On Geometry of Truss

AUTHOR(S):

SHIRAISHI, Naruhito; FURUTA, Hitoshi

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By

Naruhito SHIRAISHI* and Hitoshi FURUTA*

(Received June 27, 1979)

Abstract

It has been shown, by a number of examples, that it is possible to obtain rational designs by introducing a geometrical variation of trussed structures into the design process. To determine the effective geometrical configuration, two kinds of parameters, cross sectional areas and nodal coordinates, need to be taken into account in the optimization procedure. The treatment of nodal coordinates apparently results in the increase of the number of design variables, which often induces some difficulties in the application of mathematical programming. Large scale optimization problems generally have a poor convergency to require excessive computation time or memory capacity. For such problems, U. Kirsch proposed a decomposition approach and showed its efficiency by some design examples.

In this paper, a search for least-weight geometry is carried out by a two-step treatment based on Kirsch's approach. The cross sectional areas are assumed to be completely dependent on and determined from the set of nodal coordinates. Then, two design approaches are considered. The first approach is the deterministic method based on the allowable stress criterion. The other is the probabilistic method, where the safety is examined by the system failure probability. Also, the influence of buckling constraints and the number of employed nodes on geometry are discussed through some numerical examples.

1. Introduction

For a specified loading condition, a number of different designs are introduced, which satisfy the functional requirements and structural safety against the load. It is evident that structural systems with different configurations have different stress distributions, and each design results in a different volume or weight. To obtain a lighter truss by changing its configuration, L. Friedland[1] indicated the following four questions in selecting a configuration of truss; (1) How many nodes shall the truss have?, (2) How shall they be connected ?, (3) Where in space shall they be located?, (4) What cross sectional areas shall the members have?

The objectives of this paper are to investigate the influence of the variation

* Department of Civil Engineering

of geometry on weight, and to find the most effective geometry from the viewpoint of a minimum-weight design. Also, the last two questions will be treated herein. For simplicity, the employed assumptions are that the trusses are statically determinate, and that their connectivities are given at the beginning of the design process.

Formally, it is easy to include a change of geometry into a design formulation. This can be done by introducing the nodal coordinates into the set of design variables. Then, the design variables consist of cross sectional areas and nodal coordinates, which have different characteristics and different orders of magnitude. In a practical use of mathematical programming, the increase of design variables induced by the addition of nodal coordinates and the mixing of variables having different dimensions will cause such difficulties as a poor convergency and an excessive computation load. G. Vanderplaats and F. Moses attempted to treat this problem in two separate but dependent design spaces, one for the cross sectional areas of members and one for the nodal coordinates. They obtained good results for a considerably large transmission tower example.

In this paper, a similar treatment is employed to remove the difficulties in conjunction with computation. Cross sectional areas are considered to be completely dependent on nodal coordinates. It is not so difficult to find the optimum set of cross sectional areas if the geometry is fixed. Furthermore, for a special case, it is possible to find an effective relation or an optimality condition. Here, the search for effective geometry is carried out, using the optimality condition and mathematical programming. Although this method has no guarantee that a global optimum solution can be obtained, it is sufficient to investigate the influence of the variation of geometry, and it is also advantageous from an engineering point of view.

At first, a two-level formulation is shown based on Kirsch's approach.[3] Next, the simplification is done by using an inherent characteristic of statically determinate trusses. Based on the optimality condition derived by H. Switzky[4], the probabilistic design formulation is shown. For the deterministic and the probabilistic approaches, some numerical examples are presented to illustrate the efficiency and the applicability of the proposed methods. Some conclusions are obtained on the influence of buckling constraints and the relation between the number of nodes and weight.

2. Partitioning of Design Variables[3]

General optimum design problems can be mathematically written as follows.

$$\text{Minimize objective function} \qquad Z = f(w) \tag{1}$$

$$\text{subject to} \qquad I(w) = 0 \tag{2}$$

$$g(w) \leqq 0 \tag{3}$$

$$w^L \leqq w \leqq w^U \tag{4}$$

in which w^L and w^U are the lower and upper bounds for design variables w; and Eqs 2 and 3 are the equality and inequality conditions, respectively. In truss problems, Eq. 2 represents the relations between the variables. For instance, they are derived from the symmetry condition and the fabrication requirements. Eq. 3 represents the design requirements with respect to safety or performance. The design variables w consist of cross sectional areas, A, and nodal coordinates, X.

$$w = \begin{bmatrix} A \\ X \end{bmatrix} \tag{5}$$

Then, the problem expressed by Eqs 1～4 is treated as a two-level problem. At the first level, the nodal coordinates, represented by X are considered to be fixed and not to be variables. The initial values are given within a range where all the imposed constraints are satisfied.

$$X = X_0 \tag{6}$$

X_0 is the initial value selected for nodal coordinates. Using X_0, we find A such that

$$I(A,\ X_0) = 0 \tag{7}$$

$$g(A,\ X_0) \leqq 0 \tag{8}$$

$$A^L \leqq A \leqq A^U \tag{9}$$

$$\text{Minimize} \quad Z = f(A,\ X_0) \tag{10}$$

The next step is to solve the second level problem.

$$X^L \leqq X \leqq X^U \tag{11}$$

$$\text{Minimize} \quad f(A_0,\ X) \tag{12}$$

where A_0 represents the cross sectional areas calculated in the previous step (as the first level) and, in turn, X is treated as a variable. The alternate steps are repeated until a sufficient design is achieved. The method mentioned above is called the modal coordination method. Although this method has no security that the optimum solution can be obtained, it is advantageous from an engineering point of view, since the iteration can always be terminated with a feasible solution. It should be, of course, noted that some treatments are necessary for programming in order to improve the convergency.

3. Deterministic Approach with the Allowable Stress Criterion

3.1 Optimality Condition and Sub-Optimization of Members

The two-level approach described in the previous section is applicable to any problem if the iterative procedure is effectively used. However, we utilize the optimality condition in determining the cross sectional areas so as to make the calculation easier. It is said that in the majority of minimum-weight designs of truss test problems, a fully stressed design is an optimal one, or that the resulting design has a weight close to that of the true solution.[5] By using this condition (fully stressed) as an optimality condition, the cross sectional areas A can be expressed as a function of the nodal coordinates X.

$$A = A(X) \tag{13}$$

In a case where only the stress limitation is employed as constraints and the allowable stress is constant, the optimal set of cross sectional areas can be determined by using the member forces F.

$$A = \sigma_a^{-1} F(X) \tag{14}$$

where σ_a is the diagonal matrix of the allowable stresses of the members. In a case with buckling constraints, σ_a, too, is a function of X.

$$A = \sigma_a^{-1}(X) F(X) \tag{15}$$

In checking the safety against a buckling failure, the slenderness-ratio is a very important factor. It is said that the allowable strength for each compressive member should be determined according to its slenderness-ratio. Then, the cross sectional areas can no longer be expressed as explicit functions of the nodal coordinates.

G. Vanderplaats and F. Moses proposed an approximate formula for pipe sections.[2] (See Fig. 1)

$$\sigma_{cai} = \frac{G_i \pi E A_i}{8L_i^2} \tag{16}$$

(E: Young's modulus,
L: membr length)

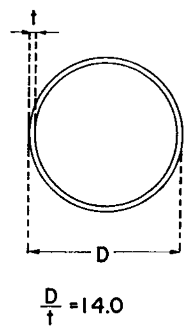

Fig. 1. Pipe Section.

where G_i is a constant calculated from the thickness and diameter of the cross section and subscript i denotes the i-th member. By using this formula, the cross sectional area of the i-th member, A_i, can be shown as

$$A_i = K_i \sqrt{F_i(X)}\, L_i(X) \tag{17}$$

in which K_i is also a constant and $K_i\sqrt{F_i(X)}\,L_i(X)$ corresponds to $\sigma_a^{-1}(X)$ in Eq. 15.

For general sections such as rectangular or square box sections, it is necessary to obtain a simple formula which relates the cross sectional areas to the nodal coordinates. This trial was already done and the results were applied to a truss design with fixed geometry.[6,7] When the length and the induced force of a member are given, the optimal cross sectional areas can be calculated with the aid of the optimality condition—"fully stressed condition". The results for pipe sections are shown in Fig. 2 and Fig. 3. Observing the results of sub-optimization for SS41 steel, the following formula can be obtained for a given length.

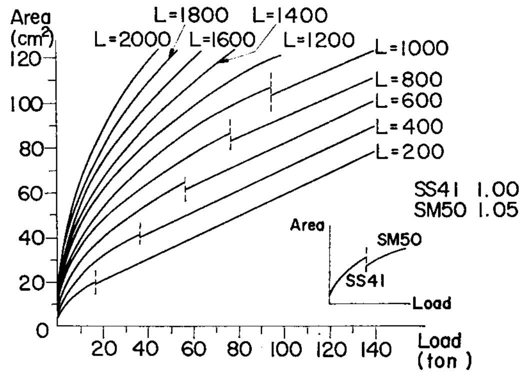

Fig. 2. Sub-optimization for SS41 Steel.

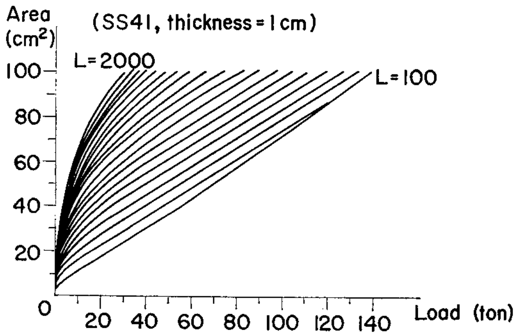

Fig. 3. Sub-optimization of Axial Members.

$$
\begin{aligned}
A(X) &= a\sqrt{F(X)}+b && 93<R \\
A(X) &= F(X)/1568+c && 20<R<93 \qquad (18)\\
A(X) &= F(X)/1400 && R<20
\end{aligned}
$$

where R denotes the slenderness-ratio, and a, b and c are constants to be obtained by the curve in Fig. 3. (See Table 1) It is noted that Fig. 2 and Fig. 3 are made by using the formula which the Japan Road Association recommends.

Table 1. Sub-optimization of SS41 steel

A / L	$A=a\sqrt{F}+b$ (93<R)		F	$A=\frac{F}{1568}+c$	F	$A=\frac{F}{1400}$
	a	b	(R=93)	(20<R<93)	(R=20)	(R<20)
100	0.110	−0.104	6778.46	4.7	62048.0	
200	0.149	1.010	14660.80	9.5	124320.0	
300	0.181	1.932	22406.72	14.2	186550.0	
400	0.208	2.786	29933.12	19.0	248766.0	
500	0.232	3.606	37428.16	23.8	310968.0	
600	0.254	4.407	45064.32	28.5	373170.0	
700	0.274	5.197	52543.68	33.3	435386.0	
800	0.293	5.980	60164.16	38.0	497588.0	
900	0.311	6.757	67627.84	42.8	559790.0	
1000	0.328	7.531	75107.20	47.6	621964.0	
1100	0.344	8.302	82712.00	52.3	684194.0	
1200	0.363	8.804	90175.68	57.1	746396.0	
1300	0.382	9.197	97796.16	61.8	808598.0	
1400	0.403	9.525	105259.04	66.6	870800.0	
1500	0.423	9.801	112707.84	71.4	933002.0	
1600	0.445	10.035	120328.32	76.1	995190.0	
1700	0.466	10.236	127792.00	80.9	1057392.0	
1800	0.488	10.407	135396.80	85.6	1119594.0	
1900	0.510	10.556	142860.48	90.4	1181796.0	
2000	0.532	10.686	150324.16	95.2	1243998.0	

A ; Sectional area (cm^2)
F ; Member force (kg)
L ; Member length (cm)
R ; Slenderness ratio

$$\sigma_{ca}=j_1\left[k_1-j_2k_4\left(\frac{L}{r}-k_2\right)\right]+(1-j_1)\frac{1.2\times10^7}{k_5+(L/r)^2} \qquad (\mathrm{kg/cm^2}) \qquad (19)$$

where

$$\begin{aligned} &k_2>L/r \qquad ;\ j_1=1,\ \ j_2=0\\ &k_2\leq L/r\leq k_3;\ j_1=1,\ \ j_2=1\\ &k_2\leq L/r \qquad ;\ j_1=0,\ \ j_2=1 \end{aligned}$$

The parameter r denotes the radius of gyration and $k_1 \sim k_5$ are constants which are dependent on the kinds of steel

3.2 Approximate Design Method

By using the realtions between cross sectional areas and nodal coordinates, the optimization problem is reduced to the unconstrained optimization problem.

$$\text{Minimize} \quad Z=\rho\sum_{i=1}^{m}A_i(X)L_i(X) \tag{20}$$

where ρ is the material density.

Then, it is obvious that the search procedure for the unconstrained problem is much easier than that for the constrained one. For three cases, Eq. 20 can be written as follows.

Case 1 Allowable Stress Limit Only

$$Z=\rho\sum_{i=1}^{m}\frac{1}{\sigma_{ai}}F_i(X)L_i(X) \tag{21}$$

For this case, the improving direction can be easily obtained by using the derivatives of Eq. 21.

$$\frac{\partial Z}{\partial X_j}=\rho\sum_{i=1}^{m}\left\{\frac{1}{\sigma_{ai}}\frac{\partial F_i(X)}{\partial X_j}L_i(X)+\frac{1}{\sigma_{ai}}F_i(X)\frac{\partial L_i(X)}{\partial X_j}\right\} \tag{22}$$

where $\dfrac{\partial F_i}{\partial X_j}=\dfrac{\partial C_i}{\partial X_j}P$

C : configuration matrix

P : applied load whose position is fixed

m : number of member

Case 2 Pipe Sections with Buckling Constraint

Using Eq. 17, the weight Z is expressed as

$$Z=\rho\{\sum_{i=1}^{m_1}\sigma_{ai}^{-1}F_i(X)L_i(X)+\sum_{i=m_1+1}^{m}K_i\{F_i(X)\}^{1/2}L_i(X)\} \tag{23}$$

where m_1 is the number of tensile members.

Case 3 Box Sections

According to Ref. 6, the following approximate formula was given.

$$Z=\rho\{\sum_{i=1}^{m_1}\sigma_{ai}^{-1}F_i(X)L_i(X)+\sum_{i=m_1+1}^{m}F_{ni}(X) \tag{24}$$

where

$$\left.\begin{aligned} F_{ni}(X)&=\left\{\frac{1}{\alpha_i}(F_i(X)-\gamma_i)^{ni}+\beta_i\right\}L_i(X), && F_i(X)\leq F_{0i} \\ F_{ni}(X)&=\frac{1}{\delta_i}(F_i(X)-\varepsilon_i)L_i(X), && F_{0i}\leq F_i(X) \end{aligned}\right\} \tag{25}$$

F_{0i} is the branching poitn of the $A-F$ curve.

α_i, β_i, γ_i, δ_i and n_i are constants to be determined from the $A-F$ curve.

Of course, it is too expensive to obtain the relations between the optimum cross sectional areas and the member forces for individual lengths of members which are determined from the geometry. Hence, the optimum cross sectional areas for the given lengths are calculated with the aid of the interpolation method.

4. Probabilistic Approach

4.1 Optimality Condition

Based on the reliability concept, the minimum-weight design can be formulated as

$$\begin{aligned} &\text{Minimize} \quad Z = \rho \sum_{i=1}^{m} A_i(X)L_i(X) \\ &\text{subject to} \quad p_f(A, X) \leq p_{fa} \end{aligned} \tag{26}$$

where p_f and p_{fa} denote the system failure probability and its allowable level, respectively. Since Eq. 26 has a single constraint with respect to the system failure probability, it is useful to employ Lagrange's multiplier method as the optimization scheme. Then, the problem can be rewritten as

$$\begin{aligned} &\Psi = Z(A, X) + \lambda(p_f(A, X) - p_{fa}) \\ &\frac{\partial \Psi}{\partial X} = \frac{\partial Z}{\partial X} + \lambda \frac{\partial p_f}{\partial X} = 0 \\ &\frac{\partial \Psi}{\partial A} = \frac{\partial Z}{\partial A} + \lambda \frac{\partial p_f}{\partial A} = 0 \\ &\frac{\partial \Psi}{\partial \lambda} = p_f - p_{fa} = 0 \end{aligned} \tag{27}$$

where Ψ and λ are the Lagrangean and Lagrange's multiplier, respectively.

For statically determinate truss problems, H. Switzky[4] presented an optimality condition which contributes to the minimum-weight design. The failure probability of statically determinate trusses can be approximated for the case where the failure probability of each member, p_{fi}, is quite small.

$$p_f = \sum_{i=1}^{m} p_{fi} \tag{28}$$

By using Eqs 27 and 28, the partial derivative of the weight with respect to the weight of the i-th member is set equal to zero. This results in

$$\frac{\partial}{\partial Z_i}\left[Z + \lambda\left(p_{fa} - \sum_{i=1}^{m} p_{fi}\right)\right] = 1 - \lambda \frac{\partial p_{fi}}{\partial Z_i} = 0 \tag{29}$$

$$\therefore \quad \frac{1}{\lambda} = \frac{\partial p_{fi}}{\partial Z_i} \tag{30}$$

Eq. 30 implies that at an over-all minimum weight, changes in the failure probability of each element are proportional to its change in weight, and that this ratio is independent of the respective elements. Furthermore, if it is assumed that the ratio of the weight of an element to the over-all weight is relatively insensitive to the over-all failure probability, the following relation can be available for reducing a valuable optimality condition.

$$\left(\frac{Z_i}{\Sigma Z_i}\right)_{p_{f1}} = \left(\frac{Z_i}{\Sigma Z_i}\right)_{p_{f2}} \tag{31}$$

Eqs 29 and 31 are satisfied if

$$\frac{p_{fi}}{p_{fa}} = \frac{Z_i}{Z} \tag{32}$$

Eq. 32 indicates that a minimum-weight design will result when the failure probability of each component is proportional to its weight.

4.2 Approximate Design Method

By using the optimality condition (i.e. Eq. 32) at each design step, the optimal geometry of the truss can be approximately sought as follows:

Step 1 Assume the initial geometry (i.e. nodal coordinates $X^{(0)}$)

Step 2 Perform the structural analysis of the truss with the geometry determined in the previous step.

Step 3 Obtain the optimum set of cross sectional areas by using the calculated values of member forces and member lengths.

Step 4 Search for the most effective direction in the design space which consists of nodal coordinates, and calculate the distance α with the aid of a one-dimensional optimization scheme.

Step 5 Then, the improved geometry can be defined as

$$X^{(I)} = X^{(I-1)} - \alpha S \tag{33}$$

Step 6 If the values of $X^{(I)}$ converge, the procedure is terminated. Otherwise, return to Step 2 and repeat Steps 3∼5.

The above process is expressed as a macro flow chart. (See Fig. 4)

The direction S can be easily calculated for the case in which the member resistances and the member forces have normal distributions. Differentiating the weight,

$$\frac{\partial Z}{\partial X_j} = \rho\left\{\sum_{i=1}^{m}\left[\frac{\partial A_i(X)}{\partial X_j}L_i(X) + A_i(X)\frac{\partial L_i(X)}{\partial X_j}\right]\right\} \tag{34}$$

Then, the cross sectional areas $A_i(X)$ can be expressed by the member force $F_i(X)$ based on Switzky's optimality condition.

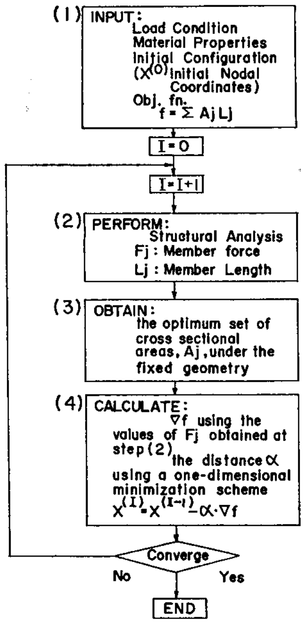

Fig. 4. Macro Flow Chart of Approximate Design Procedure.

$$A_i(X) = \frac{\mu_S\mu_{R_i}+\beta_i\sqrt{(\sigma^2_{R_i}\mu^2_S+\mu^2_{R_i}\sigma^2_S)-\beta_i\sigma^2_{R_i}\sigma^2_S}}{(\mu^2_{R_i}-\beta^2_i\sigma^2_{R_i})}F_i(X)$$
$$= D_iF_i(X) \tag{35}$$

where μ_S, μ_{R_i}: the mean values of the applied force and member resistance of the i-th member

σ^2_S, $\sigma^2_{R_i}$: the variances of the applied force and member resistance of the i-th member

$\beta_i = \Phi^{-1}(p_{fai})$

p_{fai} and $\Phi(\cdot)$ denote the allowable level assigned to the i-th member and the standard normal distribution function, respectively.

Using Eq. 35, the weight and its derivatives can be found to be

$$Z = \rho\sum_{i=1}^{m} D_iF_i(X)L_i(X) \tag{36}$$

$$\frac{\partial Z}{\partial X_j} = \rho\left\{\sum_{i=1}^{m}\left[D_i\frac{\partial_i F(X)}{\partial X_j}L_i(X)+D_iF_i(X)\frac{\partial L_i(X)}{\partial X_j}\right]\right\} \tag{37}$$

5. Numerical Examples

In this section some truss models are designed so as to investigate the characteristics of geometry and to demonstrate the applicability of the proposed methods.

5.1 Examples for the deterministic method

Example 1 2-panel Truss Model (Nein-Bar Truss)

This example is employed to investigate the relation between nodal coordinates and volume. The optimal geometry is shown in Fig. 5, with the loading condition and the selected design variables being $X_1 \sim X_3$. This truss is simply supported and its span length is 40 m. Since this truss is statically determinate and subject to a single load condition, the resulting design is fully stressed. In this case the member forces, $F_1 \sim F_9$, can be expressed by nodal coordinates.

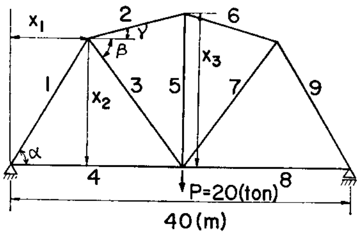

Fig. 5. Two-panel Truss Model with Optimal Geometry.

$$
\begin{aligned}
F_1 &= -\frac{10}{\sin\alpha} \\
F_2 &= \frac{1}{\sin\gamma}\left(\frac{\sin\beta\left(-\dfrac{10}{\tan\alpha}+\dfrac{10}{\tan\gamma}\right)}{\dfrac{\sin\beta}{\tan\gamma}+\cos\beta}-10\right) \\
F_3 &= -\frac{\dfrac{-10}{\tan\alpha}+\dfrac{10}{\tan\gamma}}{\left(\dfrac{\sin\beta}{\tan\gamma}+\cos\beta\right)} \\
F_4 &= \frac{10}{\tan\alpha} \\
F_5 &= -2\left(\frac{\sin\beta\left(-\dfrac{10}{\tan\alpha}+\dfrac{10}{\tan\gamma}\right)}{\left(\dfrac{\sin\beta}{\tan\gamma}+\cos\beta\right)}-10\right) \\
F_6 &= F_2, \quad F_7 = F_3, \quad F_8 = F_4, \quad F_9 = F_1
\end{aligned}
\tag{38}
$$

where the negative values indicate the compressive forces.

Unless the buckling effect is taken into consideration, the necessary cross sectional areas are found to be

$$A_i = F_i/\sigma_a \qquad (i=1, \cdots, 9) \tag{39}$$

Using Eqs 38 and 39, the total volume, V, can be obtained as follows.

$$V=\frac{1}{12000}\left[\frac{20(X_1^2+X_2^2)+400X_1}{X_2}+\frac{400(X_3-X_2)}{(20-X_1)}\right.$$
$$\left.+\frac{400\{(X_3-X_2)^2+(20-X_1)^2\}}{(20-X_1)X_3}+20\{X_2^2+(20-X_1^2)\}\frac{|20X_2-X_1X_3|}{X_2X_3(20-X_1)}\right]_{(m^3)} \quad (40)$$

where $L_1=\sqrt{X_1^2+X_2^2},\ L_2=\sqrt{(X_3-X_2)^2+(20-X_1)^2},\ L_3=\sqrt{X_2^2+(20-X_1)^2}$

$L_4=20,\quad L_5=X_3$

$\sin\alpha=X_2/L_1,\quad \cos\alpha=X_1/L_1,\quad \tan\alpha=X_2/X_1$

$\sin\beta=X_2/L_3,\quad \cos\beta=(20-X_1)/L_3,\quad \tan\beta=X_2/(20-X_1)$

$\sin\gamma=(X_3-X_2)/L_2,\quad \cos\gamma=(20-X_1)/L_2,\quad \tan\gamma=(X_3-X_2)/(20-X_1)$

$\sigma_a=1200\ \mathrm{kg/cm^2}$

When the values of $X_1 \sim X_3$ are given, the cross sectional areas and the total volume can be calculated from Eqs 38~40. It is also possible to draw the contour surface of the objective function (i.e. volume). For two cases where the values of volume are specified as $0.115\times10^6\ \mathrm{cm^3}$ and $0.120\times10^6\ \mathrm{cm^3}$, the contour surfaces are shown in Fig. 6. Fig. 7 and Fig. 8 show the projections on the X_1-X_3 plane and the X_2-X_3 plane, respectively. Fig. 7 indicates that in this model the contour surfaces have round shapes like an egg, though two ends are cut off by two planes. This is due to the side constraint imposed on the design variables, X_1 and X_2. The optimum point is located at a point near the center of these surfaces. The above observations lead to the conclusion that the problem treated here

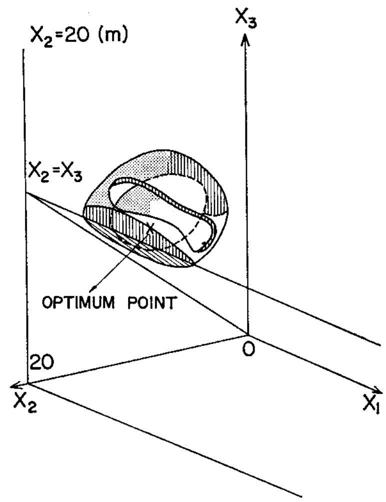

Fig. 6. Design Space of Nein-Bar Truss.

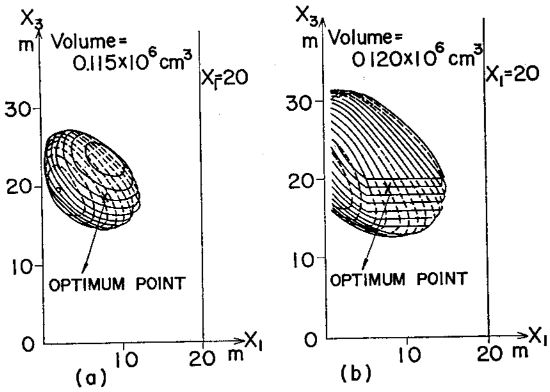

Fig. 7. Projection on X_1-X_3 Plane.

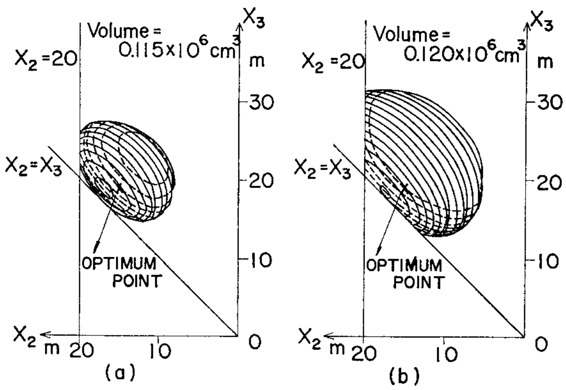

Fig. 8. Projection on X_2-X_3 Plane.

provides assurance that the employed two-level approach will give good results.

Example 2 4-panel Truss Model

Consider the 4-panel truss model shown in Fig. 9. The loading condition and other design conditions are the same as those used in the 2-panel model. The numerical results are given in Table 2 and Table 3 for two cases: the design without a buckling constraint and the design with a buckling constraint. The former case shows the same geometry as that of the 2-panel model. Through optimization, the cross sectional areas of the 4th and 5th members become zero or very small. However, when the buckling effect is taken, the optimal geometry is different from that of the former case. The height becomse smaller, and the length of each member is almost the same. This result can be easily inferred from Eq. 16, for it implies that the necessary cross sectional areas increase proportionally to the square of the member's length. Naturally, the volume of the latter design is more than two times that of the former.

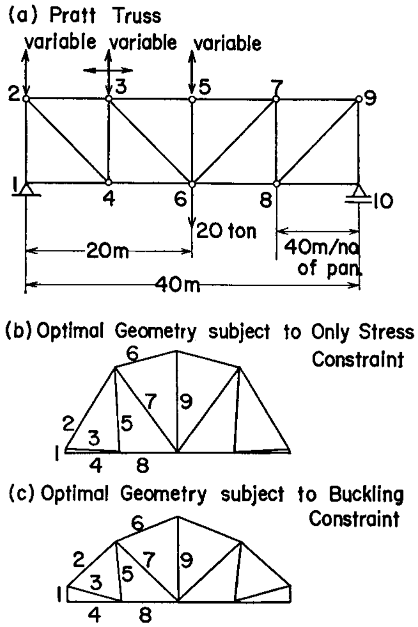

Fig. 9. 4-panel Truss Model.
(a) Pratt Truss
(b) Optimal Geometry subject to Only Stress Constraint
(c) Optimal Geometry subject to Buckling Constraint

Table 2. Numerical Results of 4-panel Truss Model

	Member area (cm²)	Member force (kg)	Member length (cm)
1	8.33	−10000.00	50.00
2	9.51	−11415.40	1706.60
3	5.02	6027.58	1001.25
4	0.00	0.00	1000.00
5	0.25	−301.67	1503.33
6	9.60	−11516.90	1140.18
7	7.20	8642.93	1860.11
8	5.00	6000.00	1000.00
9	5.05	6060.60	1800.00

Total Volum $=0.1119\times 10^6$ cm³

Table 3. Numerical Results of 4-panel Truss Model with Buckling Constraints

	Member area (cm²)	Member force (kg)	Member length (cm)
1	9.33	−10000.00	283.00
2	37.34	−11326.20	1197.70
3	28.40	8697.84	1039.27
4	0.00	0.00	1000.00
5	15.67	−2381.61	1096.05
6	41.58	−14102.50	1195.04
7	5.87	7046.27	1559.27
8	6.77	8119.27	1000.00
9	8.46	10148.70	1520.00

Total Volume $=0.2877\times 10^6$ cm³

Example 3. Some Simply Supported Truss Models

6-, 8-, 10- and 12- panel models were designed under the same condition as Example 2. These are used to investigate the relation between the number of nodes and the optimal geometry. Among them, the geometries obtained for the 6- and 8- panel models are shown in Fig. 10. Comparing them with the geometries of the 2- and 4-panel models, it can be said that in a case without a buckling constraint, an optmial geometry exists despite the number of employed nodes. Namely, it is not important how many nodes one must adopt at the beginning of design. Furthermore, some kinds of 4-panel truss models were designed in order to clarify the interaction of the panel number and the volume. The results are summarized in Fig. 11, where the design conditions are as follows:

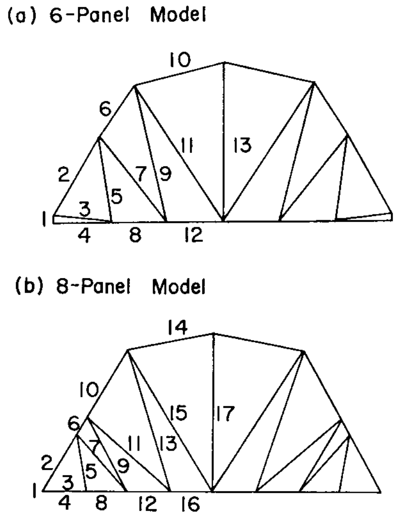

Fig. 10. Optimal Geometry.

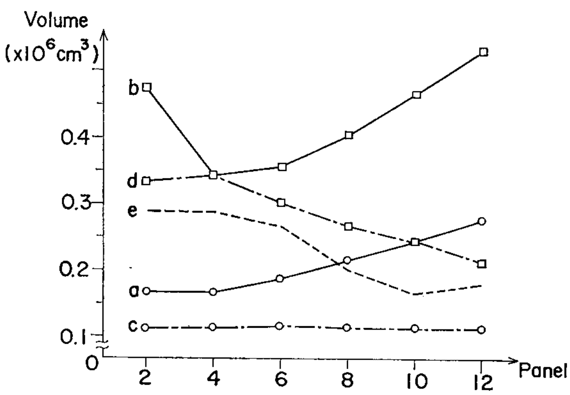

Fig. 11. Panel Number vs Volume.

case a : designs of pratt truss whose geometry is fixed, where the buckling effect is not included

case b : designs of pratt truss with buckling constraint

case c : designs of truss with variable geometries (only allowable stress limit)

case d : designs with buckling constraint, whose geometry is that given by case c

case e : designs with variable geometries, where the buckling effect is directly introduced into the optimization

From this figure, the following items can be obtained: in case a, the optimum number of panels is 2 or 4. There is no difference in volume between them. However, considering the buckling effect, the optimum number becomes 4. (See the curve of case b). The curve of case c ascertains the conclusion that there is an optimal geometry under the design condition in which only the allowable stress limit is imposed. In this curve, the value of the volume is unchanged regradless of the number of panels. The curve of case d indicates that the greater the number of panels, the lighter the truss. While the geometries given by case c have almost the same configuration with respect to the exterior members, they have different numbers of interior members. As the number of the interior members increases, the lengths of the exterior members become shorter. This is useful for a design which is considering the buckling effect. Case e shows the least volume, and then, the optimum number is obtained as 10. It may be considered that the increase caused by the addition of interior members and the decrease caused by the shortening of exterior members are balanced at this number. Also, these results lead to the conclusion that the optimum number of panel will change, if the variation of geometry is taken into account in the design process.

5.2 Examples for the probabilistic method

Example 4 4-panel Model

Consider the 4-panel model whose dimensions and other conditions are the same as Example 2. However, the resistances and the load are considered to be random. Assume now that both of them have independent normal distributions. The numerical results are given in Table 4 and Fig. 12, where the used mean values and standard deviations are $\mu_R=2040$ kg/cm^2, $\mu_S=20000$ kg, $\sigma_R=204$ kg/cm^2 and $\sigma_S=4000$ kg, respectively. Also, the employed allowable level of failure probability is 0.04. It can be found from Fig. 12 that the geometry obtained by the probabilistic method is not so different from that obtained by the deterministic method. That is, for statically determinate systems, both methods give an identical

Table 4. Numerical Results Obtained by Probabilistic Method

	Pratt Truss	Variable Geometry
A_1 (cm^2)	8.66	9.61
A_2	8.66	8.98
A_3	11.76	4.99
A_4	0.0002	0.0002
A_5	8.66	0.49
A_6	16.64	8.93
A_7	11.76	7.15
A_8	8.66	4.96
A_9	0.0002	4.41
p_f	0.04	0.04
Volume [cm^3]	169101	112581

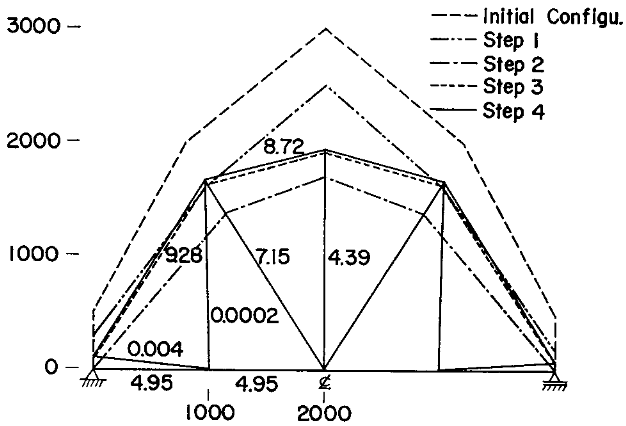

Fig. 12. Geometry of 4-panel Model at Each Design Step.

optimal geometry, though the values of the cross sectional areas differ somewhat from each other. It should be noted that it is impossible to compare both methods in volume because the employed value of the allowable level, 0.04, does not correspond to the safety factor of the deterministic approach.

From Table 4, it is obtained that the introduction of geometrical variations presents an economic design, whose volume is 34.5 per cent less than the pratt truss with a fixed geometry. The convergency is shown in Fig. 13. This shows the efficiency of the proposed method. This method requires only 7 seconds to reach the optimum point. Though the convergency depends on the initial values, the search procedure was terminated at the 5th step. The results for some design cases are given in Table 5. The truss becomes heavier as the value of the allowable level becomes smaller. However, all goemetries obtained for these conditions show no distinct difference. Next, the buckling effect is taken into account. The resulting geometry is shown in Fig. 14. This shows a tendency similar to that of the deterministic method.

Table 5. Numerical Results of 4-panel Model under Some Design Conditions

	1	2	3	4	5	6
A_1 (cm^2)	11.71	17.29	15.02	10.75	10.26	19.38
A_2	9.28	11.16	10.47	8.86	8.66	12.40
A_3	0.004	0.06	0.04	0.07	0.04	0.04
A_4	4.95	6.25	5.85	4.74	4.53	7.01
A_5	0.0002	0.04	0.04	0.04	0.04	0.04
A_6	8.92	11.00	10.23	8.46	8.25	12.25
A_7	7.15	8.68	8.13	6.81	6.66	9.68
A_8	4.95	6.30	5.79	4.67	4.54	7.01
A_9	4.39	5.44	5.06	4.19	4.05	6.06
X_2 (cm)	10.99	10.30	10.40	11.20	11.30	10.10
Y_2	19.30	18.30	18.30	19.50	19.90	17.80
X_3	950.2	950.2	950.2	950.2	950.2	950.2
Y_3	1675.4	1675.4	1675.4	1675.4	1675.4	1675.4
Y_5	1939.8	1939.7	1939.7	1939.8	1939.8	1939.7
p_{fa}	0.04	0.04	0.04	0.04	0.04	0.04
δ_p	0.2	0.02	0.2	0.2	0.2	0.3
δ_R	0.1	0.17	0.15	0.07	0.05	0.17
V (cm^3)	111768	137290	128118	106662	103823	152836
	7	8	9	10	11	12
A_1 (cm^2)	15.45	12.12	12.48	12.94	13.82	15.16
A_2	11.23	9.71	10.12	10.54	11.47	12.70
A_3	0.04	0.04	0.06	0.10	0.04	0.04
A_4	6.19	5.16	5.32	5.67	5.98	6.64
A_5	0.04	0.04	0.04	0.04	0.04	0.04
A_6	10.93	9.30	9.68	10.04	10.88	12.07
A_7	8.69	3.51	7.78	8.09	8.78	9.36
A_8	6.14	5.16	5.36	5.56	5.99	6.64
A_7	5.41	4.61	4.76	4.98	5.34	5.93
X_2 (cm)	10.60	10.90	11.00	11.20	11.10	11.20
Y_2	18.00	19.20	19.50	19.50	19.60	19.70
X_3	950.2	950.2	950.2	950.2	950.2	950.2
Y_3	1675.4	1675.4	1675.4	1675.4	1675.4	1675.4
Y_5	1939.8	1939.8	1939.8	1939.8	1939.8	1939.8
p_{fa}	0.04	0.02	0.01	0.005	0.001	0.0001
δ_p	0.3	0.2	0.2	0.2	0.2	0.2
δ_R	0.13	0.1	0.1	0.1	0.1	0.1
V (cm^3)	136829	117162	121658	126898	137105	152047

δ : coefficient of variation

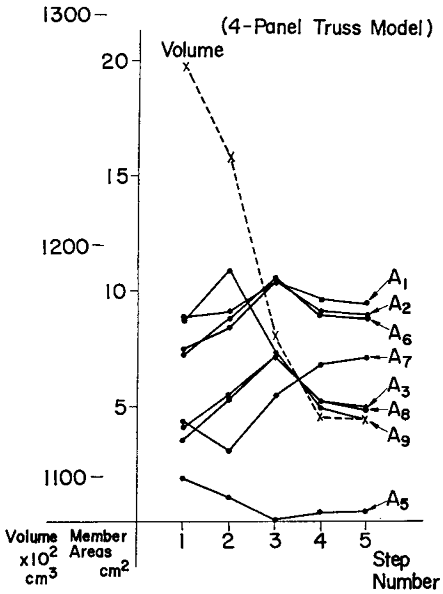

Fig. 13. Convergence of Objectives (4-pannel Model).

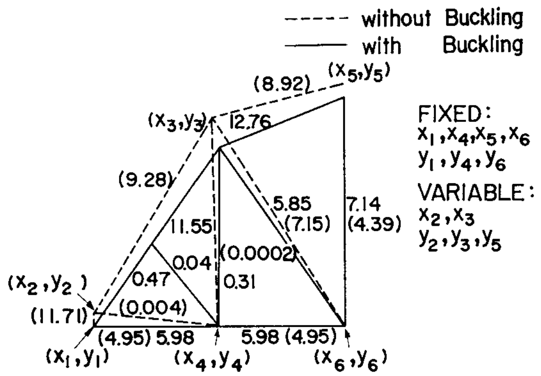

Fig. 14. Optimal Configuration of 4-panel Model with Buckling Constraint.

6. Conclusions

While the importance of geometry has been sufficiently recognized in the design of truss, its direct introduction has had such difficulties as poor convergency and excessive computation time. In this paper an approximate design method is proposed, which is applicable to both deterministic and probabilistic designs. The above problems concerned with practical computations can be removed by using the optimality conditions efficiently in the optimization procedure. Through some numerical examples, the following conclusions were reached.

1. It was confirmed that the geometry of truss can be efficiently treated with the use of the proposed method, and considerable weight reduction can often be achieved.
2. There is an optimal geometry when only the stress constraint is taken into consideration. Then, the number of nodes does not play an important role.
3. Buckling failure must be considered to specify the effective geometry. This can be done by combining the sub-optimization of the member and the usual mathematical programming.
4. When a variation of geometry is introduced into a truss design, the optimum number of panels becomes different from that of a design with fixed geometry.
5. For statically determinate trusses, the probabilistic design gives the optimal geometry which is the same as the deterministic design.
6. By using the two-level approach, the number of design variables considered at one time is reduced. Then, the convergency can be considerably improved.
7. The proposed method presented sufficient results for the employed examples. However, the rate of convergency may become slow in a case where member forces change signs as a result of geometric changes.

Acknowledgement

The authors wish to express their sincere thanks to Professor Ichiro Konishi of the Chubu Institute of Technology for his encouragement and valuable advice throughout this investigation. Also, they would like to thank Messr's Shigeki Kitazono and Singo Irie for their stimulating discussions and support.
All calculations were performed by FACOM M 190 computer in Kyoto University.

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