Fatigue Deformation Preceding Fracture Under Combined Cyclic and Steady Loads

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Abstract

In the problems of materials fatigue, the most researches have been made on fatigue brittle fracture. But when both the steady and cyclic loads work on the specimen, the fatigue deformation (creep) occurs and it often becomes larger than that under the static load of the same maximum value. So the behavior of the fatigue deformation must be investigated to clarify the mechanism of the fatigue fracture. In this paper the investigations were made to obtain and consider the fatigue deformation at the room temperature under push-pull loads with various stress amplitudes and various mean stresses on ferrous and non-ferrous metals. Opposed to the ordinary concepts, the permanent plastic tensile strain is observed to be generated even under completely reversed push-pull loads. The stress generating no fatigue deformation is in the range with the compressive mean stress. The fatigue yield point is generally different from the static yield point, and also the value of fatigue deformation under the tensile mean stress is different from the same absolute value of that under the compressive mean stress. It seems to be materials nature. The new relations between stress conditions and fatigue deformations were discovered and represented by a criterion.

1. Introduction

The safe stress range for fatigue fracture has been represented by the Goodman relation, the modified Goodman relation, the Soderberg relation, the Gerber relation, and so on[1]. These relations can be represented generally by

$$\sigma_a = \sigma_f \{1 - (\sigma_m/\sigma_u)^\alpha\} \tag{1}$$

where σ_a is the stress amplitude, σ_m the mean stress, and σ_f, σ_u and α the materials constants. Taking the yield limit σ_y into consideration, the relation

$$\sigma_a + \sigma_m \leqq \sigma_y \tag{2}$$

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is added conventionally to the safe stress range for fatigue. Eq. (2) means that the safe maximum stress, $\sigma_a+\sigma_m$, should be smaller than the static yield stress. However, as one of the authors[2] and others[3),4),5] have pointed out, sometimes the fatigue yield limit comes into a range smaller than that given by Eq. (2).

Nowadays, in many cases the fatigue strength at the limited number N cycle (strength for constant lifetime fatigue or low cycle fatigue) is commonly used for engineering design. In these cases the consideration on the fatigue deformation becomes very important, because the large fatigue deformation may occur before the fatigue fracture under the large cyclical and steady stress.

In this paper the authors clarified experimentally the relation between σ_m, σ_a and the fatigue plastic deformation at room temperature under push-pull loads, and considerations are made on the results.

2. Experimental procedure

The materials applied to the tests were a mild steel and brass. The former was taken as a sample of the ferrous metal and the latter the non-ferrous metal. The chemical compositions, heat treatment and mechanical properties of the materials are shown in Tables 1, 2 and 3.

All the fatigue tests were carried out under pushpull loads at room temperature. An electro-magnetic type fatigue testing machine was used, the load frequency being 1800 cpm. The size of the specimen is 4.5 mm in diameter and 20 mm in gage length.

To obtain fatigue deformation, the mean stress was taken at various stress levels of tension and compression. The fatigue deformation was measured at many stages in the process of fatigue by use of a microscopic comparator or two dial gages.

Table 1. Chemical composition of carbon steel

Chemical composition %					Heat treatment
C	Si	Mn	P	S	
0.17	0.27	0.53	0.016	0.018	895°C 1 hr. anneal

Table 2. Chemical composition of brass

Chemical composition %				Heat treatment
Cu	Fe	Pb	Zn	
60.75	0.02	0.01	Bal.	as-rolled

Table 3. Mechanical properties of materials tested

Materials	Mechanical properties*				
	Yield point kg/mm^2	Ultimate strength kg/mm^2	Breaking stress on final area kg/mm^2	Elongation %	Reduction of area %
Carbon steel	24.9**	42.7	88.2	39.0	65.4
Brass	34.0***	40.8	84.1	40.6	58.2

* Values obtained on specimen No.4, JIS Z 2201
** Lower yield point
*** 0.2% off set stress

To measure a fine deformation a mirror instrument was also applied. The fatigue deformation in this paper means the residual permanent strain in the gage length under no load. The fatigue tests were conducted in the range of number of cycles to fracture N over 10^4.

3. Experimental results.

3.1 Increasing behavior of fatigue deformation

The increasing behavior of the plastic strain during fatigue process is shown schematically in Fig. 1. At the stage I in Fig. 1 the plastic strain increases with the number of cycles n in the process of fatigue, and in most cases the stage ends befor $n=2\times 10^3$. At stage II the plastic strain does not increase, but occasionally in this stage the plastic strain increases stepwise as shown in Fig. 1. At stage III just before the fracture of a specimen, the plastic strain increases rapidly. In this stage

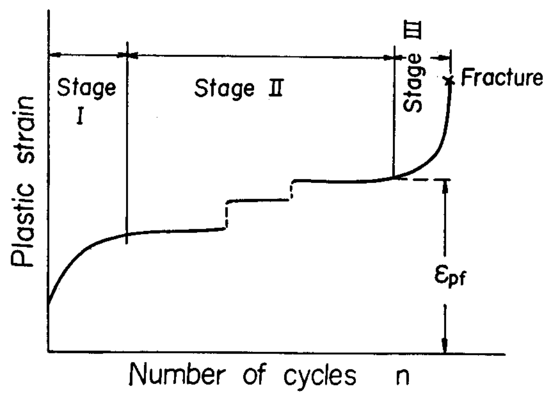

Fig. 1. Schematic representation of process of fatigue deformation

the propagation of the fatigue crack is thought to accelarete the plastic strain. The plastic strain at the end of stage II is taken as the fatigue plastic strain ε_{pf} in this paper.

3.2 Fatigue deformation and stress condition

a. Mild steel

The relation between the mean stress σ_m and the fatigue plastic strain ε_{pf} is shown in Fig. 2, taking the stress amplitude σ_a as a parameter. The curve of $\sigma_a=0$ shows the stress-strain curves under static tension and compression tests; both curves have the same yield points 26 kg/mm^2. However, while the yield range for tension holds 1% in strain, that for compression is very narrow because the nominal compressive stress increases due to the increase of the sectional area of a specimen by yielding.

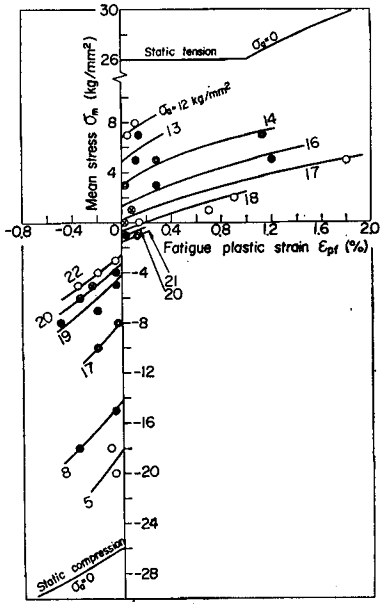

Fig. 2. σ_m–ε_{pf} diagram (Mild steel)

It is seen in Fig. 2 that the fatigue plastic strain ε_{pf} increases with the increase of the mean stress σ_m for the constant stress amplitude σ_a and the mean stress required to generate the same fatigue plastic strain ε_{pf} decreases with the increase of the stress amplitude σ_a. An attention should be paid to the results in Fig. 2 the mean stress generating no fatigue plastic strain is not zero but -2kg/mm^2, and this value seems to be independent of the stress amplitude, and that under the completely reversed stress condition, that is, $\sigma_m=0$, tensile deformation occurs.

The relation between the stress amplitude σ_a and the mean stress σ_m is shown in Fig. 3, taking the fatigue plastic strain ε_{pf} as a parameter. These curves represent the stress limits for the constant fatigue deformation when the allowable ε_{pf} is given. These fatigue deformation limit curves are not symmetrical against the ordinate axis. At $\sigma_m = -2$ kg/mm², it seems to be allowable for σ_a to become very large, because the asymptote of all the constant fatigue deformation limit curves is $\sigma_m = -2$ kg/mm². But, really, when σ_a becomes too large, stage III in Fig. 1 takes a large part of the fatigue life. Therefore, a very large deformation occurs until destruction, even if the fatigue plastic strain just before the stage III is very small. Indeed in the range of the large strain amplitude at $\sigma_a = -2$ kg/mm², the fatigue plastic strain takes a positive or negative value. Consequently it can be said that the deformation at $\sigma_m = -2$kg/mm² is somewhat unstable.

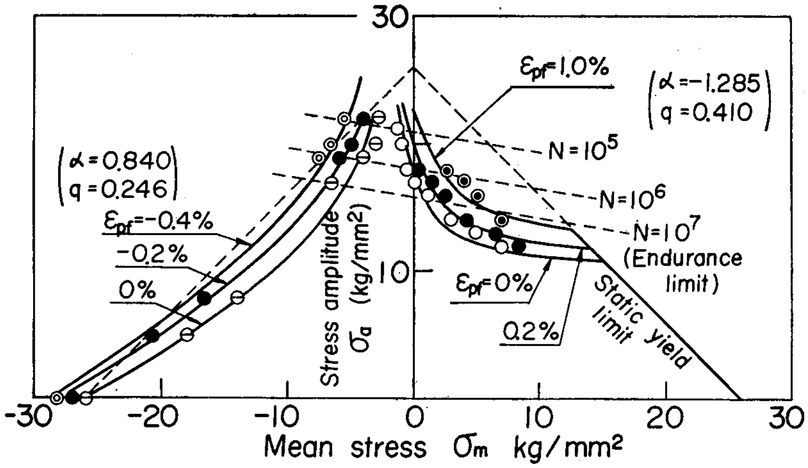

Fig. 3. Fatigue deformation limit diagrams (Mild steel)

It is apt to be thought that the genration of the tensile fatigue deformation under zero mean stress is caused by the variation of the true sectional area of the specimen, that is, the sectional area decreases for the tensile load and it increases for the compressive load, so the true stress is not completely reversed even if the nominal mean stress is zero. But the thought is not reasonable, because the variation of the sectional area is negligibly small when $\varepsilon_{pf}=0$. As seen in Fig. 3, the asymptote generating no fatigue plastic strain is independent of the value of ε_{pf}. The result is thought to indicate that the difference of the resistance of materials for the fatigue deformation between tensile and compressive loads is the substsntial nature of materials. It is quite similar to the fact that the fatigue strength under compressive mean stress is higher than that under tensile mean stress.

When the mean stress takes a large positive value, the fatigue plastic strain becomes independent of the value of σ_{pf}, that is, the limit curves do not smooth and coincide with one straight line inclined 45 degree to the abscissa as shown in Fig. 3. This straight line is the limit at which the maximum stress, $\sigma_a+\sigma_m$, is equal to the static yield stress. The result under the large positive mean stress seems to be the nature of the materials with a clear yield point, and it has been already shown by one of the authors[2]. In the case of the compressive mean stress, a similar phenomenon is not seen, but the fatigue deformation limit curves are different according to the value of ε_{pf}. It corresponds with the result that the static stress-strain curve under compression has a smaller yield range than that under tension. Moreover, in the compression side the curves showing the generation of a constant fatigue plastic strain ε_{pf} incline to abscissa with less angle than 45 degree. Therefore, the fatigue plastic strain under the compressive mean stress is larger than that under the static compressive stress of the same absolute maximum value. For reference the lines representing the limits for fatigue fracture at $N=10^5$, 10^6 and 10^7 that is the so-called endurance limit are also shown with broken lines in Fig. 3.

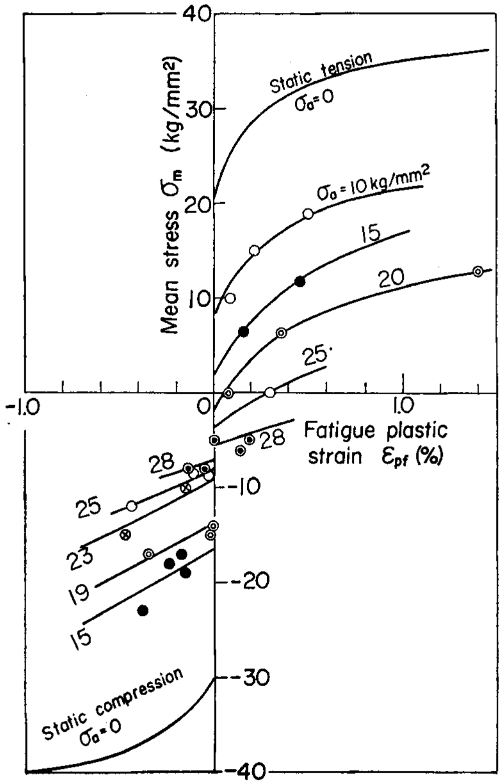

Fig. 4. σ_m–ε_{pf} diagram (Brass)

b. Brass

Fig. 4 shows the relation between σ_m and ε_{pf} on the brass similar to Fig. 2 on the steel, taking σ_a as a parameter. The brass has no clear yield point, and the static stress-strain curve under compression is different from that under tension. The elastic limit for compression is larger than that for tension. The mean stress generating no fatigue plastic strain is about $\sigma_m = -6$ kg/mm^2 for any stress amplitude, the absolute value of which is larger than that for mild steel, that is, -2 kg/mm^2. For $\sigma_m = 0$, a large tensile deformation occurs.

The relation between σ_a and σ_m is shown in Fig. 5, taking ε_{pf} as parameter. All the constant ε_{pf} curves have the asymptote $\sigma_m = -6$ kg/mm^2, and they do not coincide with one straight line inclined 45 degrees to the abscissa even for a large tensile mean stress. It is dessimilar to the mild steel and seems to be of the nature of mateials without a clear yield point. For reference the lines inclined 45 degrees to the abscissa and the lines representing the limits for fatigue fracture at $N = 10^5$, 10^6 and 10^7 with broken lines.

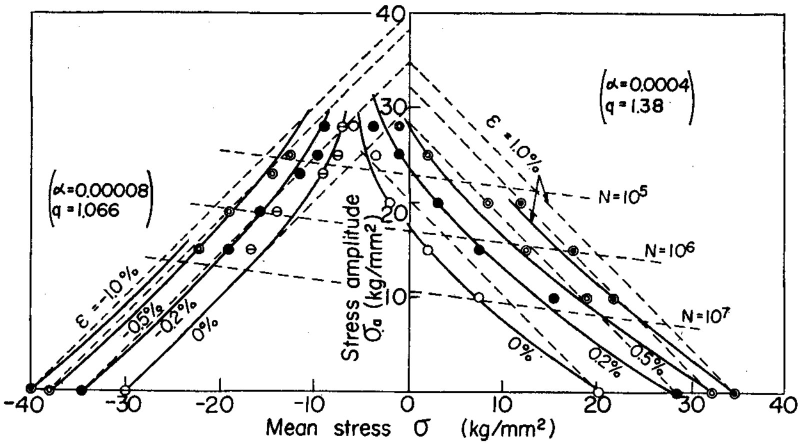

Fig. 5. Fatigue deformation limit diagrams (Brass)

4. Consideration

Consideration is made to induce the criterion which represents the relation among the mean stress σ_m, the stress amplitude σ_a, and the fatigue plastic strain ε_{pf}. In a special case when the stress amplitude is extremely small, that is, $\sigma_a = 0$, the relation between σ_m and ε_{pf} has to coincide with that between the static stress σ and the plastic strain ε_p due to the static stress. Then we derive the relation under the fatigue tests from that under the static tests. Generally the relations between σ and

ε_p in the static tests may be divided into the following two cases, depending on the presence or absence of the yield point. When the clear yield point does not exist

$$\varepsilon_p = A\,(\sigma - \sigma_e)^n, \quad \sigma \geqq \sigma_e \tag{3}$$

and when the clear yield point exists

$$\varepsilon_p = \varepsilon_y + A'\,(\sigma - \sigma_y)^{n'}, \quad \sigma \geqq \sigma_y \tag{3a}$$

σ_e being the elastic limit, σ_y the yield point and ε_y the plastic strain at the end of yield. In these equations A, n, A', and n' are the material constant determined by the static tests. Under the work of the cyclic load, the material with a clear yield point usually does not to show the clear yield point. So in the case of fatigue the materials show the same stress-strain relation as Eq. (3) instead of Eq. (3a) in the range of stress condition of the disappearance of yielding. Applying the thought to the case of fatigue we substitute $\sigma_m+k\sigma_a$ for σ in Eq. (3); that is

$$\sigma = \sigma_m + k\,\sigma_a \tag{4}$$

In Eq. (4), if $k=1$, σ becomes equal to $\sigma_a+\sigma_m$, therefore in this case it means that the fatigue plastic strain ε_{pf} is determined only by the maximum value of the applied alternating stress. However, the fatigue plastic strain cannot be considered to depend only on the maximum stress $\sigma_m+\sigma_a$, but it may be considered to be influenced by the stress amplitude. Further, since the fatigue deformation limit curves have the asymptote as shown in Figs. 3 and 5, we put k as follows:

$$k = \alpha\,(\sigma_m - \sigma_{m0})^q \tag{5}$$

where α and q are the constants, while σ_{m0} represents the asymptote. Applying the relations of Eqs. (4) and (5) into Eq. (3), the fatigue plastic strain is generally obtained as follows:

$$\varepsilon_{pf} = A\,[\sigma_m + \alpha\,(\sigma_m - \sigma_{m0})^q\,\sigma_a - \sigma_e]^n \tag{6}$$

But for the material with a clear yield point, when the mean stress takes the large positive value, the fatigue plastic strain is given by Eq. (3a) instead of Eq. (6), that is

$$\varepsilon_{pf} = \varepsilon_y + A'\,(\sigma_m + \sigma_a - \sigma_y)^{n'}, \quad \sigma_m + \sigma_a \geqq \sigma_y \tag{7}$$

In Eqs. (6) and (7), the materials constants α, q and σ_{m0} should be determined from the fatigue tests, while A, n, A' and n' should be determined from the static tests as mentioned previously. In Eq. (7), if the maximum stress $\sigma_a+\sigma_m$ is equal to σ_y, the fatigue plastic strain takes an arbitrary value smaller than ε_y, that is, in the case the constant fatigue deformation curves coincide with one straight line inclined 45 degree to the abscissa passing through the point representing σ_y.

Putting $\varepsilon_{pf}=0$ in Eq. (6), the condition in which the fatigue plastic strain begins to appear is obtained as follows:

$$\sigma_m + \alpha\,(\sigma_m - \sigma_{m0})^q\,\sigma_a - \sigma_e = 0 \tag{8}$$

If the fatigue plastic strain ε_{pf} of Eq. (6) is equal to the plastic strain generated by the static stress σ_r, the following relation can be obtained:

$$\sigma_m + \alpha\,(\sigma_m - \sigma_{m0})^q\,\sigma_a - \sigma_r = 0 \tag{9}$$

The fatigue deformation limit curves shown in Figs. 3 and 5 are drawn from calculations of Eqs. (7), (8) and (9) for some values of ε_{pf}, taking the values of the materials constants α and q as shown in those figures. We can see that the curves obtained from the calculation are in good agreement with the results of experiments. For mild steel, in the range of a large tensile mean stress, Eq. (7) should be applied instead of Eqs. (8) and (9). Thus the above criterion is proved to be appropriate to express the safe stress range for the fatigue plastic strain.

5. Conclusion

To make a contribution for clarifying the mechanism of fatigue fracture and also for engineering designs, the fatigue deformation preceding fracture under various mean stresses in addition to various cyclic stresses was measured on a ferrous and a non-ferrous metals at room temperature under push-pull loads.

Oppossed to general concepts, the permanent plastic tensile strain was discovered to be generated even under the completely reversed push-pull loads, and the stress generating no fatigue deformation was in the range of a compressive mean stress. It seems to be the materials nature and not caused by the difference of true stresses under tension and compression. A new relation between the stress conditions and the fatigue deformations was obtained and represented by a criterion.

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