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By

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Abstract

In this paper a generalized stability criterion for a system with multi-argument nonlinearities is derived. The new criterion is based on M. A. Pai's work, and is proved along B. D. O. Anderson's criterion. The new criterion enables us to construct a Lur'e type Lyapunov function in a systematic way. The new criterion is applied to a multi-machine power system with magnetic flux decays of generators. A new Lyapunov function is constructed in a well known manner established by J. L. Willems and other researchers. The new Lyapunov function is similar to the one which has already been obtained for a system without the magnetic flux decays, except for a few points which will affect a transient stability of the system.

Nomenclature

n: number of generators in a system.

$Y_{ij}\angle\phi_{ij}$: post-fault short-circuit transfer admittance between the ith and jth generator nodes (obtained after reduction of a network retaining only generator nodes).

B_{ij}: $Y_{ij}\sin(\phi_{ij})$, post-fault short-circuit transfer susceptance between the ith and jth generator nodes.

G_{ij}: $Y_{ij}\cos(\phi_{ij})$, post-fault short-circuit transfer conductance between the ith and jth generator nodes.

θ_{ij}: complement of the short-circuit transfer admittance angle ϕ_{ij}.

m_i: angular momentum constant of the ith generator.

d_i: damping power coefficient of the ith generator.

P_{mi}: mechanical power input of the ith generator.

P_{ei}: electrical power output of the ith generator.

E_{fdi}: field voltage of the ith generator.

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i_{di}: d-axis current of the ith generator.
i_{qi}: q-axis current of the ith generator.
x_{di}: d-axis synchronous reactance of the ith generator.
x_{qi}: q-axis synchronous reactance of the ith generator.
x_{di}': d-axis transient reactance of the ith generator.
T_{doi}': d-axis transient open-circuit time constant of the ith generator.
E_i: internal voltage behind transient impedance of the ith generator.
$E^0{}_i$: post-fault stable equilibrium internal voltage of the ith generator.
δ_i: angle of the rotor shaft of the ith machine in electrical radians, relative to a reference frame rotating at synchronous speed.
$\delta^0{}_i$: post-fault stable equilibrium angle of the ith generator.
δ_{ij}: δ_i-δ_j, difference between rotor angles of two generators.
ω_i: angular velocity of the rotor shaft of the ith generator, relative to the steady-state speed.
δ: n-dimensional angle vector.
δ_r: $(n-1)$-dimensional relative angle vector.
ω: n-dimensional angular velocity vector.
M: diag(m_i), system inertia constants matrix.
D: diag(d_i), system damping coefficients matrix.
α: diag(α_i), internal voltage attracting coefficients matrix.
β: diag(β_i), internal voltage disturbing coefficients matrix.
I, O: identity and null matrices.
$'$: prime implying the transposition of the vector or the matrix.

1. Introduction

There has been much progress in Lyapunov's direct method for a transient stability analysis of an electric power system in the past decade. In 1968, B. D. O. Anderson and J. B. Moore derived a generalized Popov's criterion, which made it possible to construct a Lur'e type Lyapunov function for a system with multiple non-linearities in a systematic manner.[1] In 1970, J. L. Willems applied this criterion to a model of a multi-machine power system with damping torques of generators for the first time.[2] Several researchers followed and refined his work[3-5]. The obtained Lyapunov function is suitable for estimating the transient stability region. It is important that the construction method is systematic, and that it is applicable to more complicated systems for which the construction methods based on the intuitive considerations will fail. At present, however, its application is limited to a very simple system model, because Anderson's criterion is applicable only to a system with single-argument non-linearities.

It is desired to improve a mathematical model and to construct new Lyapunov functions to include a magnetic flux decay, an automatic voltage regulator (AVR), etc. The magnetic flux decay of a generator narrows the transient stability region. On the other hand, an AVR operates so as to keep a terminal voltage of the generator constant, and improves the transient stability. Hence, their effects are important for the stability analysis. Inclusion of the flux decay and AVR to the system model needs a new stability criterion for a system with multi-argument non-linearities.

For a one-machine power system, the effects of the magnetic flux decay and AVR have been included in the Lyapunov function in several ways.[6-8] Among them, M. A. Pai and V. Rai[8] expanded Popov's criterion to a system with multi-argument non-linearities, and applied their criterion to a one-machine power system with the flux decay and voltage regulator. The Lur'e type Lyapunov function was systematically constructed. Their criterion is applicable not only to a one-machine power system but also to a multimachine power system. However, their system model is more of a particularized form, and it is convenient to derive a new criterion for a general system model before applying the criterion to a multimachine power system.

In this paper, Pai's criterion is generalized to a new stability criterion, which is proved that it is valid along Anderson's criterion. The new criterion is, of course, applicable to a system with multi-argument non-linearities. It is applied to a multi-machine power system with magnetic flux decays of generators. A Lur'e type Lyapunov function is constructed in a well-known manner established by J. L. Willems,[2] M. A. Pai and P. G. Murthy,[3] and V. E. Henner.[4] The obtained Lyapunov function has an additional term which represents the magnitude of the flux decays. If the fluxes are assumed to be constant, then the Lyapunov function reduces to the one which is constructed under an assumption of the constant flux.

The new stability criterion is also applicable to a system with an automatic voltage regulator, or to more complicated systems, which is described in another paper.

2. Stability criterion

The non-linear systems considered here are those where the system is of the form shown in Fig. 1. The Lyapunov stability is considered, and so the inputs are not indicated. The matrix $W(s)$ is an $m \times m$ matrix of stable rational transfer functions, assumed to be such that

$$W(\infty)=0 \tag{1}$$

The non-linearities $F(\sigma)$ are assumed to satisfy the following conditions[9]:

1) $F(\sigma)$ is continuous and maps R^m into R^m.
2) For some constant real matrix N,

$$F(\sigma)'N\sigma \geq 0 \qquad \text{for all } \sigma \in R^m \tag{2}$$

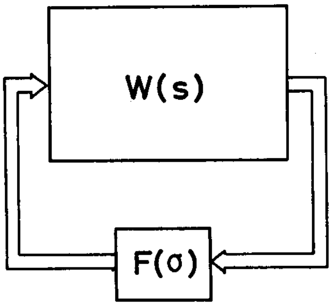

Fig. 1. Nonlinear system model

and

$$F(\sigma)=0 \qquad \text{if } \sigma=0 \tag{3}$$

3) There is a function $V_1 \in C^1$ mapping R^m into R such that

$$V_1(\sigma) \geq 0 \qquad \text{for all } \sigma \in R^m \tag{4}$$

and

$$V_1(\sigma)=0 \qquad \text{if } \sigma=0 \tag{5}$$

and for some constant real matrix Q

$$\nabla V_1(\sigma)=Q'F(\sigma) \qquad \text{for all } \sigma \in R^m \tag{6}$$

The stability criterion for the above system is given as follows:

Theorem

If there exist real matrices N and Q such that

$$Z(s)=(N+Qs)W(s) \tag{7}$$

is positive real, then the system shown in Fig. 1 is stable, where $(N+Qs)$ does not cause a pole-zero cancellation with $W(s)$.

Before proving the theorem, it is necessary to introduce the lemma by B. D. O. Anderson.[10]

Lemma (B.D.O. Anderson)

Let $Z(s)$ be a matrix of rational transfer functions such that $Z(\infty)$ is finite and $Z(s)$ has poles which lie in Re $s<0$, or are simple on Re $s=0$. Let (A, B, C) be a minimal realization of $Z(s)-Z(\infty)$. Then $Z(s)$ is positive real if and only if there exist a symmetric positive definite matrix P and matrices W_0 and L such that

$$PA+A'P=-LL'$$

$$\begin{aligned} &PB=C-LW_0 \\ &W_0'W_0=Z(\infty)+Z'(\infty) \end{aligned} \tag{8}$$

With the aid of this lemma, the theorem is proved.

Proof of Theorem

The transfer function $W(s)$ possesses a minimal realization (A, B, C), which is a set of three constant matrices satisfying

$$W(s)=C'(sI-A)^{-1}B \tag{9}$$

An expansion of $Z(s)$ in terms of A, B and C gives

$$\begin{aligned} Z(s)&=(N+Qs)\,W(s) \\ &=NC'(sI-A)^{-1}B+QC'[(sI-A)+A]\,(sI-A)^{-1}B \\ &=QC'B+(NC'+QC'A)\,(sI-A)^{-1}B \end{aligned} \tag{10}$$

and we see that since $Z(s)$ is positive real, Anderson's lemma can be applied to the triple $(A, B, CN'+A'CQ')$, which is a minimal realization of $Z(s)-Z(\infty)$. Thus, there exist a positive definite symmetric matrix P, and matrices L and W_0 such that

$$\begin{aligned} PA+A'P&=-LL' \\ PB&=CN'+A'CQ'-LW_0 \\ W_0'W_0&=QC'B+B'CQ' \end{aligned} \tag{11}$$

Consider as a tentative Lyapunov function for the system of Fig. 1:

$$V(x)=x'Px+2V_1(\sigma) \tag{12}$$

where x is the state vector of the system. Observe that the positive definiteness of P and the positive semi-definiteness of $V_1(\sigma)$ ensure that $V(x)$ is positive for every nonzero x. Differentiating $V(x)$ gives

$$\begin{aligned} \dot{V}(x)&=\dot{x}'Px+x'P\dot{x}+2\nabla V_1'(\sigma)\dot{\sigma} \\ &=(x'A'-F(\sigma)'B')Px+x'P(Ax-BF(\sigma)) \\ &\qquad +2F(\sigma)'QC'(Ax-BF(\sigma)) \\ &=x'(PA+A'P)x-2x'(PB-A'CQ')F(\sigma) \\ &\qquad -F(\sigma)'(QC'B+B'CQ')F(\sigma) \\ &=-x'LL'x+2x'LW_0F(\sigma)-2x'CN'F(\sigma)-F(\sigma)'W_0'W_0F(\sigma) \\ &=-(x'L-F(\sigma)'W_0')\,(L'x-W_0F(\sigma))-2F(\sigma)'N\sigma \end{aligned} \tag{13}$$

The first term is plainly non-positive, and the non-negative nature of $F(\sigma)'N\sigma$ ensures the non-positivity of the second term. Accordingly, $\dot{V}(x)$ proves to be non-positive. This completes the proof.

3. System equation

Consider an n-machine power system with the magnetic flux decays of the generators. Under the usual assumptions made in the power system transient stability analysis, and neglecting the short-circuit transfer conductances, the motion of the rotor of the ith generator is described by the second-order equation

$$m_i\frac{d^2\delta_i}{dt^2}+d_i\frac{d\delta_i}{dt}=\sum_{j=1}^{n}B_{ij}(E_i{}^0E_j{}^0\sin\delta_{ij}{}^0-E_iE_j\sin\delta_{ij})$$
$$\text{for } i=1,\ 2,\ \dots,\ n \tag{14}$$

and the change of the internal voltage of the ith generator is described by the first order equation

$$\frac{dE_i}{dt}=-\alpha_i(E_i-E_i{}^0)-\beta_i\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}{}^0-\cos\delta_{ij})$$
$$\text{for } i=1,\ 2,\ \dots,\ n \tag{15}$$

where α_i and β_i are defined by eq. (A9) (Appendix A).

The following $(3n-1)$ quantities are selected as the state variables.

$$\begin{aligned}
&\delta_{ri}=\delta_{1(i+1)}-\delta_1{}^0{}_{(i+1)} && \text{for } i=1,\ \dots,\ n-1\\
&\omega_i=\dot{\delta}_i && \text{for } i=1,\ \dots,\ n\\
&\Delta E_i=E_i-E^0{}_i && \text{for } i=1,\ \dots,\ n
\end{aligned} \tag{16}$$

Then the state vector is given by

$$x=[\delta_r{}',\ \omega',\ \Delta E']' \tag{17}$$

The state equation of the system is given by

$$\begin{aligned}
&\dot{x}=Ax-BF(\sigma)\\
&\sigma=C'x
\end{aligned} \tag{18}$$

where

$$A=\begin{bmatrix}0 & K'_{n(n-1)} & 0\\ 0 & -M^{-1}D_{nn} & 0\\ 0 & 0 & -\alpha_{nn}\end{bmatrix} \qquad B=\begin{bmatrix}0 & 0\\ M^{-1}T_{nm} & 0\\ 0 & \beta_{nn}\end{bmatrix}$$
$$C=\begin{bmatrix}G_{(n-1)m} & 0\\ 0 & 0\\ 0 & I_{nn}\end{bmatrix} \tag{19}$$

in which

$$K_{n(n-1)}=\begin{bmatrix}1_{1(n-1)}\\ -I_{(n-1)(n-1)}\end{bmatrix}$$

$$T_{nm}=\begin{bmatrix} 1_{1(n-1)} & 0_{1(m-n+1)} \\ -I_{(n-1)(n-1)} & T_{(n-1)(m-n+1)} \end{bmatrix} \tag{20}$$

$$G_{(n-1)m}=[I_{(n-1)(n-1)} \quad -T_{(n-1)(m-n+1)}]$$

and there is a relation as

$$T_{nm}=K_{n(n-1)}G_{(n-1)m} \tag{21}$$

The row vector $1_{1(n-1)}$ and $0_{1(m-n+1)}$ has all its elements equal to unity and zero, respectively, and $m=n(n-1)/2$.

The non-linearity $F(\sigma)$ consists of two vectors.

$$F(\sigma)=\begin{bmatrix} f_1(\sigma) \\ f_2(\sigma) \end{bmatrix} \tag{22}$$

The non-linearity $f_1(\sigma)$ is an m-vector

$$f_{1k}(\sigma)=B_{ij}\{E_iE_j\sin(\sigma_k+\delta_{ij}{}^0)-E_i{}^0E_j{}^0\sin\delta_{ij}{}^0\}$$
$$\text{for } i,j=1,2,\ldots,n,\ k=1,\ldots,m \tag{23}$$

where k is related with i and j by

$$k=(i-1)n-i(i+1)/2+j, \qquad \text{for } i<j \tag{24}$$

The non-linearity $f_2(\sigma)$ is an n-vector

$$f_{2i}(\sigma)=\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}{}^0-\cos\delta_{ij})$$
$$\text{for } i=1,2,\ldots,n \tag{25}$$

The output σ is an $(m+n)$-vector defined by

$$\sigma_k=\delta_{ij}-\delta_{ij}{}^0 \qquad \text{for } k=1,2,\ldots,m$$
$$\sigma_{m+i}=E_i-E_i{}^0 \qquad \text{for } i=1,2,\ldots,n \tag{26}$$

where k is related with i and j by eq. (24). Eq. (18) describes the multi-machine power system as a multivariable dynamical system with a linear element in the forward path, and multiple, memoryless and coupled non-linear elements in the feedback path.

4. Determination of positive real matrix

The transfer matrix $W(s)$ for the linear part of the system in eq. (18) is

$$W(s)=C'(sI-A)^{-1}B \tag{27}$$

where A, B, C in eq. (19) is the minimal realization of $W(s)$ (Appendix B). To apply the theorem derived in section 2, we have to find matrices N and Q such that

$$Z(s)=(N+Qs)\,W(s) \tag{28}$$

is positive real. The matrices N and Q also have to satisfy the conditions for the nonlinearity $F(\sigma)$. In this problem, matrix N is chosen as

$$N=0 \tag{29}$$

to satisfy eq. (2) because the term $F'(\sigma)N\sigma$ can have negative values in some region around the origin, for example, for a positive diagonal matrix N.[8] To determine matrix Q, we choose function $V_1(\sigma)$ in eq. (4) as

$$\begin{aligned} V_1(\sigma) &= \sum_{k=1}^{m}\int_0^{\sigma_k} f_{1k}(\sigma)\,d\sigma_k \\ &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\{E_iE_j(\cos\delta_{ij}{}^0-\cos\delta_{ij}) \\ &\qquad -(\delta_{ij}-\delta_{ij}{}^0)E_i{}^0E_j{}^0\sin\delta_{ij}{}^0\} \end{aligned} \tag{30}$$

where i, j and k are related by eq. (24). The partial derivative of $V_1(\sigma)$ is given as follows:

$$\begin{aligned} \frac{\partial V_1}{\partial\sigma_k} &= B_{ij}\{E_iE_j\sin(\sigma_k+\delta_{ij}{}^0)-E_i{}^0E_j{}^0\sin\delta_{ij}{}^0\} \\ &= f_{1k}(\sigma) \qquad \text{for } k=1,\ \ldots,\ m \end{aligned} \tag{31}$$

$$\begin{aligned} \frac{\partial V_1}{\partial\sigma_{(m+i)}} &= \sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}{}^0-\cos\delta_{ij}) \\ &= f_{2i}(\sigma) \qquad \text{for } i=1,\ \ldots,\ n \end{aligned} \tag{32}$$

Eq. (31) and eq. (32) are unified to

$$\varDelta V_1(\sigma)=\begin{bmatrix} I_{mm} & 0 \\ 0 & I_{nn} \end{bmatrix}\begin{bmatrix} f_1(\sigma) \\ f_2(\sigma) \end{bmatrix} \tag{33}$$

Accordingly, matrix Q in eq. (6) is

$$Q=I_{(m+n)(m+n)} \tag{34}$$

$Z(s)$ in eq. (28) with N in eq. (29) and Q in eq. (34) must be positive real. Substituting eq. (19) for matrices A, B and C into eq. (27), the transfer matrix is

$$\begin{aligned} W(s) &= \begin{bmatrix} T'\{s(sI+M^{-1}D)\}^{-1}M^{-1}T & 0 \\ 0 & I(sI+\alpha)^{-1}\beta \end{bmatrix} \\ &\equiv \begin{bmatrix} W_1(s) & 0 \\ 0 & W_2(s) \end{bmatrix} \end{aligned} \tag{35}$$

Substituting eq. (29), eq. (34) and eq. (35) into eq. (28), we obtain

$$Z(s)=\begin{bmatrix} T'(sI+M^{-1}D)^{-1}M^{-1}T & 0 \\ 0 & s(sI+\alpha)^{-1}\beta \end{bmatrix}$$

$$\equiv\begin{bmatrix} Z_1(s) & 0 \\ 0 & Z_2(s) \end{bmatrix} \tag{36}$$

where $Z_1(s)$ and $Z_2(s)$ are defined in an explicit way, and are symmetric matrices.

The conditions for $Z(s)$ to be positive real are

1) $Z(s)$ has elements which are analytic for Re $s>0$,
2) $Z^*(s)=Z(s^*)$ for Re $s>0$
3) $Z'(s^*)+Z(s)$ are positive semi-definite for Re $s>0$

The first two conditions clearly hold. For condition 3) to be satisfied, it is sufficient to show that $Z(j\omega)+Z'(-j\omega)$ are positive semi-definite for each real scalar ω. Since $Z(s)$ is a direct sum of $Z_1(s)$ and $Z_2(s)$, $Z_1(s)$ and $Z_2(s)$ are investigated independently of each other to determine whether they are positive real. Replacing s with $j\omega$ in eq. (36) for $Z_1(s)$ and $Z_2(s)$, we obtain the positive semi-definite matrices

$$Z_1(j\omega)+Z_1'(-j\omega)=T'\mathrm{diag}\left(\frac{2d_i}{m_i^2\omega^2+d_i^2}\right)T \tag{37}$$

$$Z_2(j\omega)+Z_2'(-j\omega)=\mathrm{diag}\left(\frac{2\beta_i\omega^2}{\omega^2+\alpha_i^2}\right) \tag{38}$$

Hence, the matrices $Z_1(s)$ and $Z_2(s)$ are both positive real, which guarantees that the matrix $Z(s)$ in eq. (36) is positive real, and accordingly that the system of eq. (18) is stable.

5. Construction of Lyapunov function

Since the positive real matrix $Z(s)$ is established, there exists, for the system of eq. (18), the Lyapunov function

$$V(x)=x'Px+2V_1(\sigma) \tag{12}$$

where P is a $(3n-1)\times(3n-1)$ positive definite symmetric matrix satisfying the matrix equations

$$\begin{aligned} PA+A'P&=-LL' \\ PB&=CN'+A'CQ'-LW_0 \\ W_0'W_0&=QC'B+B'CQ' \end{aligned} \tag{11}$$

where L and W_0 are $(3n-1)\times(m+n)$ and $(m+n)\times(m+n)$ auxiliary real matrices. As $Z(s)$ is positive real, there exists a matrix $Y(s)$ such that

$$Z(s)+Z'(-s)=Y'(-s)Y(s) \tag{39}$$

where $Y(s)$ is the $(m+n)\times(m+n)$ matrix, and $Y(s)$ has a minimal realization (A, B, L), i.e.,

$$Y(s)-Y(\infty)=L'(sI-A)^{-1}B \tag{40}$$

and W_0 in eq. (11) is given by

$$W_0=Y(\infty) \tag{41}$$

(Appendix C).

Since $W(s)$ in eq. (35) is a direct sum of $W_1(s)$ and $W_2(s)$, and $Z(s)$ in eq. (36) is a direct sum of $Z_1(s)$ and $Z_2(s)$, the positive definite matrix P is expressed as a direct sum of the positive definite matrices P_1 and P_2, where P_1 and P_2 are the $(2n-1)$ and nth order, respectively. The transfer matrix $W_1(s)$ is rewritten as

$$W_1(s)=C_1'(sI-A_1)^{-1}B_1 \tag{42}$$

where A_1, B_1 and C_1 are defined by

$$A_1=\begin{bmatrix}0 & K' \\ 0 & -M^{-1}D\end{bmatrix} \quad B_1=\begin{bmatrix}0 \\ M^{-1}T\end{bmatrix} \quad C_1=\begin{bmatrix}G \\ 0\end{bmatrix} \tag{43}$$

As $C_1'B_1=0$, eq. (11) reduces to

$$\begin{aligned}P_1A_1+A_1'P_1&=-L_1L_1'\\ P_1B_1&=A_1'C_1Q_1'\end{aligned} \tag{44}$$

The matrices P_1 and L_1 are partitioned as

$$P_1=\begin{bmatrix}P_{11} & P_{12} \\ P_{21} & P_{22}\end{bmatrix} \quad L_1=\begin{bmatrix}L_{11} \\ L_{12}\end{bmatrix} \tag{45}$$

where P_{11}, P_{12}, P_{21} and P_{22} are $(n-1)\times(n-1)$, $(n-1)\times n$, $n\times(n-1)$ and $n\times n$ matrices, and L_{11}, L_{12} are $(n-1)\times m$ and $n\times m$ matrices, respectively. Substituting eq. (43) for A_1, B_1 and C_1 into eq. (44), we obtain

$$0=-L_{11}L_{11}' \tag{46}$$

$$P_{11}K'-P_{12}M^{-1}D=0 \tag{47}$$

$$P_{21}K'+KP_{12}-P_{22}M^{-1}D-M^{-1}DP_{22}=-L_{12}L_{12}' \tag{48}$$

and

$$P_{12}M^{-1}T=0 \tag{49}$$

$$P_{22}M^{-1}T=T \tag{50}$$

The matrix eqs. (46)–(50) are solved after some manipulation.[4]

$$\begin{aligned}KP_{11}K'&=\rho DUD\\ KP_{12}&=\rho DUM\\ P_{22}&=M+\mu MUM\end{aligned} \tag{51}$$

where ρ is a non-negative scalar, and U is an $n\times n$ matrix with all elements equal to 1.

Substituting eq. (51) into eq. (48), we obtain

$$2D+(\mu-\rho)(MUD+DUM)\geq 0. \tag{52}$$

To satisfy the above inequality, the following inequality must be satisfied:

$$(\mu^*)^2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{(d_im_j-d_jm_i)^2}{4d_id_j}-\mu^*\sum_{i=1}^{n}m_i-1\leq 0 \tag{53}$$

that is, μ^* must lie between the two roots of the quadratic equation, where

$$\mu^*=\mu-\rho \tag{54}$$

If the damping torques are uniform, that is,

$$d_1/m_1=d_2/m_2=\ldots=d_n/m_n, \tag{55}$$

then μ^* reduces to μ_0, where[2)]

$$\mu_0=-1/\sum_{i=1}^{n}m_i. \tag{56}$$

Next, P_2 is calculated. The transfer matrix $W_2(s)$ is written as

$$W_2(s)=C_2'(sI-A_2)^{-1}B_2 \tag{57}$$

where A_2, B_2 and C_2 are defined by

$$A_2=-\alpha_{nn}, \qquad B_2=\beta_{nn}, \qquad C_2=I_{nn} \tag{58}$$

Since $Z_2(s)$ in eq. (36) is positive real, $Z_2(s)+Z_2'(-s)$ is factorized as follows:

$$Z_2(s)+Z_2'(-s)=\mathrm{diag}\left(\frac{-s\sqrt{2\beta_i}}{-s+\alpha_i}\right)\mathrm{diag}\left(\frac{s\sqrt{2\beta_i}}{s+\alpha_i}\right) \tag{59}$$

Hence, we obtain

$$Y_2(s)=\mathrm{diag}\left(\frac{s\sqrt{2\beta_i}}{s+\alpha_i}\right) \tag{60}$$

Solving eq. (40) with A_2, B_2, C_2 in eq. (58) and $Y_2(s)$ in eq. (60) we obtain L_2 as follows:

$$L_2=\mathrm{diag}\left(\frac{-\alpha_i\sqrt{2\beta_i}}{\beta_i}\right) \tag{61}$$

Substituting eq. (58) for A_2 and eq. (61) for L_2 into eq. (11), and then by solving it, P_2 is found out to be

$$P_2=\alpha\beta^{-1} \tag{62}$$

Thus we obtain P_1 and P_2, and accordingly P.

Now, we can derive an expression for the Lyapunov function. Substituting eq. (17) for x, eq. (51) for P_1 and eq. (62) for P_2 into eq. (12) for the Lyapunov function, the following expression is obtained:

$$V(x)=[\delta_r'\omega'\Delta E']\begin{bmatrix}P_{11} & P_{12} & 0\\ P_{21} & P_{22} & 0\\ 0 & 0 & P_2\end{bmatrix}\begin{bmatrix}\delta_r\\ \omega\\ \Delta E\end{bmatrix}+2V_1(\sigma)$$
$$=\delta_r'P_{11}\delta_r+2\delta_r'P_{12}\omega+\omega'P_{22}\omega+\Delta E'P_2\Delta E+2V_1(\sigma)$$
$$=\rho\delta'DUD\delta+2\rho\delta'DUM\omega+\omega'(M+\mu MUM)\omega$$
$$+\Delta E'\alpha\beta^{-1}\Delta E+2V_1(\sigma) \tag{63}$$

Substituting eq. (30) for $V_1(\sigma)$ into eq. (63), and expanding and rearranging the terms in eq. (63) we obtain the expression

$$V(x)=\sum_{i=1}^{n}m_i\omega_i^2+\mu(\sum_{i=1}^{n}m_i\omega_i)^2$$
$$+2\rho\{\sum_{i=1}^{n}d_i(\delta_i-\delta_i^0)\}(\sum_{i=1}^{n}m_i\omega_i)+\rho\{\sum_{i=1}^{n}d_i(\delta_i-\delta_i^0)\}^2$$
$$+\sum_{i=1}^{n}(\alpha_i/\beta_i)(E_i-E_i^0)^2$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}B_{ij}\{E_iE_j(\cos\delta_{ij}^0-\cos\delta_{ij})$$
$$-(\delta_{ij}-\delta_{ij}^0)E_i^0E_j^0\sin\delta_{ij}^0\} \tag{64}$$

Since $\mu=\mu^*+\rho$, the final expression for the Lyapunov function is derived after some manipulation.

$$V(x)=(1/2\sum_{i=1}^{n}m_i)\sum_{i=1}^{n}\sum_{j=1}^{n}m_im_j(\omega_i-\omega_j)^2+(\mu^*-\mu_0)(\sum_{i=1}^{n}m_i\omega_i)^2$$
$$+\rho\{\sum_{i=1}^{n}d_i(\delta_i-\delta_i^0)+\sum_{i=1}^{n}m_i\omega_i\}^2$$
$$+\sum_{i=1}^{n}(\alpha_i/\beta_i)(E_i-E_i^0)^2$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}B_{ij}\{E_iE_j(\cos\delta_{ij}^0-\cos\delta_{ij})$$
$$-(\delta_{ij}-\delta_{ij}^0)E_i^0E_j^0\sin\delta_{ij}^0\} \tag{65}$$

The first and second terms in eq. (65) represent kinetic energy. If the damping torques are uniform, then μ^* equals μ_0, and accordingly, the kinetic energy depends only on the relative angular velocities. ρ in the third term is an arbitrary non-negative scalar, but is chosen to be zero because the term narrows and complicates the estimation of the transient stability region.[11] The fourth term is the new term which represents the magnitude of magnetic flux decays. If the flux is assumed to be constant, this term disappears. The fifth term is a potential energy which is stored in the system due to the deviations of the rotor angles from their stable points. The potential energy plays an important role in defining the transient stability region.[11] Since an expression for the potential energy includes the variable internal voltages, the equi-potential curve defined by

$$V_p(\delta, E)=\text{constant} \tag{66}$$

varies with the changes of the internal voltages. Accordingly, the transient stability region changes with the internal voltages, to which we have to pay attention in applying the Lyapunov function to the transient stability analysis.

6. Damping rate of Lyapunov function

Assuming the damping torques be uniform or zero, eq. (65) for the Lyapunov function reduces to the following:

$$\begin{aligned} V(x)=&(1/2\sum_{i=1}^{n}m_i)\sum_{i=1}^{n}\sum_{j=1}^{n}m_im_j(\omega_i-\omega_j)^2 \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}B_{ij}\{E_iE_j(\cos\delta_{ij}^0-\cos\delta_{ij})-(\delta_{ij}-\delta_{ij}^0)E_i^0E_j^0\sin\delta_{ij}^0\} \\ &+\sum_{i=1}^{n}(a_i/\beta_i)(E_i-E_i^0)^2 \\ =&V_k(\omega)+V_p(\delta, E)+V_f(E) \end{aligned} \tag{67}$$

where V_k, V_p and V_f are defined in an explicit way.

The time changes of V_k, V_p and V_f are written as

$$\begin{aligned} \frac{dV_k}{dt}=&-(1/\sum_{i=1}^{n}d_i)\sum_{i=1}^{n}\sum_{j=1}^{n}d_id_j(\omega_i-\omega_j)^2 \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}B_{ij}(E_i^0E_j^0\sin\delta_{ij}^0-E_iE_j\sin\delta_{ij})(\omega_i-\omega_j) \end{aligned} \tag{68}$$

$$\begin{aligned} \frac{dV_p}{dt}=&-\sum_{i=1}^{n}\sum_{j=1}^{n}B_{ij}(E_i^0E_j^0\sin\delta_{ij}^0-E_iE_j\sin\delta_{ij})(\omega_i-\omega_j) \\ &+2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}^0-\cos\delta_{ij}) \end{aligned} \tag{69}$$

$$\frac{dV_f}{dt}=2\sum_{i=1}^{n}(dE_i/dt)(a_i/\beta_i)(E_i-E_i^0) \tag{70}$$

The first term in eq. (68) is caused by the damping torques of generators, and is non-positive. A part of the kinetic energy is dissipated by the damping torques. The second term in eq. (68) and the first term in eq. (69) are opposite signs of each other, which implies that there is an exchange of energy between the kinetic energy and the potential energy. Hence, they do not contribute to the damping of the Lyapunov function. The second term in eq. (69) and the first term in eq. (70) are summed up as follows:

$$2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)\{a_i(E_i-E_i^0)+\beta_i\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}^0-\cos\delta_{ij})\}$$
$$=-2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)^2 \tag{71}$$

This term is in proportion to squares of the (dE_i/dt)s, and accordingly, it is non-positive regardless of whether the E_is are decreasing or increasing.

As a whole, the Lyapunov function dampens according to

$$\frac{dV}{dt}=-(1/\sum_{i=1}^{n}d_i)\sum_{i=1}^{n}\sum_{j=1}^{n}d_id_j(\omega_i-\omega_j)^2-2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)^2 \tag{72}$$

while the kinetic energy and the potential energy oscillate by exchanging their energy between them.

If the magnetic flux decay is not considered, then the second term in eq. (72) does not appear. Here, we consider this new term. The internal voltages change with time according to

$$dE_i/dt=-a_i(E_i-E_i^0)-\beta_i\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}^0-\cos\delta_{ij})$$
$$\text{for } i=1,2,\ldots,n \tag{15}$$

The first term in eq. (15) prevents the internal voltage from deviating from its stable point. On the other hand, the second term causes the internal voltage to leave from the stable point. Fig. 2 shows a cos δ_{ij}—curve. The function $(\cos\delta_{ij}^0-\cos\delta_{ij})$ has positive values in section A and C, and negative values in section B. It takes its maximum value at $\delta_{ij}=\pi$ rad. Supposing only the ith generator oscillates with respect to other generators, $\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}^0-\cos\delta_{ij})$ takes its maximum value around δ_{ij}s$=\pi$ rad. This means that the internal voltage E_i decreases at a maximum rate around the stability limit because the stability limit of the δ_{ij}s usually exists near π rad. Accordingly, the damping rate of the Lyapunov function due to the changes of the

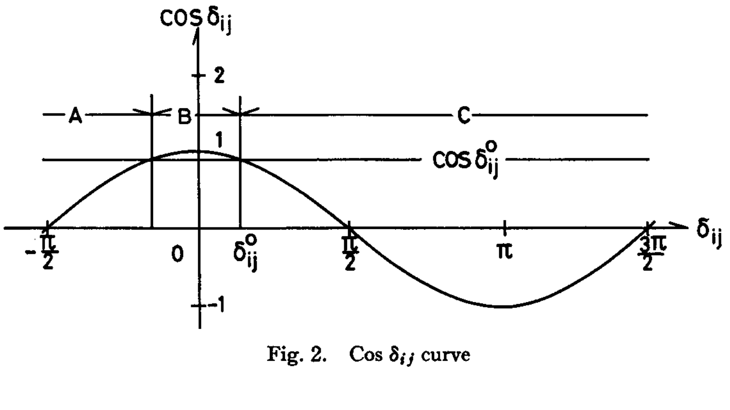

Fig. 2. Cos δ_{ij} curve

internal voltages is maximum near the stability limit. Similarly, the second term in eq. (69) is negative, and the term in eq. (70) is positive, and their absolute values are near maximum around the stability limit.

7. Conclusion

In this paper, we derived a new stability criterion for a system with multi-argument non-linearities in which power systems with magnetic flux decays and automatic voltage regulators fall. By using the new stability criterion, a Lur'e type Lyapunov function can be constructed in a systematic manner. As an example, the new criterion is applied to the multimachine power system with the magnetic flux decays for which no Lyapunov function has been constructed, and the new Lyapunov function is derived. The new Lyapunov function obtained here is similar to the one already obtained for the system with no flux decay, except for a few points.

1) The new Lyapunov function has a new term which represents a magnitude of the flux decay.

2) The potential energy is a function of the internal voltage. Accordingly, the transient stability region which is closely related to the potential energy varies with the changes of the internal voltages. This is an essential characteristic of the system with the flux decay of the generator.

3) The damping rate of the Lyapunov function is in proportion to the square of the changing rate of the internal voltage, and it is maximum around the stability limit. Hence, the flux decay has a large influence on the Lyapunov function around the stability limit.

The new stability criterion is also applicable to systems with AVRs, PSSs and speed governors. By including their effects, Lyapunov's direct method will become a very useful tool for the transient stability analysis of the electric power system, which will be reported in other papers.

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Appendix A. Derivation of system equations

The voltages and the currents of the generators are related by

$$\dot{I}_i=\sum_{j=1}^{n}\dot{Y}_{ij}\dot{E}_j=\sum_{j=1}^{n}Y_{ij}E_j\angle\delta_j+\phi_{ij} \tag{A1}$$

Assuming that $\dot{E}_i$ is parallel with the q-axis of the ith machine, then, $\dot{I}_i'$, $\dot{I}_i$ in the ith machine frame, is given by

$$\begin{aligned}\dot{I}'_i&=\dot{I}_i e^{j(\pi/2-\delta_i)}\\&=\sum_{j=1}^{n}Y_{ij}E_j\angle\pi-(\delta_{ij}+\theta_{ij})\end{aligned} \tag{A2}$$

Hence, the d- and q- axis components of the ith machine current are written as

$$\begin{aligned}i_{di}&=\mathrm{Re}\,\dot{I}_i'\\&=-\sum_{j=1}^{n}Y_{ij}E_j\cos(\delta_{ij}+\theta_{ij})\end{aligned} \tag{A3}$$

and

$$\begin{aligned}i_{qi}&=\mathrm{Im}\,\dot{I}_i'\\&=\sum_{j=1}^{n}Y_{ij}E_j\sin(\delta_{ij}+\theta_{ij})\end{aligned} \tag{A4}$$

The active power of the ith machine is given by

$$\begin{aligned}P_{ei}&=E_i i_{qi}\\&=\sum_{j=1}^{n}Y_{ij}E_iE_j\sin(\delta_{ij}+\theta_{ij})\end{aligned} \tag{A5}$$

In transient analysis the transfer conductances are usually neglected. Then, i_{di}, i_{qi} and P_{ei} are written as

$$i_{di}=-B_{ii}E_i-\sum_{\substack{j=1\\ \neq i}}^{n}B_{ij}E_j\cos\delta_{ij}$$

$$i_{qi}=G_{ii}E_i+\sum_{\substack{j=1\\ \neq i}}^{n}B_{ij}E_j\sin\delta_{ij}$$

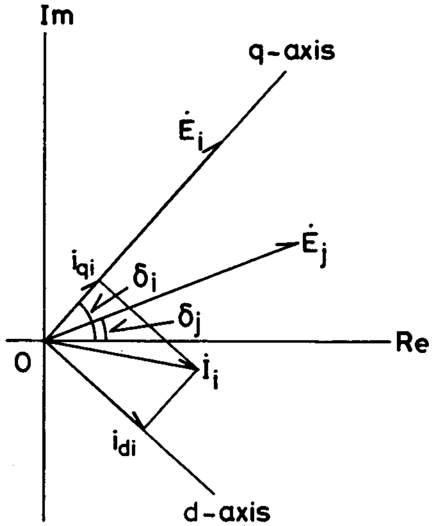

Fig. A. Relation between internal voltages and currents

$$P_{ei}=G_{ii}E_i^2+\sum_{\substack{j=1\\ \neq i}}^{n}B_{ij}E_iE_j\sin\delta_{ij} \tag{A6}$$

The time variation of the internal voltage of the ith machine is described by

$$\begin{aligned}\dot{E}_i&=\frac{1}{T_{doi}'}\{E_{fdi}-E_i-(x_{di}-x_{di}')i_{di}\}\\&=\frac{1}{T_{doi}'}[E_{fdi}-\{1-(x_{di}-x_{di}')B_{ii}\}E_i\\&+(x_{di}-x_{di}')\sum_{\substack{j=1\\ \neq i}}^{n}B_{ij}E_j\cos\delta_{ij}]\end{aligned} \tag{A7}$$

Since the time derivative of E_i is zero at the steady state,

$$\begin{aligned}0&=\frac{1}{T_{doi}'}[E_{fdi}-\{1-(x_{di}-x_{di}')B_{ii}\}E_i^0\\&+(x_{di}-x_{di}')\sum_{\substack{j=1\\ \neq i}}^{n}B_{ij}E_j^0\cos\delta_{ij}^0]\end{aligned} \tag{A8}$$

Subtracting eq. (A8) from eq. (A7) gives

$$\begin{aligned}\dot{E}_i&=-\alpha_i(E_i-E_i^0)-\beta_i\sum_{\substack{j=1\\ \neq i}}^{n}B_{ij}(E_j^0\cos\delta_{ij}^0-E_j\cos\delta_{ij})\\&\simeq-\alpha_i(E_i-E_i^0)-\beta_i\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}^0-\cos\delta_{ij})\end{aligned} \tag{A9}$$

where

$$\alpha_i=\frac{1-(x_d-x_d')_iB_{ii}}{T_{doi}'},\quad \beta_i=\frac{(x_d-x_d')_i}{T_{doi}'}$$

The dynamic equation of the ith machine is written as

$$m_i\ddot{\delta}_i+d_i\dot{\delta}_i=P_{mi}-P_{ei} \tag{A10}$$

As P_{mi} is given by

$$P_{mi}=G_{ii}E_i^{02}+\sum_{j=1}^{n}B_{ij}E_i^0E_j^0\sin\delta_{ij}^0 \tag{A11}$$

eq. (A10) is rewritten as follows:

$$\begin{aligned}m_i\ddot{\delta}_i+d_i\dot{\delta}_i&=G_{ii}(E_i^{02}-E_i^2)\\&\quad+\sum_{j=1}^{n}B_{ij}(E_i^0E_j^0\sin\delta_{ij}^0-E_iE_j\sin\delta_{ij})\\&\simeq\sum_{j=1}^{n}B_{ij}(E_i^0E_j^0\sin\delta_{ij}^0-E_iE_j\sin\delta_{ij})\end{aligned} \tag{A12}$$

where $G_{ii}(E_i^{02}-E_i^2)$ is assumed to be negligible for each i.

Summing up the above derivations, the system equations are written as

$$\begin{aligned}&m_i\ddot{\delta}_i+d_i\dot{\delta}_i=\sum_{j=1}^{n}B_{ij}(E_i^0E_j^0\sin\delta_{ij}^0-E_iE_j\sin\delta_{ij})\\&\dot{E}_i=-\alpha_i(E_i-E_i^0)-\beta_i\sum_{j=1}^{n}B_{ij}E_j(\cos\delta_{ij}^0-\cos\delta_{ij})\\&\qquad\qquad\text{for } i=1,2,\dots\dots,n\end{aligned} \tag{A13}$$

Appendix B. Minimal order of W (s)

The minimal order of the realization of the transfer matrix $W(s)$ is given by $\delta[W(s)]$, the degree of $W(s)$. It has been shown by Gilbert that if $W(s)$ has a partial fraction expansion as

$$W(s)=\sum_{i=1}^{n}W_i(s+\xi_i)^{-1} \tag{B1}$$

then $\delta[W(s)]$ is equal to the sum of the ranks of W_i.

Since $W(s)$ of eq. (34) is a direct sum of $W_1(s)$ and $W_2(s)$, the degree of $W(s)$ is written as

$$\delta[W(s)]=\delta[W_1(s)]+\delta[W_2(s)] \tag{B2}$$

$W_1(s)$ is expanded as

$$W_1(s)=\frac{1}{s}T'D^{-1}T+\sum_{i=1}^{n}\frac{1}{s+(d_i/m_i)}T'F_iT \tag{B3}$$

where

$$F_i=\mathrm{diag}\ (0,\ \dots,\ 0,\ -d_i,\ 0,\ \dots,\ 0) \tag{B4}$$

As the rank of $T'D^{-1}T$ is $(n-1)$, and that of $T'F_i\,T$ is 1 for each i,

$$\delta[W_1(s)]=2n-1 \tag{B5}$$

$W_2(s)$ is expanded as

$$W_2(s)=\sum_{i=1}^{n}\frac{1}{s+\alpha_i}G_i \tag{B6}$$

where

$$G_i=\mathrm{diag}\ (0,\ \ldots 0,\ \beta_i,\ 0,\ \ldots,\ 0) \tag{B7}$$

As the rank of G_i is 1 for each i, the degree of $W_2(s)$ is

$$\delta[W_2(s)]=n \tag{B8}$$

Substitute $\delta[W_1(s)]$ from eq. (B5) and $\delta[W_2(s)]$ from eq. (B8) into eq. (B2), then $\delta[W(s)]$ is obtained;

$$\delta[W(s)]=3n-1 \tag{B9}$$

Consequently, A, B and C in eq. (18) are the minimal realization of the transfer matrix $W(s)$.

Appendix C.

Lemma A (B. D. O. Anderson)

Let the $n\times n$ matrix $Z(s)$ be positive real, and assume that $Z(s)+Z'(-s)$ has the rank r almost everywhere. Then there exists an $r\times n$ matrix $W(s)$ such that

$$Z(s)+Z'(-s)=W'(-s)\ W(s)$$

and

(i) W has elements which are analytic for Re $s>0$, and for Re $s\geq 0$ if $Z(s)$ has elements which are analytic for Re $s\geq 0$;

(ii) rank $W=r$ for Re $s>0$,

(iii) W is unique, save for the multiplication on the left by an arbitrary orthogonal matrix.

Lemma B (B. D. O. Anderson)

Let $Z(s)$ have a minimal realization (A, B, C) and let $Z(s)$ and $W(s)$ be related as in Lemma A. Then there exists a matrix L such that (A, B, L) is a minimal realization for $W(s)$.