A Fuzzy Preference Model in Decision Making

By

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Abstract

A fuzzy preference model based on a fuzzy preference relation is presented. It is shown how this proposed fuzzy preference model can be used to identify good choices from a set of alternatives.

1. Introduction

The modeling of preferences in decision making has long been a concern in various fields like economics, management, and behavioral sciences. Preferences are commonly modeled as binary relations resulting from pairwise comparison of alternatives. The binary relation is assumed to have some minimal properties so that the alternatives can be consistently ordered [3, 6, 7]. The fuzzy binary relation may be considered as a generalization of the ordinary binary relation, in the sense thta each ordered pair is allowed a grade of membership from the interval $[0,1]$ instead of only the points $\{0,1\}$, as in the case of the ordinary binary relation. Fuzzy relations are studied under general situations and particular applications [5, 12]. In preference modeling, fuzzy relations are used in the context of social choice [2], in the concept of outranking [11], and in the aggregation of individual preferences [4].

It is the purpose of this paper to explore the use of fuzzy relations as an extension of the ordinary binary relations in preference modeling. Fuzzy preference relations have the advantage of allowing different degrees and various strengths of preference to be reflected in the preference model. In Section 2, it is shown that the fuzzy preference relations generalize the ordinary preference relations. Moreover, in Section 3, the fuzzy preference model is shown to unify some preference models given in literature. Some techniques on how to use fuzzy preference relations in identifying good choices among the alternatives are given in

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Section 4. Finally, a brief summary of the results, together with some indications for further study in order that the fuzzy models of preference may become operational, is given in Section 5.

2. Fuzzy Preference Relations

Given a set A of alternatives, a binary relation R is defined as a subset of the set of ordered pairs of alternatives. We shall presently denote the ordered pair (a, b) belonging to the binary relation R by aRb. The binary relation R may be interpreted as an association of the ordered pair (a, b) representing some modeled property. For instance, aRb can represent "a is at least preferred to b", "a is indifferent to b", etc. Some properties of the binary relation which are used in preference modeling can be found, for instance, in [3].

A fuzzy relation is a fuzzy subset of the set of ordered pairs. It is characterized by a function which associates each ordered pair, say (a, b), with a grade membership which can be interpreted as the strength of the relation of a to b [12]. To emphasize the idea of the grade membership of the relation, we shall use the notation $\mu_R(a,b)$ for the grade membership of the ordered pair (a,b) in the relation R. Thus, for instance, aRb above can be denoted as $\mu_R(a,b)=1$.

The following are some properties of fuzzy relations.

Reflexivity:	$\mu(a,a)=1 \quad a\in A$
Irreflexivity:	$\mu(a,a)=0 \quad a\in A$
Symmetry:	$\mu(a,b)=\mu(b,a) \quad a,b\in A$
Asymmetry:	$\mu(a,b)\neq\mu(b,a) \quad a,b\in A$
Transitivity:	$\mu(a,c)\geqq\max_b \min(\mu(a,b),\ \mu(b,c)) \quad a,b,c\in A$
Comparability:	$\mu(a,b)>0$ and/or $\mu(b,a)>0 \quad a,b\in A$

There are many other properties and variants of the definitions of such properties, but we require only these for our purposes.

Preference over a set of alternatives is modeled by means of relations. Relations are used to represent some mode of preference like strict preference, large preference, indifference, incomparability, etc. For instance, the relation aRb may be read as "a is preferred to b", or "a is indifferent to b", or some other preference expression depending on the situation modeled. However, preference in general varies over the different pairs of alternatives. An individual may have a clearcut preference for certain pairs, but for the other cases a less definite preference. In these vague and unclear cases, the ordinary binary relations may not capture the varying degrees of preference. Fuzzy relations, on the other hand, seem to be more approxpriate for these cases.

The fuzzy preference of the alternative a over the alternative b will be denoted by $\mu(a,b)$. This may be interpreted, for instance, as the strength of preference a over b. Moreover, it can also be understood as the degree of confidence or certainty that a is preferred to b. In order to compare two alternatives, say a and b, two evaluations are necessary, i.e., $\mu(a,b)$ and $\mu(b,a)$. This complementary pair of evaluations is represented as an ordered pair whose first component is the value of $\mu(a,b)$ and the second $\mu(b,a)$. The unit square, which includes all possible values of ordered pairs associated with preference states, is called the fuzzy preference space. Thus, a fuzzy preference on the pair a and b of the compared alternatives is represented as a point in the fuzzy preference space shown in Figure 1.

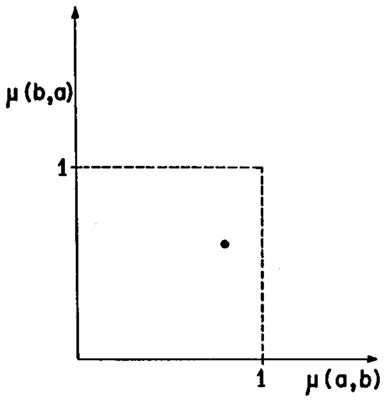

Fig. 1

Suppose the fuzzy relations μ_P and μ_I are respectively defined as $\mu_P = \max(0, \mu(a,b) - \mu(b,a))$ and $\mu_I = 1 - \max(\mu_P(a,b), \mu_P(b,a))$, where μ is a fuzzy preference relation. Then, it is obvious that μ_P and μ_I are asymmetric and symmetric, respectively, and $\mu_P(a,b) + \mu_P(b,a) + \mu_I(a,b) = 1$. The fuzzy relation μ_P, being the degree of preference dominance of one of the alternatives over the other, may be treated as the strict preference component. Moreover, μ_P is obviously asymmetric and irreflexive. Alternatives a and b are considered indifferent whenever a is as preferred as b is. The preference relation μ_I expresses this observation in relation to the strict preference relation μ_P, i.e., whenever complete comparability holds, indifference is the complement of strict preference. It is interesting to note that if μ is transitive, it can be shown that both the strict preference and indifference components are also transitive. Therefore, if μ is transitive and reflexive, then μ_I is a fuzzy equivalence relation (i.e., symmetric, reflexive and transitive), and μ_P is a strict fuzzy order (i.e., irreflexive, asymmetric, and transitive). Thus we have seen that the fuzzy indifference and strict preference relations have the same properties as their nonfuzzy counterparts.

3. Fuzzy Preference Model

We have seen in the preceding section how fuzzy preference relations generalize the ordinary preference relations. We will show that some preference models given in literature are special cases of the fuzzy preference model. We will confine our discussion to a preference modeled by a set of relations with specified properties and a set of axioms connecting these relations.

The weak order of Luce [7] is modeled by a strict preference relation P which is transitive, irreflexive, and asymmetric, an indifference relation I which is transitive, reflexive, and symmetric, and a trichotomy axiom which states that for any pair of alternatives a and b, exactly one of the following holds: aPb, bPa, aIb. In terms of fuzzy preference relations, aPb is equivalent to $\mu(a,b)=1$ and $\mu(b,a)=0$, and aIb is equivalent to $\mu(a,b)=\mu(b,a)=1$. Note that this weak order is represented by only three points in the fuzzy preference space, i.e., the points (0, 1), (1, 0), and (1, 1) which are associated with bPa, aPb, and aIb, respectively, as shown in Figure 2. The trichotomy assumption explicitly excludes the possibility of incomparability of any two alternatives.

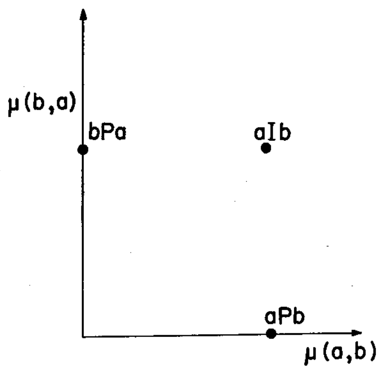

Fig. 2

It seems logical to consider also the possibility of uncertainty of preference between alternatives as another preference mode distinct from indifference. This preference mode will be called incomparability, and aQb will be used to denote "a is incomparable to b". Clearly, aQb is equivalent to $\mu(a,b)=\mu(b,a)=0$, and corresponds to the point (0, 0) in the fuzzy preference space. The reasonability of considering incomparability comes from the fact that in many practical decision situations, one cannot compare, does not want to compare, or does not even know how to compare some alternatives. By opting for incomaparability, and possibly resolving later whenever it is necessary and the available information warrants it, the risk of unacceptable errors by forcing a preference from situations which do not appear good enough to allow a conclusion is avoided.

Allowing the degree of preference to take its value from the unit interval is another interesting way of modeling preferences [1, 2, 9]. When comparing two alternatives, say a and b, the degree of preference r of alternative a over alternative b takes the value of 1 when a is definitely preferred to b, the value of 0 when b is definitely preferred to a, and the values in between for varying degrees of preference between the two extremes. Note that the degree of preference r' of b over a analogously takes the complementary values, i.e., $r'=1-r$. In addition, $r=1/2$ is usually associated with indifference between the alternatives. The degree of preference r has the interpretation in the context of group decision making as the proportion of individual preference orderings on a pair of alternatives [1, 2]. Moreover, r can also be interpreted as the probability that a is chosen over b when it is required to choose between two alternatives [9]. With respect to the fuzzy preference model, this model corrseponds to $\mu(a,b)=r$ with the condition $\mu(a,b)+\mu(b,a)=1$. The range of possible preference states is confined to the straight line segment connecting (0,1) and (1,0), as shown in Figure 3. It is noted that in this situation, the case $r=1/2$ does not exactly correspond to the interpretation of indifference between the two alternatives. Instead, it is a kind of vague preference between the alternatives. It is however possible to find another representation in the fuzzy preference model where the case $r=1/2$ corresponds to indiffreence. Assuming complete comparability, the fuzzy preference relation $\mu(a,b)$ and $\mu(b,a)$ may be defined, for instance, as follows (see Figure 4):

$$\mu(a, b) = 2r, \quad \mu(b, a) = 1 \quad \text{for } 0 \leqq r < 1/2$$
$$\mu(a, b) = 1, \quad \mu(b, a) = 2(1-r) \quad \text{for } 1/2 \leqq r \leqq 1$$

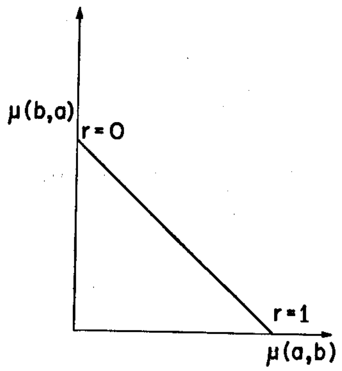

Fig. 3

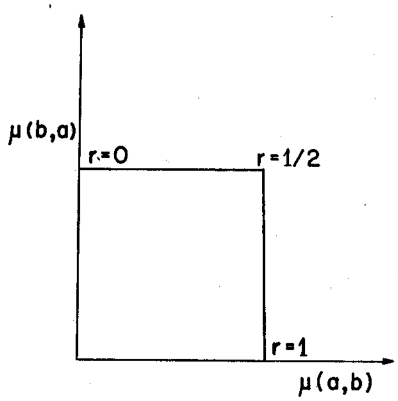

Fig. 4

The case $r=1/2$ can also be interpreted as incomparability, i.e., corresponds to (0,0). The associated fuzzy preference relation is defined analogous to the indifference intepretation. This model of preference can be considered a particular case of the fuzzy preference model where $\mu(a,b)$ and $\mu(b,a)$ are parametrized by the degree of preference r. Specifically, $\mu(a,b)$ is a non-decreasing function of r with $\mu(a,b)=0$ when $r=0$ (the case of a definite preference of b over a), and $\mu(a,b)=1$ when $r=1$ (the case of a definite preference of a over b). At the same time, $\mu(b,a)$ is a non-increasing function of r with $\mu(b,a)=1$ when $r=0$, and $\mu(b,a)=0$ when $r=1$. This is represented in the fuzzy preference space as a line parametric in r, satisfying the above mentioned properties from the point $(0,1)$ to the point $(1,0)$. This is shown in Figure 5.

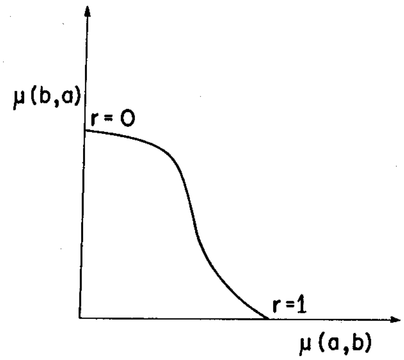

Fig. 5

Preferences have also been modeled by multilevel relations [8] where relations at different levels reflect the varying risks in the preference statements. A finite number of distinctly discriminable risk levels are indexed by integers from 1 to k. In such cases, the greater the risk, the greater is the value of the integer corresponding to it. For each risk level, there is a corresponding relation R_j which is interpreteed as the relation R_j for the j-th risk level. Meaningfulness and consistency require the relation R_j to satisfy certain properties. For instance, aR_ib implies aR_jb for all i such that $j\leqq i$, which leads to nested multilevel relations. In terms of a fuzzy preference relation, aR_jb corresponds to $\mu(a,b)\geqq\alpha_j$ for all $j=1,\cdots,k$, where α_j are threshold values such that $1>\alpha_1>\alpha_2>\cdots>\alpha_j>\cdots>\alpha_k>0$.

Finally, Roy [10] introduced a model where preferences are modeled by means of four binary relations I, P, Q and L, having the following properties:

I (indifference) : reflexive and symmetric,
P (strict preference) : irreflexive and asymmetric,

L (large preference) : irreflexive and asymmetric, and
Q (incomparability) : irreflexive and symmetric,

and a connection axiom stating that exactly one of aIb, aPb, bPa, aLb, bLa, and aQb holds for every pair of alternatives a and b. The relations I (indifference), P (strict preference), and Q (incomparability) have the same interpretations as those given previously. The large preference aLb is assumed to hold if b is not strictly preferred to, nor indifferent to b, because neither situation dominates. It is noted that in this model, a large preference cannot correspond anymore to any of the four situations resulting from the dichotomic comparison of alternatives. A possible interpretation of this model in terms of a fuzzy preference relation may be given as follows: aPb holds if $\mu(a,b)=1$ and $\mu(b,a)=0$; aLb if $\mu(a,b)>\mu(b,a)$, $\mu(a,b)\neq 1$, and $\mu(b,a)\neq 0$; aQb if $\mu(a,b)=\mu(b,a)=0$; and aIb if $\mu(a,b)=\mu(b,a)\neq 0$. The partitioned fuzzy preference space, corresponding to the five possible preference states, is shown in Figure 6. In addition, the large preference L corresponds to a region in the fuzzy preference space. This interpretation may be made more realistic by introducing thresholds in preference discrimination. For instance, the strict preference relation aPb holds, if $\mu(a,b)\geqq 1-\varepsilon_1$ and $\mu(b,a)\leqq\varepsilon_2$, where ε_1 and ε_2 are discrimination thresholds for strict preference. The other preference states may be analogously defined. Thus, instead of points and lines in the fuzzy preference space, regions correspond to the preference modes of strict preference P, indifference I, and incomparability Q. By treating preferences this way, the interpretation of preference becomes more in keeping with the sprit of the fuzzy set theory.

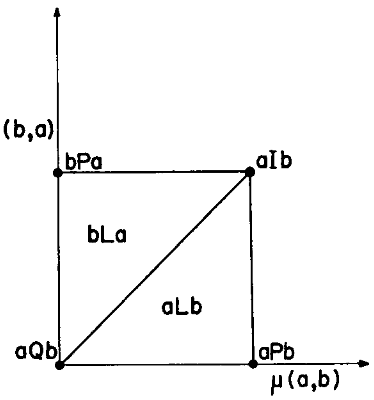

Fig. 6

4. How to Use Fuzzy Preference Relations in Choosing Alternatives

In this section it will be shown how fuzzy relations can aid the DM in his evaluation and choice of alternatives. In particular, fuzzy preference relations are used for deducing and identifying good choices among the alternatives. It will be assumed in this section that the alternatives are mutually exclusive and the fuzzy relation over these alternatives is given.

In order to aid the DM improve his decision process, it is important to clarify his desired decision output. In general, he wants to identify among the alternatives the good choices from which he will further consider for his decision. The formulation of the problem with respect to the desired decision output depends on the nature of the selection process of the alternatives. The most common decision output goal is the identification of the unique best alternative. Although this output desideratum seems to be a very natural one for a majority of decision problems, there are also other equally important formulations [10]. When the selection process involves the acceptance of alternatives, the number of which are not necessarily *a priori* determined to be only one, as well as the existence of competition among the alternatives, (e.g., admission decisions in educational institutions, choice of candidates to fill similar posts, awarding of grants and subsidies, etc.), the idea of searching for a unique best alternative becomes impractical.

Thus, one possible formulation of the decision output is a kind of trichotomy of the set of alternatives: accept all the sufficiently good alternatives, reject the very bad alternatives, and request a complementary examination (i.e., supplementary information, additional discussion, etc.) of the rest, so as to resolve the acceptability of these alternatives. A further refinement of any of these subsets (in the sense that trichotomy is again applied to the resulting subsets) may be performed whenever the desired decision output so requires, and the available information warrant it.

Moreover, a more general formulation of the decision problem involves a ranking of the alternatives. In particular, the alternatives are grouped into some kind of equivalence classes, (in the sense that the alternatives belonging to the same class are deemed equally good), which are linearly ordered. Such formulation would allow flexibility in the acceptance-rejection demarcation line which may be resolved by further analysis, subjective judgment, or even by negotiation. In short, the formulation of the problem may be a choice of one best alternative, a sorting of the alternatives into acceptable, rejected, and unresolved classes, or a ranking of the alternatives in a decreasing order of preference.

We will now discuss some concepts useful in obtaining good choices among the alternatives. An obvious candidate for a good choice would be an alternative which is at least as good as any other alternative. Such an alternative will be called dominant. In terms of relation R, an alternative $a \in S$, where S is a subset of A, is dominant in S with rsepect to the relation R, if and only if aRb holds for all $b \in S$. In order to extend this idea in terms of fuzzy preference relations and fuzzy sets, we first model the fuzzy preference relation R with the interpretation "at least preferred". The fuzzy preference relation R "at least preferred" is modeled for instance, with the membership function $\mu_R(a,b)$ as follows:

$$\mu_R(a, b) = \begin{cases} 0 & \mu(a, b) < \mu(b, a) \\ \mu(a, b) & \mu(a, b) \geqq \mu(b, a) \end{cases}$$

where $\mu(a,b)$ and $\mu(b,a)$ are the fuzzy preference relations of a over b and of b over a, respectively. Indeed, in order to say "a is at least preferred to b", it is necessary that $\mu(a,b) \geqq \mu(b,a)$, and therefore, when $\mu(a,b) < \mu(b,a)$ we must have $\mu_R(a,b) = 0$. On the other hand, when we can say that a is at least preferred to b, the degree of such preference may vary and a reasonable measure of such preference is $\mu(a,b)$. With this model of the fuzzy "at least preferred" relation, we can define the dominant set $D(S,\mu)$ of the set S with respect to the relation μ as the set with the membership function

$$\mu_{D(S,\mu)}(a) = \min_{b \in S} \mu_R(a, b) .$$

Indeed, for any fixed $b \in S$, $\mu_R(a,b)$ can be considered as the membership value of a in a set whose elements are at least preferred to b. Then, the intersection of such sets for all $b \in S$ represent the set of alternatives which are at least preferred to any other alternative in S. This set will be denoted as D when there is no confusion in the reference set S. The value $\mu_D(a)$ shows the degree that a dominates any other alternative in S.

Another useful concept for identifying good choices among alternatives in the maximal set. An alternative a is maximal in S with respect to the relation R, if there does not exist another alternative b in S which is strictly preferred to a. The set of maximal alternatives will be called a maximal set. Before going further, a model of the fuzzy strict preference relation will be constructed. A fuzzy strict preference relation μ_P is defined, for instance, as a fuzzy preference relation having the membership function

$$\mu_P(a, b) = \max (0, \mu(a, b) - \mu(b, a)) .$$

Indeed, the difference $\mu(a,b) - \mu(b,a)$ of the preference between a and b may be considered the degree of strict preference of a over b when $\mu(a,b) \geqq \mu(b,a)$, i.e., a is at least preferred to b. Moreover, when $\mu(a,b) < \mu(b,a)$ holds, we cannot say that a is strictly preferred to b, and thus $\mu_P(a,b) = 0$. Such a model intuitively fits the idea that strict preference is an irreflexive and asymmetric relation. Now consider any fixed alternative $b \in S$. The value of the function $\mu_P(b,a)$ may be considered the membership function value of the fuzzy set, which is strictly dominated by b. Then, the complement of this fuzzy set, i.e., the fuzzy set whose membership function is $1-\mu_P(b,a)$, is the set whose elements are not strictly dominated by the fixed b. Thus, the intersection of such fuzzy sets for all $b \in S$ represents the fuzzy set of alternatives which are not dominated by any other alternative in S. Therefore, the fuzzy maximal set $M(S,\mu)$ of the set S with respect to the fuzzy preference relation μ has the membreship function

$$\mu_{M(S,\mu)}(a) = \min_{b \in S} (1-\mu_P(b,a)) = 1-\max_{b \in S} \mu_P(b,a) .$$

This set will simply be denoted by M whenever there is no confusion in the reference set S.

To illustrate the notions of fuzzy dominant and maximal sets, we consider a hypothetical project with five alternatives denoted by a_1, a_2, a_3, a_4 and a_5. We will assume that the fuzzy preference relations among the alternatives have already been assessed and are given in matrix form by Figure 7. We note that the fuzzy preference relation may be conveniently represented as an $n \times n$ matrix R, whose entries r_{ij} are given by $r_{ij}=\mu(a_i,a_j)$ for $a_i,a_j \in A$. The entries of the matrix are the membership function values. For instance, the entry 0.3 in the second row third column is the membership function value $\mu(a_2,a_3)$. The fuzzy "at least preferred" relation μ_R, and the fuzzy strict preference relation μ_P derived from the fuzzy relation μ, are given in Figures 8 and 9, respectively.

	a_1	a_2	a_3	a_4	a_5
a_1	1.0	0.7	0.8	0.5	0.5
a_2	0.0	1.0	0.3	0.0	0.2
a_3	0.0	0.7	1.0	0.0	0.2
a_4	0.6	1.0	0.9	1.0	0.6
a_5	0.2	0.0	0.0	0.0	1.0

Fig. 7

	a_1	a_2	a_3	a_4	a_5
a_1	1.0	0.7	0.8	0.0	0.5
a_2	0.0	1.0	0.0	0.0	0.2
a_3	0.0	0.7	1.0	0.0	0.2
a_4	0.6	1.0	0.9	1.0	0.6
a_5	0.0	0.0	0.0	0.0	1.0

Fig. 8

	a_1	a_2	a_3	a_4	a_5
a_1	0.0	0.7	0.8	0.0	0.5
a_2	0.0	0.0	0.0	0.0	0.2
a_3	0.0	0.4	0.0	0.0	0.2
a_4	0.1	1.0	0.9	0.0	0.6
a_5	0.0	0.0	0.0	0.0	0.0

Fig. 9

The fuzzy maximal set M is derived from the fuzzy strict preference relation as $M=\{(a_1, 0.9), (a_2, 0.0), (a_3, 0.1), (a_4, 1.0), (a_5, 0.4)\}$, where each ordered pair signifies the alternative and its membership function value in M. In the ordered pair $(a_1, 0.9)$, for instance, 0.9 indicates the degree to which the alternative a_1 is not dominated by any other alternative. It is seen that a_2 does not belong to the maximal set, and is therefore a dominated alternative. On the other hand, a_4 is a maximal element in the sense of ordinary set theory. It is noted that if for some alternative a, the membership function $\mu_M(a)=a$, then this alternative is dominated by other alternatives to a degree not greater than $1-\alpha$. Similarly, from the fuzzy "at least preferred" relation, the fuzzy dominant set D is given as $D=\{(a_1, 0.0), (a_2, 0.0), (a_3, 0.0), (a_4, 0.4), (a_5, 0.0)\}$. The membership function value $\mu_D(a)=\alpha$ indicates that the alternative a is at least preferred to all the other alternatives by at least α. From these sets, we can base our selection and identification of good choices among the alternatives.

We will now show some ways of identifying good choices through the use of the fuzzy preference model. We shall successively consider the three previously mentioned problem formulations. It should be pointed out that the approach given here is not intended as a substitute for the more quantitative methods, but rather as a complementary qualitative analysis of the same problem. Indeed, due to the qualitative nature of the method, it is better suited as an initial screening method for identifying the more viable alternatives.

We will first study the problem of choosing only one alternative that seems best from the DM's point of view. We shall develop a method based on the fuzzy maximal set. An analogous procedure can also be devised using the fuzzy dominant set. Since the two concepts differ in the preference information, a complementary use of the maximal set and the dominant set seems advantageous for our purpose.

Ideally, the best choice among the alternatives for this problem formulation is a unique a^*, such that $\mu_M(a^*)=1$, if it exists. However, in general, the fuzzy preference relation μ is too weak to assure such an alternative to exixst. The set $M^*=\{a \mid \mu_M(a)=\alpha^*$, where $\alpha^*=\max \mu_M(a)\}$ seems more appropriate for such a choice among the alternatives. Indeed, M^* is the set of alternatives which are least dominated among the alternatives in the set. However, there are cases where α^* is too small a value to have a credibly good choice. Moreover, it is also possible to find alternatives which are not in M^*, but whose membership function values are so close to α^* that it becomes meaningless to have such a discriminative demarcation in the choice of alternatives.

In these cases, a very reasonable criterion may be, for instance, $\mu_M(a) \geqq \alpha^*-\varepsilon$, where $\varepsilon>0$ is a discrimination threshold value. Also, it would be meaningful to impose a condition on the minimum value of α^*, say $\bar{\alpha}$, to have a reasonably credible choice. Specifically, the set $\{a \mid \mu_M(a) \geqq \alpha^*-\varepsilon\}$, with $\alpha^* \geqq \bar{\alpha}$ and $\varepsilon>0$, would obtain a useful alternative for M^*. If no unique alternative can be found for the best choice, then a reexamination of the DM's preference (e.g., a more detailed study, reassessment of the DM's judgment) would be in order until a unique alternative is reasonably identified. It is to be noted that caution should be exercised in this elimination process so as to avoid discrimination due only to apparent differences.

We now consider the problem of sorting alternatives according to the DM's preference with respect to acceptability. The choice procedure here is to accept all the sufficiently good alternatives, reject all the very bad ones, and request supplementary information on the rest. An implementation of such a procedure with respect to the fuzzy preference model, for instance, would be to accept the set of alternatives $\{a \mid \mu_M(a)=\alpha^*\}$, where $\alpha^*=\max \mu_M(a)$, reject the alternatives in the set $\{a \mid \mu_M(a)=0\}$, and request supplementary information on the rest. Several variants of such a trichotomy procedure can be developed. For instance, different threshold values may be assigned instead of α^* and 0 as the acceptance and rejection levels, respectively. Moreover, the fuzzy dominant set may be used instead of the fuzzy maximal set. In general, an appropriately combined use of both sets may be helpful in obtaining a refinement of the resulting choices. In addition, such a trichotomic classification can be enhanced, for instance, by introducing reference sets formed from actual or fictitious alternatives with which to compare as norms for classifying the alternatives. If an alternative is sufficiently better than those alternatives in the acceptance norm set, then this alternative is accepted as a good choice. Similarly, when an alternative is dominated by the alternatives in the rejection norm set, it is rejected. The rest of the alternatives will be considered for further analysis.

In cases where the alternatives have to be arranged systematically so that the DM can conveniently pick out the preferred alternatives with respect to his preference priorities, the alternatives need to be ranked in a decreasing order of desirability or preference. The method involves a grouping of the alternatives into equivalence classes where the alternatives belonging to the same class are approximately equally desirable. The following procedure is a possible implementation of the method. Initially, we set $A_1=A$ where A is the set of alternatives to be classified. Then we construct the subset A_i^* of A sequentially as follows: $A_i^*=\{a \mid \mu_{M_i}(a)=\alpha_i^*\}$, where $M_i=M(A_i)$ is the fuzzy maximal set of A_i and $\alpha_i^*=\max \mu_{M(A_i)}(a)$. A_{i+1} is obtained from the relative complement of A_i^* with respect to A_i. The construction of such subsets is continued until all the subsequently formed subsets A_i^* result in empty sets. By convention, the last such non-empty subset will be denoted by A_m^*. The sets A_i^* for $i=1,\cdots,m$, are the equivalence classes partitioning the set of alternatives to be classified. These subsets are linearly ordered, that is, the alternatives in class A_i^* are preferred to those in class A_{i+1}^* for all the m classes. Thus, the alternatives in each $A_i^*, i=1,\cdots,m$, correspond to rank i. By interpretation, the alternatives with rank 1 are considered the most preferred alternatives, those with rank 2 are the second most preferred ones, and so on.

In the example given in Figure 7, we have seen that $M=\{(a_1, 0.9), (a_2, 0.0), (a_3, 0.1), (a_4, 0.1), (a_5, 0.4)\}$. From this set M, it is obvious that if we want the best alternative, the choice would be a_4. Moreover, if we want to sort the alternatives, i.e., classify into accepted, rejected, and indefinite sets of alternatives, such a procedure would give a_4 as automatically accepted, a_2 rejected, and the rest indefinite. When a ranking of alternatives is desired, by using the procedure described above, we obtain sequentially the alternatives a_4, a_1, a_5, a_3, and a_2 which are ranked in a decreasing order as first, second, third, fourth, and fifth, respectively.

5. Conclusion

We have seen how the fuzzy preference model extends some existing models which are based on binary preference relations. Such a fuzzy model has the advantage of robustness and flexibility in the sense that different degrees of preference can be reflected by the preference model. We have also shown some procedures for choosing good alternatives. Specifically, the concepts of dominant and maximal sets, deduced from the fuzzy preference relation among the alternatives, were utilized in identifying good choices from a set of alternatives.

An equally important aspect in the use of the fuzzy preference model for choosing good alternatives is how to derive the values used in the fuzzy preference relation. Basically, the assessment of the DM's preference involves translating his preference statements into quantitative expressions admissible in the fuzzy preference model. It is possible to devise some methods for undertaking this quantification process. A comparison of alternatives is usually done in terms of their consequences with respect to some criteria. Such a comparison is obviously affected by the subjectiveness and vagueness of the DM as he communicates his preference.

A comparison with respect to one criterion can be made more meaningful by the use of thresholds of discrimination in the criterion for the different preference modes. Moreover, when uncertainty pervades an analysis of the different intervals of the criterion, the values associated with various confidence levels could yield interesting information for the desired quantification. In addition, fuzzy consequences can be compared with respect to the overlapping of the support sets, heights of the sets, and the separation of their peaks to obtain fuzzy preference relations.

An assessment of alternatives with respect to multiple criteria is a more difficult task. When compensation and tradeoff between criteria are available, the use of measures of relative importance can be an effective approach. This could lead to some kind of concordance-discordance analysis on the consequences of the alternatives with respect to the different interacting criteria. Other approaches, like the study of the confluence of the different goals and the analysis of the domination structures in the consequences, may be explored. In summary, a further study and analysis of these mentioned approaches would contribute to make the fuzzy preference model operational for aiding the DM.

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