Structural Analysis and Design of Rigid Frames Based on Plastic Theorem

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By

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Abstract

This paper is concerned with the plastic analysis and the design problems of multistory rigid frames by use of the linear programming technique. The objective of this work is to develop the plastic analysis and design of rigid frames, paying attention to the graph theory, and to reduce the computer core size and also to save the execution time by using a decomposition technique. A few examples are presented to illustrate the method proposed here.

1. Introduction

There are three important conditions to be satisfied in the plastic analysis, namely (1) equilibrium, (2) yield criteria and (3) mechanism condition. The static method, which corresponds to the lower-bound theorem, is performed to satisfy the equilibrium and yield conditions simultaneously, with a subsequent check of the mechanism condition. On the other hand, the kinematic method, which corresponds to the upper-bound theorem, is the one which satisfies the equilibrium and mechanism conditions with a subsequent check of the yield condition. The latter method is effective for a simple structure and easily amenable to hand computation. However, its direct application for complex or large structures induces difficulties in actual formulation and computation on account of a large number of possible mechanisms. It is scarcely possible to determine all the mechanisms, and even if it could be done, it would be a tedious and prohibitive work.

Conversely, though analysis by the static method does not guarantee a true solution, the method can yield a safety-side value. Also, it can be so systematically for-

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mulated as to be suitable for use of digital computers, requiring the input data only for the pre-assigned parameters, that is, the lengths of columns and beams, the positions of joints, the plastic moment capacity of each member, etc. The critical collapse load can be easily obtained by applying it to the Linear Programming. Realization of the relation with LP has developed the plastic analysis and design, for which many papers have been presented.[3]

The method described herein is based on the static method proposed by R. K. Livesley.[1),2)] By this method, one can determine the location of the plastic hinges and the collapse load simultaneously. However, the method requires a comparatively long time and a large core size as the number of members increases. The yield conditions are examined at both ends of all members, and, also using the simple plastic theorem, the induced bending moments of member-ends must be checked for the positive and negative directions. Then, the constraints consist of inequalities whose number is four times as much as the number of members. In order to apply the simplex algorithm for the optimization tool, the number of constraints directly results in an increase of the core size and execution time. Therefore, particular treatments are called for to apply this method to large-scaled or complicated structures consisting of a large number of members.

In this paper, the authors intend to reduce the execution time for the inverse operation, which is necessary in forming the equilibrium equation, by introducing a basic relation of graph theory. In other words, the method indicates a possible application for large scale linear programming problems with respect to a structural framework analysis by selecting an appropriate determinate structural system (i.e. tree system) which can yield some simple forms of equilibrium equations. This form is suitable for using the Upper Bound Technique (UBT), which was already applied for the transportation problem with LP. The technique can shorten the calculation time and save the core size by deleting the unnecessary elements in the optimization procedure.

Although the above technique may accomplish our objectives to some extent, it will be still insufficient for a huge structure. We intend, therefore, to decrease the number of constraints and to obtain a good approximate solution, by partitioning a given system into some appropriate sub-systems. The required core size will be reduced to about $1/n^2$ times as much as those for the original problem, if one divides the given one into n sub-systems.

In this paper, a one-story frame is dealt with as a substructure to obtain the rational analysis and design methods for multistory frames, since a one-story frame is considered as a good substructure with an appropriate dimension. In the partitioning method proposed herein, the connectivity relation between individual substructures is taken into an account by introducing some appropriate redundant forces. The forces can vary and transmit the mechanical and configuration conditions of earlier stages to the treating stage.

Next, we attempt to extend the method mentioned above to the minimum weight design based on the plastic theorem. The plastic design is considered to relate reversely to the plastic analysis. The weight of a given system, which is assumed to be a function of the plastic moment capacity M_{pj} of each member, is minimized under the same constraint conditions as the plastic analysis. Then, the total weight W and unknown variables M_{pj} are sought out by using LP. But in this case, the LP algorithm becomes complex, because there can exist alternate inequalities in the constraint conditions. This is caused by considering the positive and negative yield conditions of every member-end, but it is easily treated by introducing artificial variables and by transforming it into a two-phase problem.

In the design problem, the number of design variables (M_{pj}) increases as the structure becomes large and complex, while the analysis problem had only one parameter (load factor λ) plus redundant forces as unknown variables. When the number of design variables increases by one, the calculation requires the additional core size of the number of constraints. Therefore, we should reduce the number of both the design variables and the constraints for a large structural design. The partitioning technique will remove these problems simultaneously, so that the proposed method will present a better result in saving the execution time and memory size than the analysis, when the interactions or mutual relations between divided substructures can be evaluated appropriately.

2. Plastic Analysis

2.1 Basic Equation

Based on the static approach, the equilibrium equation should be formed at first.[5)]

$$\tilde{P}' = A^t \tilde{P}, \tag{1}$$

where P' : applied external force vector at joints

P : member-end force vector

A : branch node incidence matrix

superscript t: matrix transposition

$\sim$: global coordinate system

In the local coordinate system, eq. (1) is found to be

$$P' = T_B A^t T_A^{-1} P \tag{2}$$

with the following relations

$$P' = \mathrm{T}_B \tilde{P}' \tag{3}$$

$$P = T_A \bar{P}, \tag{4}$$

where T_B: joint transformation matrix

T_A: member transformation matrix

The inverse of A^t is necessary for expressing the member force by the joint external force, which does not exist for general indeterminate systems, because the branch node incidence matrices of such systems have rectangular forms, that is, the numbers of members and joints are not same. For such a case, we need to extend the incidence matrix A by setting the appropriate number of releases on arbitrary positions. Therefore, one needs to cut off as many member-ends as the number of releases. Thus, the indeterminate system becomes the determinate system (i.e. tree system). Then, the fictitious redundant forces q are assumed to be applied for the both member-ends which are cut. This is the concept of the force method. In this case, the incidence matrix is not rectangular, but square and non-singular, and the form of eq. (2) will be established by using the extended incidence matrix $\bar{A}$.

With the aid of a basic relation of the graph theory, its inverse is easily obtained to be

$$(\bar{A}^t)^{-1} = B_T \tag{5}$$

in which B_T is the node-to-datum path matrix. Using eq. (5), the member-end force is expressed as

$$P = T_A B_T \bar{T}_B^{-1} \bar{P}', \tag{6}$$

where $\bar{T}_B$ and $\bar{P}'$ are the extended joint transformation matrix and the extended external force vector including the redundant force q, respectively. As $\bar{T}_B^{-1}$ is equal to $\bar{T}_B^t$, eq. (6) requires no inverse operation.

Since the simple plastic theory is assumed to be applied in this paper, only flexural terms should be taken into an account. This is done by multiplying the matrix H. The matrix H is a transformation matrix, and its multiplication gives the bending moments of both member-ends. H consists of H_i, which is given as

$$H = \begin{bmatrix} \nwarrow & & \\ & H_i & \\ & & \searrow \end{bmatrix} \tag{7}$$

$$H_i = \begin{bmatrix} 0 & l_i & -1 \\ 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

where $\begin{bmatrix} \nwarrow & \\ & \searrow \end{bmatrix}$ indicates that submatrices H_i, which correspond to each member, are arranged along the diagonal line and the other elements all vanish, and l_i is the length of the i-th member.

Now, let us express $\bar{P}'$ by the load factor λ and the redundant force q. Eq. (6)

yields to

$$HT_A B_T \bar{T}_B^t \bar{P}' = D\begin{bmatrix} \lambda \\ q \end{bmatrix}, \tag{9}$$

where the original property of indeterminancy is held in the matrix D by concentrating on the terms λ and q, and the dimension of its column is reduced to the number of the load factor and the redundant force.

The next step to consider is the yield condition. By using the plastic moment capacity vector M_p, the condition is shown as follows.

$$D\begin{bmatrix} \lambda \\ q \end{bmatrix} \leq M_p \tag{10}$$

The collapse load is obtained as the maximum value of λ under the constraint of inequality (10). This form is that of LP, itself. The redundant force q is found simultaneously with $\lambda_{\max}$. As all member-end forces are checked through the optimization procedure, the collapse mechanism can be obtained, and also all member-end forces are uniquely defined when the mechanism is not partial.

2.2 Application of UBT to Plastic Analysis[9)10)]

Let us consider the simple example shown in Fig. 1, in order to study the property of the determinate basic system obtained by introducing some releases. Since the structure considered is a portal frame, we can obtain the determinate basic system by cutting only one point. Then, the D matrix is given as

$$D = \begin{array}{c} \\ \bar{1} \\ \bar{2} \\ \bar{3} \\ \bar{4} \\ \bar{5} \\ \bar{6} \\ \bar{7} \\ \bar{8} \end{array} \begin{array}{c|c|c|c} \lambda & q^1 & q^2 & q^3 \\ \hline 0 & & & 1 \\ & & & -1 \\ 0 & & 0 & 1 \\ 0 & & & -1 \\ & 0 & & -1 \\ & & & 1 \\ 0 & 0 & 0 & 1 \\ 0 & & 0 & -1 \end{array} \begin{Bmatrix} \lambda \\ q^1 \\ q^2 \\ q^3 \end{Bmatrix} \leq \begin{Bmatrix} M_{P1} \\ M_{P2} \\ M_{P3} \\ M_{P4} \\ M_{P5} \\ M_{P6} \\ M_{P7} \\ M_{P8} \end{Bmatrix} \tag{11}$$

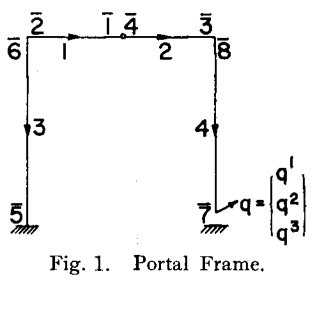

Fig. 1. Portal Frame.

where the shaded parts indicate they are occupied by an arbitrary value.

Pick up the 7-th row in the above inequality.

$$q^3 \leq M_{p7} \tag{12}$$

The inequality shows the yield condition of the 7-th member-end. For the member-ends which are released by cutting, the yield conditions are satisfied as that the introduced redundant forces are less than the specified plastic moment capacities, respectively. Such members, which are considered as having the cantilever forms, will appear after the cutting operation. Therefore, we have as many independent inequalities as the number of releases. By assembling and setting them in the lower part of inequality (11), one can easily use the General Upper Bound Technique (GUBT) or the Upper Bound Technique (UBT). By applying either technique, the pivot operations are partially done and the unnecessary elements are not stored through optimization. Thus, one can save the core size and the execution time needed for the procedure of linear programming. Also, these techniques will show their efficiency as the order of indeterminancy increases.

2.3 Approximate Method Using Partitioning Technique

In this section, the application of a partitioning technique is shown to analyze the limit collapse load. This technique is expected to give a good result, when the interactions and the mutual relations between each sub-pieced part are appreciated or estimated sufficiently.

Now, let us consider a three-story single-bay frame as an example. (Fig. 2) The D matrix of the structure is given as follows.

$$D = \begin{array}{c} \\ S1 \\ S2 \\ S3 \end{array} \begin{array}{c} \begin{array}{cccc} \lambda & q_1 & q_2 & q_3 \end{array} \\ \begin{bmatrix} A_1 & B_1 & 0 & 0 \\ A_2 & B_1' & B_2 & 0 \\ A_3 & 0 & B_2' & B_3 \end{bmatrix} \end{array}, \tag{13}$$

where $S1$, $S2$ and $S3$ indicate each story, respectively. Using eq. (13), the yield condition (i.e. inequality (10)) is found to be

$$\left.\begin{array}{llll} A_1\lambda + B_1 q_1 & & & \leq M_{p1} \\ A_2\lambda + B_1' q_1 + B_2 q_2 & & & \leq M_{p2} \\ A_3\lambda & & + B_2' q_2 + B_3 q_3 & \leq M_{p3} \end{array}\right\} \tag{14}$$

This can be rewritten as

$$\left.\begin{array}{l} A_1\lambda + B_1 q_1 \leq M_{p1} \\ A_2\lambda + B_2 q_2 \leq M_{p2} - B_1' q_2 \\ A_3\lambda + B_3 q_3 \leq M_{p3} - B_2' q_2 \end{array}\right\} \tag{15}$$

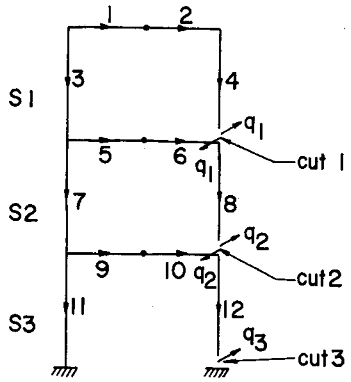

Fig. 2. Three-Story Single-Bay Frame.

Then, the linear programming is performed as the above three constraints are independent of each other. However, the inequalities, except the first one, have unknown variables (i.e. q_1 and q_2) on the right hand. The linear programming is done only when the right hand terms consist of constants. It is, therefore, needed to specify or assume the unknowns q_1 and q_2 before starting the LP algorithm. In this method, those unknowns are specified in the previous analysis steps, and they take an important role for holding the connectivity among each story. That is, the mechanical, load and configuration conditions are transmitted through the resultant forces. Moreover, the collapse modes of the stories being analyzed can be influential on the yield condition of the successive stages. Actually, this is so appeared as the plastic moment capacities M_{pj} of the member-ends, which connect the analyzed story, are altered. In inequality (15), the right hand terms are modified by reducing $B_i'q_i$.

The optimization is performed successively from the top story to the bottom one, because the top story can be considered truly independent of the other stories. For the usual multi-story frames, the procedure of this method consists of the following steps.

Step 1: Obtain the collapse load λ_1 and the redundant force q_1 of the top story by using LP.

Step 2: Obtain λ_2 and q_2 of the second story by using q_1 obtained at Step 1.

Step 3: Continue the previous step to the bottom story

Step 4: Then, the approximate collapse load λ of a given system is found to be

$$\lambda = \operatorname*{Min.}_{i=1}^{sm} \lambda_i$$

in which sm is the number of stories.

Thus, the method proposed is quite systematic, without iteration, to yield a safety-side solution, because the collapse load of each story is obtained under the condition that the previously analyzed stories have already failed. That fact can be explained by considering the relation between the collapse mechanisms of one story and those of a whole.

3. Minimum Weight Design Based on Plastic Theorem

3.1 Plastic Design

In this section, attention is particularly placed on the difference between the plastic design and the plastic analysis; and we should like to indicate the procedure of the design method in comparison with the plastic analysis.

In the plastic design, one introduces the different types of unknowns from those used in the plastic analysis. For example, one must find the rational value of M_{pj}, which is preassigned in the analysis, satisfying the equilibrium and yield conditions. The objective of the present design is the minimization of the total weight of a given system. This is a linear programming problem itself, if the total weight can be expressed as a linear function of M_{pj}.

The formulation of the design is expressed as

$$\left.\begin{array}{ll} \text{Minimize} & W=K\sum_{i=1}^{m} l_i M_{pi} \\ \text{subject to} & M_p \geq P \quad , \end{array}\right\} \tag{16}$$

where m, K and l_i are the number of members, constant and the length of the i-th member, respectively.

However, in this case, the employed linear programming algorithm will become complex, since the constant terms may have both positive and negative values. That is, the alternate inequalities will occur in the constraints, if we make all constants positive according to the requirement of LP. Then, this mixed type linear programming problem can be solved by introducing artificial variables and transforming it into a two-phase problem.[7]

3.2 Plastic Design Using Partitioning Technique

One of the main difficulties in the design of a large-scale structure is due to the fact that design variables increase in addition to the increase of constraints. The optimization problem with many design variables generally tends to reach a solution of poor convergency in the mathematical treatment, in spite of enormous calculation time, which characteristics often appear in non-linear programming problems. LP problems may provide such difficulties, even though they are not so frequent as those of NP problems. Furthermore, the errors can be accumulated through many runnings, which are required by treating the system as a whole.

In that case, the partitioning method may take the place of the design of a system as a whole, and the method shows its efficiency, because it can decrease the numbers of both design variables and constraints. However, the interactions between divided subsystems must be evaluated exactly. Using the iterative procedure for the connectivity of the successive stories, the solution may be obtained with a good approximation. However, its success depends on the treating problem itself. Thus, it concludes that the application of the piecewise technique, instead of the direct method, may give good results as far as the treatment of the connecting of the subdivided systems succeeds.

Here, the partitioning method without any iteration is proposed in order to reduce the execution time efficiently. The estimation or specification of the connectivity will take a key role for the method. Paying attention to the characteristics of multistory frames, it is done by introducing the fictitious redundant forces.
For the same example used in the previous section, the design formulation is given to be

$$\left.\begin{array}{ll} \text{Maximize} & -K\sum_{i=1}^{m} l_i M_{pi} \\ \text{subject to} & -M_{p1}+B_1 q_1 \leq -A_1\lambda \\ & -M_{p2}+B_2 q_2 \leq -A_2\lambda - B_1' q_1 \\ & -M_{p3}+B_3 q_3 \leq -A_3\lambda - B_2' q_2 \end{array}\right\} \tag{17}$$

The above equations are rewritten as the form of the maximizing problem, in order to have the same form in the analysis. The piecewise design can be performed by using the same procedure as the piecewise analysis. Its procedure is shown in a macro flow-chart in Fig. 3.

By partitioning a given system into some sub-systems, the design space is also divided. In the proposed method, some subspaces considered important are taken

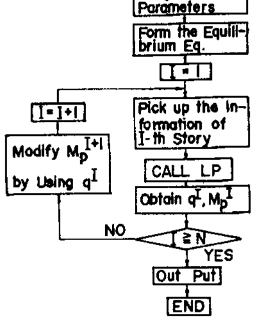

Fig. 3. Macro Flow Chart of Piecewise Plastic Design.

into account and they are successively treated one after another, affected by the results obtained at earlier stages. Thus, if the original system is complex, the design space is also complex. Also, the simple partitioning of the space becomes impossible, because by any partitioning of the system, one space can not be treated independently of the other spaces. In this case, the iterative method becomes necessary, even if the number of iteration is limited to only a few times.

However, in the case of a multi-story building, its configuration has a distinguishing feature, that is, every story is connected only to the upper and lower stories. This kind of configuration may give useful information for finding the collapse mechanisms of the system. That is, there are many independent (corresponding to only one story) modes in the possible mechanisms. If such independent modes govern the whole structural collapse, only the constraint subspaces corresponding to them are required for the constraints of the whole design, and the modification due to the effects of the intersections is no longer necessary. In this case, the usual partitioning method will give a good approximation. If an independent mode does not agree with the true mode, it will show a very safety-side value. The proposed method is considered to give good solutions for both cases, by introducing the variable fictitious forces. Since the forces can transmit the effects of the collapse mode which occurred in the upper story, the method can correspond to a variety of modes to some extent.

4. Numerical Results

Example 1. Analysis of Portal Frame

To illustrate the validity and the efficiency of the method proposed in this paper, let us take the portal frame in Fig. 4 as an example. The load and configuration conditions are presented in the same figure. Since the method depends on the simple plastic theorem, a proportional loading is employed. The plastic moment capacity of each member-end is specified as having the same value 40. This example was used in the study by Baker and et al,[4] and it may be suitable for considering the effects

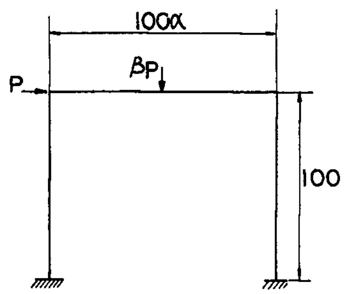

Fig. 4. Geometrical and Load Conditions.

of various parameters.

The results are presented in Fig. 5. The parameter α is the ratio of the lengths of column and beam, and β is the ratio of the vertical load and horizontal load. In the figure, it is observed that the collapse mechanism of the structure takes three different forms as the load parameter β varies. They are the panel, composite and beam mechanisms. The obtained load factor λ shows a consistency as long as the panel mechanism is dominant, and the value decreases gradually from the range of the composite one. Also, this decreasing continues over the range of beam one.

Table 1 illustrates the results in accordance with several kinds of methods, including the proposed one. The collapse loads obtained by all the methods, except Livesley II, are completely equal to the exact solution given by the kinematic method. The one by Livesley II gives a slightly different value. This is due to the fact that the method includes the assumption that the plastic hinges do not occur at the member-ends but locate at $l_i/10$ from them.

The Baker method requires the least calculation time among them all. However,

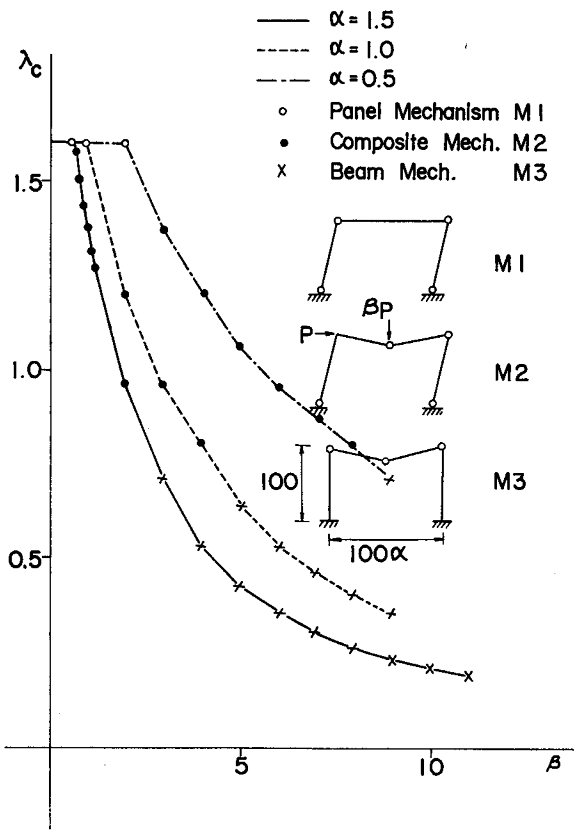

Fig. 5. λ_c-β Diagram.

Table 1. Limit load factor.

Method	L.L.F. (λ_c)	Cal. Time
Baker et. al	1.92	1.1 sec
Livesley (I)	1.92	1.8 sec
Livesley (II)	1.95	1.8 sec
Proposed M.	1.92	1.7 sec
Strict Sol.	1.92	—

the method includes the additional work of the pre-operation for the input of the initial tableau. Therefore, the value of 1.1 sec. is considered as the execution time of the linear programming itself. The proposed method requires less time than the Livesley method. The difference between them may depend on whether the inverse operation is needed or not for forming the equilibrium equation.

Example 2. Analysis of Three-Story Single-Bay Frame

To demonstrate the piecewise method, let us consider a three-story single-bay frame. The results are presented in Table 2 and Table 3 comparing them with the whole system analysis and Ridha's method. In Ridha's storywise method,[8] some different treatments are applied for the connectivity between each story. That is, the forces considered in the analysis of each story include the story loads, the horizontal loads for the upper stories and moments equal to the plastic moment capacities of the columns directly above the story. On the other hand, in the proposed method, the connectivity is taken into consideration by introducing unknown redundant forces. Those forces may vary and be influenced by the collapse mechanism type of the upper story, while Ridha's method applies the constant moment capacities of the upper story to the lower story. Thus, it can be said that the proposed method gives a better ap-

Table 2. Results of three-story single-bay frame (I).

M_p; 40 in all members

	$\beta=0.2$			$\beta=2.0$			$\beta=5.0$		
	WS	RS	PS	WS	RS	PS	WS	RS	PS
1-St.	3.20	3.20	3.20	1.92	1.92	1.92	0.854	0.854	0.854
2-St.	1.60	0.80	1.20	1.33	0.82	1.20	0.762	0.560	0.760
3-St.	0.96	0.54	0.80	0.96	0.72	0.80	0.648	0.480	0.648
$\lambda_{\min}$	0.96	0.54	0.80	0.96	0.72	0.80	0.648	0.480	0.648
cal. time (sec) 2-St.	2.6	2.0	2.0	2.6	2.0	2.0	2.6	2.0	2.0
cal. time (sec) 3-St.	4.5	2.8	2.8	4.5	2.8	2.8	4.5	2.8	2.8

WS ; Solution of whole system analysis
RS ; Solution of Ridha's method
PS ; Solution of proposed method

Table 3. Results of three-story single-bay frame (II).

M_p; 40, 60, 80

	$\beta=0.2$			$\beta=2.0$			$\beta=5.0$		
	WS	RS	PS	WS	RS	PS	WS	RS	PS
1-St.	3.20	3.20	3.20	1.92	1.92	1.92	0.85	0.85	0.85
2-St.	2.12	1.60	2.00	1.74	1.60	1.73	0.85	0.98	0.95
3-St.	1.73	1.08	1.68	1.60	1.60	1.60	0.85	1.07	1.02
λ_{min}	1.73	1.08	1.68	1.60	1.60	1.60	0.85	0.85	0.85
cal. time (sec) 2-St.	2.6	2.0	2.0	2.6	2.0	2.0	2.6	2.0	2.0
cal. time (sec) 3-St.	4.5	2.8	2.8	4.5	2.8	2.8	4.5	2.8	2.8

proximation than the Ridha's.

Table 3 presents the results of a case where each story has different moment capacities. In this case, we use the values of 40, 60 and 80 for three stories, respectively. Giving these values to the constraints consequently succeeded in this analysis; and these values may be appropriate by taking into account the characteristics of the multistory frame under the load shown in Fig. 8. This success in the piecewise analysis may be brought to the rational design problem, using the procedure proposed herein. From the same tables, it is confirmed that the piecewise (storywise) method can reduce the execution times by 0.6 sec. and 1.7 sec. for the two-story and three-story frames as compared with the times of the direct analysis of the whole system, respectively. Thus, the method will be the more efficient and powerful, the larger and more complex the structure.

Example 3. Design of Portal Frame[6]

Here, we show the design procedure by using a portal frame example. The loading and other conditions are shown in Fig. 6. As its geometrical symmetricity is assumed, the formulation by the kinematic method can be shown as follows.

Minimize the objective function

$$W=2(lM_{p1}+hM_{p2}) \tag{18}$$

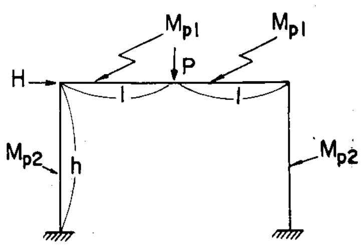

Fig. 6. Design of Portal Frame.

subject to

$$4M_{p1} \geq Pl \tag{19a}$$
$$2M_{p1}+2M_{p2} \geq Hh \tag{19b}$$
$$2M_{p1}+2M_{p2} \geq Pl \tag{19c}$$
$$4M_{p2} \geq Hh \tag{19d}$$
$$4M_{p1}+2M_{p2} \geq Pl+Hh \tag{19e}$$
$$2M_{p1}+4M_{p2} \geq Pl+Hh \tag{19f}$$

Since the problem is a simple one with only two unknown variables M_{p1} and M_{p2}, it can be solved graphically. Now, we specify the pre-assigned parameters as follows.

$$\left.\begin{aligned} l &= 75 \\ h &= 100 \\ P &= 8 \\ H &= 4 \end{aligned}\right\} \tag{20}$$

Then, we can obtain the design space shown in Fig. 7. Observing it, the minimum value of the total weight is 58,333 with $M_{p1}=M_{p2}=500/3$. In this case, the inequalities (19e) and (19f) are active only as the constraints. The proposed method, which is based on the lower-bound theorem, presents a solution equal to that obtained by the kinematic method. Also, in this case, the execution of the method requires only 1.8 sec.

Example 4. Design of Three-Story Single-Bay Frame

This example demonstrates the efficiency of the partitioning design method

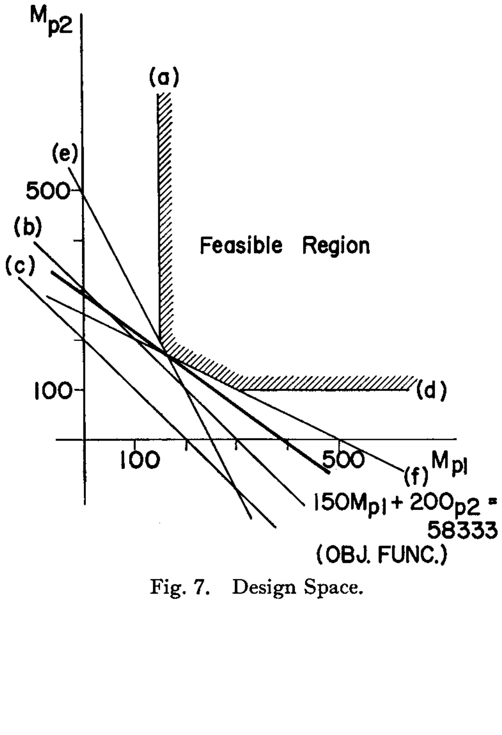

Fig. 7. Design Space.

Table 4. Design of three-story single-bay frame.

	$\beta=0.2$		$\beta=2.0$		$\beta=5.0$	
	Whole M.	Proposed M.	Whole M.	Proposed M.	Whole M.	Proposed M.
X_1	100	100	300	167	375	550
X_2	100	100	100	167	375	200
X_3	300	300	300	366	575	516
X_4	200	200	200	200	200	316
X_5	500	500	500	400	584	660
X_6	300	300	300	299	384	344
weight	255000	255000	285000	288000	421900	430900
Limit F	4.0	4.0	4.0	4.0	4.0	4.0
Cal. Time	6.2 sec	3.0	5.6	2.9	5.7	2.9

described in this paper. The numerical results are presented in Table 4 along with those of the whole system design. For the sake of generality, three loading systems are applied. The horizontal load is equal to H $(=4)$ for these cases, while the vertical load P is varied. The parameter β indicates the ratio of P and H. It is, then, expected that the panel, composite and beam type collapse mechanisms are dominant for $\beta=0.2$, $\beta=2.0$ and $\beta=5.0$, respectively.

Table 4 indicates the following items. For $\beta=0.2$, the piecewise design completely agrees with the whole system design. In this case, the collapse of the given system occurs as panel one. This leads to the conclusion that the piecewise treatment is justified in this case. In the mechanisms, all columns are led to collapse, and the introduced redundant forces are equal to the plastic moment capacities, being transmitted to the lower story. For $\beta=5.0$, there are little differences between the sets of design variables. The differences are due to the fact that a beam mode yields the collapse. At a glance, one may fear whether the partitioning method can give good results, but the obtained total weight is very close to the optimum weight. While only the load factor attracts our attention in the plastic analysis, the set of design variables, itself, is important in the plastic design. Therefore, the design problem seems to be sensitive to the change of the constraint region.

For $\beta=2.0$, the values of the top story are different from those of the whole system design. In the proposed method, the top story is designed independently and is not affected by the other stories. However, the differences disappear in the design of the second story. The shortness of rigidity is compensated in the second story as its beam is designed stronger than the one of the whole system design. The design of the third story shows good agreement with the true optimum design.

The differences between the piecewise and whole system designs are concerned with the corresponding collapse mechanisms. Many mechanisms correspond to the

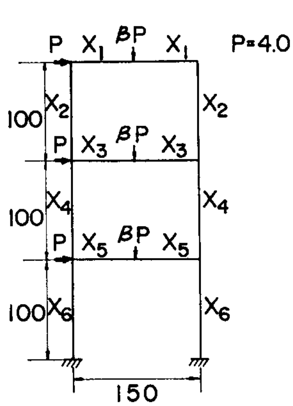

Fig. 8. Design Variables.

same collapse load of the obtained design. Fig. 8 presents some of them. The panel mechanisms CP1, CP2 and CP3 are all governing ones for the whole system design, but CP3 can not be taken in the piecewise design. That is, the piecewise design has less plastic hinges than the design as a whole. In that sense, the piecewise method is considered to be more safe, though the load factors of both designs are equal.

In each case, the total weights by use of the piecewise method are heavier, but their differences are so small, compared with the true values, as to be acceptable. The largest difference in weight, which is seen in the case $\beta=5.0$, is about 2%.

It is remarkable that the execution time of each case is about half of that required for the whole system design. While the execution time of the whole system design increases as to be proportional to the square of the number of members, the proposed method will require time which increases linearly. Therefore, it may be concluded that the proposed partitioning method will become more available and effective as the structure becomes more complex and/or large.

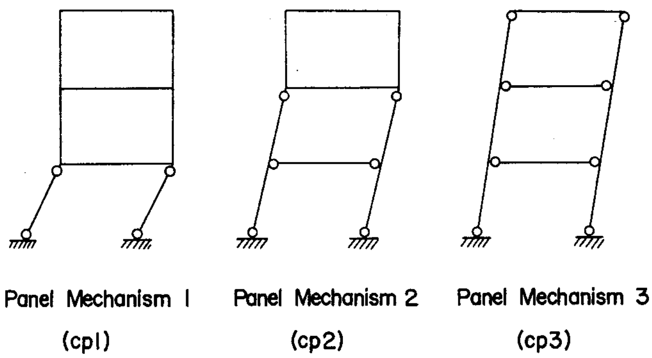

Fig. 9. Possible Collapse Mechanisms.

5. Concluding Remarks

With the aid of linear programming, the analysis and design methods based on the plastic theorem are discussed in this paper. In order to obtain a rational method, we attempt to introduce the basic relation of the graph theory and the concept of decomposition of systems. Observing some examples, it may be concluded as follows.

1) The static method is quite suitable for the use of linear programming because of its simple and systematic formulation. Also, the method gives a safety-side value, even if it leads to an approximate solution.

2) Paying attention to the graph theory, the inverse operation of the matrices can be removed in the stage of constructing the equilibrium equation. Therefore, the execution time can be reduced.

3) Introducing the concept of the force method and the selected determinate basic system, the plastic analysis and design methods can be improved for the use of LP. The UBT and the GUBT, which are considered as the simple forms of the decomposition algorithm proposed by G. B. Dantzig, can take an important role to save the core size and the execution time.

4) The partitioning analysis method presents good approximate solutions with smaller computer memory size and less execution time than the whole system analysis requires. Moreover, a better agreement of the approximation with the exact one is expected, when it is applied for the system where the moment capacities are given with balance.

5) The plastic design can be performed by the same way as proposed for the analysis. The complexity of treatment in LP can be easily removed by using appropriate artificial variables.

6) The concept of partitioning of a system could be introduced in the design problem for a multistory framed structure so as to give an approximate solution. Its accuracy depends on the correspondence between the collapse type of one story and that of the whole. The weight of the piecewise design may be acceptable compared with that of the whole system design. For the complex and/or large structures, the proposed method will succeed in saving the computer core size and the execution time. Then, one can have the possibility that the small-sized computer may be available in the structural design.

7) By using the properties of configurations of structures, the partitioning method will become more efficient and powerful for other types of structures. Then, piecewise patterns other than storywise will offer good results.

8) In order to improve the simple plastic method, one should consider the axial force effect, and also the p-Δ effect which can not be neglected in actual designs. The axial

force effect can be included, if a yield surface is used for constraints to be given as

$$g_1(M)+g_2(N)\leq C,$$

where g_1 and g_2 are appropriate functions, and M, N and C are the bending moment, axial force and constant, respectively.

In that case, a nonlinear programming is generally needed for the optimization tool. But, linearizing the yield surface appropriately, the case can be also dealt with as a linear programming problem.

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