A Theoretical Analysis of Neutral Beam Probe for Measuring Ion Temperature of a Plasma

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Abstract

A theoretical basis of the neutral beam probe for measuring the local ion temperature of a plasma is given. The results of numerical calculations for some typical plasmas show that it is desirable to choose the beam energy in the region of 10 keV to 30 keV for plasmas of current and of next generation tokamaks. It is pointed out that the method of least squares can be applied to determine the local ion temperature from the measured data.

1. Introduction

Parameters of a high temperature plasma can be measured by analyzing the energy spectrum of the fast neutral particles emerging from the plasma column. This technique involves two methods, i.e. the passive and the active method.

In the passive method, neutral particles emerging from a plasma are generated by the charge-exchange reaction between the plasma ions and the slow neutrals which immerge into the plasma from the vacuum. Usually, the mean free path of the slow neutrals in a plasma is only a few centimenters. When the plasma becomes considerably large and dense, the slow neutrals are hardly able to reach the central part of the plasma. Consequently, it is difficult to measure the parameters of the central part of a large and dense plasma with the passive method. Moreover, the whole plasma volume visible from the analyzer through the collimator contributes to the fast neutral flux detected by the spectrum analyzer. Therefore, the passive method can not be used for measuring the local parameters of a plasma.

Both difficulties can be solved by injecting a well-collimated beam of fast neutral particles into the plasma. This is called the active method or the neutral beam probe

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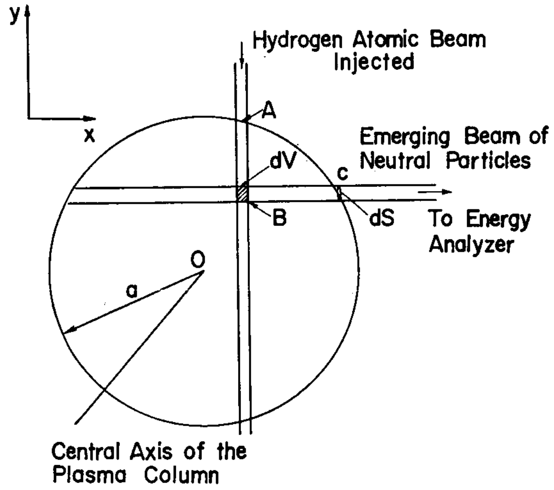

Fig. 1. Conceptional diagram of neutral beam probe for measuring local ion temperature of a plasma.

method. Brusati has proposed to measure the local ion distribution and the neutral density profile in a toroidal plasma with this method.[1] Some experimental reports have been written by Russian scientists.[2,3] In this paper, we will give a theoretical basis to the neutral beam probe for measuring the local ion temperature of a plasma. In addition, we try to find out the adequate energy of the injected hydrogen atomic beam and propose a method for determining the ion temperature from the neutral spectrum.

Figure 1 shows the conceptional diagram of the neutral beam probe which can be used to measure the local parameters of a plasma. A well-collimated beam of neutral particles is injected into a plasma column from A. The diagnostic set-up is supposed to be arranged in such a way that the axis of the collimator for energy analysis perpendicularly intersects the injected beam at B. The test volume dV is then a small volume of plasma that is passed by the injected beam, and is visible through the collimator aperture. Among the fast neutral particles which are generated by the charge-exchange reaction between the injected beam and the plasma ions within dV, those passing through the collimator at C will reach the energy analyzer, with which their energy spectrum is measured.

2. General Theory

In this investigation, we take into account the ionization of neutrals by electrons and by ions, as well as the charge-exchange between the neutral and the plasma ions.

The current density of the injected beam at the test volume dV around point B is given by

$$j_b(B, E_b) = j_b(A, E_b) p_{in}(A, B) \tag{1}$$

with $p_{in}(A, B) = \exp[-h(A, B)]$, which is the transport kernel for the injected neutral particle beam, while $h(A, B)$ is the optical thickness of the path from A to B and is given by the integral $h(A, B) = \int_A^B \frac{1}{\lambda_{bt}} dy$, where λ_{bt} is the mean free path for all the concerned sunk processes of the injected neutral particles, and is dependent on both the beam energy and the plasma parameters.

In the test volume, the number of the fast neutral particles, which are generated in unit time and in an energy interval $(E, E+dE)$ by the charge-exchange reaction between the injected beam and the plasma ions, is given by

$$N_n = \sigma_{cx}(|v_r|)|v_r| n_i(B) F_B(E) dE dV \frac{j_b(B, E_b)}{v_b}, \tag{2}$$

where σ_{cx} is the cross-section for the charge-exchange reaction, v_r is the relative velocity of the reacting particles and is expressed as $v_r = \sqrt{v_i^2 + v_b^2}$, and v_i as well as v_b are the velocities of the plasma ion and the injected hydrogen atom, respectively, $n_i(B)$ and $F_B(E)$ are, respectively, the ion density and the ion energy distribution functions at B.

In the analysis below, the velocity distribution of the ions is assumed to be Maxwellian and isotropic. Then the distribution function $F_B(E)$ is

$$F_B(E) = (E/\pi k T_i(B))^{1/2} \frac{2}{kT_i(B)} \exp\left[\frac{-E}{kT_i(B)}\right].$$

Assuming the aperture of the collimator at C to be dS, then the solid angle of dS at point B is dS/BC^2. On the other hand, the probability for a fast particle to fly from B to C can be written as $p_{out}(B, C) = \exp[-h(B, C)]$, with $h(B, C) = \int_B^C \frac{1}{\lambda_{it}} dx$,

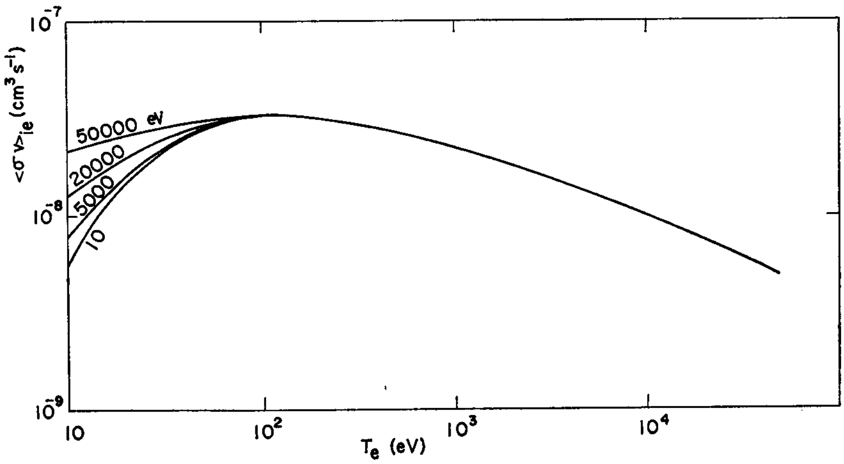

Fig. 2. Rate coefficient for ionization of hydrogen atoms by electrons. Energy of neutrals is shown as a parameter.

where λ_{it} is the mean free path of the sunk process of the fast neutrals. Hence the counting rate of the fast neutral particles in an energy interval $(E, E+dE)$ is given by

$$J(C, E)=\sigma_{cx}(|v_r|)|v_r|n_i(B)F_B(E)dEdV\frac{j_b(B, E_b)}{v_b}\frac{dS}{4\pi BC^2}p_{out}(B, C). \qquad (3)$$

In order to write Eq. (3) in a simpler form, we put $z=dV/(4\pi BC^2)$, which has the dimension of length. Then we write Eq. (3) in a current density form $j_{out}(C, E)=J(C, E)/dS$, and normalize it with the injected beam current $j_b(A, E_b)$, to obtain

$$\gamma(E)=\frac{j_{out}(C, E)}{j_b(A, E_b)}=\frac{\sigma_{cx}(|v_r|)|v_r|z}{v_b}n_i(B)F_B(E)dEp_{in}(A, B)p_{out}(B, C). \qquad (4)$$

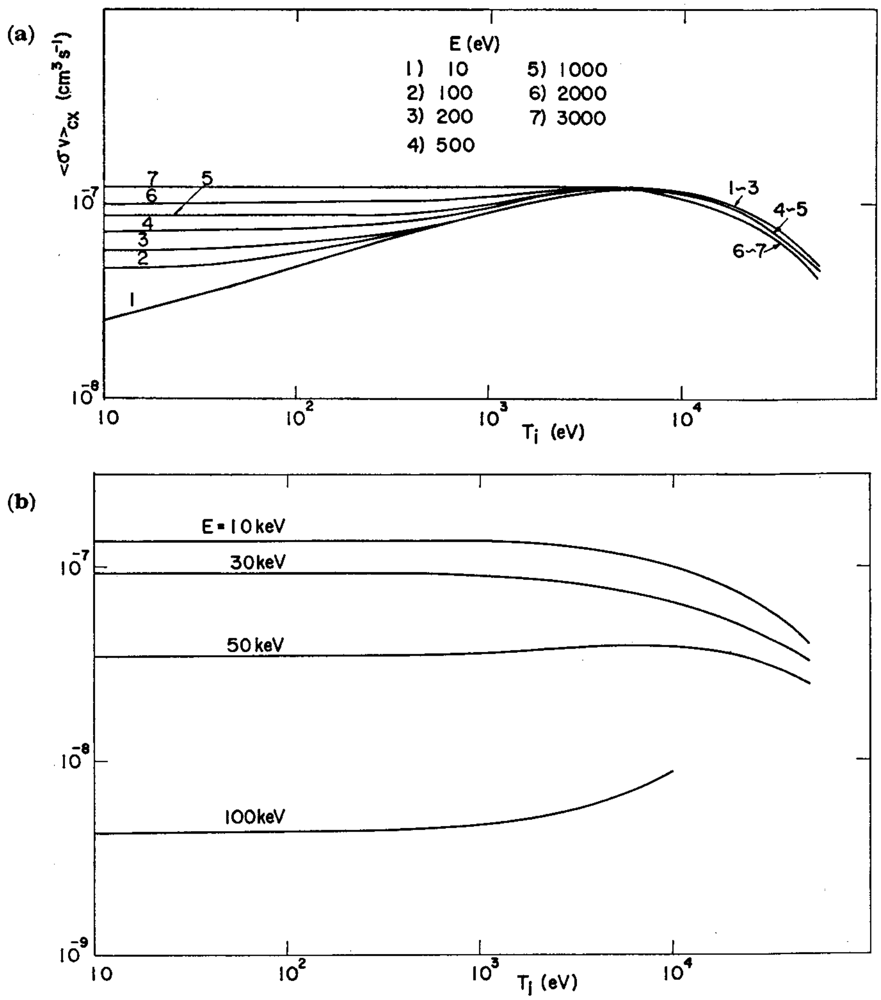

Fig. 3(a)–(b). Rate coefficient for charge-exchange. Energy of neutrals is shown as a parameter.

The mean free paths λ_{bt} and λ_{it} which are involved in the transport kernels $p_{in}(A, B)$ and $p_{out}(B, C)$, respectively, can be calculated with the cross-section data summarized by Riviere[4]. They can be expressed as follows:

$$\frac{1}{\lambda_{bt}} = \frac{1}{\lambda_{b,ie}} + \frac{1}{\lambda_{b,cx}} + \frac{1}{\lambda_{b,ii}} \tag{5}$$

$$\frac{1}{\lambda_{it}} = \frac{1}{\lambda_{i,ie}} + \frac{1}{\lambda_{i,cx}} + \frac{1}{\lambda_{i,ii}} \tag{6}$$

and

$$\frac{1}{\lambda_{b,ie}} = \frac{\langle\sigma v\rangle_{ie}}{v_b} n_e \tag{7}$$

$$\frac{1}{\lambda_{b,cx}} = \frac{\langle\sigma v\rangle_{cx}}{v_b} n_i \tag{8}$$

$$\frac{1}{\lambda_{b,ii}} = \frac{\langle\sigma v\rangle_{ii}}{v_b} n_i \tag{9}$$

$$\frac{1}{\lambda_{i,ie}} = \frac{\langle\sigma v\rangle_{ie}}{v_i} n_e \tag{10}$$

$$\frac{1}{\lambda_{i,cx}} = \frac{\langle\sigma v\rangle_{cx}}{v_i} n_i \tag{11}$$

$$\frac{1}{\lambda_{i,ii}} = \frac{\langle\sigma v\rangle_{ii}}{v_i} n_i \tag{12}$$

where $\langle\sigma v\rangle$'s are the rate coefficients which are dependent on the plasma temperature and hence are dependent on place. Figures 2–4 show their dependence on the plasma

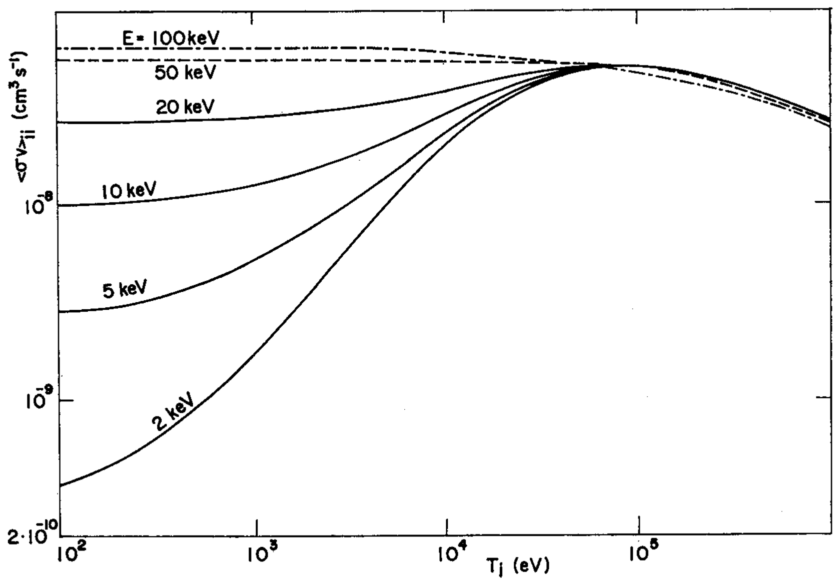

Fig. 4. Rate coefficient for ionization of hydrogen atoms by protons. Energy of neutrals is shown as a parameter.

temperature with the energy of the hydrogen atoms as a parameter.[5] The subscripts denote the reactions concerned, for instance, *ie* denotes the reaction of ionization by electrons, and so forth.

3. Application of the Theory to the Central Test Volume of a Plasma

During the numerical calculation, we have assumed that the spatial distributions of the plasma density and the ion temperature are as follows:

$$n_e(\xi)=n_i(\xi)=n(0)(1-\xi^2), \tag{13}$$

and

$$T_i(\xi)=T_{i0}(1-\xi^4)^2+10, \tag{14}$$

where $\xi=x/a$, and a is the radius of the plasma. The electron temperature is assumed to be twice as high as the ion temperature, i.e.

$$T_e(\xi)=2T_i(\xi). \tag{15}$$

On performing the numerical calculation, we set the test point B at the centre of the plasma column for the sake of simplicity. Since the rate coefficients $\langle\sigma v\rangle$'s are not sensitive to the plasma temperature, on carrying out the integrals in p_{in} and p_{out} of the transport kernels, we fix them to their values calculated for the average plasma temperature along AB or BC. With the explicit expression of the Maxwellian distribution $F_B(E)$, Eq. (4) becomes

$$\gamma(E)=\beta(E/kT_i(B))^{3/2}\exp\left[-\frac{E}{kT_i(B)}-\frac{c\bar{n}a\langle\sigma v\rangle_{it}}{\sqrt{E}}\right] \tag{16}$$

where $c=7.2245\times10^{-7}$, E is the energy of the detected particles, in eV, $\bar{n}$ is the line averaged plasma density, and

$$\beta=\frac{\sigma_{cx}(|v_r|)|v_r|z}{v_b}n_i(B)\frac{2}{\sqrt{\pi}}\frac{\Delta E}{E}\exp\left[-\frac{\bar{n}a\langle\sigma v\rangle_{bt}}{v_b}\right]. \tag{17}$$

In Eq. (17), $\Delta E/E$ is considered as the energy resolution of the neutral spectrometer, and can be assumed to be independent of energy E.

4. Numerical Results for some Typical Plasmas

We have calculated the spectrum $\gamma(E)$ for some typical plasmas with the injected beam of several different energies. The plasma parameters and the beam energies are assumed as shown in Table 1.

Table 1. Plasma Parameters and Beam Energies Assumed in the Test Problems.

a (cm)	25	50	100		
E_b (keV)	10	30	50	80	
$n(0)$ (10^{13} cm^{-3})	1.5	3.0	5.0	10	
$T_i(0)$ (eV)	500	1000	2000	5000	10000

Here, we just show some typical results of the test problems. With the beam energy as a parameter, Fig. 5 shows the emerging neutral spectra $\gamma(E)$ for a plasma whose parameters are $a=100$ cm, $n(0)=10\times10^{13}$ cm^{-3}, and $T_i(0)=5$ keV. We can see from Fig. 5 that the differences between the intensities of the emerging neutral fluxes are very little for the beam energies of 30 keV, 50 keV and 80 keV. As was expected, the beam energy above 30 keV would not bring much merit to the intensity of the emerging neutral flux, since the rate coefficient for charge-exchange decreases rapidly at the energy above 30 keV. Therefore, it is desirable to choose the energy of the neutral beam in the region of 10 keV to 30 keV for the plasmas of current and of next generation tokamaks.

Figure 6 shows the emerging neutral spectra from a plasma whose maximum ion temperature is 1 keV with the density and the radius of the plasma as parameters. In Fig. 6, the ordinate represents the logarithm of the emerging neutral spectra, log $\gamma(E)$, while the abscissa represents the energy of the emerging neutral particles measured in unit of the maximum ion temperature $T_i(0)$ of the plasma.

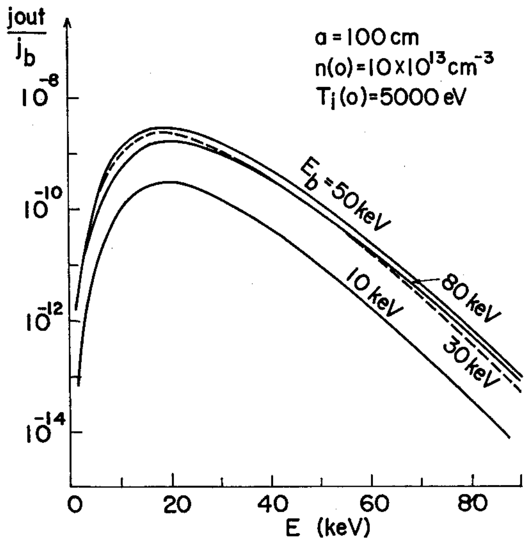

Fig. 5. Emerging neutral fluxes from a plasma. Energy of the neutral beam probe is shown as a parameter.

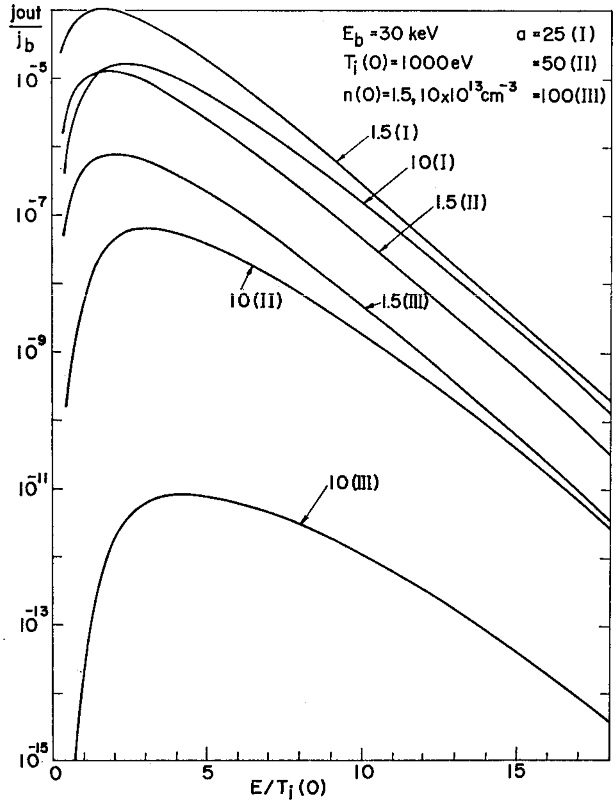

Fig. 6. Emerging neutral fluxes from a plasma. Plasma density and radius are shown as parameters.

5. Method for Experimentally Determining Ion Temperature

Taking the natural logarithms of both sides of Eq. (16), we obtain

$$\log \gamma(E) = \log \beta - \frac{3}{2} \log (kT_i) + \frac{3}{2} \log E - \frac{E}{kT_i} - \frac{\bar{c} n a \langle \sigma v \rangle_{it}}{\sqrt{E}} \tag{18}$$

where $kT_i = kT_i(0)$. Introducing the following substitutions into Eq. (18)

$$A = \log \beta, \qquad B = \bar{c} n a \langle \sigma v \rangle_{it} \tag{19}$$

and solving it for kT_i, we obtain

$$kT_i = \frac{E}{A - \frac{3}{2} \log (kT_i) + \frac{3}{2} \log E - BE^{-1/2} - \log \gamma(E)}. \tag{20}$$

If all the plasma parameters except kT_i are known, we can straightforwardly determine the ion temperature kT_i by measuring $\gamma(E)$ at only one point of the energy E.

However, in usual cases, it is difficult to predict the plasma parameters in determining kT_i by Eq. (20). In such a situation, we can apply the method of least squares to determine the ion temperature kT_i. Firstly, we put

$$C=kT_i \tag{21}$$

and then rewrite Eq. (18) as

$$\log \gamma(E)=y(E)=A-\frac{3}{2}\log C+\frac{3}{2}\log E-\frac{B}{\sqrt{E}}-\frac{E}{C}. \tag{22}$$

We can measure a set of n data points (E_i, y_i), (here $y_i=\log \gamma(E_i)$, $i=1, 2, 3, \ldots, n$, and $n>4$), and fit the expression (22) to these data points by the method of least squares. Then the coefficients A, B and C $(=kT_i)$ in Eq. (22) can be determined with their confidence limits.

6. Conclusions

1) The beam probe of fast hydrogen atoms can be used to measure the local ion temperature of a plasma. For the plasmas of current and of next generation tokamaks, it is desirable to choose the beam energy in the region of 10 keV to 30 keV.
2) The method of least squares can be applied to determine the ion temperature of a plasma from the measured data.

Acknowledgement

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