;

$$\theta_{el} = 0 \quad ; \text{ longitudinal wave} \cdots\cdots (43)$$

$$\theta_{et} = \frac{1}{2}\cos^{-1}\frac{-\beta_t}{2+\beta_t} \quad ; \text{ transverse wave} \cdots\cdots (44)$$

Then, the maximum pipe strain is given by the value of e_S corresponding to these angles, which yields

$$e_{el} = \frac{\varepsilon_l}{1+\beta_l} \quad ; \quad \text{longitudinal wave} \cdots\cdots (45)$$

$$e_{et} = \frac{\gamma_t}{2\sqrt{1+\beta_t}} \quad ; \quad \text{transverse wave} \cdots\cdots (46)$$

As β_l and β_t usually assume small values relative to unity, these results justify the often employed approximations assuming $e_{el} \simeq \varepsilon_l$ and $e_{et} \simeq \gamma_t/2$, and also $\theta_{et} \simeq \pi/4$.

The limited values for the ground strains ε_l and γ_t, up to which the elastic solutions apply, are obtained by substituting ε_{Gs} in Eq. (16') into ε_G in Eqs. (6) and (6'), and solving for ε_l and γ_t. These calculations are made with $\theta = \theta_{el}$ and θ_{et}, respectively. These limiting ground strains are represented by

$$\varepsilon_{lt}=\frac{2}{\pi}(1+\beta_l)\,e_{lw} \quad ; \quad \text{longitudinal wave} \qquad (47)$$

$$\gamma_{tt}=\frac{4}{\pi}\sqrt{(1+\beta_t)(2+\beta_t)}\,e_{tw} \quad ; \quad \text{transverse wave} \qquad (48)$$

in which β_l and β_t are given by Eq. (42), and

$$\left.\begin{aligned} e_{lw}&=\frac{L_l\,K\,\Delta u_{cr}}{4Ed}=\frac{L_lG\gamma_{cr}}{4Ed} \\ e_{tw}&=\frac{L_t\,K\,\Delta u_{cr}}{4Ed}=\frac{L_tG\gamma_{cr}}{4Ed} \end{aligned}\right\} \qquad (49)$$

The non-dimensional parameter e_{lw} coincides with the upper bound solution, Eq. (18), when a longitudinal wave propagates with $\theta=0$. The parameters e_{lw} and e_{tw} will be used conveniently for expressing the approximate soluitons in 4. 3.

4.2 Plastic Solution

Under partial slippage conditions, it is difficult to derive a closed-form solution for the maximum value of e_S, since the transcendental equation, Eq. (13) for determining ξ involves θ. This justifies the development of the approximate solutions in the next section. Herein the general features of the maximum pipe strains, under slippage will be discussed, using the numerical results based on the exact solution.

Fig. 7 shows examples of the pipe strain amplitude under slippage conditions plotted against the angle of incidence θ. The exact solution has been obtained from

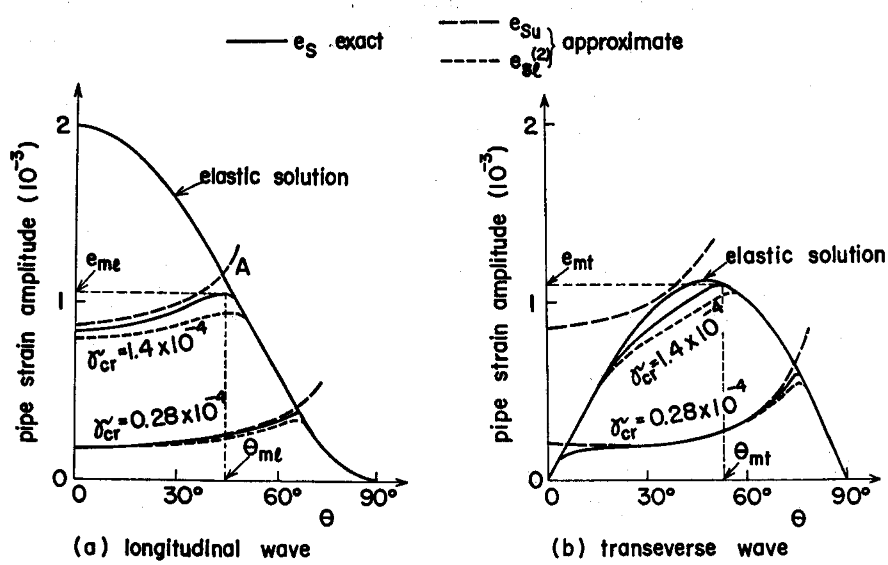

Fig. 7 Pipe Strain Amplitude. ($\varepsilon_l=\gamma_t=2\times10^{-3}$, $L_l/D=L_t/D=10^3$, $h/D=15$, $D/d=100$, $G/E=2.5\times10^{-4}$)

Eq. (17). The upper and lower bound solutions can be obtained by using Eqs. (18) and (22), or Eqs. (34) and (40). The elastic solution is based on Eq. (17′), or Eqs. (41) and (41′).

Fig. 7 demonstrates a widely recognized idea that slippage between the soil and the pipe will reduce the pipe strain. As the slippage condition γ_{cr} decreases (implying that slippage takes place at a smaller ground strain), the maximum pipe strain represented by e_{ml} and e_{mt} decreases.

It should be noted that many previous works on the estimation of axial pipe strains have assumed the propagation of longitudinal waves in the direction of the pipe axis, claiming that it is a conservative assumption. However, it is clear from Fig. 7 that this is not necessarily true when slippage occurs. In each of the two cases of γ_{cr} shown in Fig. 7, the maximum pipe strain e_{mt} obtained for the transverse waves is larger than the maximum pipe strain e_{ml} for the longitudinal waves.

This feature can be seen more clearly in Fig. 8 which shows the maximum pipe strains plotted against input free field strains. Observe that the pipe strain e_{mt} for the transverse waves tends to be larger than e_{ml} for the longitudinal waves as the free field strain increases, causing slippages.

It may be observed that the values of θ corresponding to the maximum pipe strains e_{ml} and e_{mt}, denoted by θ_{ml} and θ_{mt} respectively, are fairly close when slippage occurs, whereas in the elastic solutions there is a difference of about 45° between them. Therefore, under slippage conditions, the maximum pipe strains induced by the longitudinal and transverse waves may be combined. If we assume independent ramdom phases for these two types, of waves, a conservative estimate of the combined

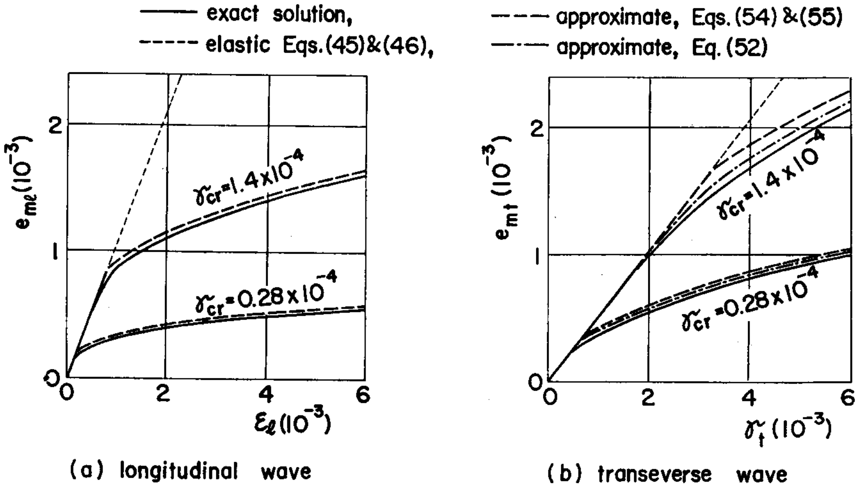

Fig. 8 Maximum Pipe Strain with Unknown Direction of Incidence. (Same parameter values as Fig. 7.)

maximum pipe strain e_{mc} may be obtained from

$$e_{mc} \cong \sqrt{e_{ml}^2 + e_{mt}^2} \quad \cdots\cdots (50)$$

4.3 Simple Approximate Formula for Maximum Pipe Strains under General Slippage Conditions

Iterative computation is the only way to obtain exact numerical results for the maximum pipe strains e_{ml} and e_{mt} under slippage conditions. Herein, an approximate closed-form solution is derived. The resulting formulas are very simple, and they should be useful for practical purposes.

The general idea comes from Fig. 7, in which the upper bound solution e_{S_u} is a close approximation of the exact solution e_S. Then, the intersection of the upper bound solution and the elastic solution, for example point A in Fig. 7, will give a good approximation of the maximum pipe strain. Such points are determined by equating Eq. (17′) and Eq. (18). By virtue of Eqs. (4) and (5), and noting that α_1 is approximately equal to unity under normal conditions, we obtain the following approximations of the maximum pipe strains under slippage conditions.

For longitudinal waves :

$$e_{pl} = \sqrt[3]{e_{lw}^2 \varepsilon_l} \quad \cdots\cdots (51)$$

For transverse waves :

$$e_{pt}^{(1)} = e_{tw} \Big/ \left[1 - \frac{4}{3}\cos^2\left\{\frac{\pi}{2} + \frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}}{2}\frac{e_{tw}}{\gamma_t}\right)\right\}\right]^{1/2} \quad \cdots\cdots (52)$$

The parameters e_{lw} and e_{tw} have been defined by Eq. (49).

Furthermore, from an asymptotic behavior of $e_{pt}^{(1)}$ with an increase in γ_t, Eq. (52) can be replaced by

$$e_{pt}^{(2)} = \sqrt{e_{tw}\gamma_t} \quad \cdots\cdots (53)$$

The approximate formulas for general slippage conditions may be developed by adopting the formula assuming a larger value from the elastic solution (Eqs. (45) and (46), or from the approximate plastic solution (Eqs. (51) and (53)). The results are as follows ;

longitudinal waves :

$$e_{ml} \cong \begin{cases} e_{el} = \dfrac{\varepsilon_l}{1+\beta_l}, & \varepsilon_l \leq \varepsilon_{ld} \\ e_{pl} = \sqrt[3]{e_{lw}^2 \varepsilon_l}, & \varepsilon_l > \varepsilon_{ld} \end{cases} \quad \cdots\cdots (54)$$

transverse waves :

$$e_{mt} \cong \begin{cases} e_{et} = \dfrac{\gamma_t}{2\sqrt{1+\beta_t}}, & \gamma_t \leq \gamma_{td} \\ e_{pt}^{(2)} \doteq \sqrt{e_{tw}\gamma_t}, & \gamma_t > \gamma_{td} \end{cases} \quad \cdots\cdots (55)$$

in which

$$\left. \begin{aligned} \varepsilon_{ld} &= \sqrt{(1+\beta_l)^3}\, e_{lw} \\ \gamma_{td} &= 4(1+\beta_t)\, e_{tw} \end{aligned} \right\} \quad \cdots\cdots (56)$$

These results are plotted in Fig. 8. Observe that their agreement with the exact solution is satisfactory, and particularly good for small γ_{er}.

A remarkable aspect of Eqs. (54) and (55) is that the maximum pipe strains e_{ml} and e_{mt} are not bound by any fixed values, but will increase infinitely with the ground strains ε_l and γ_e, whereas the pipe strain amplitude with a fixed value of the angle of incidence has a bound given by Eq. (18).

Conclusions

From the results of this study, the following conclusions may be derived.

(1) A coprehensive analysis has been performed for axial strains developed in buried straight pipes by horizontally propagating seismic waves, including longitudinal and transverse waves.

(2) Pipe strain amplitude under slippage between soil and pipe has been obtained for an arbitary direction of incident waves. Upper and lower bound solutions were also derived.

(3) Maximum pipe strains with unknown direction of incident waves have been discussed extensively. It has been pointed out that under slippage conditions, the maximum pipe strain induced by transverse waves tends to be larger than that induced by longitudinal waves as the free field strains increase.

(4) Simple closed-form approximate solutions have been developed for the maximum pipe strains. They imply that the maximum pipe strain under slippage for the longitudinal waves increases as a cubic root of the free field normal strain. For the transverse waves it increases as a square root of the free field shear strain.

(5) It has been suggested that the maximum pipe strains from the longitudinal and transverse waves be combinecl in the form of Eq. (50), on the basis of the results under slippage conditions.

Finally it may be pointed out that the analytical results obtained in this study should be discussed in the light of experimental results, particularly in estimating the stiffness of soil relative to the pipe, and in evaluating the slippage conditions. Some preliminary discussion is made concerning this in Appendix A.

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Appendix A. On Estimation of Spring Constant K and Slippage Conditions Δu_{cr} and γ_{cr}

In determining the pipe strains under partial slippage conditions, the estimation of the spring constant K and the critical relative displacement Δu_{cr}, or alternatively critical shear strain γ_{cr}, is a key question. Herein, a simple method is proposed for this purpose on the basis of the experimental results presented by Kuribayashi et. al.[4].

It is expected that the values of K for the shallow pipes will depend on their depth h from the ground surface, Fig. A. 1, but the values for the deeply buried pipes will be independent of h. Kuribayashi et. al.[4] performed shake-table tests on cast iron pipes with an outer diameter of 16 cm buried at various depths in a sand box, as shown in Fig. A. 2. The shear modulus of the sandy soil used for the tests was found to be 580 kgf/cm² on the average. From the axial force and the soil displacement relative to the pipe in Ref. (4), the spring constant K can be roughly estimated by

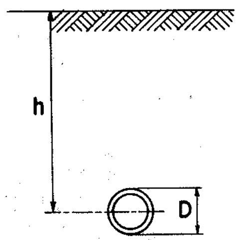

Fig. A. 1 Location of Buried Pipes.

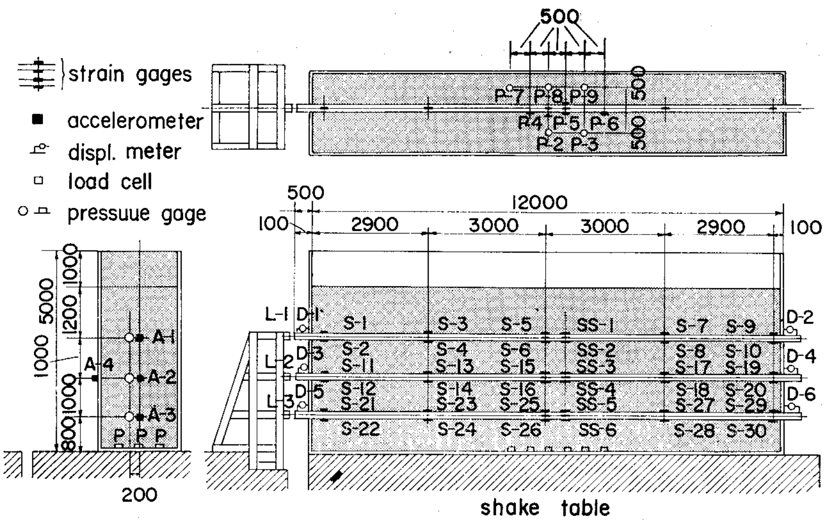

Fig. A. 2 Dynamic Tests of Buried Pipes by Kuribayashi et. al.[4]

$$K \cong \begin{cases} 1.4 \text{ kg f/cm}^3 & ; \ h/D=7.5 \\ 2.8 \text{ kg f/cm}^3 & ; \ h/D=13.75,\ 20 \end{cases} \qquad \cdots\cdots \text{(A. 1)}$$

Plots of the these values in Fig. A. 3 would suggest that the value of K is proportional to the pipe depth h, when h/D is less than some 13, whereas it is independent of h for larger values of h/D. It is interesting to note that these experimental results on the effect of h/D on K agree qualitatively with the elastic solution obtained by Parmelee and Ludtke[7]. It should, however, also be pointed out that the numerical values of the elastic spring constant, obtained from the result of Ref. (7) for the case of the model tests in Ref. (4), are excessively larger than the test results by two decimal orders.

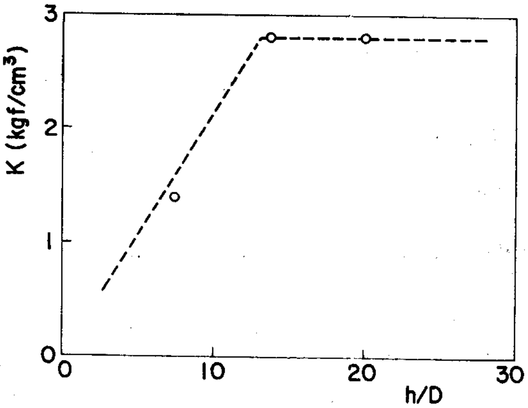

Fig. A. 3 Spring Constant Estimated from Experimental Results in Ref. (4).

Ugai and Yamaguchi[12] made a theoretical analysis of the dynamic values of the spring constant K, assuming straight pipes buried in infinite elastic media, whose results were found to be in fairly good agreement with the experimental results. They concluded that the spring constant per unit area is roughly inversely proportional to the diameter D of the pipe, and proportional to the shear modulus G of the soil. Combining this with the results in Eq. (A. 1), K may be represented by

$$K \cong \begin{cases} 0.059 \dfrac{G}{D} \dfrac{h}{D} & ; \ 0 < \dfrac{h}{D} \leq r_d \\ 0.059\, r_d \dfrac{G}{D} & ; \ \dfrac{h}{D} > r_d \end{cases} \qquad \text{(in kg}f/\text{cm}^3) \cdots\cdots \text{(A. 2)}$$

in which $r_d = 13$ is used for the numerical computation in this study.

Likewise, the frictional force between the soil and the pipe in slippage has been shown to be proportional to h/D for $h/D < 13$, and constant for $h/D > 13$. From this, we may put

$$\Delta u_{cr}=\begin{cases}\dfrac{\mu w_s h}{K} & ; \quad \dfrac{h}{D}<r_d \\ \dfrac{\mu w_s r_d D}{K} & ; \quad \dfrac{h}{D}>r_d\end{cases} \qquad \text{(A. 3)}$$

where w_s= the unit weight of the soil above the pipe, and μ= the equivalent coefficient of friction. As for the experimental results in Ref. (4), where $r_d=13$, the value of μ has been found to be approximately 0. 5.

It is hoped that Eqs. (A. 2) and (A. 3) will provide rough estimates for the spring constant K and the critical relative displacement Δu_{cr} for cast iron pipes buried in sandy soils. For a practical estimation of K and Δu_{cr} for more general cases including other types of soil and other pipe materials and diameters, further experimental works will be of great value.

The critical shear strain γ_{cr} for slippage condition may be estimated from the above results. By using the second part of Eq. (A. 3) and Eq. (33), we have

$$G\gamma_{cr}=K\,\Delta u_{cr}=\mu w_s r_d D$$

Therefore, γ_{cr} is expressed as

$$\gamma_{cr}=\frac{\mu w_s r_d D}{G} \qquad \text{(A. 4)}$$

For sandy soils, the values of w_s and G will vary in the range $w_s=1.6\sim1.8\ t/\mathrm{m}^3$ and $G=300\sim1500$ kg f/cm². The pipe diameter D will be in the range 0. 1~1. 5 m. The equivalent coeffient of friction μ will depend not only on the soil properties, but also on the finish of the pipe surface. Here, we may assume that $\mu=0.1\sim1.0$. Then, from Eq. (A. 4), γ_{cr}, with $r_d=13$, will vary in the range $\gamma_{cr}=0.014\times10^{-3}\sim11.7\times10^{-3}$. Since μ is expected to increase with G, the above range for γ_{cr} may include some unrealistic extreme cases. In the case of the experimental results in Ref. (4), with $\mu=0.5$, $w_s=1.7\ t/\mathrm{m}^3$, $D=16$ cm and $r_d=13$, we have $\gamma_{cr}=0.3\times10^{-3}$.

Appendix B. Notations

D = outer diameter of pipe ;
d = thickness of pipe wall ;
E = Young's modulus of pipe material ;
e = generally represents pipe strain ;
$e(z)$ = pipe strain at location z ;
e_{sl}, e_{st} = maximum pipe strains without slippage for longitudinal and transverse waves, respectively ;
$e_l^{(2)}$, $e_l'^{(2)}$ = lower bounds on $e(z)$;
e_{lw} = $L_l\,K\,\Delta u_{cr}/(4Ed)=L_l G\gamma_{cr}/(4Ed)$;

e_{mc} = maximum pipe strain with combined effects of longitudinal and transverse waves ;

e_{ml}, e_{mt} = maximum pipe strains induced by longitudinal and transverse waves, respectively, with arbitrary angles of incidence ;

e_{pl} = approximate maximum pipe strain under slippage for longitudinal waves ;

$e_{pt}^{(1)}, e_{pt}^{(2)}$ = approximate maximum pipe strains under slippage for transverse waves ;

e_S = pipe strain amplitude for a fixed angle of incidence ;

$e_{Sl}^{(1)}, e_{Sl}^{(2)}, e_{Sl}'^{(1)}, e_{Sl}'^{(2)}$ = lower bounds on e_S ;

e_{Su}, e'_{Su} = upper bounds on e_S ;

e_{tw} = $L_t K \Delta u_{cr}/(4Ed) = L_t G\gamma_{cr}/(4Ed)$;

f = $f(z)$ = shearing stress acting on pipe surface as seismic load ;

G = shear modulus of soil ;

h = depth of buried pipe ;

K = equivalent spring constant ;

L_a, L_l, L_t = apparent wave length, wave length of longitudinal waves, and wave length of transverse waves, respectively ;

r_d = maximum value of h/D with influence of ground surface ;

$U(x)$ = displacement profile function ;

u = generally represents displacement ;

u_a = apparent free field displacement along pipe axis ;

u_G = apparent free field displacement amplitude ;

u_{Gs} = value of u_G on initiation of slippage ;

u_l, u_t = free field displacements due to longitudinal and transverse waves, respectively ;

u_p = axial displacement of pipe ;

$u_v(x, z)$ = soil displacement in the vicinity of pipe ;

w_s = weight per unit volume of soil ;

x = distance from pipe axis ;

z, z' = absolute and relative positions along pipe, respectively ;

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ = parameters ;

β_l, β_t = $(2\pi/\lambda L_l)^2$, $(2\pi/\lambda L_t)^2$, respectively ;

γ = generally represents ground shear strain ;

γ_{cr} = critical shear strain for slippage ;

γ_t = free field shear strain due to transverse waves ;

γ_{td} = $4(1+\beta_t)e_{tw}$;

γ_{ts} = limiting free field strain for elastic solution of maximum pipe strain ;

Δu_{cr} = critical relative displacement for slippage ;

ε = generally represents ground normal strain ;

ε_G = apparent free field normal strain along pipe axis ;

$\varepsilon_{Gs}, \varepsilon'_{Gs}$ = value of ε_G on initiation of slippage ;
ε_l = free field normal strain due to longitudinal waves ;
ε_{ld} = $\sqrt{(1+\beta_l)^3}\, e_{lw}$;
ε_{ls} = limiting free field normal strain for elastic solution of maximum pipe strain ;
ζ = $(2\pi/L_e)(G/Ed)$;
θ = angle of incidence ;
θ_{el}, θ_{et} = values of θ corresponding to e_{el} and e_{et}, respectively ;
θ_{ml}, θ_{mt} = values of θ corresponding to e_{ml} and e_{mt}, respectively ;
λ = $\sqrt{K/Ed}$;
μ = equivalent coefficient of friction ;
ξ, ξ' = parameters defining boundaries of elastic and plastic regions along pipes ;
ξ_p = position along direction of wave propagation ;
ϕ_0 = $[dU/dx]_{x=D/2}$; and
ω = circular frequency.