An Analysis of Traveling-wave Amplification of Surface Magnetostatic Wave

By

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Abstract

The amplification of a surface magnetostatic wave is analyzed which propagates on the contacting surface of ferrite and semiconductor layers magnetized transversely. A hydrodynamical and collision dominant model is adopted for carriers in the semiconductor. It is explained that neither diffusion effect nor surface charges induced by the Hall effect have to be considered in this case. Maxwell's equation under adequate boundary conditions is solved to give a complex characteristic equation of real ω and complex k. The following results have been obtained from the equation. 1) When the carrier density or the thickness of the semiconductor is small, the amplification factor is proportional to both of them. 2) There is an optimum value of the carrier density or the thickness to obtain the maximum amplification factor. 3) The amplification factor depends on the direction of static magnetic field.

§ 1. Introduction

Since the propagation velocity of a magnetostatic wave is as low as 10^{-4} of light velocity, it may be used for a slow wave structure of a traveling wave amplifier. We can, on the other hand, obtain semiconductors with a great mobility and a great saturation velocity due to the progress of the semiconductor manufacturing technique. The two facts above seems to promise the construction of a solid-state traveling wave amplifier utilizing the magnetostatic wave and such a semiconductor. However, the

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drifting carrier in semiconductors has a very great collision frequency compared with the electron beams in a vacuum, so that it is difficult to propagate carrier waves with low losses, unless some particular conditions are imposed, Hence, it will be difficult to realize such a type of traveling wave amplifier as mentioned above.

It is known that the energey carried by a wave becomes negative on a moving observer, when the moving velocity of the observer exceeds the phase velocity of the wave. Therefore, if the drift velocity of the carriers in the semiconductor is larger than the phase velocity of the magnetostatic wave, the wave "suffers" negative losses, that is, obtains a positive gain. Schloemann[1] and Robinson *et al*[2] employed the idea, and analyzed the interaction between surface magnetostatic waves propagating on the surface of a transversely magnetized semi-infinite ferrite and drifting carriers in an adjacent semi-infinite semiconductor. But their semi-infinite ferrite system has no propagating magnetostatic wave solution when the semiconductor is removed. Hence, the amplifying (perturbed) mode cannot meet the physical requirement that it should coincide with the propagating (unperturbed) mode if the carrier density of the semiconductor tends zero.

Unlike their analysis, we have investigated the wave in a ferrite slab with a finite thickness, in which case propagating wave modes exist even when a semiconductor is not placed on the ferrite.[3] From the analyses of a composite structure of a ferrite slab and a semi-infinite semiconductor, the following propagation characteristics have become known. The net amplification occurs when the carrier drift velocity is larger than the phase velocity of the wave. If we make the carrier density tend to zero, the amplifying mode coincides with the propagating mode without a semiconductor.

We shall investigate in this paper a case in which both the ferrite and the semicondutor are plates of finite thickness. Then, the effect of the finite thickness will be examined for the semiconductor as well as for the ferrite. It will be explained that the surface charge induced by the Hall effect on the semiconductor surfaces does not contribute to the amplification factor very much, and that the diffusion effect of carriers is not necessary to be considered. As a consequence, the characteristic equation becomes so simple that the explicit expression for the amplification factor is obtained under some conditions. A strict numerical calculation is also performed and compared with the above analytical solution to show the valid range of the approximation.

§ 2. Ferrite and semiconductor system to be analyzed

Surface magnetostatic waves propagating in a $+y$ direction are analyzed in the ferrite and semiconductor system infinitely extending to a y and z direction, as

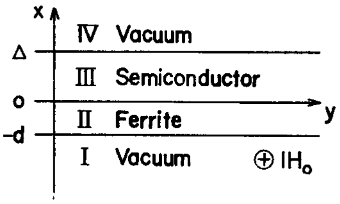

Fig. 1. A crossview of the geometrical configuration of the semiconductor and ferrite slabs. Static magnetic field $\boldsymbol{H}_0$ is parallel to the z axis.

shown in Fig. 1. A static magnetic field is applied in a +or $-z$ direction and a static electric field is also applied so that chaged carriers drift in a $+y$ direction. All quantities are assumed to be independent of z, *i.e.*, $\partial/\partial z=0$.

Writing down Maxwell's equation into each rectangular component, we know that TE and TM modes exist independently in this system. Since the TM mode is not related with the components of a permeability tensor which characterizes magnetized ferrite, we do not deal with the TM but with the TE mode which has H_x, H_y, and E_z components. It is known that only a magnetostatic mode (TE mode) propagates in such a thin ferrite slab as we are considering.[4] Therefore, we adopt the magnetostatic approximation from the first in order to simplify the analysis.

The following simplifications are taken for the drifting carriers in the semiconductor.

1) They are treated macroscopically according to the hydrodynamical approximation.
2) The mobility is isotropic and independent of the electric field.
3) The carrier is of single kind, and its density does not change due to creation or annihilation.
4) The collision frequency is much greater than the excitation frequency.

§ 3. Rf charge and current density in semiconductor

If the diffusion effect can be neglected, the following equations hold for the drifting charges in a semiconductor, where we distinguish the dc and rf quantities by the subscripts 0 and 1 respectively.

$$\frac{dv}{dt}=\frac{e}{m}(\boldsymbol{E}+v\times\boldsymbol{B})-\nu_c v \tag{1}$$

$$\boldsymbol{i}_1=\rho_0 v_1+\rho_1 v_0, \tag{2}$$

$$\mathrm{div}\ \boldsymbol{i}=-\frac{\partial\rho_1}{\partial t}, \tag{3}$$

where ν_c is the collision angular frequency.

All the rf quantities are assumed to vary with time and space according to the function $\exp[j(\omega t-ky)-f_s x]$, and hence

$$\frac{d\boldsymbol{v}}{dt}=\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)\boldsymbol{v}=\{j\omega-f_s v_x-jk(v_0+v_y)\}\boldsymbol{v}_1\simeq j(\omega-kv_0)\boldsymbol{v}_1, \tag{4}$$

where the y component of the carrier dc velocity is v_0. Equation (1) is divided into dc and rf parts

$$\frac{e}{m}(\boldsymbol{E}_0+\boldsymbol{v}_0\times\boldsymbol{B}_0)-\nu_c\boldsymbol{v}_0=0 \tag{5}$$

$$\frac{e}{m}(\boldsymbol{E}_1+\boldsymbol{v}_0\times\boldsymbol{B}_1+\boldsymbol{v}_1\times\boldsymbol{B}_0)-j(\omega-kv_0-j\nu_c)\boldsymbol{v}_1=0 \tag{6}$$

From now on, the subscripts 1 will be omitted from the rf electromagnetic field components for the sake of simplicity.

With reference to Maxwell's equation

$$\nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t}, \tag{7}$$

we eliminate $\boldsymbol{B}_1$ in eq. (6), and obtain

$$j(\omega-kv_0-j\nu_c)\boldsymbol{v}_1=\frac{e}{m}\begin{pmatrix} E_x-\frac{kv_0}{\omega}E_x-j\frac{f_s v_0}{\omega}E_y+v_{1y}B_0 \\ E_y-v_{1x}B_0 \\ E_z-\frac{kv_0}{\omega}E_z \end{pmatrix} \tag{8}$$

Letting $\omega-kv_0-j\nu_c=\Omega$ and $eB_0/m=\omega_c$, eq. (8) leads to

$$\boldsymbol{v}_1=\frac{e}{m}\frac{1}{\omega_c{}^2-\Omega^2}\begin{pmatrix} j\Omega\left(1-\frac{kv_0}{\omega}\right) & \omega_c+\frac{f_s v_0}{\omega}\Omega & 0 \\ -\omega_c\left(1-\frac{kv_0}{\omega}\right) & j\left(\Omega+\frac{f_s v_0}{\omega}\omega_c\right) & 0 \\ 0 & 0 & -j\frac{\omega_c{}^2-\Omega^2}{\Omega}\left(1-\frac{kv_0}{\omega}\right) \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{9}$$

Since we are interested in the TE mode with $E_x=E_y=0$, we have

$$v_{1x}=v_{1y}=0 \tag{10}$$

according to eq. (9). If, moreover, we recall the assumption in §2 (a collision-dominated model), we arrive at

$$v_{1z}\simeq\frac{e}{m\nu_c}\left(1-\frac{kv_0}{\omega}\right)E_z=\mu^*\left(1-\frac{kv_0}{\omega}\right)E_z, \tag{11}$$

where μ^* is the mobility of the charges and given as $e/m\nu_c$.

Now, the elimination of $\boldsymbol{i}_1$, from eqs. (2) and (3) yields

$$\rho_1=\frac{j\rho_0\,\mathrm{div}\,\boldsymbol{v}_1}{\omega-kv_0} \tag{12}$$

Since all the quantities are assumed to be uniform in a z direction, eq. (12) reduces to

$$\rho_1 = j\frac{\rho_0}{\omega - kv_0}\left(\frac{\partial v_{1x}}{\partial x} + \frac{\partial v_{1y}}{\partial y}\right) \tag{13}$$

and, then we finally obtain

$$\rho_1 = 0 \tag{14}$$

by virtue of eq. (10). It is concluded that carrier bunching does not occur in our case.

By eqs. (11) and (14) the rf current in the semiconductor defined as eq. (2) reduces to

$$\boldsymbol{i}_1 = \sigma\left(1 - \frac{kv_0}{\omega}\right)E_z\boldsymbol{z} \tag{15}$$

where $\sigma(=\rho_0\mu)$ is the conductivity and $\boldsymbol{z}$ is the z-directed unit vector. From this equation, it is seen that the rf current flows only in a z direction and that the effective conductivity becomes $1-(v_0/v_p)$ times that of the no dc drift field, where v_p is the phase velocity of the magnetostatic wave. Thus, it is understood that instability may be induced when v_0 exceeds v_p, and the effective conductivity becomes negative.

§ 4. The effect of diffusion term

The diffusion effect can be taken into account by adding the term $(\nu_c D/\rho_0)\nabla\rho_1$, to the right hand side of eq. (1).[5] Since ρ_1 is a linear combination of v_{1x} and v_{1y} by eq. (13) and $\nabla\rho_1$ has only x and y-components $(\partial/\partial z=0)$, the diffusion term only gives quantities proportionate to v_{1x} and v_{1y} onto the x and y parts of eq. (8).

The relation of $\boldsymbol{v}_1$ and $\boldsymbol{E}$, consequently, is the same as eq. (9) in the sense that the mobility tensor does not have xz, yz, zx and zy components. Hence, the same result is obtained for the rf current, carrying through an analysis along lines from eq. (9).

Since the whole effect of the carrier motion is expressed as an effective dielectric tensor via this rf current, it can be concluded that the diffusion term has no effect on the wave, considering that $\boldsymbol{i}_1$ does not change with the diffusion term.

§ 5. Characteristic equation

We will derive the electromagnetic field of the surface magnetostatic wave propagating in the structure shown in Fig. 1. All rf quantities are assumed to vary according to the function $\exp[j(\omega t-ky)]$, which, however, for convenience will be deleted.

Region I $x \leq -d$ (vacuum)

$$\left.\begin{aligned} &H_{x1}=fa_1e^{fx},\\ &H_{y1}=-jka_1e^{fx},\\ &E_{z1}=\omega\mu_0a_1e^{fx}=\frac{\omega}{k}B_{x1}, \end{aligned}\right\}\quad(16)$$

$$f^2=k^2, \quad (17)$$

where a_n and b_n are arbitrary constants.

Region II $-d\leq x\leq 0$ (ferrite)

$$\left.\begin{aligned} &H_{x2}=f(a_2e^{fx}-b_2e^{-fx})\\ &H_{y2}=-jk(a_2e^{fx}+b_2e^{-fx})\\ &E_{z2}=\omega\mu_0\left\{(\mu+\nu s)a_2e^{fx}-(\mu-\nu s)\,b_2e^{-fx}\right\}=\frac{\omega}{k}B_{x2}, \end{aligned}\right\}\quad(18)$$

where μ and ν are the components of a relative permeability tensor which is given as

$$[\mu]=\begin{pmatrix} \mu & j\nu & 0\\ -j\nu & \mu & 0\\ 0 & 0 & 1 \end{pmatrix} \quad (19)$$

$$\mu=1+\frac{\omega_0\omega_m}{\omega_0^{\,2}-\omega^2},\qquad \nu=\frac{\omega\omega_m}{\omega_0^{\,2}-\omega^2} \quad (20)$$

$$\omega_0=\gamma H_0,\quad \omega_m=\gamma 4\pi M \quad (21)$$

The symbol s in eq. (18) takes the value ± 1 according as the static magnetic field points to the $\pm z$ direction respectively.

Region III $0\leq x\leq\Delta$ (semiconductor)

$$\left.\begin{aligned} &H_{x3}=k(a_3e^{f_sx}+b_3e^{-f_sx})\\ &H_{y3}=-jf_s(a_3e^{f_sx}-b_3e^{-f_sx})\\ &E_{z3}=\omega\mu_0(a_3e^{f_sx}+b_3e^{-f_sx})=\frac{\omega}{k}B_{x3} \end{aligned}\right\}\quad(22)$$

$$f_s^2=k^2-\omega^2\mu_0\varepsilon_0\varepsilon_s\varepsilon_{33} \quad (23)$$

$$\varepsilon_{33}=1-\frac{\omega_p^2(\omega-kv_0)}{\omega^2(\omega-kv_0-j\nu_c)} \quad (24)$$

$$\omega_p^2=\frac{ne^2}{m\varepsilon_0\varepsilon_s} \quad (25)$$

where ω_p : plasma angular frequency

n : carrier density

ε_s : relative permittivity of lattice

Region IV $\Delta\leq x$ (vacuum)

$$\left.\begin{aligned} &H_{x4}=fb_4e^{-fx}\\ &H_{y4}=jkb_4e^{-fx}\\ &E_{z4}=\omega\mu_0b_4e^{-fx}=\frac{\omega}{k}B_{x4} \end{aligned}\right\}\quad(26)$$

The electromagnetic field given above must satisfy the following boundary conditions

$$\left.\begin{array}{lll} H_{y1}=H_{y2}, & B_{x1}=B_{x2} & at\ x=-d \\ H_{y3}-H_{y2}=i_s^-, & B_{x2}=B_{x3} & at\ x=0 \\ H_{y4}-H_{y3}=i_s^+, & B_{x3}=B_{x4} & at\ x=\Delta \end{array}\right\} \tag{27}$$

where i_s^- and i_s^+ are the effective current densities at the semiconductor surface. They are calculated in the Appendix and lead to

$$i_s^+=-i_s^-=\varepsilon_0\varepsilon_s v_0\mu^* B_0\left(1-\frac{kv_0}{\omega}\right)E_{z3} \tag{28}$$

Then, eq. (16) ~ (28) give the following characteristic equation

$$e^{-2kd}=\frac{\{1+G(\mu+\nu s)\}(1+\mu-\nu s)}{\{1-G(\mu-\nu s)\}(1-\mu-\nu s)}, \tag{29}$$

where

$$G=\frac{(\gamma_s-jR)(\gamma_s+1+jR)-(\gamma_s+jR)(\gamma_s-1-jR)e^{-2f_s\Delta}}{\gamma_s+1+jR+(\gamma_s-1-jR)e^{-2f_s\Delta}} \tag{30}$$

$$\gamma_s=f_s/k \tag{31}$$

$$R=\mu_0\varepsilon_0\varepsilon_s v_0\mu^* B_0\left(\frac{\omega}{k}-v_0\right)=\frac{v_0 v_p\mu^* B_0}{c_s^2}\left(1-\frac{v_0}{v_p}\right) \tag{32}$$

c_s : light velocity in a semiconductor lattice

The term R in eq. (30) arises from i_s^+ shown in eq. (28) as the surface charge effect accumulated on the semiconductor surfaces due to the Hall effect. The term γ_s which is given as

$$\gamma_s=\sqrt{1-\frac{\omega^2\mu_0\varepsilon_0\varepsilon_s}{k^2}+\frac{\omega_p^2\varepsilon_0\varepsilon_s\mu_0(\omega-kv_0)}{k^2(\omega-kv_0-j\nu_c)}} \tag{33}$$

by eqs. (23), (24) and (31), is simplified as

$$\gamma_s=\sqrt{1+j\frac{\sigma\mu_0}{k^2}(\omega-kv_0)} \tag{34}$$

according to the collision-dominant and the magnetostatic approximation, where

$$\sigma=ne^2/m\nu_c \tag{35}$$

is the conductivity of the semiconductor.

§ 6. Approximate expression for amplification factor

Since eq. (29) is a transcendental equation of complex variables and cannot be treated analytically as it is, one should derive an approximate equation under some proper conditions.

In these experiments, since we excite the wave with a fine wire or strip type antenna which locaized spacially, it is adequate to solve the characteristic equation for ω real and k complex.[6] Let

$$k=\beta+j\alpha \tag{36}$$

and the $\omega-\beta$ relation is the dispersion relation of the wave, and α is the amplification factor of the travelling wave amplification.

If we can suppose

$$|(\sigma\mu_0/k^2)(\omega-kv_0)|\equiv|2\Gamma|\ll 1 \tag{37}$$

in eq. (34), γ_s reduces to

$$\gamma_s=\sqrt{1+j2\Gamma}\simeq 1+j\Gamma \tag{38}$$

and eq. (30) is simplified as

$$G\simeq 1+j(\Gamma-R)(1-e^{-2\beta d}) \tag{39}$$

where Γ is the bulk effect of the interaction.

On the other hand, transforming eq. (29) gives

$$G(\mu^2-\nu^2+\nu s)-\nu s+1+(G+1)\mu\coth kd=0 \tag{40}$$

The following two equations are obtained as the real and imaginary parts of eq. (40), using eqs. (36) and (39).

$$\mu^2-\nu^2+1+2\mu\coth\beta d=0 \tag{41}$$

$$(\Gamma-R)(\mu^2-\nu^2+\nu s+\mu\coth\beta d)+2\alpha d\mu\,\mathrm{cosech}^2\,\beta d=0 \tag{42}$$

Equation (41) is just the dispersion relation of the DE mode (Damon-Eshbach mode).[7] This is as it should be, for the DE mode is the magnetostatic mode propagating in the system without the semiconductor shown in Fig. 1, and the inequality (37) shows that the conductivity of the semiconductor is too small to perturb the dispersion relation.

Now, by eqs. (41) and (42)

$$\alpha=\frac{\mu}{d(\mu-\nu s-1)(\mu+\nu s+1)}\left(\frac{\omega\sigma\mu_0}{2\beta^2}-\frac{v_0v_p\mu^*B_0}{c_s^2}\right)\left(1-\frac{v_0}{v_p}\right)\left(1-e^{-2\beta d}\right) \tag{43}$$

The term $\omega\sigma\mu_0/2\beta^2$ on the right hand side represents the bulk effect of the carrier, and the term $v_0v_p\mu^*B_0/c_s^2$ represents the surface effect. While the latter acts on the former additively when the carriers are electrons, it acts subtractively when they are holes. This is because electrons accumulate at the bottom surface of the semiconductor to increase the interaction with the wave, but holes acts contrarily to that. Then, let us estimate the magnitude of the bulk and surface effects. If we calculate using the values of the next section for the necessary parameters, c_s is 2 or 3 figures larger than v_0 and v_p which are roughly of the same order, and μ^*B is smaller than unity, so that $v_0v_p\mu^*B_0/c_s^2<10^{-5}$. If we let $n=10^{14}\sim10^{16}/\mathrm{cm}^3$, $\mu^*=5\times10^4\mathrm{cm}^2/\mathrm{V\cdot sec}$, $\omega/2\pi=$ 3GHz and $\beta=10^3/\mathrm{cm}$, we have $\omega\sigma\mu_0/\beta^2=10^{-4}\sim10^{-2}$. Then, the surface current effect can be neglected compared with the bulk effect.

By the calculation above, we know also the valid range of the inequality (37). Since $|\Gamma'|<10^{-2}$ for $n<10^{16}/\mathrm{cm}^3$, the inequality safely holds as far as $\sigma\lesssim 10^2$℧/cm, being translated into the conductivity.

The third term on the right hand side of eq. (43) changes its sign at $v_0=v_p$. The first term is always negative, while the second and the fourth terms are always positive in sign, and consequently $\alpha\gtrless 0$ according as $v_0\gtrless v_p$. This conclusion has already been anticipated by the analyses in §3 and in references 1) and 3).

The term $1\text{-}e^{-2\beta d}$ appears as the effect of a finite semiconductor thickness, which is discussed in reference 7) in detail.

§ 7. Numerical calculation and discussion

Since the approximate equation (43) for the amplification factor is valid only for a comparatively small conductivity, we will examine the characteristic equation (29) for the arbitrary σ in this section. We adopt YIG as the ferrite and n-type InSb as the semiconductor, and calculate the equation using the following parameters.

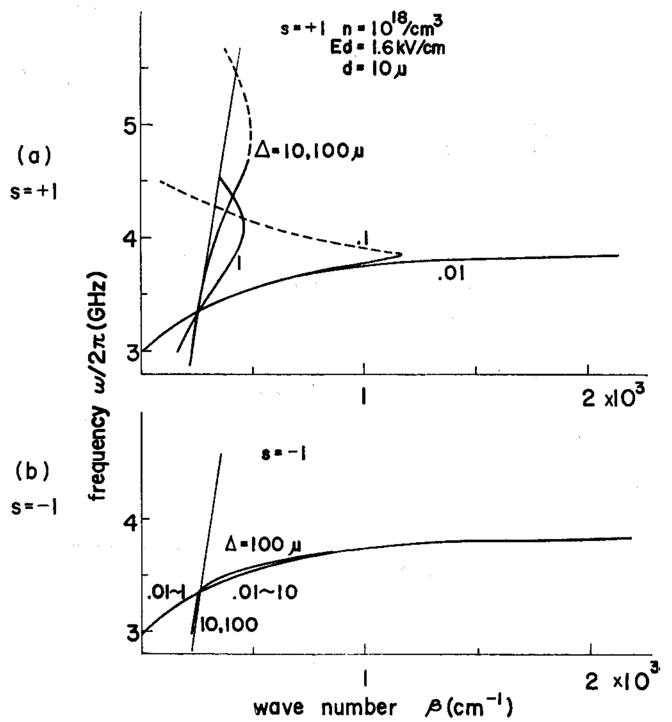

Fig. 2. Dispersion of the magnetostatic wave for several values of semiconductor thickness. The thin straight line indicates the carrier dispersion $\omega=kv_0$.

YIG	saturation magnetization	$4\pi M_s=1750\text{G}$
	thickness	$d=10\mu$
InSb	relative dielectric constant	$\varepsilon_s=17.9$
	electron mobility	$\mu^*=5\times10^4\text{cm}^2/\text{V}\cdot\text{sec}$
applied magnetic field		$H_0=500\text{Oe}$

(1) Dispersion curves

It is illustrated in Fig. 2 how the dispersion curve varies depending on the thickness of the semiconductor in the case of $n=10^{18}/\text{cm}^3$, the value of which may be the upper limit of the actual n-InSb. Translating the value into conductivity, it is $8\times10^3\mho/\text{cm}$ and fairly near to those of metals. However, the semiconductor does not act as a metal on the magnetostatic wave, but as vacuum, as long as the thickness is small. This fact is seen from the dispersion curves being the same as that of the DE mode.

When the static magnetic field is applied in a $+z$ direction ($s=+1$), the effect

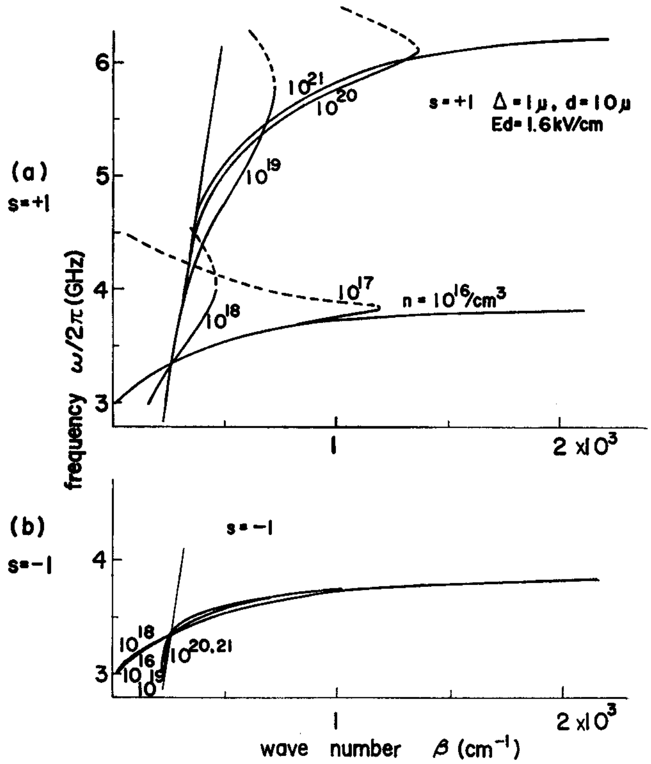

Fig. 3. Dispersion of the magnetostatic wave for several values of carrier density. The thin straight line indicates the carrier dispersion $\omega=kv_0$.

of the charged carriers on the wave increases as the semiconductor thickness increases, and the dispersion curve is distorted from that of the DE wave. Since, however, the value of σ is not large enough in Fig. 2, the dispersion curve does not reach that of the FM mode (Ferrite-Metal mode).[9]

The dotted lines in Figs. 2 and 3 are evanescent modes with a large attenuation constant. Though they have little physical importance, they are drawn as the high-frequency continuation of the amplifying modes.

Figure 3, contrary to Fig. 2, shows the dispersion curves for several electron densities, holding the semiconductor thickness constant. When $s=+1$, it is well shown that the curve varies from the DE mode with an increasing n. Comparing Figs. 2 and 3, one can understand that the semiconductor thickness and the carrier density have qualitatively the same effect on the dispersion characteristics of the wave. That is, the magnetostatic wave can approach either the DE or the FM mode by controlling Δ or n. However, if the n is too small, the wave cannot approach the FM mode, however large Δ may be.

When $s=-1$, the dispersion curve depends on neither Δ nor n very much, as shown in Figs. 2(b) and 3(b). This is because the wave energy concentrates on the bottom surface of the ferrite so that the interaction decreases.

(2) Amplification factor

Now, examine the imaginary part of the wave number, that is, the amplification

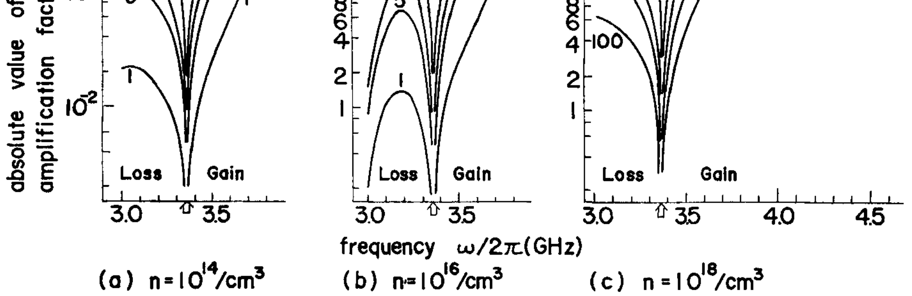

Fig. 4. Amplification factor for several values of semiconductor thickness and three carrier density. The mark ⇑ on the abscissa stands for the crossing point of the line $\omega=kv_0$ and dispersion curves in Figs. 2 and 3; Ferrite thickness $d=10\mu$, Drift field $E_d=1.6$kV/cm.

(or attenuation if negative) constant. The frequency characteristics of α are drawn in Fig. 4 for the constant n and the variable Δ. (Curves for $s=-1$ are omitted.) All α's become zero at 3.36 GHz at which the dispersion curves and the line $\omega=kv_0$ intersect. On the left side of the mark ⇑, the wave attenuates because $\alpha<0$, and vice versa. Consequently, it is concluded also in these numerical calculations that the amplification occurs when the drift velocity exceeds the phase velocity of the wave.

In Figs. 4(a) and (b), the amplification factor is shown to increase monotonically with Δ in the case of a small n as expected from eq. (43). The reason is as follows: Δ is the larger, and the greater part of the wave energy is contained in the semiconductor region, so that the interaction increases.

However, when n is large enough as shown in Fig. 4(c), the above theory does not hold, for just the wave fields are affected by the semiconductor thickness. That is the reason why $|\alpha|$ decreases with an increasing Δ. There is a certain value of Δ which gives the maximum amplification factor in this case.

In the next place, let n vary, holding Δ constant. Figure 5 explains that $|\alpha|$ increases in proportion to n when n is small enough, as predicted by eq. (43), and that it becomes maxmum at $n=10^{18}/\text{cm}^3$. Consequently, the electron density must be selected properly if one wants to make the amplification factor as large as possible.

In Figs. 4 and 5(a), the frequency range of $|\alpha|$ broadens as n becomes large,

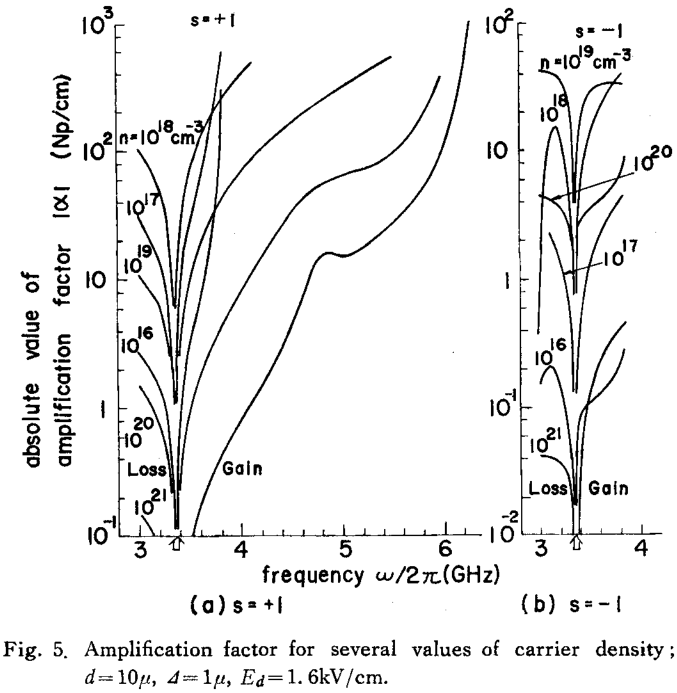

Fig. 5. Amplification factor for several values of carrier density; $d=10\mu$, $\Delta=1\mu$, $E_d=1.6\text{kV/cm}$.

because the magnetostatic wave approaches gradually to the FM mode as the conductivity becomes larger.

Figure 5(b) shows the amplification factor at the reverse static magnetic field ($s=-1$), and indicates that it is smaller than in the case where $s=1$ for almost all the values of n. The reason is as mentioned before.

It is a desirable characteristic for the construction of an amplifier that the amplification factor increases with frequency (decreases of the attenuation factor when $\alpha<0$), for the magnetostatic waves actually have an intrinsic propagation loss caused by a magnetic relaxation which is nearly a growing function of frequency. The gain and loss may be added to compensate the frequency characteristics of α, and one may obtain a flat and broad amplifier in frequency.

Lastly, we shall show the drift field dependence of the amplification factor. In Fig. 6, it is shown that α is a linear function of E_d for both $s=\pm1$ in the case where $n=10^{16}/\mathrm{cm}^3$. This result is consistent with the amplification characteristics of the TE waves in conductive moving media[10] and is also anticipated from eq. (43) considering that $v_0=\mu^* E_d$. On the other hand in the case where $n=10^{18}/\mathrm{cm}^3$, α is saturating as E_d increases as shown in Fig. 7. The reason is understood in the following way. The semiconductor has an effective conductivity $\sigma_{eff}=\sigma(1-v_0/v_p)$ and the wave is attenuated by the conduction current loss in the semiconductor (or amplified if σ_{eff} is negative).[8] When n and v_0 are large, σ_{eff} becomes large enough

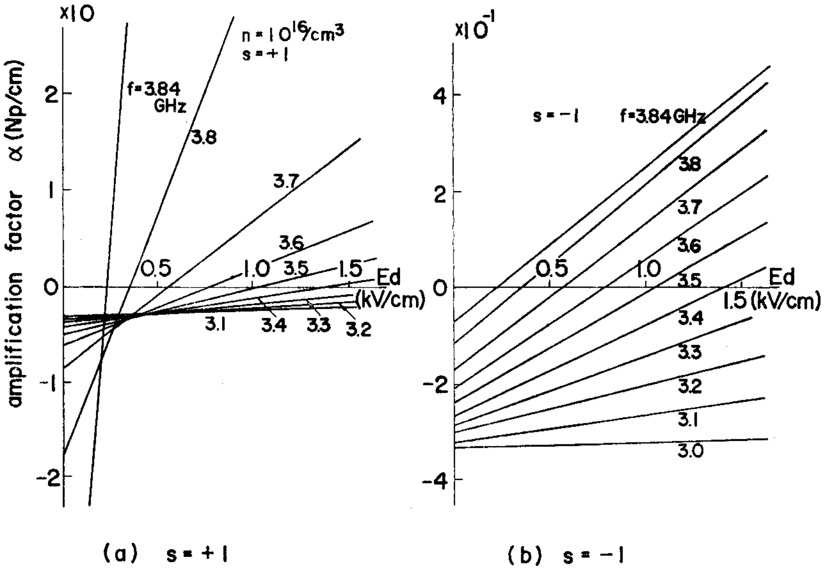

Fig. 6. Drift-field dependence of amplification factor for small carrier density; $n=10^{16}/\mathrm{cm}^3$, $d=10\mu$, $\Delta=1\mu$.

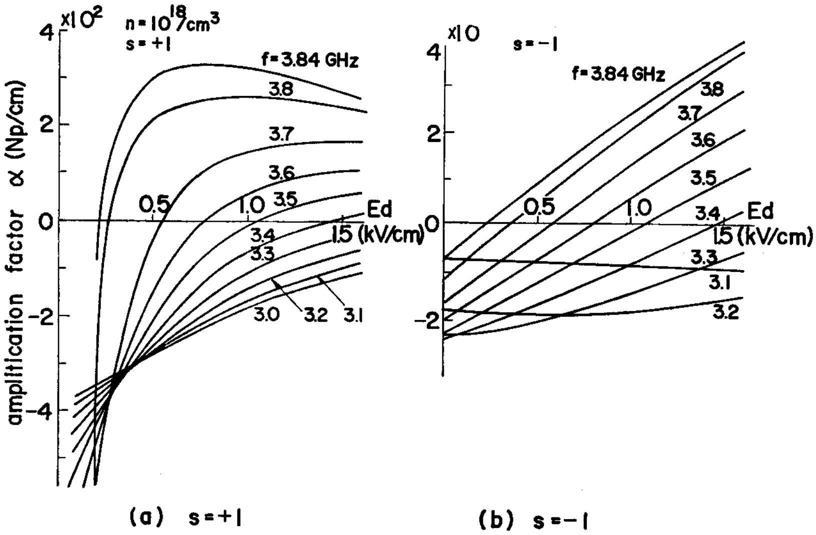

Fig. 7. Drift-field dependence of amplification factor for great carrier density; $n=10^{18}/\mathrm{cm}^3$, $d=10\mu$, $\Delta=1\mu$.

to diminish the electric field in the semiconductor, and lessens the interaction at high drift field.

§8. Conclusion

We have investigated theoretically the interaction of surface magnetostatic waves and drifting carriers propagating in a transversely magnetized ferrite/semiconductor slab system. Analysing the carrier motion macroscopically, it has been shown that diffusion effects and surface current effects do not contribute to the interaction in our configuration.

On these grounds, the amplification factor has been calculated both analytically and numerically, and the following important characteristics have been derived.

(1) The amplification takes place when the drift velocity of the carriers exceeds the phase velocity of the wave.

(2) When the conductivity and the thickness of the semiconductor slab are small, the amplification factor is in proportion to them and, moreover, to the drift field.

(3) In order to obtain the maximum amplification factor, one must choose a semiconductor slab with a proper conductivity and thickness.

(4) The amplification factor decreases if the direction of the static magnetic field is reversed in a $-z$ direction.

(5) The amplification factor is nearly a monotonically increasing function of frequency.

Concerning the value of the amplification factor, it can become very large since the value 10 on the ordinates in Figs. 4 and 5 stands for 86.9 dB/cm.

We can utilize this system as a solid state amplifier if it is possible to tailor the frequency characteristics and to treat the surface of the semiconductor and ferrite satisfactorily.

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Appendix Derivation of Surface current

At the surface of the semiconductor, shown in Fig. 1, the law of charge conservation eq (3) reduces to

$$\frac{\partial \rho_{1s}}{\partial t}+\frac{\partial i_{1sy}}{\partial y}-i_{1x}=0 \qquad \text{(A. 1)}$$

where the subscript s means "surface". By eq. (2), we obtain

$$\left.\begin{aligned} i_{1x}&=\rho_0 v_{1x}\\ i_{1sy}&=\rho_{0s}v_{1sy}+\rho_{1s}v_0, \end{aligned}\right\} \qquad \text{(A. 2)}$$

and on substitution into (A. 1) there results:[11]

$$j(\omega-kv_0)\rho_{1s}-jk\rho_{0s}v_{1sy}-\rho_0 v_{1x}=0 \qquad \text{(A. 3)}$$

Since $v_{1x}=0$ by eq. (10) and a similar relation with eq. (9) holds for surface fields, we have

$$v_{1sy}=0 \qquad \text{(A. 4)}$$

and, hence

$$\rho_{1s}=0 \qquad \text{(A. 5)}$$

by eq. (A. 3).
Substituting (A. 4) and (A. 5) into (A. 2), we obtain

$$i_{1sy}=0 \qquad \text{(A. 6)}$$

The surface current, therefore, has a single component and leads to

$$i_{1sz}=\rho_{0s}v_{1sz}+\rho_{1s}v_{0s}=\rho_{0s}v_{1sz} \qquad \text{(A. 7)}$$

Now, ρ_{0s} is left to be determined. The carriers in the semiconductor experience the x-directed force ev_0B_0 by virtue of the Hall effect. The carriers may have an x-component of the dc drift velocity by this force, but it is in fact cancelled out by the electric field of the dc charges induced on the semiconductor surfaces. Therefore, let the x-component of eq. (5) be equal to zero, and we have

$$E_{0x} = -v_0 B_0 \tag{A. 8}$$

The surface charge density is derived according to the Gauss Theorem and leads to

$$\left.\begin{aligned} \rho_{0s}^{+} &= D_n = \varepsilon E_n = -\varepsilon E_{0x} = \varepsilon v_0 B_0 \\ \rho_{0s}^{-} &= \varepsilon E_{0x} = -\varepsilon v_0 B_0 \end{aligned}\right\} \tag{A. 9}$$

where the superscripts $\pm$ represent the top and bottom surface of the semiconductor respectively. Consequently, by eq. (11), (A. 7) and (A. 9) we obtain

$$i_{1s_z}^{+} = -i_{1s_z}^{-} = \varepsilon_0 \varepsilon_s v_0 \mu^* B_0 \left(1 - \frac{k v_0}{\omega}\right) E_z \tag{A.10}$$

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