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On the Frequency and Altitude Resolutions in Incoherent-Scatter-Radar Measurements

By

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Abstract

Systematic distortions of the spectrum of the signal scattered from diffusive media, and limitations on the resolutions in frequency and altitude due to the finite receiver bandwidth and the finite transmitted-pulse duration are estimated, using the radar-ambiguity function. The requirements for frequency and altitude resolutions are incompatible in the filter-bank method (FBM) which directly measures the power spectrum. With the actual parameters of the ionosphere, the FBM is shown to be inconvenient for pulse radar measurements except those of higher altitudes and/or with higher radar frequencies. In the correlation-function method (CFM) which calculates the cross-correlation or autocorrelation function, the resolutions are independently determined in frequency and altitude. The pulse duration is concerned only with the resolution in altitude. The minimum duration is limited by the sensitivity of the radar system, while the maximum is limited by the characteristic time of fluctuations in the medium which scatter the radio waves. Various methods for measuring the autocorrelation function are also described.

1. Introduction

Ionosphere is virtually transparent to radio waves which are of frequencies considerably higher than the maximum plasma frequency in the ionosphere. A small fraction of these waves, however, is scattered by density fluctuations in the ionospheric plasma which are caused either by plasma instabilities or, in the absence of such instabilities, by the thermal motion of the plasma.[1] The scattering produced

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by the thermal fluctuations is called incoherent (or Thomson) scattering (IS), and is used as a diagnostic tool for studying the different regions of the ionosphere.[2),3)] An appropriate theoretical curve of the power spectrum or the autocorrelation function of the scattered signal can be calculated for any desired set of ionospheric parameters.[4)] In the IS radar measurements, the ionospheric parameters are analyzed by comparing a measured power spectrum or autocorrelation function with a set of theoretical curves and finding the best fit.

Non-thermal scattering, produced by the ionization fluctuations or by the atmospheric-temperature fluctuations induced by an atmospheric turbulence, is proved to be a powerful tool for studying the middle atmosphere (10-100 km).[5)] The whole shape of the autocorrelation function, however, has not yet been used because of a lack of sufficient understanding about the upper atmospheric turbulence. Only the total power and the doppler shift have been used.[6),7)]

There are two methods processing the scattered signals.[8)] One method is to measure the power spectrum directly, using a bank of filters, and called the filter-bank method (FBM). The other method is to calculate the autocorrelation function which is the Fourier transform of the power spectrum, and called the correlation-function method (CFM). In both methods, however, estimates of the power spectrum or the autocorrelation function will differ from the true one because of random statistical errors and also because of systematic distortions due to the finite length of the transmitted pulse and the finite receiver bandwidth. Our object is to calculate these errors with the radar-ambiguity function, and to study the frequency and altitude resolutions obtainable for both the FBM and CFM, and for the various forms of the transmitted pulses.

A modeling of the received signal is performed in section 2. In section 3, the power spectrum which is smeared by systematic distortions is given for the FBM. It is shown that the requirements for frequency and altitude resolutions are incompatible, and that the sufficient resolutions are not expected below the ionospheric F region and/or for lower radar frequencies. Section 4 is devoted to the formulation of the errors associated with the CFM. There are three methods of measurements in the CFM, i. e., single-, double- and multiple-pulse methods, for which the autocorrelation function is estimated. In the CFM, the resolutions are independently determined in frequency and altitude, and the pulse duration is concerned only with the resolution in altitude. The maximum duration is limited by the characteristic time of the fluctuations, while the minimum is limited by the sensitivity of the radar system. The implications of these descriptions are briefly discussed in section 5.

2. Modeling of signal

The input signal at the receiver terminal at time t, $\tilde{x}(t)$, consists of three components; i. e.

$$\tilde{x}(t)=\tilde{s}(t)+\tilde{c}(t)+\tilde{n}(t), \tag{1}$$

where $\tilde{s}(t)$, $\tilde{c}(t)$ and $\tilde{n}(t)$ are the complex envelopes of the scattered signal, clutters and noises, respectively. A tilde ($\sim$) denotes the value to be random.

The broadening of the spectrum is macroscopically described by the amplitude modulation of the scattered waves caused by a fluctuation of the refractive index. We define function $\tilde{f}_s(t, r)$ as the fluctuation of the signal scatterd from altitude r. Then $\tilde{s}(t)$ is expressed by the sum of the signals scattered from all altitudes,

$$\tilde{s}(t)=\int_0^{\infty} u\left(t-\frac{2r}{c}\right)\tilde{f}_s\left(t-\frac{r}{c},\ r\right)dr, \tag{2}$$

where $u(t)$ is the complex envelope of the transmitted wave. The statistical properties of $\tilde{f}_s(t, r)$ are assumed to be

$$<\tilde{f}_s(t, r)>=0, \tag{3}$$

and

$$<\tilde{f}_s(t_1, r_1)\tilde{f}_s^*(t_2, r_2)>=P_s\left(\frac{r_1+r_2}{2}\right)$$

$$\cdot R_{ss}\left(t_1-t_2, \frac{r_1+r_2}{2}\right)\Theta_s(r_1-r_2), \tag{4}$$

where the angular brackets denote the ensemble average or expected value. $P_s(r)$ is the power of the scattered signal and $R_{ss}(\tau, r)$ is the autocorrelation function of the fluctuation. $\Theta_s(r_1-r_2)$ indicates the spatial correlation function of signals scattered from altitude r_1 and r_2. If the scattered signals from the two altitudes, the distance of which should be larger than the radar wavelength, are uncorrelated, $\Theta_s(r_1-r_2)$ can be replaced by $\delta(r_1-r_2)$, where $\delta(r)$ is Dirac's delta function. This condition is always satisfied above the ionospheric E region, where the transmitted wave is scattered by the thermal fluctuation of the plasma. On the other hand, considerable correlation exists between the signals scattered from the middle atmosphere, where the scattering is caused by the non-thermal fluctuation arising from the turbulent mixing of the neutral atmosphere. In this case, $\Theta_s(r_1-r_2)$ is finite within the characteristic length of the turbulence.

We can also define $\tilde{f}_c(\tau, r)$ as well as P_c, R_{cc} and Θ_c for the clutter. The ensemble average of $\tilde{f}_c$, however, does not vanish because the clutter comes from hard targets such as ground and mountains. In this case, the power is proportional to r^{-4}, so that the clutter virtually has no effect upon the signals scattered from the upper atmosphere, especially above an altitude of 100 km. Since the width of the spectrum, $S_{cc}(\omega, r)$, is very narrow or the autocorrelation function R_{cc} has a large correlation time compared with that of the scattered signal, it is possible to assume that $S_{cc}(\omega, r) \sim \delta(\omega)$.

The noise $\tilde{n}(t)$ consists mainly of the cosmic noise and the thermal noise of the receiving system. Since the widths of $\tilde{s}(t)$ and $\tilde{c}(t)$ are sufficiently narrow in comparison with the radar frequency, we assume that $\tilde{n}(t)$ is virtually white around the radar frequency. Thus $\langle \tilde{n}(t_1)\tilde{n}^*(t_2) \rangle = \sigma_n^2 \delta(t_1 - t_2)$, where σ_n^2 is the noise power per unit frequency band. Assuming that both Θ_s and Θ_c are approximated by the δ function, the signal model is expressed by the superposition of multiplicative transmission lines as shown in Fig. 1.

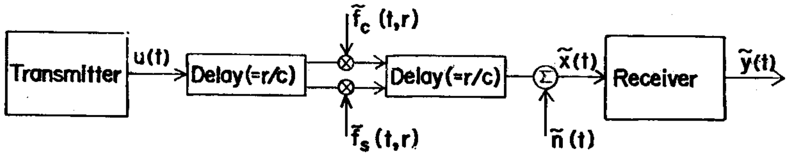

Fig. 1 Received signal model represented by equivalent multiplicative transmission lines. The condition $\Theta(r_1 - r_2) = \delta(r_1 - r_2)$ is assumed for signal and clutter.

3. Filter-bank method (FBM)

It is possible to analyze the signal spectra directly, using a bank of narrow-band filters (Fig. 2). The outputs of the filters are rectified by means of square-law-envelope detectors, and then sampled at the delay $2r_0/c$, where r_0 is the altitude at which

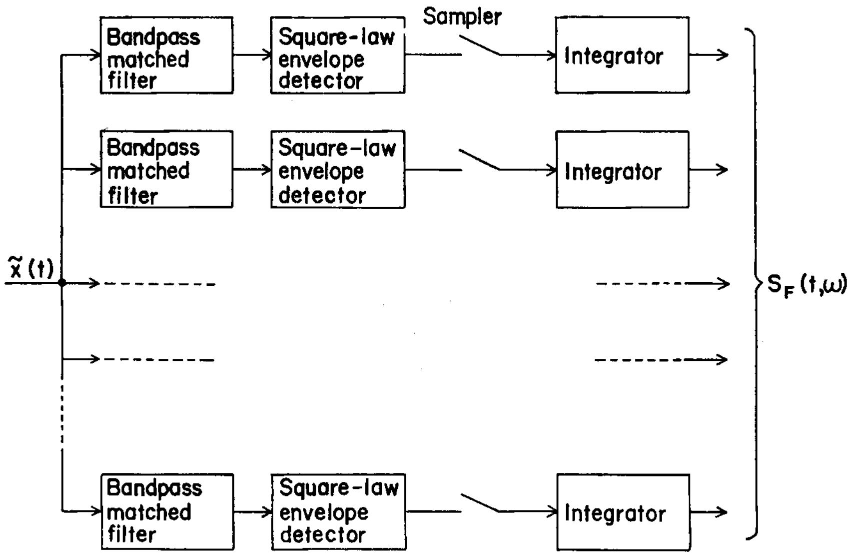

Fig. 2 Filter-bank receiver matched to transmitted-pulse duration.

the signal spectrum is explored. The integration is performed over many successively transmitted pulses to improve the signal-to-noise ratio (SNR).

The output signal spectrum, $S_F(t, \omega)$, is given by the radar ambiguity function determined by the input signal $\tilde{x}(t)$, and the receiver response function $v(t)$[9] (ω being the frequency displacement from transmitted frequency). Taking an ensemble average, $S_F(t, \omega)$ is expressed as follows;

$$S_F(t,\omega) = <|\chi_{\tilde{x}v}(-t, \omega)|^2>, \tag{5}$$

where $\chi_{\tilde{x}v}$ is the ambiguity function defined by

$$\chi_{\tilde{x}v}(\tau, \omega) = \int_{-\infty}^{\infty} \tilde{x}(s) v(s+\tau) e^{-j\omega s} ds. \tag{6}$$

With the aid of Eqs. (2), (4) and (6), the spectral component S_{FS} which contains the true (or explored) spectrum of the scattering medium at altitude r, $S_{ss}(\omega, r)$, is given by

$$S_{FS}(t, \omega) = \int\int_{-\infty}^{\infty}\int\int_{0}^{\infty} dr_1 dr_2 ds_1 ds_2 < \tilde{f}_s\left(s_1 - \frac{r_1}{c}, r_1\right) \tilde{f}_s^*\left(s_2 - \frac{r_2}{c}, r_2\right) >$$
$$\cdot u\left(s_1 - \frac{2r_1}{c}\right) v^*(s_1 - t) u^*\left(s_2 - \frac{2r_2}{c}\right)$$
$$\cdot v(s_2 - t) e^{-j\omega(s_1 - s_2)}, \tag{7}$$
$$= \int\int_0^{\infty} dr_1 dr_2 \cdot P_s\left(\frac{r_1 + r_2}{2}\right) \Theta_s(r_1 - r_2) e^{-j\omega\left(\frac{r_1 - r_2}{c}\right)}$$
$$\cdot S_{ss}\left(\omega, \frac{r_1 + r_2}{2}\right) \otimes_\omega \chi_{uv}\left(-t + \frac{2r_1}{c}, \omega\right)$$
$$\cdot \chi_{uv}^*\left(-t + \frac{2r_2}{c}, \omega\right) e^{-j2\omega\left(\frac{r_1 - r_2}{c}\right)}, \tag{8}$$

where $\otimes_\omega$ means the convolution with respect to ω. As χ_{uv} is nearly zero for $|t-(2r/c)| > d$, where d is the pulse duration, the integration over r can be extended from 0 to $-\infty$ without any significant change in the integral. Using the relation $\chi_{uv}(-\tau, -\omega) = e^{j\omega\tau} \cdot \chi_{vu}^*(\tau, \omega)$, Eq. (8) becomes

$$S_{FS}(t, \omega) = \int\int_{-\infty}^{\infty} dr_1 dr_2 \cdot P_s\left(\frac{r_1 + r_2}{2}\right) \Theta_s(r_1 - r_2) e^{-j\omega\left(\frac{r_1 - r_2}{2}\right)}$$
$$\cdot S_{ss}\left(\omega, \frac{r_1 + r_2}{2}\right) \otimes_\omega \chi_{vu}^*\left(t - \frac{2r_1}{c}, -\omega\right)$$
$$\cdot \chi_{vu}\left(t - \frac{2r_2}{c}, -\omega\right). \tag{9}$$

In a case where only the ionospheric IS is considered, $\Theta_s(r_1 - r_2) = \delta(r_1 - r_2)$, so that

$$S_{FS}(t,\ \omega)=\int_{-\infty}^{\infty} P_s(r)\, S_{ss}(\omega,\ r) \otimes_\omega |\chi_{vu}\left(t-\frac{2r}{c},\ -\omega\right)|^2 dr. \tag{10}$$

Equation (10) means that the true spectrum is convolved with the ambiguity function in the frequency domain when processed by the FBM. The width of χ_{vu} along the ω axis determines the frequency resolution Δf and that along the τ axis, the altitude (or range) resolution Δh. The increase of the pulse duration results in the decrease of Δf and the increase of Δh, and *vice versa*. This is due to the law of conservation of the ambiguity, i. e., $\int\int |\chi(\tau,\ \omega)|^2 d\omega d\tau=$constant. One of the two could not be improved without worsening the other. The values of altitude and frequency resolutions are minimized when the filter matched to the duration of the transmitted pulse is employed (i. e., $u=v$).

If Δf is sufficiently small compared with the width of $S_{ss}(\omega,\ r)$, the convolution has no serious effect upon the explored $S_{ss}(\omega,\ r)$. We will estimate the values of Δf and Δh for the ionospheric scattering. If we assume that both $P_s(r)$ and $S_{ss}(\omega,\ r)$ are independent of r near the observed altitude r_0, Eq. (10) becomes

$$S_{FS}(t,\ \omega)=P_s(r_0)\, S_{ss}(\omega,\ r_0) \otimes_\omega j(\omega), \tag{11}$$

where $j(\omega)=\int |\chi_{vu}(z,\ -\omega)|^2 dz$.

When a simple rectangular pulse with a duration d is transmitted and a matched filter is used, $j(\omega)$ is expressed as $j(\omega)=(2/\pi\omega^2 d)$ $[1-\{\sin(\omega d)/\omega d\}]$. Based on this equation, we obtain $\Delta f=1.2$ kHz for the commonly employed duration of 500 μsec ($\Delta h=75$ km). On the other hand, the width of the spectrum is about 2 and 10 kHz at the E and F regions, respectively, and becomes wider at higher altitudes (for radar frequency $f_0=400$ MHz). The scale heights of these regions are respectively about 5-10 and 50-100 km. Thus, the sufficient altitude and frequency resolutions are not expected below the F region by the FBM. Since the width of the spectrum, however, is proportional to f_0, the FBM is preferably suitable for UHF radars.

The component which convolves the spectra of clutter and noise can be expressed as

$$\int\int_{-\infty}^{\infty} dr_1 dr_2 \cdot P_c\left(\frac{r_1+r_2}{2}\right) \Theta_c(r_1-r_2)\, \chi^*_{vu}\left(t-\frac{2r_1}{c},\ -\omega\right)$$

$$\cdot \chi_{vu}\left(t-\frac{2r_2}{c},\ -\omega\right)+\sigma_n^2 \int_{-\infty}^{\infty} |V(\omega)|^2 d\omega, \tag{12}$$

where $V(\omega)$ is the Fourier transform of $v(t)$. Equation (12) indicates that the filter matched to the transmitted pulse maximizes the clutter-to-noise ratio. The spectral width of the clutter is that of χ_{vu}, so that we can also reduce the smearings which the explored spectrum suffers from the clutter, by using longer durations of the pulse. In addition, the noise power is proportional to the width of the filter or the inverse of the pulse duration. Therefore, we should conclude that the FBM is suitable for longer durations of the pulse, especially, for the continuous waves.

4. Correlation-function method (CFM)

The autocorrelation function is shown to be identical with the Fourier transform of the power spectrum by the Wiener-Khinchin theorem. In the CFM, the autocorrelation function of the input signal is explored instead of measuring the spectrum directly. In the following, it is shown that in the CFM, the range resolution (Δh) is uniquely determined by the pulse duration (d) and that the frequency resolution (Δf) is independent of d, although Δh and Δf were dependent upon each other in the FBM. Since the signal power is proportional to $\Delta h (\propto d)$, the minimum value of Δh is limited by the sensitivity of the radar system in the CFM.

The receiver used in the CFM has only one IF filter which is usually matched to the transmitted pulse. The output signal of the receiver, $\tilde{y}(t)$, is the convolution of $\tilde{x}(t)$ and $v(t)$; *i. e.*

$$\tilde{y}(t)=\int_0^\infty\int_{-\infty}^\infty \mathrm{dsdr}\cdot u\left(s-\frac{2r}{c}\right)\tilde{f}_s\left(s-\frac{r}{c},\ r\right)v^*(s-t)$$

$$+\int_0^\infty\int_{-\infty}^\infty \mathrm{dsdr}\cdot u\left(s-\frac{2r}{c}\right)\tilde{f}_c\left(s-\frac{r}{c},\ r\right)v^*(s-t)+\int_{-\infty}^\infty \tilde{n}(s)v^*(s-t)ds. \quad (13)$$

With the use of a notation such as $R_G(t_1,\ t_2)=\langle\tilde{y}(t_1)\tilde{y}^*(t_2)\rangle$, the component R_{GS} which contains the signal scattered from the medium is

$$R_{CS}(t_1,\ t_2)=\int_0^\infty\int_{-\infty}^\infty\iint ds_1ds_2dr_1dr_2\left\langle\tilde{f}_s\left(s_1-\frac{r_1}{c},\ r_1\right)\tilde{f}_s^*\left(s_2-\frac{r_2}{c},\ r_2\right)\right\rangle$$

$$\cdot u\left(s_1-\frac{2r_1}{c}\right)v^*(s_1-t_1)\,u^*\left(s_2-\frac{2r_2}{c}\right)v(s_2-t_2). \quad (14)$$

Equation (14) indicates that there is no phase shift proportional to the doppler shift which appeared in the FBM (cf. Eq. (7)). Substituting Eqs. (2) and (4) in (14), we arrive at the following expression;

$$R_{CS}(t_1,\ t_2)=\int_0^\infty\int_{-\infty}^\infty\int d\omega dr_1dr_2\cdot P_s\left(\frac{r_1+r_2}{2}\right)S_{ss}\left(\omega,\ \frac{r_1+r_2}{2}\right)e^{j\omega(t_1-t_2)}$$

$$\cdot\Theta_s(r_1-r_2)\,e^{-j\omega\left(\frac{r_1-r_2}{c}\right)}\chi_{vu}^*\left(t_1-\frac{2r_1}{c},\ \omega\right)\chi_{vu}\left(t_2-\frac{2r_2}{c},\ \omega\right). \quad (15)$$

If we further assume $\Theta_s(r_1-r_2)=\delta(r_1-r_2)$, Eq. (15) becomes

$$R_{CS}(t+T,\ t)=\iint_{-\infty}^{\infty} dr d\omega \cdot P_s(r)\, S_{ss}(\omega,\ r)\, e^{j\omega T} \chi_{vu}^*\Big(t+T-\frac{2r}{c},\ \omega\Big)\chi_{vu}\Big(t-\frac{2r}{c},\ \omega\Big), \tag{16}$$

where $T(=|t_1-t_2|)$ is the time lag and $t=t_2$. Equation (16) shows that the explored spectrum of the scattered signal is simply multiplied by, and not convolved with, the ambiguity function in the frequency domain.

The clutter and noise components are

$$\iint_{-\infty}^{\infty} dr dz \cdot P_c(r)\, \Theta_c(z)\, \chi_{vu}^*\Big(t-\frac{2r}{c}+T-\frac{2z}{c},\ 0\Big)\chi_{vu}\Big(t-\frac{2r}{c},\ 0\Big) + \sigma_n^2\int_{-\infty}^{\infty} |V(\omega)|^2 e^{j\omega T} d\omega, \tag{17}$$

where we assume $S_c(\omega,\ r)=\delta(\omega)$, $z=r_1-r_2$ and $z<<r_1,\ r_2$. The second term contains the autocorrelation function of the noise. Although $\tilde{n}(t)$ is virtually white, the finite bandwidth of the receiver makes the noise correlated within the finite duration.

Letting $T=0$ in Eq. (16), we obtain the variance of the scattered signal, i. e. the signal power, as

$$\iint_{-\infty}^{\infty} dr d\omega \cdot P_s(r)\, S_{ss}(\omega,\ r)\, \Big|\chi_{vu}\Big(t-\frac{2r}{c},\ \omega\Big)\Big|^2. \tag{18}$$

If the bandwidth of the spectrum S_{ss} is wider than that of χ_{vu}, we lose some amount of information on the explored spectrum. We must use either a shorter duration of the pulse or a wider bandwidth of the IF filter.

The conditions that the bandwidth of S_{ss} be sufficiently narrow compared with that of χ_{vu} and that S_{ss} be independent of r around r_0 further simplify the above result to yield

$$R_{CS}(t+T,\ t)=R_{ss}(T,\ r_0)\int_{-\infty}^{\infty} P_s(r)\, \chi_{vu}^*\Big(t+T-\frac{2r}{c},\ 0\Big)\chi_{vu}\Big(t-\frac{2r}{c},\ 0\Big)\, dr, \tag{19}$$

which indicates that the measured autocorrelation function is simply proportional to that of the scattering medium.

There are three schemes of measurements,[8] which we will describe in the following.

4.1 Single-pulse method

In this method, only a single pulse with a long duration d is transmitted. The

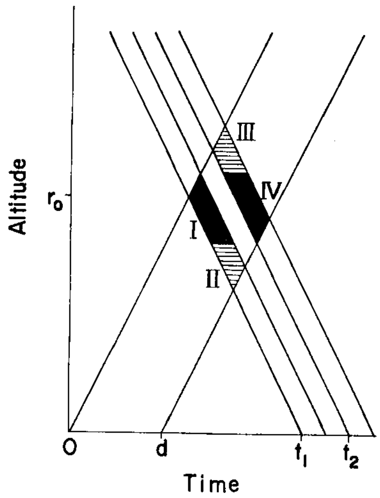

Fig. 3 Altitude-time diagram for single-pulse method. d and T are respectively the pulse duration and time lag. $t_1=2r_0/c$ and $t_2=t_1+T$.

receiver output is sampled at times $t_1=2r_0/c$ and $t_2=(2r_0/c)+T$ (see Fig. 3). The whole shape of the autocorrelation function is measured by changing T. The receiver gate has a finite width in time, but is not matched to the transmitted pulse. The altitude range from which a signal is received at a given time is shown by the shaded areas on the diagram. In the ionospheric IS, where the condition $\Theta_s(r_1-r_2)=\delta(r_1-r_2)$ is virtually satisfied, only contributions from the darkly shaded portions (I & IV) are correlated. Signals from different altitudes (II & III) are uncorrelated and add to the noise, being considered as self-clutter. The approximate resolution in altitude Δh is determined by the extent of the darkly shaded portions. No correlation is measured at the time lag $T>d$, because the correlated portions (I & IV) vanish. These properties are embodied in the term $\chi^*_{vu}(t+T-(2r/c),\ \omega)\ \chi_{vu}(t-(2r/c),\ \omega)$ in Eq. (16). The major drawbacks of the single pulse method are that the measurable maximum time lag, T_{max}, should be smaller than the pulse duration d, and that the SNR becomes poorer as $T\rightarrow d$. This method seems preferably suitable for the observation above the ionospheric F region, in which a long duration of pulse may be employed to improve the SNR.

4.2 Double-pulse method

In this method, two pulses having the same duration are transmitted at times 0 and Δ (Fig. 4). The receivers are usually matched to the transmitted pulses. There are two cases according to the polarization of the pulses. First, if the two pulses have the same polarization and only one receiver is used, signals from two altitude

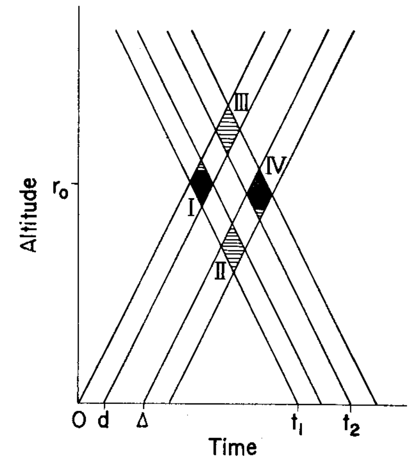

Fig. 4 Altitude-time diagram for double-pulse method. Δ is the pulse spacing and d and T are the same as those in Fig. 3. $t_1=2r_0/c$ and $t_2=t_1+T(T\neq\Delta)$.

ranges are contained in each sample. Only the signals from altitudes in the vicinity of r_0 are correlated and the signals from the other two altitudes contribute to the noise (self-clutter). The maximum SNR obtainable is unity. Secondly, if the polarizations of the two pulses are orthogonal and two separate receivers are used, the signals from a particular pulse enter only one receiver. The SNR could be made arbitrarily large. In the double-pulse method, the time lag T is chosen by the interval of the two pulses and is independent of the resolution in altitude which is uniquely determined by the duration of the transmitted pulse.

The waveform of the double pulses with spacing Δ is expressed as $u(t)=b(t)+b(t-\Delta)$, where $b(t)$ is the waveform of a single rectangular pulse. Then the ambiguity function χ_{vu} is given by the superposition of the two χ_{vb} 's, so that the term $\chi_{vu}^*\chi_{vu}$ in Eq. (16) becomes

$$\begin{aligned}\chi_{vu}^*\left(t+T-\frac{2r}{c},\ \omega\right)\chi_{vu}\left(t-\frac{2r}{c},\ \omega\right)&=\chi_{vb}^*\left(t+T-\Delta-\frac{2r}{c},\ \omega\right)\chi_{vb}\left(t-\frac{2r}{c},\ \omega\right)\\&+\chi_{vb}^*\left(t+T-\frac{2r}{c},\ \omega\right)\chi_{vb}\left(t-\frac{2r}{c},\ \omega\right)\\&+\chi_{vb}^*\left(t+T-\Delta-\frac{2r}{c},\ \omega\right)\chi_{vb}\left(t-\frac{2r}{c}-\Delta,\ \omega\right)\\&+\chi_{vb}^*\left(t+T-\frac{2r}{c},\ \omega\right)\chi_{vb}\left(t-\frac{2r}{c}-\Delta,\ \omega\right).\end{aligned}\qquad(20)$$

Since $\chi(\tau,\ \omega)$, is zero for $d<|\tau|$, the above relation will lead to simpler expressions under appropriate conditions. For examples,

a) if $|T-\Delta|<d$, $\Delta\geq 3d$, and $T\geq 2d$, i. e., if some portions of the region I and IV

are correlated, Eq. (20) reduces to

$$\chi_{vu}^{*}\chi_{vu}=\chi_{vb}^{*}\left(t+T-\Delta-\frac{2r}{c},\ \omega\right)\chi_{vb}\left(t-\frac{2r}{c},\ \omega\right). \tag{21}$$

This scheme is illustrated in Fig. 4.

b) If $T=\Delta\geq 2d$, i. e., if the intervals of transmission and gating are the same,

$$\chi_{vu}^{*}\chi_{vu}=\left|\chi_{vb}\left(t-\frac{2r}{c},\ \omega\right)\right|^{2}. \tag{22}$$

Substituting Eq. (22) into (16), we obtain the signal component $R_{CS}(t+T,\ t)$ as

$$R_{CS}(t+T,\ t)=\iint_{-\infty}^{\infty}drd\omega\cdot P_{s}(r)\,S_{ss}(\omega,\ r)e^{j\omega T}\left|\chi_{vb}\left(t-\frac{2r}{c},\ \omega\right)\right|^{2}. \tag{23}$$

The bandwidth of the spectrum $S_{ss}(\omega,\ r)$ is approximately the inverse of the characteristic time of the fluctuations $\tau(r)$. If we use the matched filter (the decorder for compressed pulses) for the case $d>\tau(r)$, the bandwidth of χ_{vb} becomes narrower than that of $S_{ss}(\omega,\ r)$, and we would lose some amount of information on the explored spectrum. Therefore, in a case where a matched receiving system is employed, the pulse duration must satisfy the condition that $d<\tau(r)$. Since $\tau(r)$ decreases with altitudes for the ionospheric IS, we would suffer from the reduction of the SNR at higher altitudes. On the other hand, if the bandwidth of χ_{vb} is wider than that of $S_{ss}(\omega,\ r)$, Eq. (23) becomes

$$R_{CS}(t+T,\ t)=\int_{-\infty}^{\infty}P_{s}(r)\,R_{ss}(T,\ r)\left|\chi_{vb}\left(t-\frac{2r}{c},\ 0\right)\right|^{2}dr. \tag{24}$$

This condition is virtually valid below the ionospheric E region. If $d=50\mu$ sec, the bandwidth of $|\chi_{vb}|^{2}$ will be nearly equal to 20 kHz, while that of $S(\omega,\ r)$ will be 1 kHz. The observed correlation function is found to be the simple superposition of the correlation functions of all altitudes, and we can neglect the effect of the systematic error, if we use sufficiently short pulses. In a case where $P_{s}(r)$ and $R_{ss}(T,\ r)$ are assumed to be independent of r around r_{0}, Eq. (24) further reduces to

$$R_{CS}(t+T,\ t)=R_{ss}(T,\ r_{0})P_{s}(r_{0})j(0). \tag{25}$$

In the IS from the ionospheric F region, where the plasma parameters of the medium change slowly, Eq. (23) is expressed as

$$R_{CS}(t+T,\ t)=P_{s}(r_{0})\int_{-\infty}^{\infty}S_{ss}(\omega,r_{0})\,e^{j\omega T}j(\omega)\,d\omega. \tag{26}$$

4.3 Multiple-pulse method

As indicated above, the shorter pulse $d<<\tau(r)$ is desired to be used for a reduction of the systematic errors and the improvement of the altitude resolution, but it simultaneously reduces the SNR. For example, in the case of a simple rectangular pulse, the signal power is $S \propto d \propto \Delta h$ and the noise power is $N \propto 1/\Delta h$, so that the SNR is proportional to $(\Delta h)^2$. The minimum value of Δh is determined by the sensitivity of the radar system. Integration over many successive samples improves the SNR but it reduces the resolution in time. There are two ways, however, to improve the SNR without worsening the resolution in time. One way is to increase equivalently the transmitted peak power, using the pulse compression techniques.[10] The other way is to increase the number of samples obtained per unit time.[11] The multiple-pulse method is categorized to the latter method which simultaneously studies many time lags (10 or more). A sequence of pulses is transmitted with particular irregular spacings and gated at the same intervals (Fig. 5). Each sample

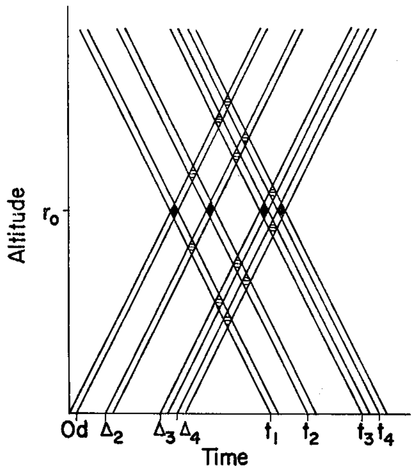

Fig. 5 Altitude-time diagram for multiple-(quadruple-) pulse method. Δ_i (i=2, 3, 4) is the spacing between the i-th and the first pulses. $t_1 = 2r_0/c$ and $t_i = t_1 + \Delta_i$.

contains signals from many altitudes, but no pair of these samples is correlated except in the vicinity of the altitude r_0. The signals from other altitudes add to the noise as self-clutter. The multiple-pulse method is an extension of the concepts of the double-pulse method. The waveform of the transmitted multiple-pulse $u(t)$ may be written as

$$u(t) = \sum_{i=1}^{n} b(t - \Delta_i) \tag{27}$$

with spacings at which a sequence of n pulses are transmitted, $\Delta_1(=0)$, Δ_2, ……, Δ_n. Therefore the relation $\chi_{vu}(\tau, \omega) = \sum_{i=1}^{n} \chi_{vb}(\tau - \Delta_i, \omega)$ is given for the ambiguity function.

If $\min|\Delta_i - \Delta_j| \geq 2d$ and $T = |\Delta_i - \Delta_j|$ $(i, j = 1, 2, \cdots n)$, $\chi^*_{vu}\chi_{vu}$ is simplified to yield Eq. (22), so that the description for the double-pulse method following Eq. (23) may also be applicable to the multiple-pulse method.

The number of self-clutters contained in each sample is $n-1$. Let S, C and N be the power of the signal, clutter and noise, respectively. Assuming that the scattered power is constant over the whole ranges explored, $C \simeq (n-1)S$. The SNR is, therefore, given by $S/(C+N) = 1/\{n-1+(N/S)\}$. The *r. m. s.* error of the measured correlation function, e, is

$$e^2 \propto (1/K)\ (S+C+N)^2/S^2, \tag{28}$$

where K is the number of samples. Alternatively, it may be rewritten as $K \propto \{n + (N/S)\}^2$ for some preset value of e. On the other hand, the number of samples available from n pulses is ${}_nC_2 = n(n-1)/2$. Thus, the resolution in time Δt is given by the ratio of K to ${}_nC_2$ as

$$\Delta t \propto \{n + (N/S)\}^2/n(n-1), \tag{29}$$

so that Δt is improved with an increasing n, especially in the case SNR$\ll$1.[11]

4.4 Elimination of clutter and noise

The clutter and noise (Eq. (17)) should be subtracted from the measured correlation function $R_C(t_1, t_2)$ before analyzing the spectra of the scattered signal.

The noise is measured at the range where the scattered signal is virtually negligible. It is negligible for the product at the lags much greater than the inverse of the IF bandwidth, but adds to the value of the autocorrelation function at $T=0$ or the signal power. Although the noise is expected to be a stationary process, one actually received changes with time because of the heating of the TR switch caused by a transmission of pulses, etc. Thus, the noise power should always be monitored, and usually be observed at the frequency near the radar frequency.

The average of the scattered signal $<\tilde{s}(t)>$ is assumed to be zero. However, there remains some amount of offset in the average of the output of the receiver, $<\tilde{y}(t)>$, which is caused by the clutter and the biases of the AD converter, etc. These components are subtracted either by

$$R_C(t+T,\ t) - <\tilde{y}(t+T)><\tilde{y}^*(t)> \tag{30}$$

or by

$$R_C(t+T,\ t) - R_C(t+T_\infty,\ t), \tag{31}$$

where $T_\infty \gg T$. T_∞ is taken to be smaller than 1/PRF (usually$<$10 msec for ionospheric measurements). In Eq. (31), the characteristic time of the clutter is assumed to be much larger than that of the medium. However, in the scattering from below the ionospheric *D* region which is mingled by the clutter in all existing radars, the characteristic time of the medium is of the order of 1 sec. In this case, some amount of information on the explored autocorrelation function will be lost in the calculation of Eq. (30) or (31).

5. Concluding remarks

In this paper, we have estimated the systematic distortions of the spectrum associated with IS radar measurements and the limitations on resolutions in frequency and altitude due to the finite receiver bandwidth and transmitted-pulse duration, using the radar-ambiguity function. The power spectrum of the scattering medium estimated by the FBM is convolved with the ambiguity function in the frequency domain, so that the requirements for frequency and altitude resolutions are incompatible. It was indicated that the FBM is inconvenient for pulse radar measurements, except for those of higher altitudes and/or with higher radar frequencies.

The spectrum estimated by the CFM was discussed for single-, double- and multiple-pulse methods. It was shown that in the CFM, the explored spectrum of the scattering medium is simply multiplied by, and not convolved with, the ambiguity function in the frequency domain. Therefore, the resolution in frequency is independent of the pulse duration which is concerned only with the resolution in altitude. We could employ comparably smaller values of duration. However, the minimum value is limited by the sensitivity of the radar system concerned, and the maximum is limited by the characteristic time of the fluctuations of the medium which scatter the radio waves, when the IF filter matched to the transmitted pulse is used.

Since signals scattered incoherently from the ionospheric heights are considered to be virtually independent of altitudes, we were restricted to the case $\Theta_s(r_1-r_2)=\delta(r_1-r_2)$. The case where signals from different altitudes are correlated will be discussed in a future paper.

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