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# Active Suppression of Chatter by Programed Variation of Spindle Speed

By

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## Abstract

By a programed variation of the spindle speed of machine tools, it is possible to control the onset of regenerative-type self-excited chatter, which restricts the maximum depth of cut allowable in rough cutting operations. This paper presents a theoretical basis of this phenomenon, together with experimental proofs demonstrated in turning operations.

## 1. Introduction

Among various types of chatters encountered in metal cutting and grinding operations, the regenerative-type self-excited chatter is the one which occurs owing to an excessive stiffness of the cutting process. Onset of this particular type of chatter is most commonly experienced which limits the maximum depth of cut possible to be taken in roughing operations by a given combination of the machine tool, the workpiece and the tool.

In the present paper, a study was made to evaluate the practical potentials of cutting metal under a continually varying spindle speed, as a measure to suppress the regenerative-type self-excited chatter. The idea is based on the fundamental knowledge of the mechanism of the chatter.

The present paper first describes the theoretical principle of chatter suppression by the spindle speed variation from the viewpoint of the energy flow. Next, a series of turning tests are carried out under varying spindle speeds to identify the parameters affecting the effectiveness of chatter suppression.

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## 2. Theory of Chatter Suppression by Varying Spindle Speed

### 2.1 Mechanism of Regenerative Chatter with Respect to Energy Balance

According to previous studies<sup>(1,2)</sup> on the dynamics of cutting process under the effect of the regenerative feedback, the transfer function of the cutting force variation in response to the harmonic variation in the depth of cut, is represented by the point  $T_c$  on a circle drawn in the stiffness vector diagram, as shown in Fig. 1.

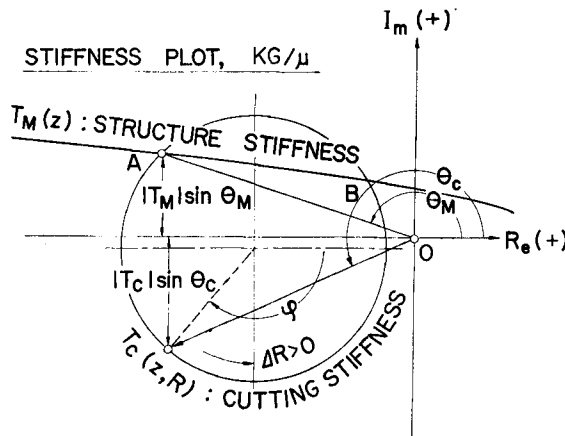


Fig. 1 Theoretical illustration of energy flow per vibration cycle.  
 $\varphi$ : Phase difference between inner and outer modulations.  
 $\Delta R$ : Rate of spindle speed variation.

The diameter of the circle is proportional to the cutting stiffness, namely to the average depth of cut. The angular position  $\varphi$  of the response point  $T_c$  is the phase difference between inner and outer modulations, and is given by the following equations:

$$\left. \begin{aligned} \varphi &= 2\pi\epsilon \\ \frac{Z}{R} &= N(\text{integer}) + \epsilon(\text{fraction}) \end{aligned} \right\} \dots\dots\dots(1)$$

In the above equations,  $Z$  denotes the frequency of the harmonic variation in the depth of cut, and  $R$  is the rotational speed of the spindle for the case of turning. Therefore,  $\varphi$  is dependent on the two parameters  $Z$  and  $R$ .

When a deep cut is taken, such that the circle is large enough to intersect with the stiffness response curve  $T_M(z)$  of the structure supporting the work and the tool, the process is unstable and a self-excited chatter develops. The amplitude of the vibration builds up as the mechanical energy accumulates in the structure after

the start of the cutting. The chatter amplitude grows until a balance between the supplied and dissipated energies is reached. This state of energy balance is reached, when both transfer functions,  $T_O$  and  $T_M$ , of the cutting process and the structure respectively, attain either of the two intersection points A or B.

In this case, the amount of energy  $W_M$  dissipated in the structure per cycle of vibration is,

$$W_M = \int T_M x dx = \pi X^2 |T_M| \sin \theta_M \dots \dots \dots (2)$$

where  $x$  denotes the harmonic variation in the depth of cut,  $X$  being its amplitude. As illustrated in Fig. 1,  $W_M$  is proportional to the projectile of the vector  $\vec{OA}$  to the Im-axis. In the same way, the amount of energy  $W_O$  supplied to the structure by the cutting process per cycle of vibration is as follows:

$$W_O = \int T_O x dx = \pi X^2 |T_O| \sin \theta_O \dots \dots \dots (3)$$

It is understood that the same amount of energy as dissipated in the structure is supplied to the structure by the cutting process since the points  $T_M$  and  $T_O$  are coincident in this case.

## 2.2 Energy Flow under Spindle Speed Variation

If we could arbitrarily regulate the angular position  $\varphi$  of the cutting process dynamics  $T_O$  by providing a very sensitive control on the spindle speed  $R$ , the amount of energy supply  $W_O$  from the cutting process could be controlled.

The energy increment  $\Delta E$  accumulated in the structure per vibration cycle is given by following equation:

$$\Delta E = W_O - W_M = \pi X^2 \{ |T_O| \sin \theta_O - |T_M| \sin \theta_M \} \dots \dots \dots (4)$$

Thus, by fixing  $T_O$  at a point on the upper arc between A and B, we could supply more energy to the structure than it dissipates. In such a case,  $\Delta E$  is positive, indicating that the vibration amplitude of the structure increases. Or, we could supply less energy to the structure than it dissipates, or even take out energy from the structure by fixing  $T_O$  at a point on the lower arc between A and B. In this case,  $\Delta E$  is negative, resulting in the reduction of the vibration amplitude.

Instead of trying to fix  $T_O$  at one particular point, which is in reality impossible to achieve merely by speed control, we can drive the point  $T_O$ , always going around the circle by continually varying the spindle speed. In this case, when the rate of spindle speed variation is higher, the angular position  $\varphi$  varies faster, as shown later in section 2.3. Thus, the upper arc between the two intersections A and B is passed in a shorter time, so that a smaller amount of energy is accumulated. As a result, the chatter eventually has no chance to build up. Supposing the angular position  $\varphi$  is varied at a constant rate, the shorter will

be the time for the point  $T_O$  to pass the upper arc between A and B than the lower arc. Therefore, the structure is always likely to lose more energy than it accumulates when the cutting process point  $T_O$  completes a circling. The above theory leads us to expect following points:

- 1) A higher rate of spindle speed variation is more effective for the chatter suppression.
- 2) The narrower the intersection area is between the two stiffness response curves of  $T_O$  and  $T_M$ , the easier is it to suppress the chatter by spindle speed variation.

### 2.3 Simulation based on a Simplified Model.

The vibration amplitude  $X$  of the structure is related to the total amount of energy  $E$  accumulated in it by the following equation:

$$E = \frac{1}{2} k X^2 \quad \dots\dots\dots(5)$$

In the equation,  $k$  denotes the equivalent stiffness of the structure for the mode of resonance concerned. Thus, the energy increment  $\Delta E$  of the structure per vibration cycle, given by the equation (4), is related to the increment  $\Delta X$  of the vibration amplitude during a cycle as follows:

$$\Delta E = \pi X^2 \{ |T_O| \sin \theta_O - |T_M| \sin \theta_M \} = k X \Delta X$$

Therefore,  $\Delta X$  is given by the following equation:

$$\Delta X = \frac{\pi X}{k} \{ |T_O| \sin \theta_O - |T_M| \sin \theta_M \} \quad \dots\dots\dots(6)$$

According to this equation, the vibration amplitude can be subsequently computed in every cycle, if the angular position  $\varphi$  of the cutting process dynamics  $T_O$  is given for every cycle. Computation is made on a simplified model based on the following suppositions:

- 1) After a steady state chattering of a frequency  $Z$  is reached at a cutting condition, the spindle speed is made to vary from time zero at a constant rate  $\Delta R$  per second.
- 2) Even after the start of the speed variation, the frequency of the vibration can be considered to stay constant at the same frequency  $Z$ .
- 3) Differences in the amplitude and the wave length of the surface undulations produced in two successive rotations are little. Hence, the phase difference  $\varphi$  between the inner and outer modulations is given by the phase lag of the peak point of the outer modulation to that of the inner modulation, as illustrated in Fig. 2.

Based on these suppositions, the wave length  $\lambda_n$  of the chatter mark on the work

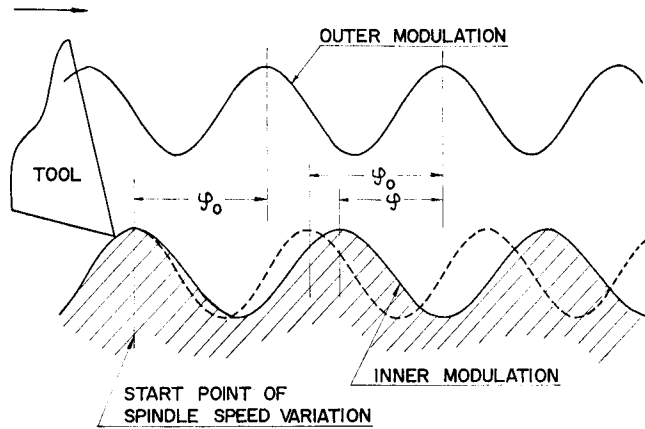


Fig. 2 Illustration of phase difference  $\varphi$  between inner and outer modulations after start of spindle speed variation.  
 $\varphi_0$ : Initial value of  $\varphi$

surface after  $n$  cycles of vibration from time zero, is given as follows:

$$\lambda_n = \lambda_0 \left\{ 1 + \frac{\Delta R \cdot \tau \left( n - \frac{1}{2} \right)}{R_0} \right\}$$

where the suffix 0 denotes the values at time zero, and  $\tau$  denotes the period of the chatter vibration, which is equal to  $1/Z$ . From this equation  $\varphi_n$  is obtained as follows:

a) During the first revolution after the start of the spindle speed variation.

In this case, the wave length of the outer modulation is constant at  $\lambda_0$ , but that of the inner modulation varies in the subsequent cycles, so that the following equation is established:

$$\frac{\varphi_n}{2\pi} \lambda_0 = \frac{\varphi_0}{2\pi} \lambda_0 - \sum_{i=1}^n (\lambda_i - \lambda_0) \dots\dots\dots(7)$$

Accordingly,  $\varphi_n$  is successively given as follows.

$$\varphi_n = \varphi_0 - \frac{\pi \cdot \Delta R \cdot \tau \cdot n^2}{R_0}, \text{ for } n=1, 2, \dots, N \dots\dots\dots(8)$$

b) After the first revolution

In this case, both the wave length of the outer modulation and that of the inner modulation vary in the subsequent cycles, so that the following equation can be established:

$$\frac{\varphi_n}{2\pi} \lambda_{n-N} = \frac{\varphi_0}{2\pi} \lambda_0 - \sum_{i=n-N+1}^n (\lambda_i - \lambda_0)$$

$\varphi_n$  is then given as follows,

$$\varphi_n = \frac{R_0\varphi_0 - \pi \cdot \Delta R \cdot \tau \cdot N(2n - N)}{R_0 + \Delta R \cdot \tau \left(n - N - \frac{1}{2}\right)}, \text{ for } n = N + 1, \dots \dots \dots (9)$$

In the above equations,  $N$  is the integer number of chatter cycles per revolution of the spindle rotating at the initial speed.

From the above equations (8) and (9), we can obtain the angular position  $\varphi_n$  of  $T_O$  in every cycle of vibration. By using  $\theta_O = 2\pi - \varphi_n$  in the following equation, the vibration amplitude  $X$  can be calculated in successive cycles:

$$X_n = X_{n-1} + \Delta X_{n-1} = \left\{ 1 + \frac{\pi}{k} (|T_C| \sin\theta_C - |T_M| \sin\theta_M) \right\} X_{n-1} \dots \dots \dots (10)$$

Based on this finite equation, a digital computation was made to simulate the variation of chatter amplitude after a steady state chattering of 50  $\mu$  amplitude was reached at a cutting condition just on the stability boundary, and the spindle speed was made to increase from time zero by a constant rate. The results shown in Fig. 3 indicate that the amplitude reduces rapidly after the start of speed increase, and that the amplitude reduces more rapidly at a higher rate of spindle speed variation  $\Delta R$ .

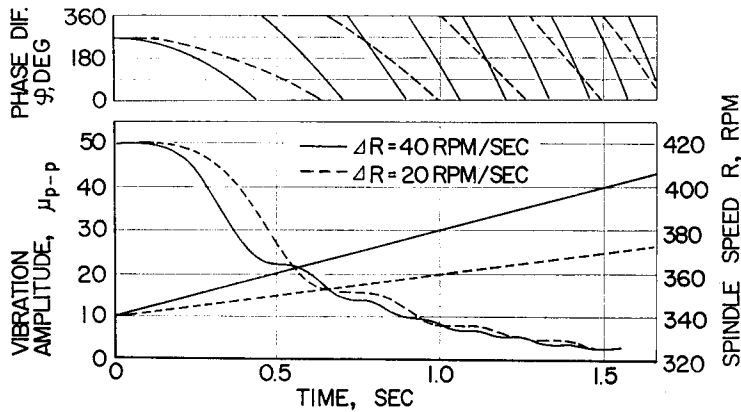


Fig. 3 Typical results of simulation based on simplified model. Equivalent stiffness of structure  $k=50 \text{ kg}/\mu$ , resonance frequency 60 Hz, damping ratio  $\zeta=0.005$ .

### 3. Experimental Equipments and Procedures

#### 3.1 Structural Models for Turning Tests.

##### a) Flexible Arbor.

A long arbor with an attached vibrating mass is mounted between the chuck and the center of a lathe, as shown in Fig. 4. A work disc is attached to one end

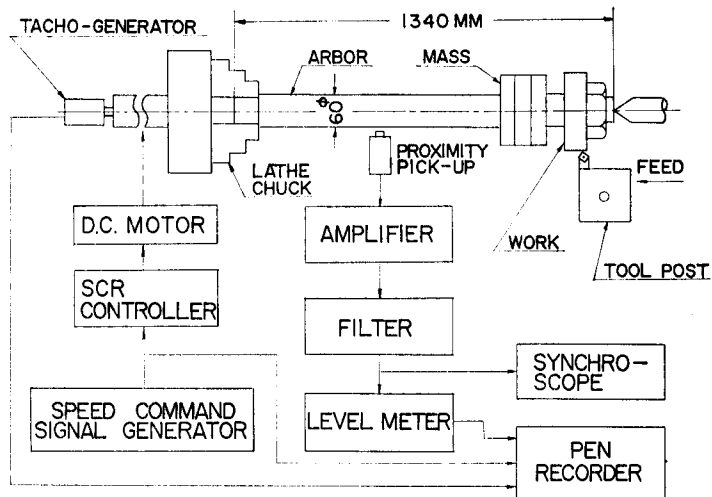


Fig. 4 Illustration of experimental set-up using the flexible arbor.

of arbor, whose periphery is turned by a single point tool. Owing to the flexuous resonance of the arbor, the system is liable to chatter above 0.2 mm depth of cut, when the cutting speed is above 50 m/min and held constant. The main dimensions of the test equipments are as follows:

- length of the arbor: 1340 mm,
- diameter of the arbor: 60 mm,
- resonance frequency of the work system: 60 Hz,
- work diameter: 100 to 180 mm,
- thickness of the work disc: 50 mm,
- work material: steel A (0.45% C class plain steel).

b) Flexible Tool Bar.

A tool bar supporting a single point tool is used to turn the outer periphery of a work bar rigidly supported by the lathe chuck. In this case, the system is liable to chatter due to the flexuous resonance of the tool bar above 0.2 mm depth of cut, when the cutting speed is above 60 m/min and held constant. The main dimensions of this test setup are as follows:

- overhang of the tool bar from the clamp end: 250 mm,
- diameter of the tool bar: 40 mm,
- resonance frequency of the tool system: 276 Hz,
- work diameter: 50 to 60 mm,
- work material: steel B (0.45% C class plain steel).

The chemical contents and hardness of the work steels are listed in Table 1.



Table 1. List of 0.45% C class plain steels used for cutting tests

work materials	C	Si	Mn	P	S	Cu	Cr	Bhn	Used for tests with
STEEL A	0.49	0.38	0.68	0.022	0.024	0.14	0.12	225	flexible arbor
STEEL B	0.45	0.24	0.73	0.031	0.014	0.10	0.04	191	flexible tool bar

### 3.2 Spindle Drive System

The lathe spindle is powered by a variable speed D. C. motor rated at 11kw, regulated by an S. C. R. speed controller. The D.C. motor is designed to run at a speed proportional to the input voltage applied to the speed controller. The motor speed can be controlled from 0 to 3600 rpm by varying the input voltage from 0 to 6 volts. Through belts, this D. C. motor is connected to the intake pulley of the spindle gear box of a conventional engine lathe.

The transfer function of the spindle speed variation, in response to the harmonic variation of the input voltage applied to the S. C. R. controller, is shown in Fig. 5. It shows that the transfer function is that of a first order delay system with a time constant of 0.64 sec.

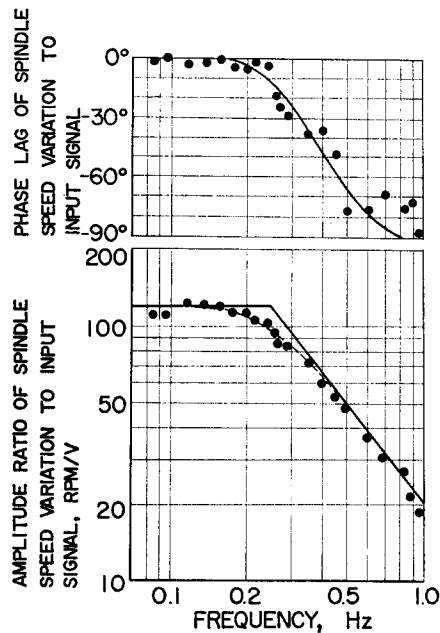


Fig. 5 Frequency response of spindle speed control system. Sinusoidal input signal of  $2V_{p-p}$  is applied to S.C.R. motor speed controller, mean spindle speed 400 rpm.

### 3.3 Triangular Wave Signal Generator

A spindle speed variation in a triangular wave form is commanded by a triangular wave signal generator connected to the S. C. R. controller. In this type of speed variation, the rate of the speed variation is held constant, except at the top and bottom points of the wave.

By setting dials on the front panel, adjustments are made of the mean voltage of the triangular wave, which gives the average speed of the spindle, the range of voltage variation, and the increasing and decreasing slopes of the wave, which are kept equal to each other throughout the experiments in this paper. The main part of this signal generator is an integrator, whose input is switched from a positive voltage to a negative voltage, and vice versa alternately under the control of several comparators.

### 3.4 Dimensions of the Tools used for Turning Tests

- a) Cutting Tests with an Arbor
- |                         |             |
|-------------------------|-------------|
| end cutting edge angle  | 45 deg.     |
| side cutting edge angle | 45 deg.     |
| side rake angle         | -5 deg.     |
| relief angle            | 5 deg.      |
| nose radius             | 0.8 mm      |
| tool material           | carbide P20 |
- b) Cutting Tests with a Tool Bar
- |                         |             |
|-------------------------|-------------|
| end cutting edge angle  | 15 deg.     |
| side cutting edge angle | 15 deg.     |
| side rake angle         | -6 deg.     |
| relief angle            | 6 deg.      |
| nose radius             | 0.4 mm      |
| tool material           | carbide P20 |

### 3.5 Monitoring System

During the turning tests with the flexible arbor, the amplitude of vibration is monitored by a proximity pickup located at the middle point of the arbor. The amplitude monitored at this point is known to be 3.4 times greater than that at the workpiece for the resonance mode at 60 Hz. During the tests with the flexible tool bar, the vibration amplitude is monitored by an acceleration pickup located at the end of the tool bar.

The variation of the vibration amplitude, the spindle speed measured by the tachogenerator and also the speed command signal are recorded on a pen recorder during the experiments.

## 4. Experimental Results

### 4.1 Typical Effect demonstrated

After the regenerative chatter reached a constant amplitude during the cutting of the workpiece attached to one end of the flexible arbor at a constant speed, the spindle speed was made to vary by a triangular speed command signal. The variation of the vibration amplitude measured is shown in Fig. 6. It shows clearly that the amplitude reduces rapidly soon after the start of the speed variation.

### 4.2 Parameters affecting the Effectiveness of Chatter Suppression

The turning tests were carried out to evaluate the influence of the following parameters on chatter suppression.

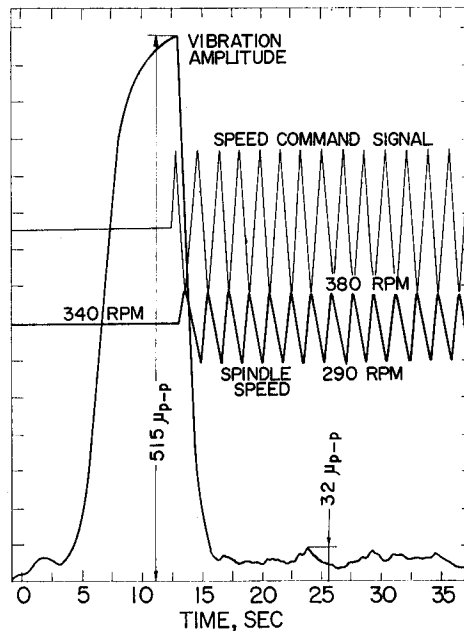


Fig. 6 Typical effect of chatter suppression by programed spindle speed variation.

Speed command signal triangular wave, depth of cut 0.5mm, feed 0.2mm/rev, work diameter 147.5mm, average cutting speed 154 m/min, rate of spindle speed variation 95rpm/sec, work material steel A, test with the flexible arbor.

### a) Rate and Range of Spindle Speed Variation

The periphery of the workpiece, attached to one end of the flexible arbor, was turned at various rates of spindle speed variation across various ranges commanded by triangular wave signals. The vibration amplitudes measured are shown in Fig. 7. It shows that the range of the speed variation has little influence on the effectiveness, as long as the speed is varied across a range wider than 20% of the average speed. It also shows that a higher rate of speed variation is more effective.

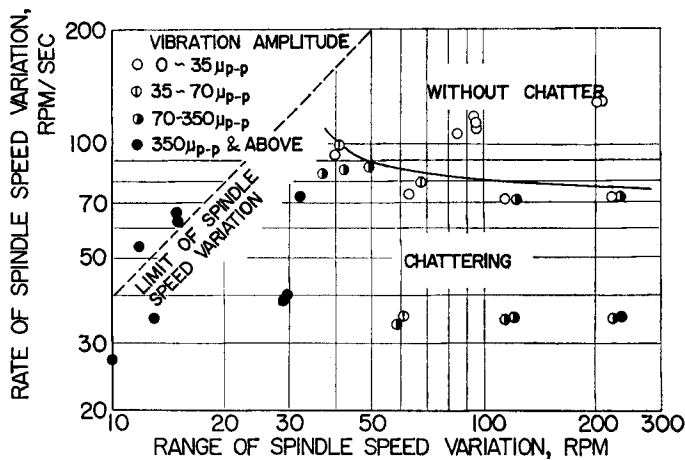


Fig. 7 Vibration amplitude at various rates and ranges of spindle speed variation. Speed command signal triangular wave, depth of cut 0.5mm, feed 0.2mm/rev, work diameter 145 to 155mm, average cutting speed 160m/min, average speed of spindle 340rpm, work material steel A, test with the flexible arbor.

### b) Depth of Cut

Turning tests with the flexible arbor were also carried out at various depths of cut and rates of triangular speed variation. As shown in Fig. 8, the vibration amplitudes under the effect of speed variation are less for a shallower cut.

Since the circle of the dynamic cutting stiffness is larger at a deeper cut, it is expected that the stiffness response loci of the cutting process and the structure will have a greater area of mutual intersection. For this reason, it generally results in a greater amplitude of vibration even under a varying spindle speed.

The figure also shows that a higher rate of spindle speed variation is more effective to suppress the chatter. Thus, a higher rate of speed variation is needed to suppress the chatter at a deeper cutting.

### c) Cutting Speed

The vibration amplitudes of the flexible tool bar were measured under the

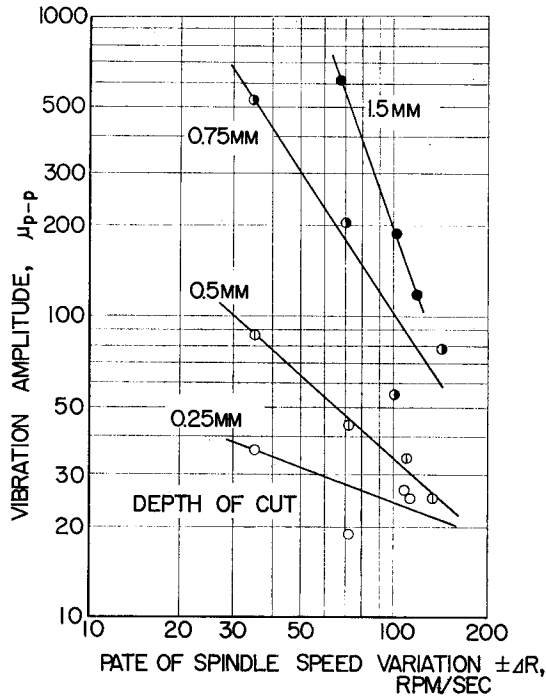


Fig. 8 Suppression of chatter amplitude at various depths of cut. Speed command signal triangular wave, feed 0.2mm/rev, work diameter 150 to 180mm, average cutting speed 135m/min, average speed of spindle 260rpm, range of spindle speed variation 150 to 370rpm, work material steel A, test with the flexible arbor.

effect of a triangular speed variation at various average cutting speeds and rates of speed variation. As seen in Fig. 9, the maximum chatter amplitude tends to be greater at a higher cutting speed. Thus, a higher rate of speed variation is also needed to suppress the chatter at a higher cutting speed.

#### d) Wave Form of Speed Command Signal

Fig. 10 shows that either the rectangular wave or the multi-stepped wave used as the speed command signal, is also effective to suppress the chatter. Furthermore, the rectangular wave is more effective than the multi-stepped wave. To show the practical effectiveness of the speed variation, the vibration amplitudes at constant speeds are illustrated in the same figure.

The chatter suppression by a rectangular signal has the following advantages.

- 1) A rectangular wave is easy to generate.
- 2) The speed variation by a rectangular signal is most effective, because the speed is varied at a maximum rate of variation possible with the given set of the spindle drive system.

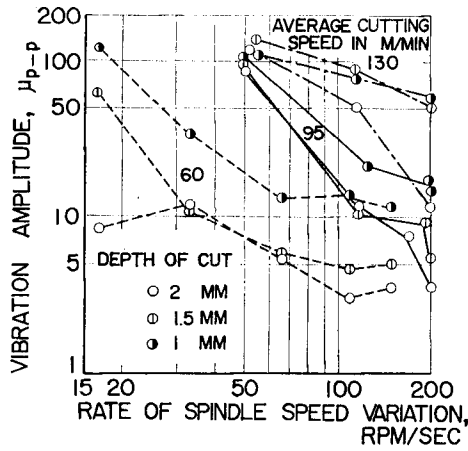


Fig. 9 Suppression of chatter amplitude at various cutting speeds. Speed command signal triangular wave, feed 0.1mm/rev, work diameter 55mm, average cutting speed 60m/min (for 350rpm), 95m/min (for 550rpm), and 130 m/min (for 750rpm), widths of spindle speed variation 200rpm, work material steel B, test with the flexible tool bar.

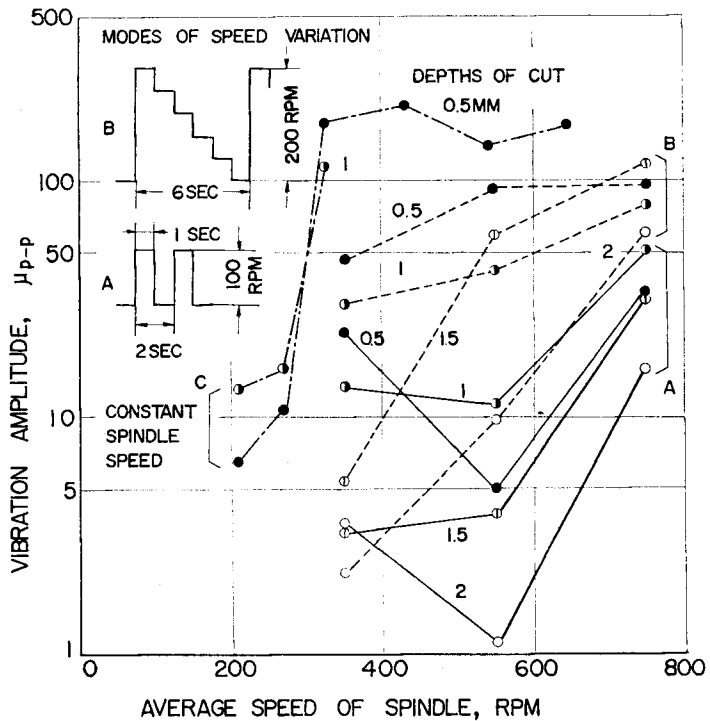


Fig. 10 Suppression of chatter amplitude by rectangular and multi-stepped signals. Feed 0.1mm/rev, work diameter 55mm, work material steel B, test with the flexible tool bar.

## 5. Conclusions

The following conclusions are obtained in this paper.

- a. For the turning operation at an unstable condition, where the regenerative-type self-excited chatter occurs, the vibration amplitude is suppressed by varying the spindle speed.
- b. A higher rate of speed variation is more effective for chatter suppression.
- c. The range of the speed variation has little influence on the effectiveness, as long as the speed is varied across a range wider than 20% of the average speed.
- d. Generally speaking, a higher rate of speed variation is needed to suppress the chatter at a deeper cutting, or at a higher cutting speed.
- e. The spindle speed variation commanded by any one of the triangular, rectangular, and multi-stepped signals is effective. In particular, the rectangular wave signal has the best performance, because the spindle speed can be commanded to vary at the highest rate.

Compared to several other control-type methods attempted to date<sup>(3-11)</sup> toward the active suppression of chatter, the present method seems to have particular potentials for practical applications due to following features:

- (1) the only major hardware required is a continuous speed drive D. C. motor system, which is already accommodated with many machine tools,
- (2) because of its open-loop principle, the effect of the control appears directly and exhibits no instability due to the control system,
- (3) and finally, the method is versatile because no additional hardware materials, such as sensors or actuators, are located in the vicinity of the chip producing area.

The present method is expected to find applications in the roughing operations of turning, vertical boring, radial drilling and boring.

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