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CITATION:

UMOTO, Jūrō...[et al]. Analysis of a Thyristor Chopper Circuit of Forced Commutation Type. *Memoirs of the Faculty of Engineering, Kyoto University* 1970, 32(2): 156-174

ISSUE DATE:

1970-09-10

URL:

<http://hdl.handle.net/2433/280815>

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# Analysis of a Thyristor Chopper Circuit of Forced Commutation Type

By

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(Received December 23, 1969)

In this paper, we investigate theoretically and experimentally a thyristor chopper circuit of forced commutation type, which is named the oscillation circuit type chopper. Considering the power source inductance and the internal resistances of circuit elements, we analyze the chopper circuit strictly, compare the digital computation results with the experimental ones and find numerically the optimum circuit conditions.

## 1. Introduction

As in recent years we can produce the thyristors of good characteristics and big capacity, chopper circuits with the thyristors are used extensively as variable d.c. power source, especially for speed control of d.c. motors<sup>1)</sup>.

Thyristor chopper circuits are the ones which control the d.c. power that is supplied to the load, by interrupting it by means of a thyristor. We can classify the circuits into the self-commutation- and the forced-commutation-types. The former is represented by the Morgan circuit<sup>2,3)</sup>, and the latter by the Jones circuit and the oscillation circuit type chopper<sup>4,5)</sup>. As the Morgan circuit does not need an auxiliary commutation thyristor, the trigger circuit can be simplified, but since the on-time of the main thyristor is determined by the circuit constants, in the case of the fixed repetition frequency, the on-time control is complicated. The Jones circuit is stable for load fluctuations, but it has the defect that the saturable auto-transformer conducts main current and so the optimum operating frequency becomes low. On the other hand, the oscillation circuit has the advantages that the main thyristor only conducts the main current, the circuit can operate till the relative high frequency and it presents stable transient performances.

Now as strict analyses of the oscillation circuit type chopper have not been done, in which are considered all the modes to appear in the circuit, in this paper

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considering the power source inductance and the internal resistance of every element, etc. with respect to the thyristor chopper, we derive theoretically exact solutions of the fundamental circuit equations, compare the digital computation results with the experimental ones and discuss the characteristics of the circuit.

## 2. Theoretical Analysis

### 2.1 Working Modes

Fig. 1 shows a chopper circuit, which is called the oscillation circuit type chopper. In this circuit,

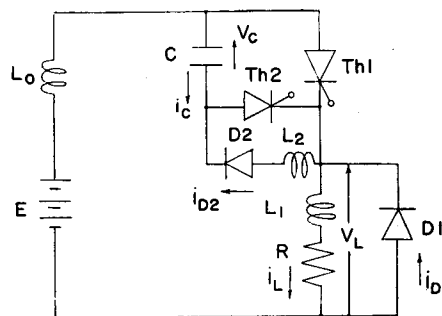


Fig. 1. Oscillation circuit type chopper.

*Th1* and *Th2* : main and auxiliary commutation thyristors,

*D1* and *D2* : diodes,

$L_0$  : power source inductance,

$L_2$  : inductance,

$L_1$  and  $R$  : load inductance and resistance,

$C$  : capacitance,

$E$  : d.c. power source voltage,

$v_L$  and  $i_L$  : load voltage and current,

$v_C$  and  $i_C$  : condenser voltage and current,

$i_{D1}$  and  $i_{D2}$  : currents through *D1* and *D2*.

Next Fig. 2 (a) to (h) illustrate every circuit mode, which the circuit in Fig. 1 gives. Namely

Mode I : *Th1* is gated on in the situation that  $C$  is charged positively, the d.c. power is supplied to the load from the source and in this duration the polarity of  $v_C$  is reversed.

Mode II :  $v_C$  is kept negative by *D2*, and meanwhile the source is supplying the power to the load through *Th1*.

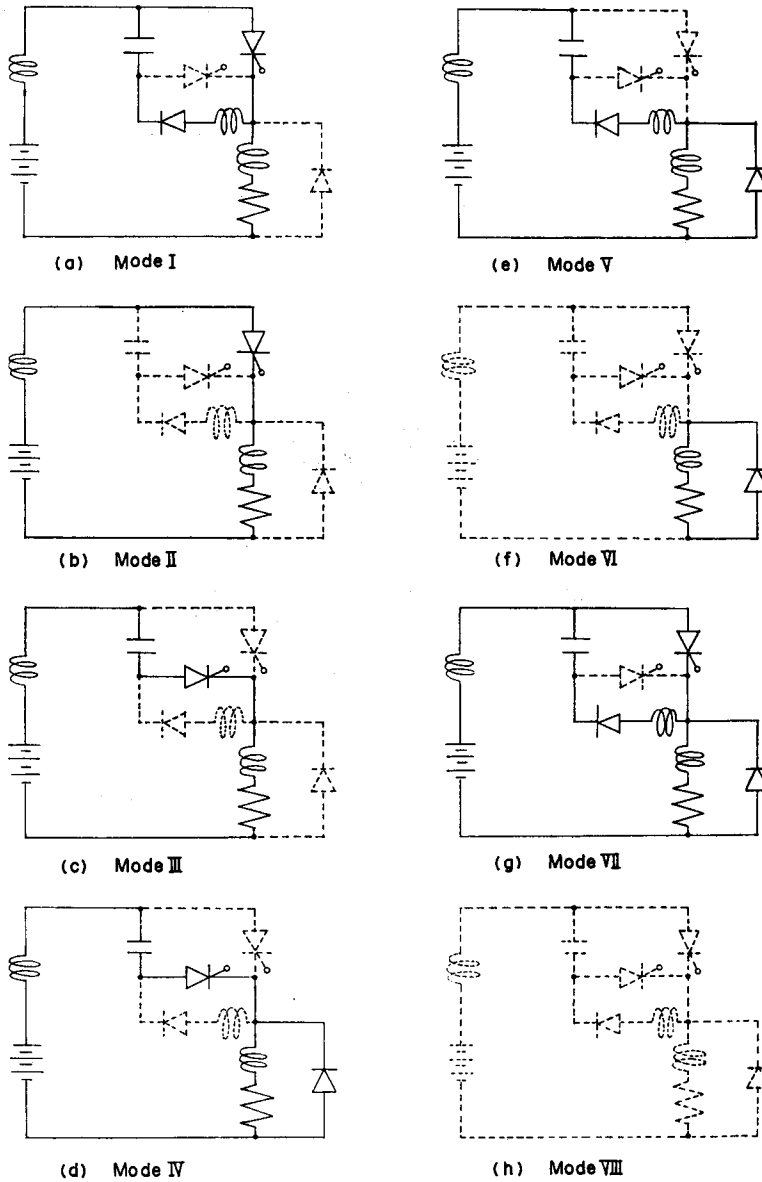
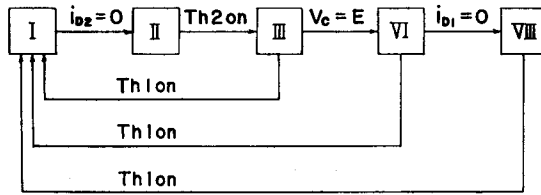


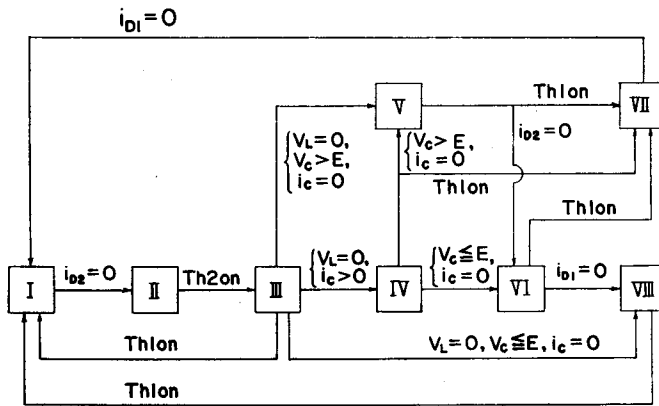
Fig. 2. Situation of every circuit mode.

Mode III :  $Th 2$  is triggered,  $v_c$  appears as inverse voltage across  $Th 1$ , hence  $Th 1$  is turned off and  $C$  is charged positively again.

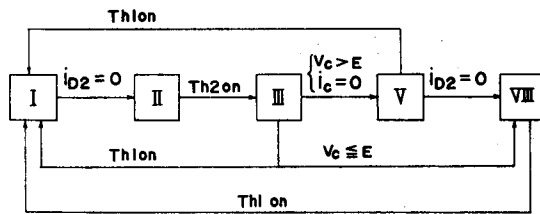
Mode IV : if in mode III  $v_L$  decreases to zero and  $D 1$  conducts the current, mode IV comes about while  $i_c$  is flowing. This mode exists in the case, where the conductive portion in mode III becomes



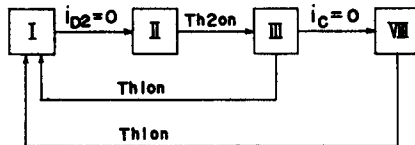
(a)  $L_0=0, L_1 \neq 0$



(b)  $L_0 \neq 0, L_1 \neq 0$



(c)  $L_0 \neq 0, L_1=0$



(d)  $L_0=0, L_1=0$

Fig. 3. Mode transitions and criteria.

oscillatory by  $L_0$  and  $C$ . In transition from mode III to IV,  $i_c$  can not drop to zero discontinuously due to  $L_0$ . In this interval,  $v_c$  is higher than  $E$ .

Mode V : in consequence that  $v_c$  is higher than  $E$  and the circuit oscillatory by  $L_0$  and  $C$ ,  $i_c$  flows toward the source inversely.  $Th 2$  turns off because of its inverse voltage. At the end of this mode,  $v_c$  becomes lower than  $E$ .

Mode VI : charging current  $i_c$ , which is going to flow once again, is checked by  $D 2$  and  $i_c$  circulates through  $D 1$  due to the existence of  $L_1$ .

Mode VII : in the case where  $Th 1$  is gated on in mode IV, V or VI, mode VII presents itself as the transient state before moving to mode I.

Mode VIII: the situation that all thyristors and diodes are off.

Next, we show the mode transitions and the criteria in Fig. 3.

## 2.2 Fundamental Circuit Equations and Their Solutions

The fundamental differential equations for all circuit modes shown in Fig. 2 become as follows:

for mode I:

$$\left. \begin{aligned} E &= (L_0 + L_1) \frac{di_{L1}}{dt} + (R + r_1)i_{L1} + r_6 i_{D21}, \\ i_{C1} &= C \frac{dv_{C1}}{dt}, \\ v_{C1} &= L_2 \frac{di_{D21}}{dt} + r_2 i_{D21} + r_6 i_{L1}, \\ i_{C1} + i_{D21} &= 0, \\ i_{D11} &= 0, \end{aligned} \right\} \quad (1)$$

for mode II:

$$\left. \begin{aligned} E &= (L_0 + L_1) \frac{di_{L2}}{dt} + (R + r_1)i_{L2}, \\ i_{C2} &= 0, \\ C \frac{dv_{C2}}{dt} &= 0, \\ i_{D12} &= 0, \\ i_{D22} &= 0, \end{aligned} \right\} \quad (2)$$

for mode III:

$$\left. \begin{aligned} E &= (L_0 + L_1) \frac{di_{L3}}{dt} + (R + r_3)i_{L3} + v_{C3}, \\ i_{L3} - i_{C3} &= 0, \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} i_{C3} &= C \frac{dv_{C3}}{dt}, \\ i_{D13} &= 0, \\ i_{D23} &= 0, \end{aligned} \right\}$$

for mode IV :

$$\left. \begin{aligned} E &= L_0 \frac{di_{C4}}{dt} + r_3 i_{C4} - r_4 i_{D14} + v_{C4}, \\ i_{C4} &= C \frac{dv_{C4}}{dt}, \\ Ri_{L4} + L_1 \frac{di_{L4}}{dt} + r_4 i_{D14} &= 0, \\ i_{D14} + i_{C4} - i_{L4} &= 0, \\ i_{D24} &= 0, \end{aligned} \right\} \quad (4)$$

for mode V :

$$\left. \begin{aligned} E &= -(L_0 + L_2) \frac{di_{D25}}{dt} - r_4 i_{D15} - r_5 i_{D25} + v_{C5}, \\ i_{C5} &= C \frac{dv_{C5}}{dt}, \\ Ri_{L5} + L_1 \frac{di_{L5}}{dt} + r_4 i_{D15} &= 0, \\ i_{C5} + i_{D25} &= 0, \\ i_{D15} - i_{D25} - i_{L5} &= 0, \end{aligned} \right\} \quad (5)$$

for mode VI :

$$\left. \begin{aligned} (R + r_4) i_{L6} + L_1 \frac{di_{L6}}{dt} &= 0, \\ i_{C6} &= 0, \\ C \frac{dv_{C6}}{dt} &= 0, \\ i_{D16} - i_{L6} &= 0, \\ i_{D26} &= 0, \end{aligned} \right\} \quad (6)$$

for mode VII :

$$\left. \begin{aligned} E &= L_0 \frac{d}{dt} (i_{L7} - i_{D17}) + r_1 (i_{L7} - i_{D17}) - r_4 i_{D17} + r_6 i_{D27}, \\ i_{C7} &= C \frac{dv_{C7}}{dt}, \\ Ri_{L7} + L_1 \frac{di_{L7}}{dt} + r_4 i_{D17} &= 0, \\ i_{C7} + i_{D27} &= 0, \\ v_{C7} &= L_2 \frac{di_{D27}}{dt} + r_2 i_{D27} + r_6 (i_{L7} - i_{D17}), \end{aligned} \right\} \quad (7)$$

for mode VIII :

$$\left. \begin{aligned} i_{L8} &= 0, \\ i_{C8} &= 0, \\ C \frac{dv_{C8}}{dt} &= 0, \\ i_{D18} &= 0, \\ i_{D28} &= 0, \end{aligned} \right\} \quad (8)$$

where

added suffixes 1 to 8: show the mode number,

$$r_1 = r_{Th1} + r_{L0}, \quad r_2 = r_{Th2} + r_{D2} + r_C + r_{L2},$$

$$r_3 = r_{Th2} + r_{L0} + r_C, \quad r_4 = r_{D1},$$

$$r_5 = r_{D2} + r_{L0} + r_C + r_{L2}, \quad r_6 = r_{Th1},$$

$r_{Th1}$  and  $r_{Th2}$ : internal resistance of  $Th1$  and  $Th2$ ,

$r_{D1}$  and  $r_{D2}$ : internal resistances of  $D1$  and  $D2$ ,

$r_{L0}$  and  $r_{L2}$ : internal resistances of  $L_0$  and  $L_2$ ,

$r_C$ : resistance for measurement of  $i_C$ ,

and after solving  $i_L(t)$  with the fundamental equations, the load voltage  $v_L(t)$  is obtained by

$$v_L(t) = L_1 \frac{di_L(t)}{dt} + R i_L(t). \quad (9)$$

Next applying the Laplace transformation to the fundamental circuit equations (1) to (8) and considering the mode changes illustrated in Fig.3 and the stage number, the solutions of voltages and currents are given in the following matrix forms.

$$[w_{rn}(s)] = [\varphi_r(s)] + [X_r(s)][w_{rn}^{-0}], \quad (10)$$

where

$$\left. \begin{aligned} [w_{rn}(s)] &= \begin{pmatrix} i_{Lrn}(s) \\ i_{Crn}(s) \\ v_{Crn}(s) \\ i_{D1rn}(s) \\ i_{D2rn}(s) \end{pmatrix} \\ [w_{rn}^{-0}] &: \text{initial value matrix of} \\ &\quad \text{first kind of } [w_{rn}(t)], \\ r &= 1, 2, \dots, 8: \text{mode number,} \\ n &= 1, 2, \dots: \text{stage number,} \\ s &: \text{Laplace operator.} \end{aligned} \right\} \quad (10)$$

Then by applying the inverse Laplace transformation to Eq. (10), we have

$$[w_{rn}(t)] = [\varphi_r(t)] + [X_r(t)][w_{rn}^{-0}] \quad (11)$$



As it is troublesome to describe  $[\varphi_r(t)]$ 's and  $[\mathcal{X}_r(t)]$ 's of all modes, let us show only the theoretical results for the case of  $\lambda < 1$  in mode III. For mode III, Eq. (11) is given by

$$\begin{pmatrix} i_{L3}(t) \\ i_{C3}(t) \\ v_{C3}(t) \\ i_{D13}(t) \\ i_{D23}(t) \end{pmatrix} = [\varphi_3(t)] + [\mathcal{X}_3(t)] \begin{pmatrix} i_{L3}^{-0} \\ i_{C3}^{-0} \\ v_{C3}^{-0} \\ i_{D13}^{-0} \\ i_{D23}^{-0} \end{pmatrix}, \tag{12}$$

where suffix  $r$  is omitted and

$$\left. \begin{aligned} [\varphi_3(t)] &= \begin{pmatrix} \varphi_{13}(t) \\ \varphi_{23}(t) \\ \varphi_{33}(t) \\ 0 \\ 0 \end{pmatrix}, \\ [\mathcal{X}_3(t)] &= \begin{pmatrix} \mathcal{X}_{113}(t) & 0 & \mathcal{X}_{133}(t) & 0 & 0 \\ \mathcal{X}_{213}(t) & 0 & \mathcal{X}_{233}(t) & 0 & 0 \\ \mathcal{X}_{313}(t) & 0 & \mathcal{X}_{333}(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \end{aligned} \right\} \tag{12}'$$

in which for the case, in which  $\lambda = \frac{R+r_3}{2} \sqrt{\frac{C}{L_0+L_1}} < 1$ , namely the circuit is oscillatory,

$$\left. \begin{aligned} \varphi_{13}(t) &= \frac{E}{L_0+L_1} \frac{\varepsilon^{-\lambda t}}{\nu\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t), \\ \varphi_{23}(t) &= \varphi_{13}(t), \\ \varphi_{33}(t) &= E \left[ 1 - \varepsilon^{-\lambda t} \left\{ \frac{\lambda}{\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t) + \cos(\nu\sqrt{1-\lambda^2} t) \right\} \right], \\ \mathcal{X}_{113}(t) &= \varepsilon^{-\lambda t} \left\{ \cos(\nu\sqrt{1-\lambda^2} t) - \frac{\lambda}{\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t) \right\}, \\ \mathcal{X}_{213}(t) &= \mathcal{X}_{113}(t), \\ \mathcal{X}_{313}(t) &= \frac{\varepsilon^{-\lambda t}}{C\nu\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t), \\ \mathcal{X}_{133}(t) &= -\frac{1}{L_0+L_1} \frac{\varepsilon^{-\lambda t}}{\nu\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t), \\ \mathcal{X}_{233}(t) &= \mathcal{X}_{133}(t), \\ \mathcal{X}_{333}(t) &= \varepsilon^{-\lambda t} \left\{ \cos(\nu\sqrt{1-\lambda^2} t) + \frac{\lambda}{\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t) \right\}, \\ \nu &= 1/\sqrt{C(L_0+L_1)}. \end{aligned} \right\} \tag{12}''$$

In this connection,

$$\begin{aligned}
 v_{L3}(t) &= L_1 \frac{di_{L3}(t)}{dt} + Ri_{L3}(t) \\
 &= \frac{EL_1}{L_0 + L_1} \varepsilon^{-\lambda t} \left\{ \cos(\nu\sqrt{1-\lambda^2} t) + \frac{-\lambda\nu + R/L_1}{\nu\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t) \right\} \\
 &\quad + i_{L3}^{-0} \varepsilon^{-\lambda t} \left[ \left\{ R - \frac{L_1(R+r_3)}{L_0 + L_1} \right\} \cos(\nu\sqrt{1-\lambda^2} t) + \frac{\lambda}{\sqrt{1-\lambda^2}} \right. \\
 &\quad \times \left. \left\{ -R + \frac{L_1(R+r_3)}{L_0 + L_1} - \frac{2L_1}{C(R+r_3)} \right\} \sin(\nu\sqrt{1-\lambda^2} t) \right. \\
 &\quad \left. + v_{C3}^{-0} \varepsilon^{-\lambda t} \frac{L_1}{L_0 + L_1} \left\{ -\cos(\nu\sqrt{1-\lambda^2} t) + \frac{\lambda\nu - R/L_1}{\nu\sqrt{1-\lambda^2}} \sin(\nu\sqrt{1-\lambda^2} t) \right\} \right].
 \end{aligned} \tag{13}$$

In the same way we can get  $[\varphi(t)]$ 's and  $[\chi(t)]$ 's of the other modes.

Next the initial value matrix  $[w_{rn}^{-0}]$  can be determined by means of the analytical method of the periodically interrupted electric circuit of the third genus. Namely from Eq. (11) and Fig. 4, where  $t_r$ 's ( $r=1, 2, \dots, 7$ ) express the duration of the interval of the  $r$ -th mode, we can derive the following equations,

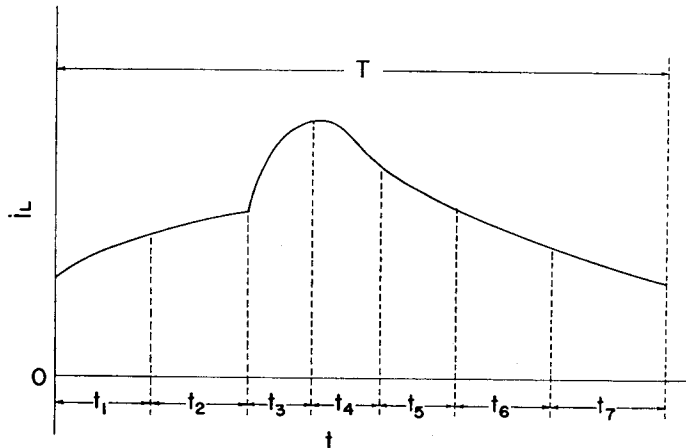


Fig. 4. Duration of interval of each mode.

$$\begin{aligned}
 [w_{rn}^{-0}] &= [\varphi_{r-1, n}] + [\chi_{r-1, n}] [\varphi_{r-2, n}] + [\chi_{r-1, n}] [\chi_{r-2, n}] [\varphi_{r-3, n}] + \dots \\
 &\quad \dots + [\chi_{r-1, n}] [\chi_{r-2, n}] \dots [\chi_{1, n}] [\chi_{7, n-1}] \dots [\chi_{r, n-1}] [w_{r, n-1}^{-0}], \\
 [w_{r, n-1}] &= [\varphi_{r-1, n-1}] + [\chi_{r-1, n-1}] [\varphi_{r-2, n-1}] + \dots \\
 &\quad \dots + [\chi_{n-1, n-1}] [\chi_{r-2, n-1}] \dots [\chi_{1, n-1}] [\chi_{7, n-2}] \dots [\chi_{r, n-2}] [w_{r, n-2}^{-0}] \\
 &\quad \dots \\
 &\quad \dots \\
 [w_{r, 2}^{-0}] &= [\varphi_{r-1, 2}] + [\chi_{r-1, 2}] [\varphi_{r-2, 2}] + \dots \\
 &\quad \dots + [\chi_{r-1, 2}] [\chi_{r-2, 2}] \dots [\chi_{12}] [\chi_{71}] \dots [\chi_{r1}] [w_{r1}^{-0}],
 \end{aligned} \tag{14}$$

where

$$\left. \begin{aligned} [w_{r1}^{-0}] &= [\varphi_{r-1,1}] + [\chi_{r-1}][\varphi_{r-2,1}] + \dots + [\chi_{r-1,1}][\chi_{r-2,1}] \dots [\chi_{21}][w_{11}^{-0}], \\ [w_{12}^{-0}] &= [\varphi_{71}] + [\chi_{71}][\varphi_{61}] + \dots + [\chi_{71}][\chi_{61}] \dots [\chi_{21}][w_{11}^{-0}], \\ [\varphi_{jk}] &= [\varphi_j(t_{jk})], \\ [\chi_{jk}] &= [\chi_j(t_{jk})], \\ j &= 1, 2, \dots, r, \dots, 7, \\ k &= 1, 2, \dots, n, \dots \end{aligned} \right\} (14)$$

Therefore if we give numerically the initial value matrix of the first stage and the first mode, we can calculate the numerical values of the voltages and currents at an arbitrary instant by using Eq. s (11) and (14).

### 3. Numerical Calculations and Experimental Results

As described in Section 2.2, we can numerically calculate the values of the voltages and currents at an arbitrary time, when we give the numerical values of the elements of  $[w_{11}^{-0}]$ , namely the starting values of the voltages and currents. But since the duration of each mode is affected by the magnitudes of voltages and currents, it is impossible to get theoretically the duration of each mode. So let us use the iteration method, which is very practical for a digital computer. It is the method, in which after providing numerically the elements of  $[w_{11}^{-0}]$ , at each calculation time step the criterion of a mode transition are checked, and if the criterion is satisfied, the final values of every voltage and current in the mode are used as the initial ones in the next mode, and if such a way is continued and every voltage and current reach the state, which we can regard as the steady one, the numerical computation is stopped. By this method the circuit in Fig. 1 can be simulated with a digital computer, and we can numerically obtain the values of the voltages and currents at an arbitrary instant and for an arbitrary circuit conditions. Fig. 5 illustrates the digital computer flow-chart.

Now we carry out the numerical calculations with the following combinations of circuit parameters,

- i)  $C = 1 \sim 30 \mu\text{F}$   
 $L_0 = 0 \text{ mH}$ ,  
 $L_1$  : parameter,  
 $r_{L_0} = 0 \Omega$ ,
- ii)  $C = 5 \mu\text{F}$ ,  
 $L_0 = 0 \sim 40 \text{ mH}$ ,  
 $L_1$  : parameter,  
 $r_{L_0}$  : depends on  $L_0$

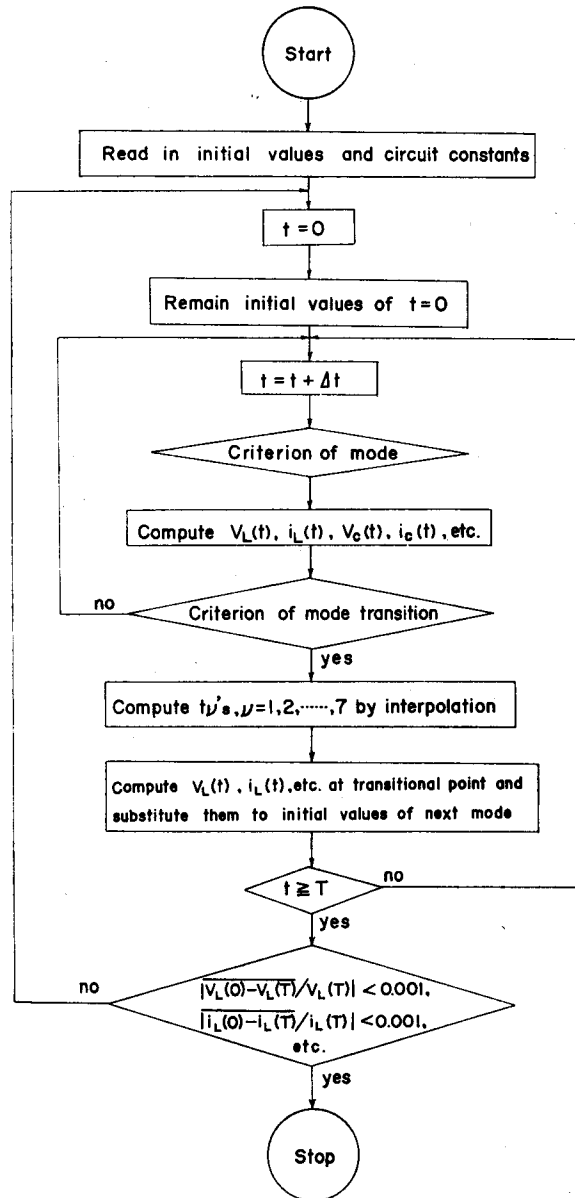


Fig. 5. Digital computer flow-chart.

where the values of the following quantities are assumed as constant, namely

$$\begin{array}{lll}
 E=50 \text{ V}, & T=5 \text{ ms}, & T_{on}=2 \text{ ms}, \\
 R=50 \Omega, & L_2=10 \text{ mH}, & r_{Th1}=1.7 \Omega, \\
 r_{Th2}=1.2 \Omega, & r_C=0.5 \Omega, & r_{D1}=1.7 \Omega \\
 r_{D2}=1.7 \Omega & \text{and} & r_{L2}=2.0 \Omega.
 \end{array}$$

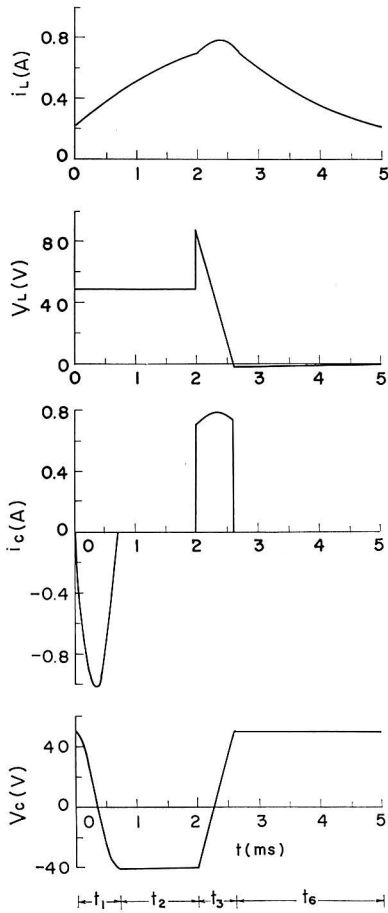


Fig. 6. Calculated waveforms of  $i_L$ ,  $v_L$ ,  $i_C$  and  $v_C$  when  $C=5 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=100 \text{ mH}$  and  $r_{L0}=0 \Omega$ .

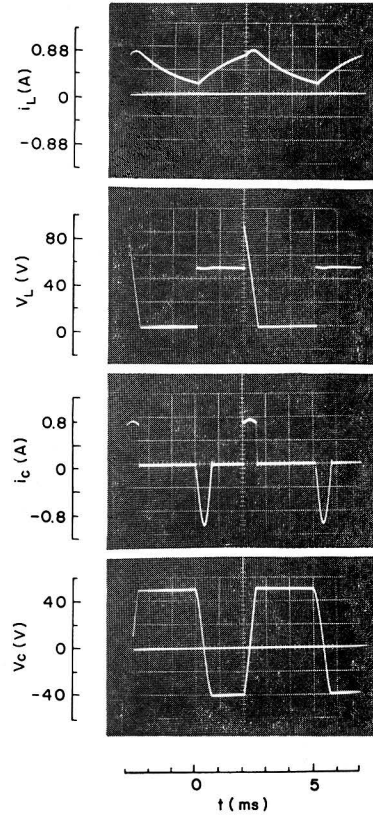


Fig. 6'. Experimental oscillograms of  $i_L$ ,  $v_L$ ,  $i_C$  and  $v_C$  when  $C=5 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=10 \text{ mH}$  and  $r_{L0}=0 \Omega$ .

Next we show several examples of numerical calculations in Figs 6, 7, 8 and 9, where

Fig. 6 :  $C=5 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=100 \text{ mH}$ ,  $r_{L0}=0 \Omega$ ,

Fig. 7 :  $C=10 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=100 \text{ mH}$ ,  $r_{L0}=0 \Omega$ ,

Fig. 8 :  $C=5 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=20 \text{ mH}$ ,  $r_{L0}=0 \Omega$ ,

Fig. 9 :  $C=5 \mu\text{F}$ ,  $L_0=10 \text{ mH}$ ,  $L_1=100 \text{ mH}$ ,  $r_{L0}=1.9 \Omega$ .

Also in Figs 6', 7', 8', and 9', we present the experimental oscillograms corresponding to the calculated waveforms shown in Figs 6, 7, 8 and 9 respectively.

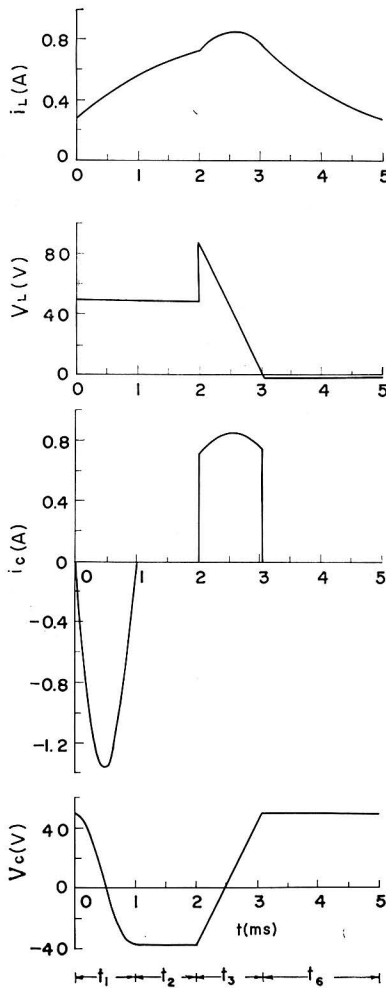


Fig. 7. Calculated waveforms of  $i_L$ ,  $v_L$ ,  $i_C$  and  $v_C$  when  $C=10 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=100 \text{ mH}$  and  $r_{L0}=0 \Omega$ .

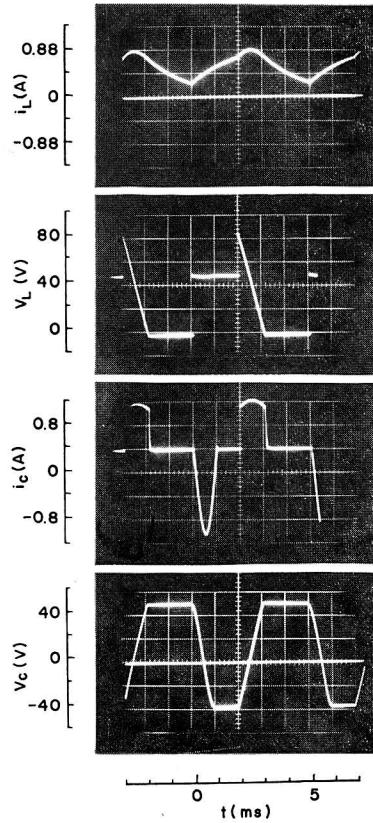


Fig. 7'. Experimental oscillograms of  $i_L$ ,  $v_L$ ,  $i_C$  and  $v_C$  when  $C=10 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=100 \text{ mH}$  and  $r_{L0}=0 \Omega$ .

#### 4. Discussion

Comparing the waveforms by numerical calculations with ones from experimental results, we see that both correspond very much as shown in Figs 6 to 9 and 6' to 9'. The results considering the internal resistance of each element present themselves by the fact that in mode II the value of  $v_{C2}$  decreases to about 40 V, in modes I and II  $v_L$  decreases very slightly and in modes IV, V and VI  $v_L$  becomes negative.

Now in Fig.10 we show the calculated relations between the condenser capacity

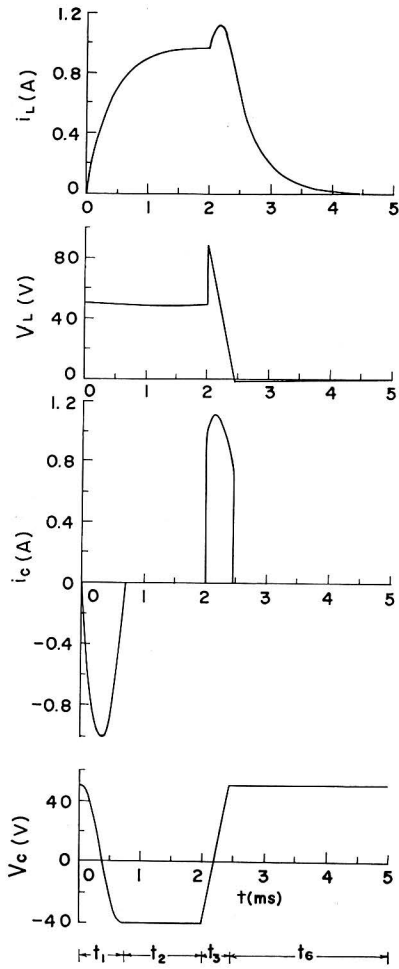


Fig. 8. Calculated waveforms of  $i_L$ ,  $v_L$ ,  $i_C$  and  $v_C$  when  $C=5 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=20 \text{ mH}$  and  $r_{L0}=0 \Omega$ .

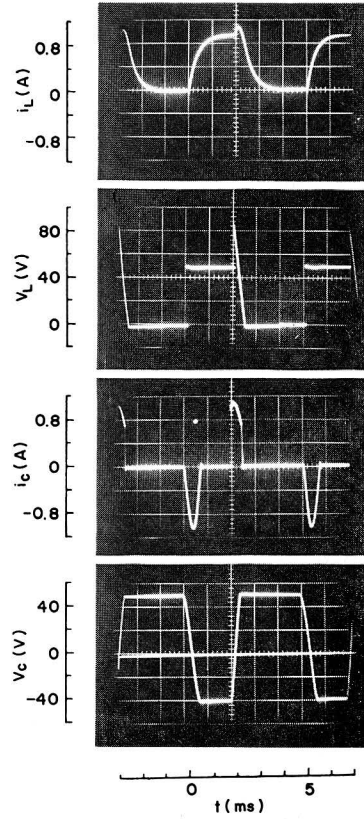


Fig. 8'. Experimental oscillograms of  $i_L$ ,  $v_L$ ,  $i_C$  and  $v_C$  when  $C=5 \mu\text{F}$ ,  $L_0=0 \text{ mH}$ ,  $L_1=20 \text{ mH}$  and  $r_{L0}=0 \Omega$ .

$C$  and the duration  $t_3$  of the mode III in the case where  $L_0=0$ . As the load voltage waveform of an ideal chopper circuit should be a complete rectangular wave, we must make  $t_3$  as short as possible. As mode III is the one in which  $Th1$  is turned off, the following condition must be satisfied to make turn-off of  $Th1$  possible, namely

$$\Delta t > t_{off}, \tag{15}$$

where

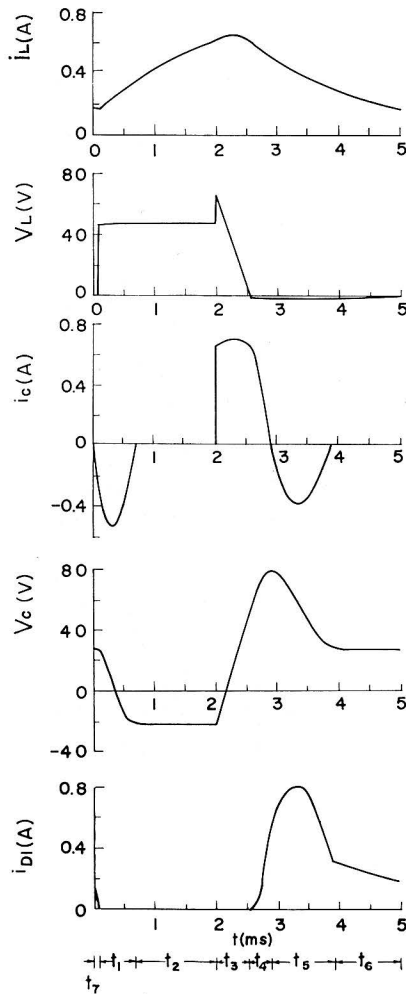


Fig. 9. Calculated waveforms of  $i_L$ ,  $v_L$ ,  $i_C$ ,  $v_C$  and  $i_{D1}$  when  $C=5\ \mu\text{F}$ ,  $L_0=10\ \text{mH}$ ,  $L_1=100\ \text{mH}$  and  $r_{L0}=1.9\ \Omega$ .

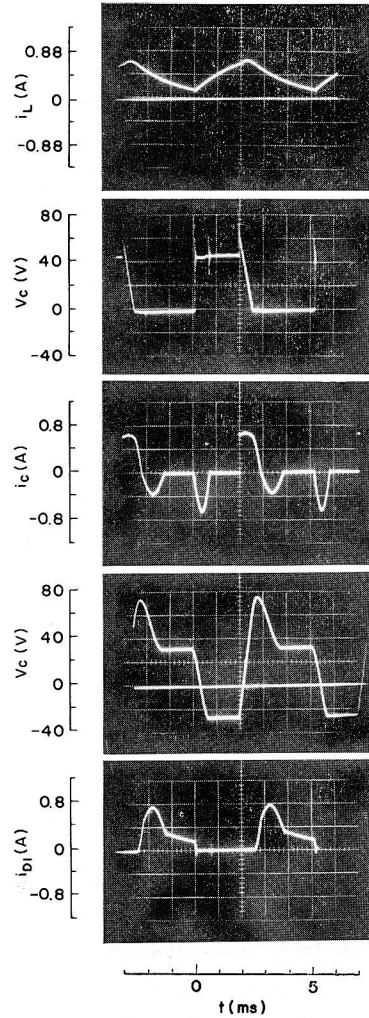


Fig. 9'. Experimental oscillograms of  $i_L$ ,  $v_L$ ,  $i_C$ ,  $v_C$  and  $i_{D1}$  when  $C=5\ \mu\text{F}$ ,  $L_0=10\ \text{mH}$ ,  $L_1=100\ \text{mH}$  and  $r_{L0}=1.9\ \Omega$ .

$$\left. \begin{aligned} t_{off} &: \text{the turn-off time of } Th\ 1, \\ \Delta t &: \text{see Fig. 11.} \end{aligned} \right\} \quad (15)$$

In the figures,  $v_{Ci}$  and  $v_{Cm}$  are the initial and peak values, respectively, of  $v_C$  in a stage.

On the other hand, from Fig. 10 we can find that  $t_3$  increases when  $C$  does and also it varies with  $L_1$ . From numerical calculation results we arrive at a relation



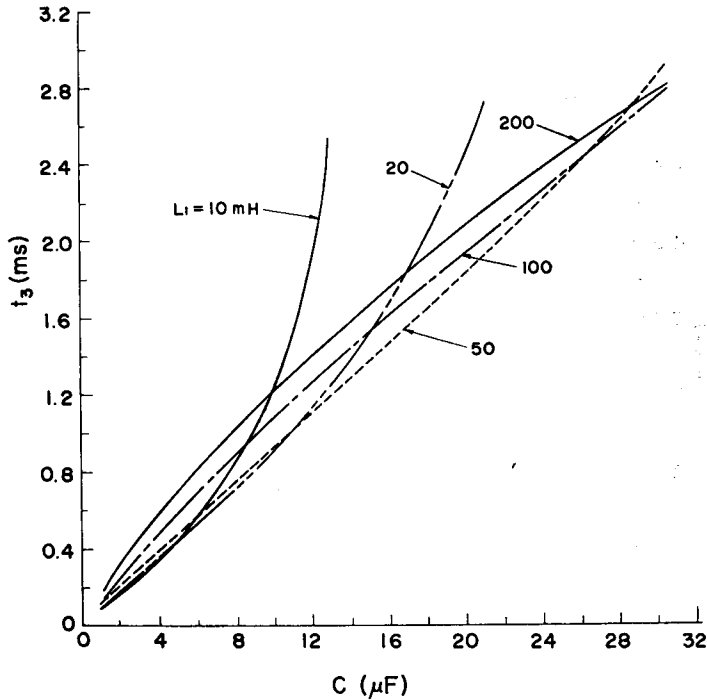


Fig. 10. Relations of  $t_3$  vs.  $C$ .

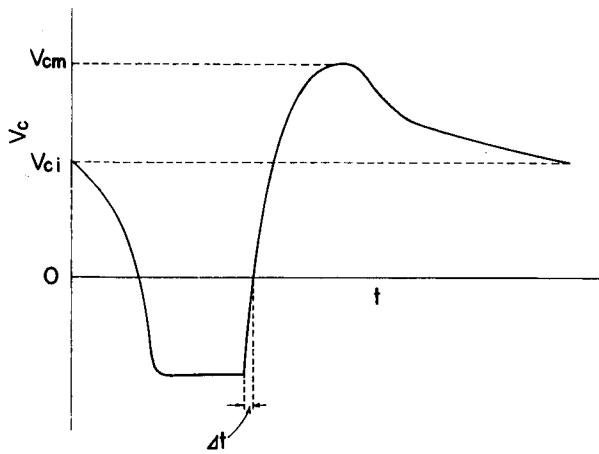


Fig. 11.  $u_c-t$  curve.

that  $t_3$  is about twice  $\Delta t$ , accordingly by the characteristic curves in Fig. 10 we can determine the values of  $L_1$  and  $C$  which makes  $t_3$  shortest in the range, in which Eq. (15) is satisfied. In this connection, the condition of  $\lambda < 1$  must be satisfied, because in the case of  $\lambda \geq 1$  mode VI doesn't exist.

Now we show the calculated relations of the average load voltage  $V_L$  and current  $I_L$  vs.  $C$  in Figs 12 and 13 respectively. According to those figures, both  $V_L$  and  $I_L$  increase when  $L_1$  and  $C$  do, because the larger  $C$  is, the larger  $t_s$  and so  $V_L$

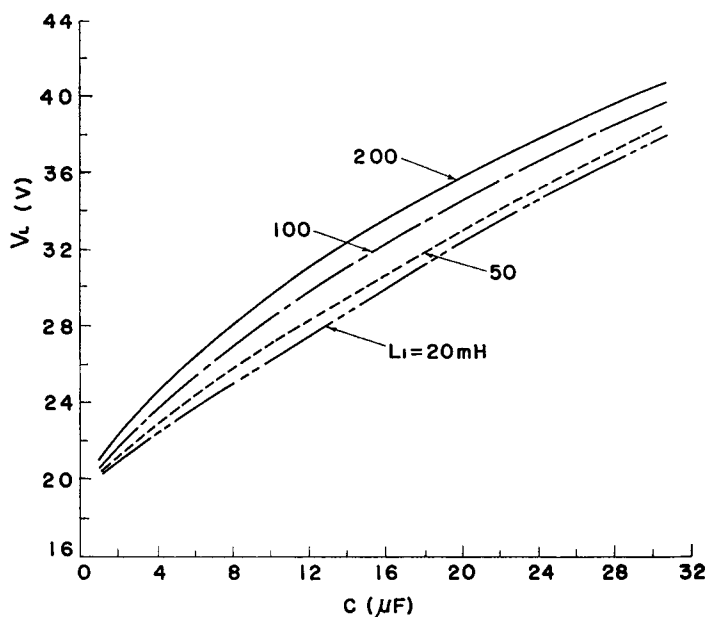


Fig. 12. Relations of  $V_L$  vs.  $C$ .

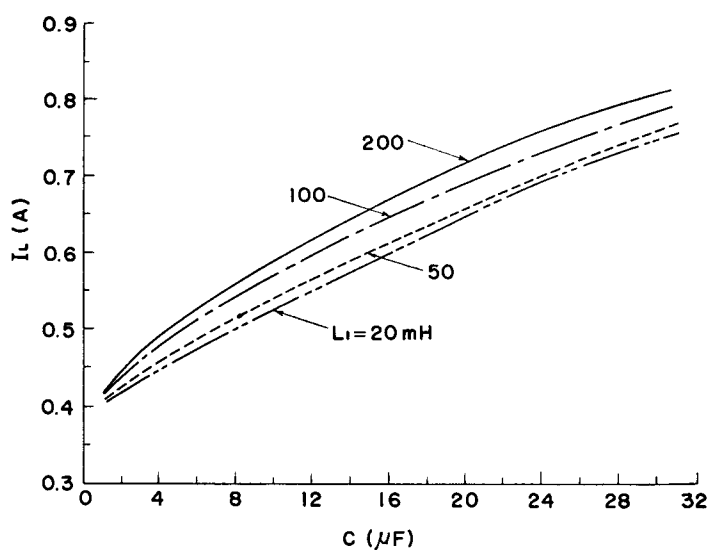
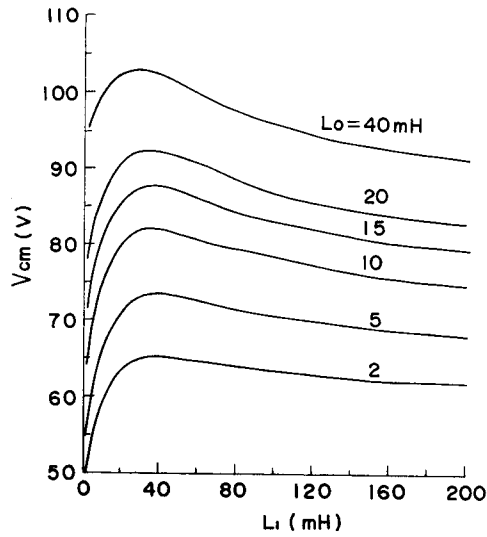


Fig. 13. Relations of  $I_L$  vs.  $C$ .

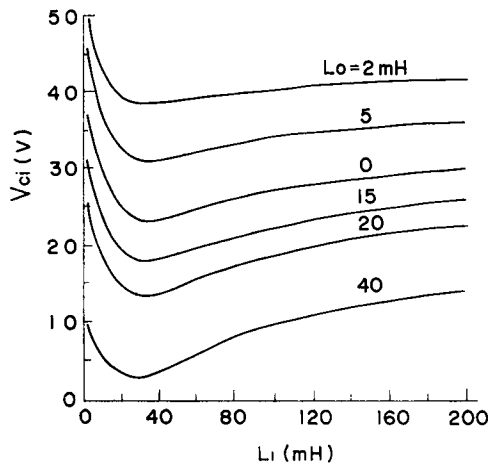
and  $I_L$  become and the larger  $L_1$  is, the larger  $v_{c3}^{+0}$  therefore  $V_L$  becomes, and if  $L_1$  increases, the increment of  $i_{L6}$  becomes larger than the decrements of  $i_{L1}$  and  $i_{L2}$  and therefore  $I_L$  increases. If the waveform of  $v_L$  is ideal, it is satisfied theoretically that  $V_L=20V$ , where  $V_L$  is given by the following conventional and simple relation,

$$V_L = ET_{on}/T. \tag{16}$$

However the practical values of  $V_L$  and  $I_L$  must be acquired from Fig. 12 and Fig. 13.



(a)  $V_{cm} - L_1$



(b)  $V_{ci} - L_1$

Fig. 14. Relations of  $v_{cm}$  and  $v_{ci}$  vs.  $L_1$ .

Next, when the source consists of transformer and rectifiers, we can not avoid containing inductance in the power source, and the waveforms of the voltages and currents in such a case are shown in Fig. 9 and Fig.9'. According to the figures, when  $L_0$  is contained, the initial inclination of  $v_L$  in mode I is not so sharp and an oscillatory phenomenon due to  $L_0$  and  $C$  appears upon the waveform of  $v_c$ . Then we show the quantitative relations between  $L_1$  and  $v_{cm}$ , and  $L_1$  and  $v_{ci}$  in Fig.14 (a) and (b) respectively. According to Fig. 14,  $v_{cm}$  increases and  $v_{ci}$  decreases when  $L_0$  increases, and concerning  $L_1$  the tendency is most remarkable near about  $20mH$  because of the resonance of  $L_1$  and  $C$ , though the value is a little changed by the value of  $C$ . Moreover the inversed voltage of  $Th 1$  must be higher than  $v_{cm}$ , and as  $v_{ci}$  decreases with the increase of  $L_0$ , commutating failures may be sometimes induced unless enough charging voltage is given. In order to prevent them, we must adopt the means of using a thyristor instead of  $D 1$  or connect large smoothing condensers to the power source.

## 5. Conclusion

In this paper, we have performed strict analysis about the oscillation circuit type chopper, investigated numerically and experimentally under various circuit conditions, obtained the various useful results of optimum circuit conditions and others as previously described and given several guides for chopper circuit designs.

## Acknowledgement

The authors wish to express their appreciation to Mr. T. Andō in Department of Electrical Engineering.

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