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Abstract

This paper investigates the method for identifying the unknown parameters in the dynamics of an aircraft from flight data affected by random disturbances due to wind gusts. Two general algorithms suitable for applying to such stochastic environments, i.e. the method of maximum likelihood (ML) estimation and the extended Kalman filter (EKF) technique, are examined for capability by numerical simulations. The advantages and shortcomings of each algorithm are discussed in detail, which leads to the conclusion that the combined use of the two algorithms provides a powerful on-line technique, insensitive to initial parameter estimates.

Nomenclature

a_n	=normal acceleration, g
l_z, l_α	=distances from c.g. to accelerometer and angle of attack vane
M_α, M_q, M_{δ_e}	=pitching moment derivatives divided by moment of inertia, rad/sec^2
V	=mean velocity, m/sec
w_g	=vertical turbulence velocity, m/sec
Z_α, Z_{δ_e}	=normal force derivatives divided by mass and velocity, rad/sec
α	=angle of attack, rad
α_g	=angle of attack induced by vertical gust, rad
δ_e	=elevator deflection, rad
θ	=pitch angle, rad
σ_{w_e}	=rms value of vertical turbulence velocities, m/sec
$\sigma_{v_1}, \cdots, \sigma_{v_4}$	=rms values of measurement noises
tr.	=matrix trace function
$\|\|\cdot\|\|_{B^{-1}}$	=vector norm weighted with respect to B^{-1}

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Superscripts

• =time derivative of superscripted variable

^ =estimate of superscripted variable

I. Introduction

Techniques for identifying the stability and control derivatives of an aircraft from its response to known control inputs have received considerable attention for a long time. Recently, the progress of adaptive control technology has presented a new demand for developing digital techniques suitable for on-line processing by using airborn computers. There have been significant advances in analysis techniques, and numerous reports have been published on the topic during the last decade.

Most of the well-established techniques are designed by assuming that the tests are performed in smooth air. But the responses of an aircraft generally are affected by random disturbance inputs such as atmospheric gusts. Moreover, the observed data are contaminated with measurement noises in the majority of cases. Several attempts have been made to establish a technique which will be effective under such circumustances.

The computational algorithms developed may be classified into two groups according to an analysis technique based upon the maximum likelihood (ML) and the extended Kalman filter (EKF) methods, respectively. The former is a processing method for a bacth of data, and many successful results, including a case in atmospheric turbulence, have been reported.[1),2)] On the contrary, as for the latter, there are few satisfactory results in practical applications, since it is highly sensitive to the choice of the initial estimates.[3)] However, it is especially important because it is suitable for an on-line state estimation/ parameter identification required in adaptive control systems. The advantages and shortcomings of these algorithms are fully discussed in the following sections.

II. Statement of the Aircraft Dynamic System Model

The model of the system to be studied is linearized dynamic, and observation equations of an aircraft in a stochastic environment, which can be described as follows:

$$x_t = \boldsymbol{F}(\theta)x_t+\boldsymbol{G}(\theta)u_t+w_t \tag{1}$$

$$y_t = \boldsymbol{H}(\theta)x_t+\boldsymbol{D}(\theta)u_t+v_t \tag{2}$$

where θ is the vector of unknown parameters and w_t and v_t are the vector-valued uncorrelated Gaussian white noise process with a zero mean value and covaraince matrix $\boldsymbol{Q}$ and $\boldsymbol{R}$, respectively.

We select the longitudinal short period dynamics subjected to the external disturbances due to atmospheric gusts as a typical example of the objective system for the parameter identification. The complete perturbed equations of motion for this mode, including the gust shaping filter dynamics, are given in terms of four state variables; i.e. $x_t=[\alpha, \theta, \dot{\theta}, \alpha_g]^T$. The state measurements are provided by angle-of-attack vane, pitch attitude gyro and pitch rate gyro, respectively. The vertical accelerometer measurements are assumed to be available, i.e. $y_t=[\dot{\theta}_m, \theta_m, a_n, \alpha_m]$. The matrices $\boldsymbol{F}$, $\boldsymbol{G}$, $\boldsymbol{H}$ and $\boldsymbol{D}$ are given by[4),5)]

$$\boldsymbol{F}=\begin{pmatrix} Z_\alpha & 0 & 1 & Z_\alpha \\ 0 & 0 & 1 & 0 \\ M_\alpha & 0 & M_q & M_\alpha \\ 0 & 0 & 0 & -\omega_c \end{pmatrix}, \quad \boldsymbol{G}=\begin{pmatrix} Z_{\delta_e} \\ 0 \\ M_{\delta_e} \\ 0 \end{pmatrix},$$

$$\boldsymbol{H}=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \dfrac{l_z M_\alpha - V Z_\alpha}{g} & 0 & \dfrac{l_z M_q}{g} & \dfrac{l_z M_\alpha - V Z_\alpha}{g} \\ 1 & 0 & -l_\alpha/V & 1 \end{pmatrix}, \quad \boldsymbol{D}=\begin{pmatrix} 0 \\ 0 \\ \dfrac{l_z M_{\delta_e} - V Z_{\delta_e}}{g} \\ 0 \end{pmatrix}$$

and the noise covariance matrices are described as

$$\boldsymbol{Q}=\begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & \dfrac{2\sigma_{w_g}^2 \omega_c}{g} \end{pmatrix}, \quad \boldsymbol{R}=\begin{pmatrix} \sigma_{v_1}^2 & & & \\ & \sigma_{v_2}^2 & & \\ & & \sigma_{v_3}^2 & \\ & & & \sigma_{v_4}^2 \end{pmatrix}.$$

When θ is known, the optimal estimate of the state vector in the sense of the minimal error variance can be obtained by using the ordinary Kalman filter equations. However, needless to say, the state estimate is not optimal when θ, used in the filter equations, is different from the true value.

In the following sections, we investigate the algorithms for identifying the unknown parameters

Table 1. Inertial and aerodynamic data of the airplane.

Total mass	$m=38000$ Kg
Mean aerodynamic chord	$\bar{c}=4.434$ m
Wing area	$S=120.5$ m^2
Moment of inertia	$I_B=940800$ Kg·m^2
Air density	$\rho=0.652$ Kg/m^3
Cruising velocity	$V=173.0$ m/sec
Aerodynamic derivatives	$C_{L\alpha}=5.13$ 1/rad
	$C_{m\alpha}=-1.25$ 1/rad
	$C_{m_q}=-20.2$ 1/rad
	$C_{L\delta_e}=0.390$ 1/rad
	$C_{m\delta_e}=-1.36$ 1/rad

from the computer-generated flight data by using the above described dynamical system model. The numerical data of the example aircraft to be employed in simulation runs are listed in Table 1, and the supplementary data for simulation are shown in Table 2.

Table 2 Supplementary data for simulation

Real values of the parameters	
$Z_\alpha = -0.9167$	1/rad sec
$M_\alpha = -6.923$	1/rad sec^2
$M_q = -1.434$	1/rad sec
RMS velocity of vertical gust	
$\sigma_{w_g} = 2.7$	m/sec
RMS measurement noise intensity	
(NOISE-I)	(NOISE-II)
$\sigma_{v_1} = 0.0002528$	0.001264
$\sigma_{v_2} = 0.0002236$	0.001118
$\sigma_{v_3} = 0.0021240$	0.010620
$\sigma_{v_4} = 0.00014306$	0.0007153

III. ML Algorithm

We first discuss the method of maximum likelihood estimation. The marginal ML estimate of θ is defined as the estimate that maximizes an *"a posteriori"* probability density of θ based on the observation set Y^N $(=\{y_1, y_2, \cdots, y_N\})$. If no *"a priori"* distribution of θ is given and if the distribution is Gaussian, the ML estimate can be obtained by minimizing the following log-likelihood function $L(\theta, N)$ with respect to θ.

$$L(\theta, N) = \frac{1}{2}\sum_{k=1}^{N}(\|\nu_k\|^2 \boldsymbol{B}_k^{-1} + \ln. |\boldsymbol{B}_k|) \tag{3}$$

where ν_k and $\boldsymbol{B}_k$ are the conditional estaimate of the measurement residuals and their covariance matrix, respectively:

$$\nu_k = y_k - E[y_k | Y^{k-1}, \theta] \tag{4}$$

$$\boldsymbol{B}_k = E[\nu_k \nu_k^T | \theta] \tag{5}$$

If the parameter values are not bounded, the ML estimate is computed by setting

$$\nabla L(\theta, N) = \frac{\partial L}{\partial \theta} = 0 \tag{6}$$

where

$$\nabla L(\theta, N) = \sum_{k=1}^{N}[\nabla \nu_k^T \boldsymbol{B}_k^{-1} \nu_k + \frac{1}{2}\text{ tr. }\{(\nu_k \nu_k{}^T \boldsymbol{B}_k^{-1} - I)\ (\nabla \boldsymbol{B}_k \boldsymbol{B}_k^{-1}\}] \tag{7}$$

Since above equation is nonlinar in θ, the Newton-Raphson interative optimization technique is employed. The provided estimate is assured to be asymptotically unbiased, efficient and consistent.

One difficulty with this algorithm is that the number of difference equations to be solved increases rapidly with the dimensions of θ. Therefore, it is preferable to make some simplifications in view of practical use. We adopt the quasi

ML algorithm by neglecting the second term of Eq. (7) for a large N when θ does not include any element of the noise covariance matrix $\boldsymbol{Q}$ or $\boldsymbol{R}$. The Newton-Raphson iterative algorithm based on this quasi ML criterion is summarized as follows:

$$\hat{\theta}(i) = \hat{\theta}(i-1) - [\nabla^2 L(\hat{\theta}(i-1), N)]^{-1} \nabla L(\hat{\theta}(i-1), N) \tag{8}$$

$$\nabla L(\theta, N) = \sum_{k=1}^{N} \nabla \nu_k^T \boldsymbol{B}_k^{-1} \nu_k \tag{9}$$

$$\nabla^2 L(\theta, N) = \sum_{k=1}^{N} \nabla \nu_k^T \boldsymbol{B}_k^{-1} \nabla \nu_k \tag{10}$$

where $\nabla \nu_k$ are computed with the sensitivity equations for the Kalman filter.

This algorithm may easily be converted into a recursive form by defining

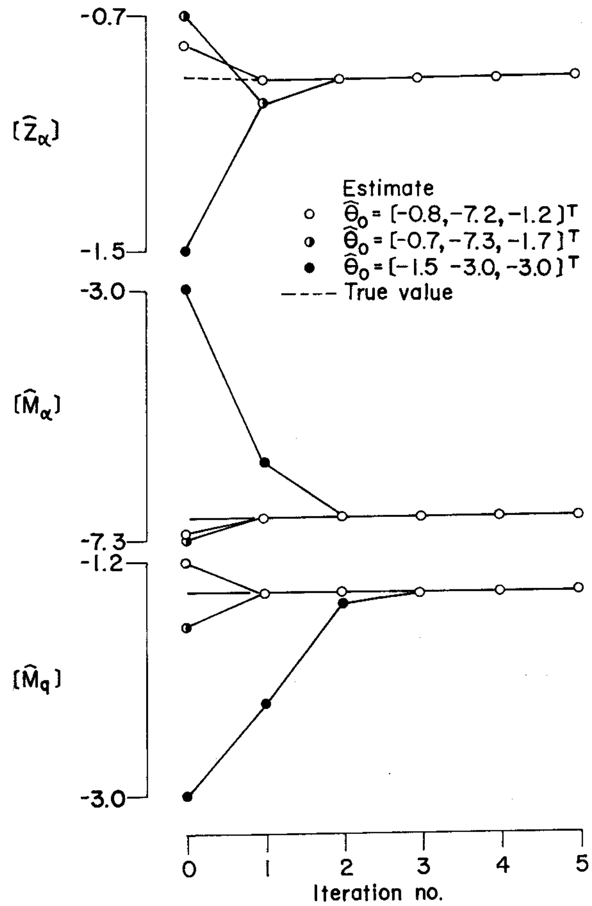

Fig. 1. Convergence of stability derivatives for 3.0 seconds of data starting at three sets of initial estimates.

$\boldsymbol{P}_\theta(k)$ as $[\nabla^2 L(\theta, k)]^{-1}$ and applying the matrix inversion lemma:

$$\begin{aligned}\boldsymbol{P}_\theta(k) &= [\boldsymbol{P}_\theta^{-1}(k-1)+\nabla\nu_k^T\boldsymbol{B}_k^{-1}\nabla\nu_k]^{-1}\\ &= [I+\boldsymbol{K}_\theta(k)\nabla\nu_k]\boldsymbol{P}_\theta(k-1) \qquad (11)\end{aligned}$$

$$\boldsymbol{K}_\theta(k) = -\boldsymbol{P}_\theta(k-1)\nabla\nu^T{}_k[\nabla\nu_k\boldsymbol{P}_\theta(k-1)\nabla\nu_k^T+\boldsymbol{B}_k]^{-1} \qquad (12)$$

The Newton-Raphson correction term $\Delta\theta(k)$ can be provided recursively

$$\Delta\theta(k) = [I+\boldsymbol{K}_\theta(k)\nabla\nu_k]\Delta\theta(k-1)+\boldsymbol{K}_\theta(k)\nu_k\,, \quad \Delta\theta(0)=0\,. \qquad (13)$$

By means of computer simulation, we examined the quasi ML algorithm for capability in the practical application to the aircraft parameter identification. The parameter vector to be identified was assumed to consist of three unknown stability derivatives: i.e. $\theta=[Z_\alpha, M_\alpha, M_q]$.

A systematic examination showed that this algorithm had excellent convergence characteristics. The results of the simulation runs are summarized in Fig. 1, which is the illustration of the estimated parameter values Z_α, M_α and M_q as functions of the iteration number starting at three sets of initial values. We see that the initial choice of the parameter values does not affect the converged value. Moreover, the estimates finally obtained are in good agreement with the true values which are indicated by the dotted lines.

Figure 2 shows the relation between the size of the sampled batch and the

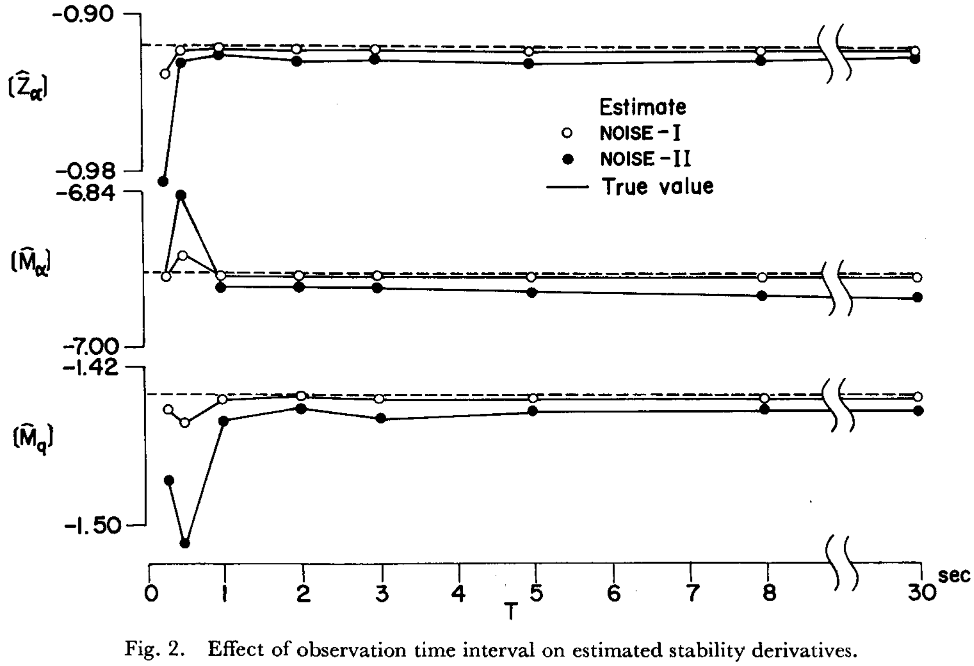

Fig. 2. Effect of observation time interval on estimated stability derivatives.

accuracy of the estimated value by taking the length of the observed data in seconds, T, as the abscissa. Since the sampling cycle for the measurements is taken as 100 Hz, the number of data points $N=100T$. The initial parameter values are selected as $\theta(0)=[-1.5, -3.0, -3.0]$. If T is larger than 1 second, there is scarcely any improvement in the estimated value, and the final accuracy depends on the measurement of the S/N values.

Another simulation run is made in order to show the capability of this algorithm in identifying the noise covariance parameters. The strict ML criterion is adopted in this case since the quasi ML approximation is not acceptable for a noise covariance parameter identification. The result of the σ_{w_g} identification is shown in Fig. 3 as a typical example.

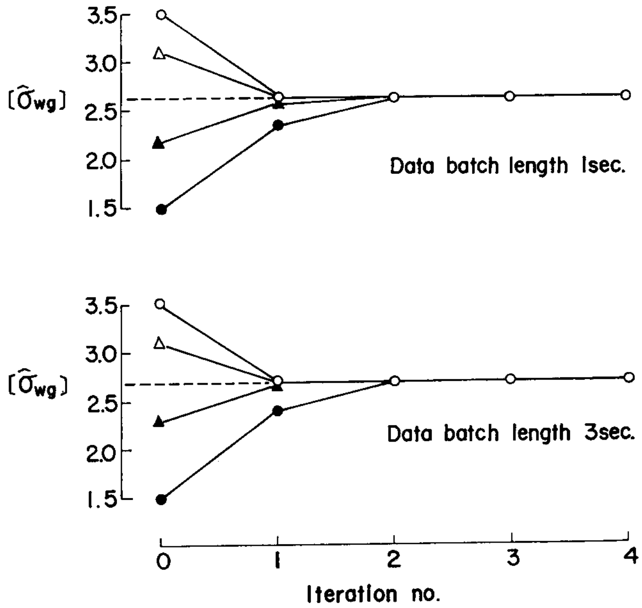

Fig. 3. Convergence of RMS gust velocity for 1.0 and 3.0 seconds of data.

IV. EKF Algorithm

The second algorithm consists essentially of the application of the standard extended Kalman filter technique to the nonlinear system, derived by augumenting the state vector with the parameters to be iddentified:

$$\begin{bmatrix} \dot{x}_t \\ \cdots \\ \dot{\theta}_t \end{bmatrix} = \begin{bmatrix} f(x_t, \theta_t, u_t, t) \\ \cdots\cdots\cdots\cdots \\ 0 \end{bmatrix} + \begin{bmatrix} w_t \\ \cdots \\ 0 \end{bmatrix}$$

$$y_t = h(x_t, \theta_t, u_t, t)$$

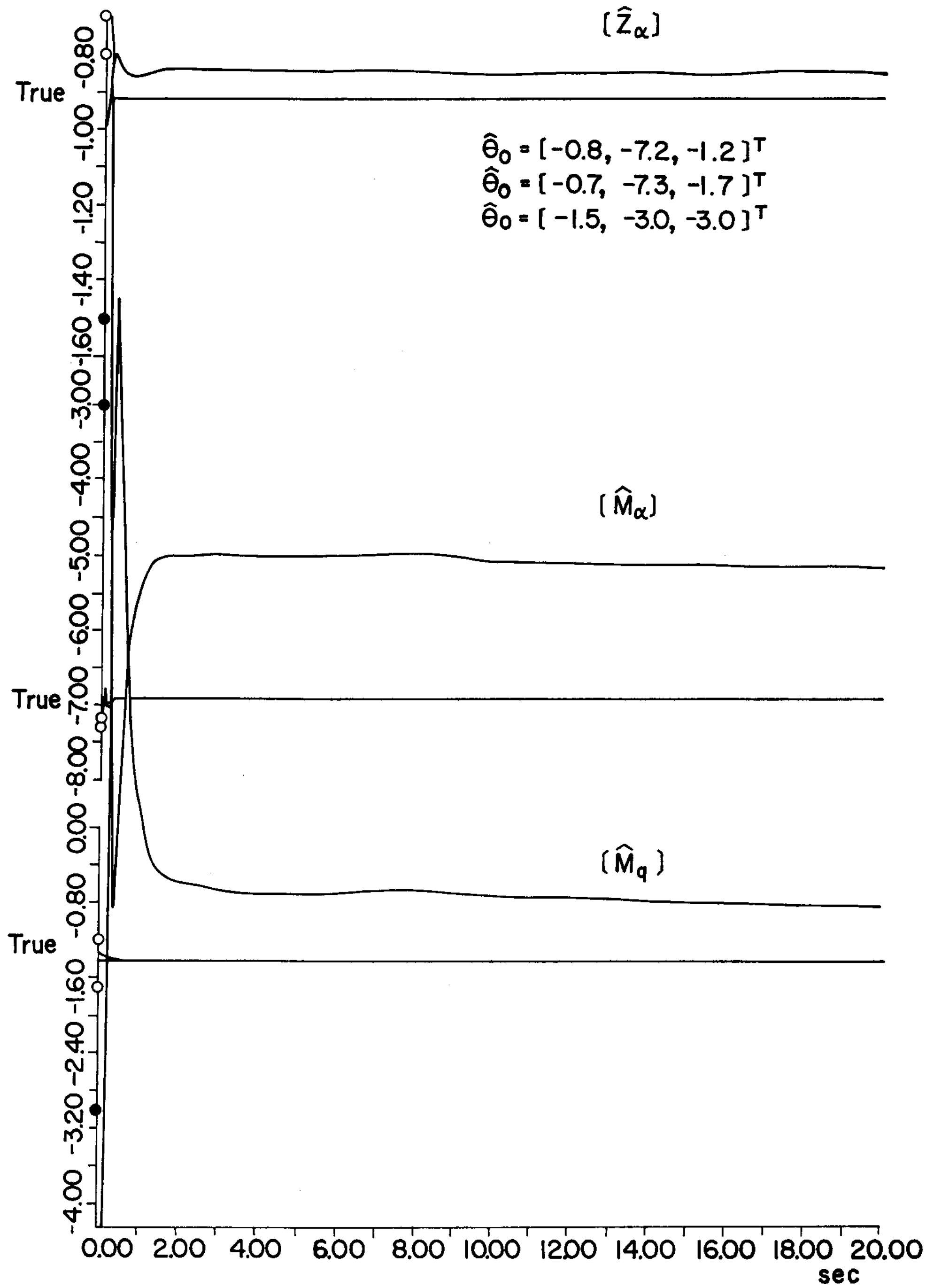

Fig. 4. Effect of starting values of parameter estimates on the convergence of the EKF algorithm.

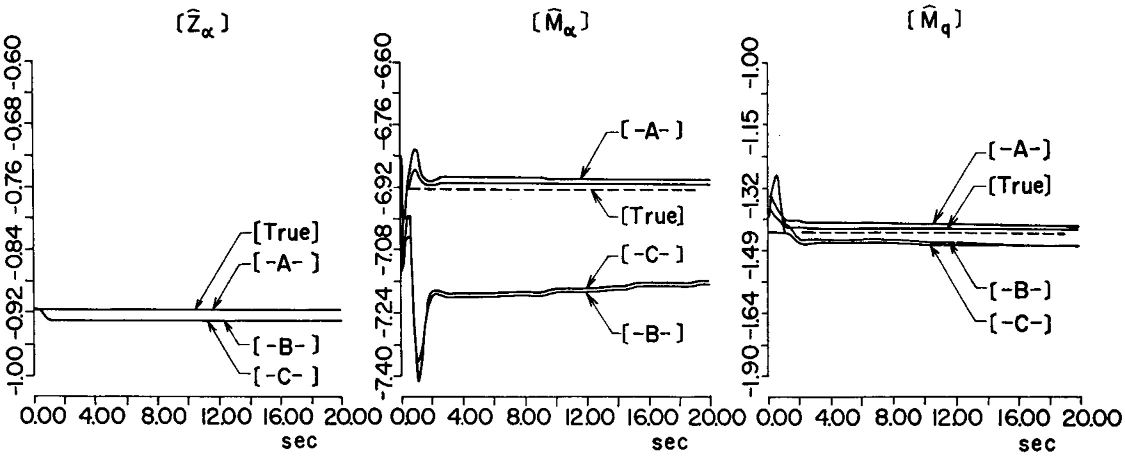

Fig. 5. Effect of starting values of parameter variances on the convergence of the EKF algorithm.

After the linear filtering theory is extended to nonlinear systems shown in Ref. 6, we can readily obtain the EKF algorithm for this augmented system. It provides on-line (sequential) estimates of the state and parameter vectors simultaneously. A pair of Kalman gain matrices, corresponding to those for updating the state and parameter estimates, respectively, are computed either simultaneously through the perturbed covariance equations for the augmented system, or separately by using the Fridland's bias filtering algorithm.

One of the main difficulties of the EKF algorithm in practice is that the selection of the initial values of the parameter estimates and their variances has a significant effect on the convergence characteristics. Figure 4 shows the estimates of each parameter as a function of time, starting at the same three sets of initial estimates that have been employed in the previous runs with the ML algorithm. It can be recognized from the figure that the results highly depend on the starting values; i.e. a good selection of the initial values provides a satisfactory result, whereas a poor one yields a biased estimate. This exhibits a striking contrast to the ML algorithm.

The starting values of the parameter variances also have a significant effect on the result of the estimation. Figure 5 illustrates the results of the estimation of each parameter with four different sets of initial variances; one set is true ([0.047, 0.142, 0.07]) and the other sets are erroneous (A=[0.02, 0.01, 0.03], B=[0.1, 0.6, 0.2] and C=[1.1, 1.9, 1.0].)

Combined ML/EKF Algorithm

The preceeding examinations for the EKF algorithm have shown that an inadequate selection of the starting values results in a fatal error in the estimated values. Therefore, there is a need for employing a start-up procedure which is insensitive to the initial estimates.

We investigate here the combined ML/EKF algorithm, in which the iterative ML solution is obtained by using the initial batch of the provided data, followed by processing the remaining data with the EKF in order to miprove the estimates sequentially. A combination of the two algorithms gives a powerful method for parameter identification because the weak points of the original ones compensate each other. As a result, the new algorithm has advantages in that it requires no "*a priori*" information on the initial parameter estimate, and it gives an on-line estimate in real time with less computational time and a higher degree of accuracy than the ML identification on overall data. Moreover, as has been suggested by Tanaka[7], since these two algorithms include essentially the same computational structures, a large simplification may be possible by using a single computer program jointly, and switching over from one to the other. However, it uses an extensively approximated ML algorithm, and the convergence of such an algorithm is not

Table 3 Convergence of stability derivatives of EKF algorithm, using ML algorithm for 3.0 second of data as start-up procedure

	T sec	[True variance]			[(True variance)×4]			[(True variance)/4]		
		1.2×10^{-6}	9.0×10^{-6}	4.0×10^{-6}	4.8×10^{-6}	3.6×10^{-5}	1.6×10^{-5}	3.0×10^{-7}	2.3×10^{-6}	1.0×10^{-6}
		Z_α	M_α	M_q	Z_α	M_α	M_q	Z_α	M_α	M_q
NOISE-I	0	−0.9178	−6.926	−1.436	−0.9178	−6.926	−1.436	−0.9178	−6.926	−1.436
		(0.12%	0.07%	0.14%)						
	10.0	−0.9166	−6.925	−1.435	−0.9166	−6.925	−1.435	−0.9168	−6.926	−1.435
	20.0	−0.9166	−6.924	−1.435	−0.9166	−6.924	−1.434	−0.9168	−6.925	−1.435
		(0.01%	0.01%	0.07%)	(0.01%	0.01%	0.00%)	(0.01%	0.03%	0.07%)
	True value	−0.9167	−6.923	−1.434	−0.9167	−6.923	−1.434	−0.9167	−6.923	−1.434

	T sec	[True variance]			[(True variance)×4]			[(True variance)/4]		
		3.0×10^{-5}	2.0×10^{-4}	1.0×10^{-5}	1.2×10^{-4}	8.0×10^{-4}	4.0×10^{-4}	7.5×10^{-6}	5.0×10^{-5}	2.5×10^{-5}
		Z_α	M_α	M_q	Z_α	M_α	M_q	Z_α	$_\alpha M$	M_q
NOISE-II	0	−0.9224	−6.937	−1.446	−0.9224	−6.937	−1.466	−0.9224	−6.937	−1.446
		(0.62%	0.20%	0.84%)						
	10.0	−0.9165	−6.934	−1.438	−0.9166	−6.933	−1.437	−0.9172	−6.936	−1.441
	20.0	−0.9167	−6.928	−1.438	−0.9164	−6.926	−1.437	−0.2173	−0.931	−1.440
		(0.00%	0.07%	0.28%)	(0.03%	0.04%	0.20%)	(0.07%	0.12%	0.42%)
	True value	−0.9167	−6.923	−1.434	−0.9167	−6.923	−1.434	−0.9167	−6.923	−1.434

assured.

A typical result of simulation runs for the combined ML/EKF algorithm is shown in Table 3. The initial 3.0 seconds' data are employed in the batch-iterative ML identification, and the remaining data are processed by using the EKF technique. We see in the left column the result obtained by using the true initial variance, which exibits the excellent ability of this algorithm. The final estimates, obtained after 23.0 seconds in all, have a remarkable accuracy for three or four digits, which is superior to the result of the batch ML based on over 30.0 seconds' data. The results obtained by using erroneous initial variances, i.e. four times and a quarter of the true values, are also shown in the same table. The influence of the initial variance values on the estimated results is not apparent within this range of variations.

V. Conclusions

We investigated the problem of aircraft parameter identification when there were external random disturbance inputs such as atmospheric gusts and stochastic measurement noises. Two general algorithms, i.e. the maximum likelihood (ML) method and the extended Kalman filter (EKF) technique, as well as a combination of them are examined by means of numerical simulations. The conclusions reached are summarized as follows.

The strength of the ML algorithm lies in that it requires no *"a priori"* information on the initial parameter estimates. With the strict version of the criterion, it also is adopted for the identification of the noise covariance parameters, such as the rms velocity of wind gusts or rms measurement noises. However, it is not suitable for airborn rapid computation since it is a batch processing system by nature.

The EKF algorithm computes an on-line estimate of the parameter in real time. It has a smaller program size and requires less computational time and memories than the former. But the convergence characteristics of this algorithm are largely affected by the starting values of the parameter estimate and its variance. When *"a priori"* information on the initial values in poor, the practical use of this algorithm is doubtful.

The combined use of the above two algorithms gives a useful method for an on-line parameter estimation. It uses the ML algorithm as a start-up procedure and processes the remaining data sequentially with the EKF technique. The results of simulation runs assured that this combined ML/EKF algorithm has a great capability in application to the aircraft parameter identification problem.

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