Stability Calculation of Power Systems Having Shunt Loads

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(Received June 23, 1969)

The small signal performance of a multimachine power system is described by a set of differential equations of the form $\dot{x}=Ax$. The analysis includes the effect of local shunt loads and margin of stability defined in the eigenvalue-plane. The construction of the A-matrix involves an equivalent circuit of a transmission network, a hybrid reference frame and an axis transformation based on Park's transformation.

Once the A-matrix is obtained, standard computer programs may be used for studying the dynamic stability characteristics of the power system. Root-locus analysis is adapted to get information on the dynamic stability of a sample model.

1. Introduction

Owing to the progress of digital computers with great memory capacity and quick processing ability, many excellent works[1),2)] on the dynamic stability of electric power systems have been done recently.

We have already reported on the relation between the stability margin of a power system and its disturbed motion[3),4)]. In that report the analysis and the computation were simplified by neglecting the local shunt loads and by making certain assumptions.

The present paper describes the inclusion of local shunt loads in the analysis of the power-system stability. The model system contains an arbitrary number of synchronous machines and a transmission network of arbitrary topological form. Using linear graph theory, the transmission network is transformed into an equivalent circuit of the Lagrangian tree form. Even if the network is complicated, the necessary set of network equations can be easily obtained from the equivalent circuit.

Transmission lines and impedance loads are expressed in the relation between the phase currents and the phase voltages of the transmission network. Synchronous machines are expressed with Park's quantities. After both sets of equations are obtained, the quantities of the transmission network are projected into the frames

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fixed to the rotors of every synchronous machine. This set of axis-transformations projects the phase quantities onto the direct and the quadrature axes of a synchronous machine and enables the whole system to be expressed with Park's quantities.

In addition the displacement angle of a rotor, δ_j, and the difference of the displacement angles, δ_{ij}, are introduced in order to represent the mechanical performance of synchronous machines and the power-flow relations in the system, respectively. In this way, the representation of the effect of damper windings and the governor action, as well as the axis transformation, mentioned above, becomes simpler.

Once a general linearized set of differential equations describing the small signal performance of a multimachine system with local shunt loads is obtained, the dynamic stability of the system is studied by the root-locus analysis.

A typical use of this procedure is demonstrated by a simple study of a 3-machine system.

2. Development of Equations for Multimachine Systems

Equations describing the performance of the entire system are derived on the basis of a hybrid reference frame. Each individual synchronous machine is described by Park's equations in the frame fixed to its rotor. The complete description includes governor and excitation systems. The interconnecting network and local impedance loads are stated in terms of phase quantities. Machines are connected to the network at the specified nodes, where voltages and currents in the two reference frames are related to one another by axis transformation.

Axis Transformation

The axis transformation used in the present paper is mainly based on Park's transformation and its inverse transformation. The angular position of the j-th rotor-pole with respect to a stationary reference is expressed with θ_j as follows:

$$\theta_j = \omega_0 t + \delta_j \tag{1}$$

$$\omega_j = p\theta_j = \omega_0 + p\delta_j \tag{2}$$

$$\delta_{ij} = \theta_i - \theta_j = \delta_i - \delta_j \tag{3}$$

where ω_0 is the synchronous speed.

In steady state, the operation angle δ_j does not change, the operation angle difference δ_{ij} holds constant, and $p\delta_j$ becomes zero. Therefore, the angular velocity of the j-th machine ω_j equals ω_0. In transient state, the operation angle δ_j changes depending on the unbalance of torque, and ω_j differs from ω_0.

Let $\boldsymbol{w}_{aj}$ be the column vector of phase quantities at the j-th bus, and $\boldsymbol{w}_{dij}$ be the column vector of their Park's quantities with respect to the rotating frame of the i-th machine. Then, the relationships between $\boldsymbol{w}_{aj}$ and $\boldsymbol{w}_{djj}$ become:

$$\boldsymbol{w}_{djj} = \boldsymbol{P}(\theta_j)\boldsymbol{w}_{aj} \tag{4}$$

$$\boldsymbol{w}_{aj} = \boldsymbol{P}^{-1}(\theta_j)\boldsymbol{w}_{djj} \tag{5}$$

A combined transformation matrix and its derivative are introduced as:

$$\boldsymbol{T}(\delta_{ij}) = \boldsymbol{P}(\theta_i)\boldsymbol{P}^{-1}(\theta_j) \tag{6}$$

$$\boldsymbol{T}'(\delta_{ij}) = -\frac{\partial}{\partial \delta_{ij}}\boldsymbol{T}(\delta_{ij}) = \boldsymbol{P}(\theta_i)\frac{\partial}{\partial \theta_j}\boldsymbol{P}^{-1}(\theta_j) \tag{7}$$

The elements of the foregoing transformation matrices are shown in Appendix A.

The application of the transformation matrix $\boldsymbol{T}(\delta_{ij})$ to $\boldsymbol{w}_{djj}$ gives:

$$\boldsymbol{w}_{dij} = \boldsymbol{T}(\delta_{ij})\boldsymbol{w}_{djj} \tag{8}$$

By using this set of axis-transformations, the phase quantities are projected onto the rotating frames.

Network Equations

Any interconnecting network can be transformed into the equivalent circuit that has the simplest form of the Lagrangian tree, as shown in Fig. 1. (See Appendix B.) All nodes, to which no shunt loads nor power sources are connected may be eliminated, and only those nodes that should be formulated in the necessary set of equations may remain in the equivalent circuit. A shunt load consists of a resistor, an inductor and a capacitor, as shown in Fig. 2, and conventionally includes the capacitance between the transmission lines and the ground.

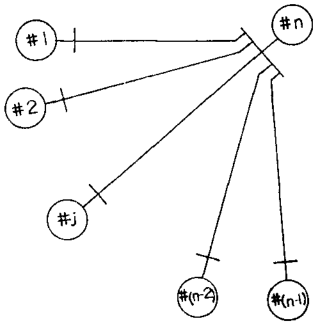

Fig. 1. Equivalent Circuit of Lagrangian Tree Form

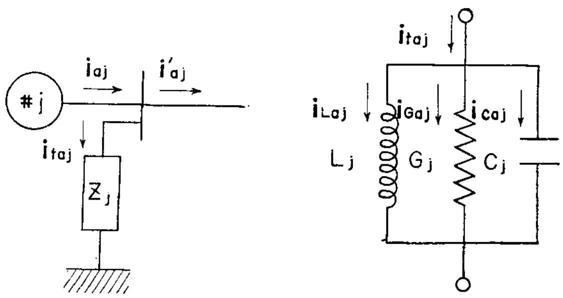

Fig. 2. Local Shunt Load

Choosing the n-th node as a voltage reference node, the equivalent circuit of Fig. 1 yields the following set of equations:

$$\boldsymbol{v}_{aj}-\boldsymbol{v}_{an}=\sum_{k=1}^{n-1}(R_{jk}+L_{jk}\cdot p)\boldsymbol{i}'_{ak} \tag{9}$$

$$\sum_{j=1}^{n}\boldsymbol{i}'_{aj}=\boldsymbol{0} \tag{10}$$

An additional shunt load at each bus may be expressed as:

$$\boldsymbol{i}_{aj}=\boldsymbol{i}'_{aj}+\boldsymbol{i}_{Laj}+\boldsymbol{i}_{Caj}+\boldsymbol{i}_{Gaj} \tag{11}$$

$$L_j p\boldsymbol{i}_{Laj}=\boldsymbol{v}_{aj} \tag{12}$$

$$\boldsymbol{i}_{Caj}=C_j p\boldsymbol{v}_{aj} \tag{13}$$

$$\boldsymbol{i}_{Gaj}=G_j\boldsymbol{v}_{aj} \tag{14}$$

The application of the axis transformations gives:

$$\boldsymbol{v}_{dj}-\boldsymbol{T}(\delta_{jn})\boldsymbol{v}_{dn}=\sum_{k=1}^{n-1}[\{R_{jk}\ \boldsymbol{T}(\delta_{jk})+\omega_k\ L_{jk}\ \boldsymbol{T}'(\delta_{jk})\}\boldsymbol{i}'_{dk}+L_{jk}\ \boldsymbol{T}(\delta_{jk})p\boldsymbol{i}'_{dk}] \tag{15}$$

$$\sum_{j=1}^{n}\boldsymbol{T}(\delta_{nj})\boldsymbol{i}'_{dj}=\boldsymbol{0} \tag{16}$$

$$\boldsymbol{i}_{dj}=\boldsymbol{i}'_{dj}+\boldsymbol{i}_{Ldj}+\boldsymbol{i}_{Cdj}+\boldsymbol{i}_{Gdj} \tag{17}$$

$$\omega_j L_j\boldsymbol{T}'(0)\boldsymbol{i}_{Ldj}+L_j\boldsymbol{T}(0)p\boldsymbol{i}_{Ldj}=\boldsymbol{v}_{dj} \tag{18}$$

$$\boldsymbol{i}_{Cdj}=\omega_j C_j\boldsymbol{T}'(0)\boldsymbol{v}_{dj}+C_j\boldsymbol{T}(0)p\boldsymbol{v}_{dj} \tag{19}$$

$$\boldsymbol{i}_{Gdj}=G_j\boldsymbol{v}_{dj} \tag{20}$$

Now eqns. (15)∼(20) are a set of first-order differential equations describing the behavior of the balanced-phase network. Their transient solution, by the definition of ω_j as the angular velocity of the j-th machine, depends on the transient performance of the rotor.

These equations contain zero-sequence equations, but because the model system is restricted to the case of balanced-phase parameters, their extra complexity is omitted from the analysis. Therefore the order of all vectors and transformation matrices $\boldsymbol{T}(\delta_{ij})$ and $\boldsymbol{T}'(\delta_{ij})$ are reduced by one.

Further, the transients occuring on the transmission network and on the shunt loads are of little interest in dynamic stability studies because of their short duration in comparison with even the shortest lived transients occuring in the machine windings. Therefore, for the purpose of obtaining a boundary condition here, their transient solutions may be neglected to give:

$$\boldsymbol{v}_{dj}-\boldsymbol{T}(\delta_{jn})\boldsymbol{v}_{dn}=\sum_{k=1}^{n-1}\{R_{jk}\boldsymbol{T}(\delta_{jk})+\omega_k L_{jk}\boldsymbol{T}'(\delta_{jk})\}\boldsymbol{i}'_{dk}$$

$$j=1\sim(n-1) \tag{21}$$

$$\sum_{j=1}^{n}\boldsymbol{T}(\delta_{nj})\boldsymbol{i}'_{dj}=\boldsymbol{0} \tag{22}$$

$$\boldsymbol{i}_{dj} = \boldsymbol{i}'_{dj} + \boldsymbol{Y}(\omega_j)\boldsymbol{v}_{dj} \tag{23}$$

where

$$\boldsymbol{Y}(\omega_j) = \begin{bmatrix} G_j & 1/\omega_j L_j - \omega_j C_j \\ -1/\omega_j L_j + \omega_j C_j & G_j \end{bmatrix} \tag{24}$$

It is evident that some of the quantities equal to zero allow eqns. (21)~(23) to be used to describe the performance of several networks of simpler form. Let $n=2$, $R_{jk}=G_j=1/L_j=C_j=0$, and let the 2-nd node be the infinite bus. Then, the familiar equations for a one-machine infinite bus system may be given:

$$\begin{bmatrix} v_{d1} \\ v_{q1} \end{bmatrix} = \begin{bmatrix} \cos\delta_{12} & \sin\delta_{12} \\ -\sin\delta_{12} & \cos\delta_{12} \end{bmatrix} \begin{bmatrix} 0 \\ v_{q2} \end{bmatrix} + \omega_1 L_{11} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \tag{25}$$

For convenience in later manipulations of equations, let $\boldsymbol{Y}'(\omega_j)$ be introduced as:

$$\boldsymbol{Y}'(\omega_j) = \frac{\partial}{\partial \omega_j} \boldsymbol{Y}(\omega_j) = \begin{bmatrix} 0 & -1/\omega_j^2 L_j - C_j \\ 1/\omega_j^2 L_j + C_j & 0 \end{bmatrix} \tag{26}$$

Description of Synchronous Machine

Complete description of the small signal performance of the synchronous machine requires consideration of its electrical and mechanical characteristics as well as those of associated control systems. Restricting the modes of operation to the cases that do not require zero-axis variables, Park's model describing the dynamic characteristics of a synchronous machine in per-unit form is summarized in the following paragraphs. Time is scaled by multiplying $2\pi f_0$ so as to make ω_0 be unity. Explanation of the symbols is listed at the end of this paper.

Induced Voltages in Armature Circuits:

$$v_{dj} = -(r_j + L_{dj} p) i_{dj} + (\psi_{fqj} + L'_{qj} i_{qj}) p\delta_j + M_{fj} p i_{fj} \tag{27}$$
$$v_{qj} = -(r_j + L_{qj} p) i_{qj} + (\psi_{fdj} - L'_{dj} i_{dj}) p\delta_j + M_{gj} p i_{gj} \tag{28}$$

Change of Flux-linkage in Field Circuits:

$$p\psi_{fdj} = \{E_{fdj} - \psi_{fdj} - (L_{dj} - L'_{dj}) \cdot i_{dj}\} / T'_{d0j} \tag{29}$$
$$p\psi_{fqj} = \{-\psi_{fqj} + (L_{qj} - L'_{qj}) \cdot i_{qj}\} / T'_{q0j} \tag{30}$$

Mechanical Equations:

$$M_j \cdot p\omega_j = T_{Mj} - (\psi_{dj} \cdot i_{qj} - \psi_{qj} \cdot i_{dj}) - D_j \cdot \omega_j \tag{31}$$

Voltage Regulator: A widely used model for the continuously acting voltage regulator may be described by:

$$\Delta E_{fdj} = -K_{fj} \cdot \Delta v_{tj}/(1+T_{fj} \cdot p) \tag{32}$$

where

$$\Delta v_{tj} = (v_{q0j} \cdot \Delta v_{qj} + v_{d0j} \cdot \Delta v_{dj})/v_{t0j}$$

Speed Governor: The effect of the conventional speed governor may be represented by:

$$\Delta T_{Mj} = -K_{gj} \cdot \Delta \omega_j/(1+T_{gj} \cdot p) \tag{33}$$

The foregoing equations are similar to those used in other papers.[5]

If desired, any other model for the voltage regulator or speed governor can be easily introduced.

3. Linearized Equations for Perturbed Motions

First, network equations (21)$\sim$(23) are rewritten for small perturbations from a fixed operating point. The absence of zero-sequence equations allows voltage vectors and current vectors to be denoted by the complex variables. Choosing the rotor-pole axis as the real axis, transformation matrices $\boldsymbol{T}(\delta_{ij})$, $\boldsymbol{T}'(\delta_{ij})$, and the admittance matrix $\boldsymbol{Y}(\omega_j)$ together with its derivative $\boldsymbol{Y}'(\omega_j)$ are denoted with complex values T_{ij}, T'_{ij}, Y_j and Y'_j, respectively, for their representation in steady state:

$$T_{ij} = \exp\,(-j\delta_{ij0}) \tag{34}$$

$$T'_{ij} = \exp\,\{-j(\delta_{ij0}-\pi/2)\} = jT_{ij} \tag{35}$$

$$Y_j = G_j - j(\omega_0 C_j - 1/\omega_0 L_j) \tag{36}$$

$$Y'_j = j(1/\omega_0^2 L_j + C_j) \tag{37}$$

Then the equations for the perturbed motions become:

$$\begin{aligned} \Delta \boldsymbol{v}_{dj} - T_{jn} \cdot \Delta \boldsymbol{v}_{dn} + jT_{jn} \cdot \boldsymbol{v}_{d0n} \cdot \Delta\delta_{jn} &= \sum_{k=1}^{n-1} \{Z_{jk} \cdot T_{jk}(\Delta \boldsymbol{i}_{dk} - Y_k \cdot \Delta \boldsymbol{v}_{dk}) - j\bar{Z}_{jk} \cdot T_{jk} \cdot \boldsymbol{i}'_{d0k} \cdot (\Delta\delta_{jn} - \Delta\delta_{kn}) \\ &\quad + (j\omega_0 L_{jk} \cdot \boldsymbol{i}'_{d0k} - Z_{jk} \cdot Y'_k \cdot \boldsymbol{v}_{d0k}) \cdot T_{jk} \cdot \Delta\omega_k\} \quad j = 1 \sim (n-1) \end{aligned} \tag{38}$$

$$\sum_{j=1}^{n} (-T_{nj} \cdot Y_j \cdot \Delta \boldsymbol{v}_{dj} + T_{nj} \cdot \Delta \boldsymbol{i}_{dj} - T_{nj} \cdot Y'_j \cdot \boldsymbol{v}_{d0j} \cdot \Delta\omega_j + jT_{nj} \cdot \boldsymbol{i}'_{d0j} \cdot \Delta\delta_{jn}) = \boldsymbol{0} \tag{39}$$

$$\Delta \boldsymbol{i}'_{dj} = \Delta \boldsymbol{i}_{dj} - Y_j \cdot \Delta \boldsymbol{v}_{dj} - Y'_j \cdot \boldsymbol{v}_{d0j} \Delta\omega_j \quad j = 1 \sim (n-1)$$

where

$$Z_{ij} = R_{ij} + j\omega_0 L_{ij}, \; \bar{Z}_{ij} = R_{ij} - j\omega_0 L_{ij}$$

Similarly, machine equations are rewritten for small perturbations in the case of the j-th machine.

From eqns. (27), (28):

$$\Delta v_{dj} = \Delta\psi_{fqj} + x'_{qj}\cdot\Delta i_{qj} + v_{d0j}\cdot\Delta\omega_j \tag{40}$$

$$\Delta v_{qj} = \Delta\psi_{fdj} - x'_{dj}\cdot\Delta i_{dj} + v_{q0j}\cdot\Delta\omega_j \tag{41}$$

From eqns. (2), (3), (29)~(33):

$$p\Delta\delta_{jn} = \omega_j - \omega_n \tag{42}$$

$$p\Delta\psi_{fdj} = \{\Delta E_{fdj} - \Delta\psi_{fdj} - (x_{dj} - x'_{dj})\cdot\Delta i_{dj}\}/T'_{d0j} \tag{43}$$

$$p\Delta\psi_{fqj} = \{-\Delta\psi_{fqj} + (x_{qj} - x'_{qj})\cdot\Delta i_{qj}\}/T'_{q0j} \tag{44}$$

$$p\Delta\omega_j = \{\Delta T_{Mj} - \psi_{d0j}\cdot\Delta i_{qj} - i_{q0j}\cdot\Delta\psi_{dj} + \psi_{q0j}\cdot\Delta i_{dj} + i_{d0j}\cdot\Delta\psi_{qj} - D_j\cdot\Delta\omega_j\}/M_j \tag{45}$$

$$p\Delta E_{fdj} = -\{\Delta E_{fdj} + K_{fj}(v_{q0j}\cdot\Delta v_{qj} + v_{d0j}\cdot\Delta v_{dj})/v_{t0j}\}/T_{fj} \tag{46}$$

$$p\Delta T_{Mj} = -(\Delta T_{Mj} + K_{gj}\cdot\Delta\omega_j)/T_{gj} \tag{47}$$

Finally a set of equations (38)~(47) are rearranged to give matrix equations. Let a pair of vectors $\boldsymbol{x}$, $\boldsymbol{y}$ be defined as:

$$\begin{aligned}\boldsymbol{x} = [&\Delta\delta_{1n}, \Delta\psi_{fd1}, \Delta\psi_{fq1}, \Delta E_{fd1}, \Delta\omega_1, \Delta T_{M1}, \cdots,\\ &\Delta\delta_{(n-1)n}, \Delta\psi_{fd(n-1)}, \Delta\psi_{fq(n-1)}, \Delta E_{fd(n-1)}, \Delta\omega_{(n-1)}, \Delta T_{M(n-1)}, \cdots,\\ &\Delta\psi_{fdn}, \Delta\psi_{fqn}, \Delta E_{fdn}, \Delta\omega_n, \Delta T_{Mn}]^T\end{aligned}$$

$$\boldsymbol{y} = [\Delta v_{d1}, \Delta v_{q1}, \Delta i_{d1}, \Delta i_{q1}, \cdots, \Delta v_{dn}, \Delta v_{qn}, \Delta i_{dn}, \Delta i_{qn}]^T$$

where $\boldsymbol{x}$ has the order of $(6n-1)$ and $\boldsymbol{y}$ the order of $(4n)$.

Then, eqns. (38)~(41) become:

$$A_3\boldsymbol{x} = A_4\boldsymbol{y} \tag{48}$$

Similarlly eqns. (42)~(47) are rearranged to give:

$$p\boldsymbol{x} = A_1\boldsymbol{x} + A_2\boldsymbol{y} \tag{49}$$

Inversion of A_4 and insertion of $\boldsymbol{y}$ to eqn. (49) gives:

$$p\boldsymbol{x} = A\boldsymbol{x} \tag{50}$$

where $A = A_1 + A_2\, A_4^{-1}\, A_3$

4. Root-locus Analysis and Margin of Stability

Once the system is described by a set of differential equations in the state-space form as eqn. (50), the small performance of the whole system may be studied via several standard computer programs.

In the present paper, the coefficients of the characteristic polynomials are calculated through the Frame method after constructing the A-matrix. Then the Newton method or the Hitchcock-Bairstow method is used to compute the roots of the characteristic equation.

The eigenvalues of a linear dynamical system correspond to its natural modes of response, with each real part giving the reciprocal decay-time constant or damping coeffecient of a mode, and with each pair of imaginary parts giving the natural angular frequency.

The necessary and sufficient condition for dynamic stablity is that all the eigenvalues have negative real parts. The forcing frequencies which could lead to hunting problems may be determined by examination of the imaginary parts of the eigenvalues. Thus, the dynamic stability may be directly checked with the real parts of the eigenvalues.

Further, a form of quantitative information on the relative stability of the system may be obtained by plotting the variation of the eigenvalues as system parameters are varied. Such plots are usually made on the complex plane so that the requirement for stability is that all the eigenvalues fall on the left half-plane. Those states of operation, however, that are close to the critical states can easily lose their stability. In actual fact, the practical operation of a power system always requires some margin of stability.

For the inclusion of a stability margin in the analysis, the eigenvalues are restricted so that all of them may lie on the left half-plane apart from the imaginary axis. The eigenvalue nearest to the imaginary axis, which corresponds to the dominant mode in the performance of the disturbed system is forced to fall within the left half-domain restricted by the line $\alpha=1/T_D$, as shown in Fig. 3. By this restriction of the area of the eigenvalues, the critical condition of the operation contains the dominant mode whose decay time is T_D at longest.

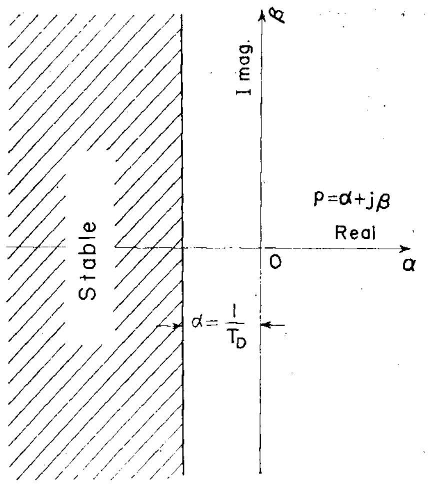

Fig. 3. Domain of Eigevalues Restricted by $\alpha=1/T_D$

5. Initial Conditions

Before the dynamic stability of the system is studied, it is necessary to find the initial values of pertinent variables. Prior to a disturbance, either the active power out-put and the terminal voltage, or the active power out-put and the reactive power out-put are known for each machine.

After load flow calculation, the operation angle δ_{j0} is determined according to the phasor diagram of Fig. 4. Once the angle δ_{j0} is known, initial values of other variables may be determined and transformed into the rotor-pole frame of every machine by multiplying exp $(-j\ \delta_{ij0})$.

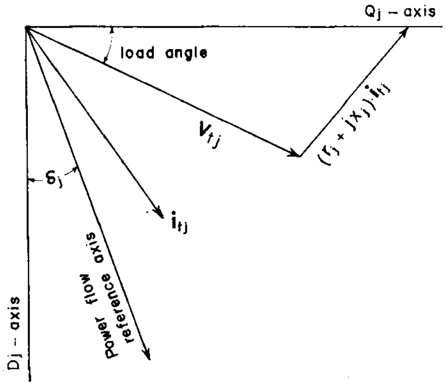

Fig. 4. Phasor Diagram for Initial Values

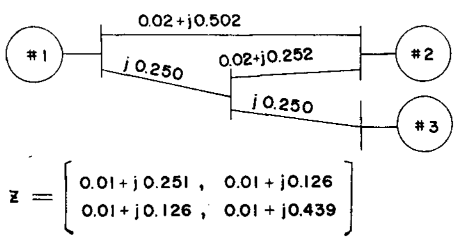

Fig. 5. Model of 3-machine System

6. Sample Application to a 3-machine Problem

A multimachine power system contains an extraordinarily large amount of system-parameters. It would be confusing and also outside the scope of this paper to study all their effects. Hence, a simple model of a 3-machine system as shown in Fig. 5 is studied to demonstrate the effects of the load flow and of the local shunt loads on the stability. In this example, #3 machine is equipped with neither voltage regulators nor governors. The parameters of the synchronous machines, the transmission network and the shunt loads are shown in Table 1. The data for the machines are taken from Kimbark[6], and typical values are used for the control loops. The equivalent circuit yields the impedance matrix of the order of 2×2 as shown in Table 1.

Underexcited or leading power factor operating points have been chosen. They are the conditions where small signal performance and asymptotic stability are of most interest.

Table 1. System Date for Sample System.

Machine constants			
	#1-Machine	#2-Machine	#3-Machine
x_d	1.10	1.10	1.10
x_d'	0.23	0.23	0.23
x_q	1.08	1.08	1.08
x_q'	0.23	0.23	0.23
r	0.05	0.05	0.05
T_{d0}'	9.5 sec.	9.5 sec.	9.5 sec.
T_{q0}'	1.7 sec.	1.7 sec.	1.7 sec.
M	5.0 sec.	5.0 sec.	5.0 sec.
D	1×10^{-6}	1×10^{-6}	1×10^{-6}
AVR constants			
	#1-Machine	#2-Machine	
K_f	40	40	
T_f	4 sec.	4 sec.	
Governor constants			
	#1-Machine	#2-Machine	
K_g	3	3	
T_g	1 sec.	1 sec.	

Table 2. Typical Listing of Eigenvalues for Sample System.

$P=0.35$ $Q=0.55$					
Real:	−0.05202 (−5.10 sec.)	−0.20536 (−1.29 sec.)	−0.25590 (−1.04 sec.)	−0.26896 (−0.99 sec.)	−0.38629 (−0.72 sec.)
Complex:	$-0.04507 + j0.14328$	(−5.89 sec., 0.086 Hz)			
	$-0.13228 + j0.10199$	(−2.00 sec., 0.061 Hz)			
	$-0.05322 + j0.23066$	(−4.99 sec., 0.14 Hz)			
	$-0.07523 + j3.36466$	(−3.53 sec., 2.02 Hz)			
	$-0.10565 + j3.64286$	(−2.51 sec., 2.10 Hz)			

Table 2 shows a typical listing of the eigenvalues for the model system and their corresponding values in seconds and in Hz in brackets. The eigenvalues are listed in order of increasing magnitude, so the mode associated with the slow permanent droop action of the governors and with the rotor oscillations appear first. The other groups of the rapidly damped high frequency modes are associated with the armature circuits.

Fig. 6 shows the loci of two dominant eigenvalues as the power out-put of #1 machine, $W_1=P_1+jQ_1$ is varied. Either of the two eigenvalues approaches the

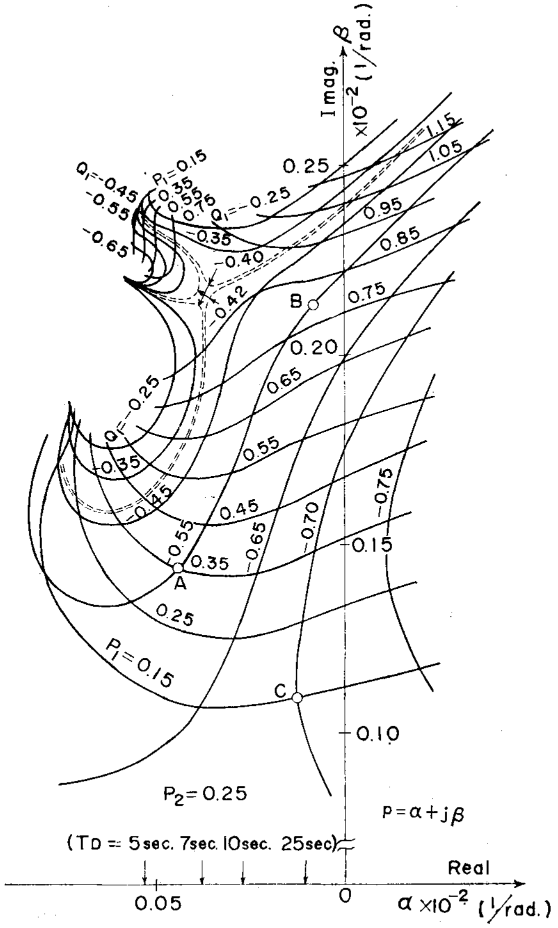

Fig. 6. Loci of Two Dominant Eigenvalues as W_1 varies

imaginary axis as $(-Q_1)$ increases. Consequently the system becomes less stable. On the other hand, its frequency rises as P_1 increases. The other eigenvalue almost moves in the opposite direction.

The domain of stability on the P_1-Q_1 plane may be obtained by knowing the values of P_1 and Q_1 at the intersection of the root-loci and the imaginary axis. In the same way, the domains of operation allowing for margin are obtained against various values of T_D, as shown in Fig. 7. The boundary of the domain takes rather diverse shapes depending on the value of T_D. This fact points out the difficulty of finding some physical meaning in the widely used margin, which is specified only with critical power out-put[7]. Fig. 8 shows the similar domains of operation while the active power out-put of ♯2 machine is varied.

Holding all the parameters fixed, only the shunt load at the terminal of ♯1

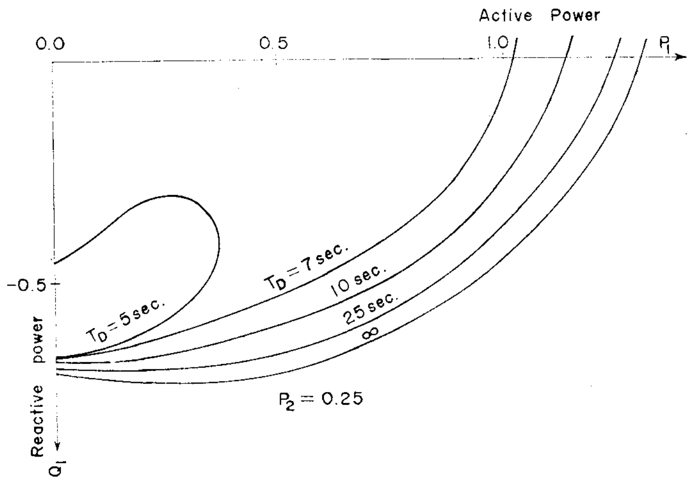

Fig. 7. Domains of Operation as Margin T_D varies

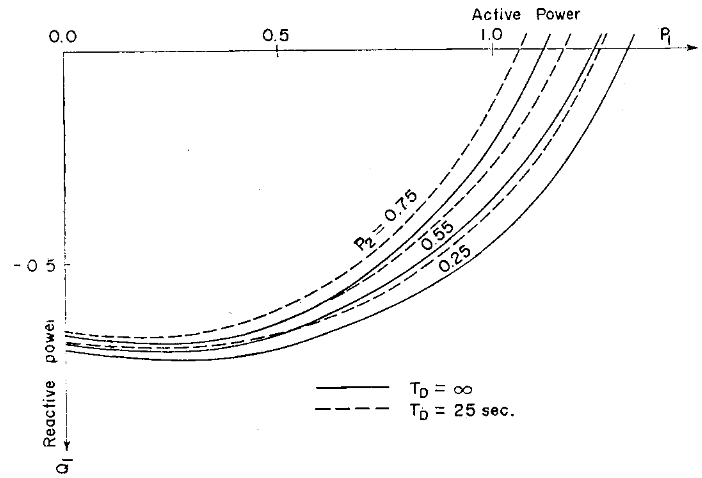

Fig. 8. Domains of Operation as P_2 varies

machine is varied to know its effect on the stability. The power factor ($\cos \varphi$) and the percent consumption (p.c.) of the load with respect to the absolute value of the power out-put from the #1 machine are varied at the operating points A, B and C of Fig. 6. Fig. 9 shows the loci of the two dominant eigenvalues as the shunt load varies. It is clear from the figure that leading power consumption at #1 generator, which is feeding leading power to the system, makes the system less stable, and that excess lagging reactive power consumption also makes the system less stable.

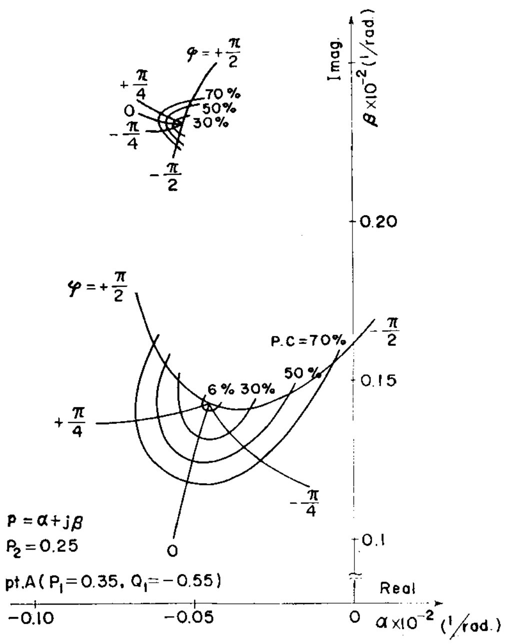

Fig. 9(a) Loci of Two Dominant Eigenvalues as Shunt Load varies

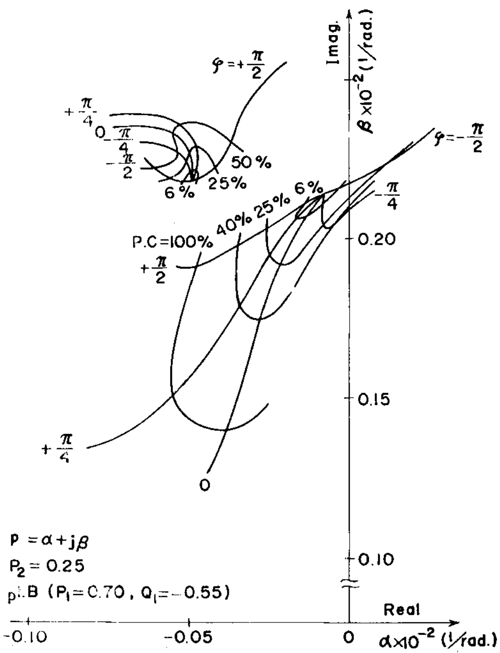

Fig. 9(b) Loci of Two Dominant Eigenvalues as Shunt Load varies

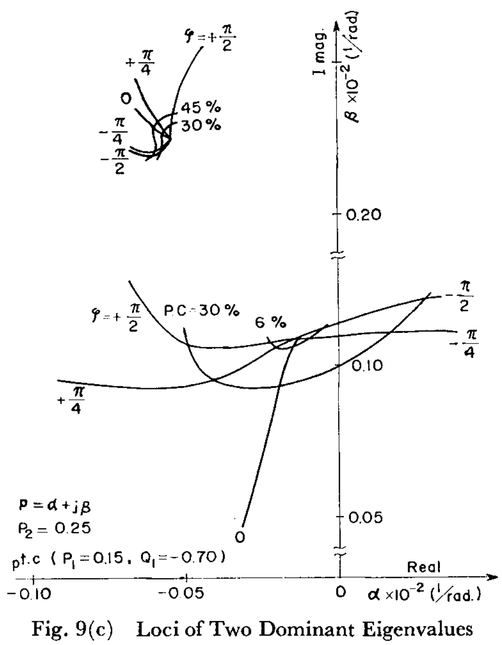

Fig. 9(c) Loci of Two Dominant Eigenvalues as Shunt Load varies

p.c.: percent power consumption of shunt load (c.f. section 6)

φ : power factor angle (c.f. section 6)

7. Conclusions

This paper has presented the method of calculating the small signal dynamic stability of a multimachine electric power system. This method has the following advantages over other analysis of power system small signal dynamics.

1) It is not limited to single machine or pairs of machines but can handle a number of machines connected to a network of any form. It also considers the effect of local shunt loads. It is limited only by the memory capacity of the digital computer used in its implementation.

2) It uses the model of a salient pole synchronous machine and includes representation of the governors and the voltage regulators. Further, it allows the inclusion of any alternative governor or voltage regulator that acts continuously.

3) The "state space form" of eqn. (50) enables the use of any technique of modern multivariable linear control theory such as Liapunov functions, frequency response, or state space formula.

In addition, this paper has demonstrated the "margin of stability" by restricting the domain of the eigenvalues. It has been shown that the margin specified only with the critical values of power out-put has less physical meaning than the margin proposed here.

Nomenclature

v_{dj} =direct axis voltage of #j machine
v_{qj} =quadrature axis voltage of #j machine
i_{dj} =direct axis current of #j machine
i_{qj} =quadrature axis current of #j machine
ψ_{fdj} =direct axis field flux linkage of #j machine
ψ_{fqj} =quadrature axis field flux linkage of #j machine
x_{dj} =direct axis synchronous reactance of #j machine
x_{qj} =quadrature axis synchronous reactance of #j machine
x'_{dj} =direct axis transient reactance of #j machine
x'_{qj} =quadrature axis transient reactance of #j machine
T'_{d0j} =open-circuit field time constant in direct axis of #j machine
T'_{q0j} =open-circuit field time constant in quadrature axis of #j machine
M_j =inertia constant of #j machine
D_j =damping coefficient of #j machine
T_{Mj} =prime mover torque of #j machine
K_{fj} =voltage regulator gain of #j machine
T_{fj} =voltage regulator time constant of #j machine

K_{gj} =governor gain of #j machine
T_{gj} =governor time constant of #j machine
Subscript zero denotes initial operating condition.

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Appendix A Transformation Matrices

$$\boldsymbol{P}(\theta_j) = \frac{2}{3}\begin{pmatrix} \cos\theta_j & \cos(\theta_j-120^\circ) & \cos(\theta_j+120^\circ) \\ -\sin\theta_j & -\sin(\theta_j-120^\circ) & -\sin(\theta_j+120^\circ) \\ 1/2 & 1/2 & 1/2 \end{pmatrix}$$

$$\boldsymbol{P}^{-1}(\theta_j) = \begin{pmatrix} \cos\theta_j & -\sin\theta_j & 1 \\ \cos(\theta_j-120^\circ) & -\sin(\theta_j-120^\circ) & 1 \\ \cos(\theta_j+120^\circ) & -\sin(\theta_j+120^\circ) & 1 \end{pmatrix}$$

$$\boldsymbol{T}(\delta_{ij}) = \begin{pmatrix} \cos\delta_{ij} & \sin\delta_{ij} & 0 \\ -\sin\delta_{ij} & \cos\delta_{ij} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{T}'(\delta_{ij}) = \begin{pmatrix} \sin\delta_{ij} & -\cos\delta_{ij} & 0 \\ \cos\delta_{ij} & \sin\delta_{ij} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Appendix B Equivalent Circuit of 3-phase Transmission Network in State space

Only those cases of transmission networks are considered here which consist of 3-phase lumped parameter transmission lines interconnecting buses, and which have no capacitive elements. Capacitances between the lines and the ground and series capacitances for the compensation of reactive power or those for the improvement of transient stability must therefore be approximately expressed with capacitances at the buses in order that the inductive and conductive elements can be separated from the capacitive elements for consideration as a group.

A 3–phase transmission line is separated into inductive and conductive parts. Two parts are represented with oriented single lines named inductor-edge and resistor-edge respectively. A set of vectors and matrices corresponding to the edges is introduced:

$V=[v_a, v_b, v_c]^T$; edge voltage vector consisting of phase voltages
$I=[i_a, i_b, i_c]^T$; edge current vector consisting of phase currents
R_i ; 3×3 diagonal matrix consisting of phase resistances of the i-th line
L_i ; 3×3 symmetric matrix consisting of self- and phase to phase mutual inductances of the i-th line
M_{ij} ; 3×3 matrix consisting of the j-th line to the i-th line mutual inductances, since the mutual inductances are reciprocal, $M_{ij}=M^T_{ji}$ holds where $i\neq j$.

Supplementarily, let:

J-bus ; bus to be contained in the equivalent circuit
J-edge ; additional edge inserted in the *D*-graph for the formation of the equivalent circuit
D-graph ; graph formed by single line representation of the transmission network
ND-graph ; composite graph of the *D*-graph and the *J*-edges
n_J ; the number of *J*-buses

These notations are from Reed[8]. Also let the stored energy of an inductor-edge be a vector whose components are the stored energies of phase inductors.

First the equivalent circuit of a tree-formed transmission system is considered. The stored energy of an inductor-edge is a function of the inductor-edge current. Thus, if the stored energies of a set of inductor-edges are dependent, the inductor-edge currents are necessarily dependent, and the inductor-edges must therefore form a cutset. Conversely, if a set of inductor-edges does not form a cutset, their stored energies are independent. Thus, only when a transmission network does not contain any loops, that is, only when its form is a tree, is it possible to insert *J*-edges across the *J*-buses in such a way that any combination of the *J*-edges with inductor-edges forms a cutset, whereas any combination of the *J*-edges alone does not form a cutset. (where the number of the *J*-edges becomes (n_J-1))

Let an arbitrary positive orientation be allocated to the current through each edge of the *ND*-graph. Then take the corresponding positive orientations for the edge voltages to be such that simultaneously positive current and positive voltage for an inductor-edge corresponds to the absorption of power by the edge. A set of fundamental independent loops may be formed by insertion of the *J*-edges into the *D*-graph one at a time. (*D*-graph considered here is a tree.) For the *D*-graph, let a correspondent transformation matrix $\boldsymbol{T}$ be defined as having elements:
$T_{ij}=+\mathbf{1}$; if the positive orientation of current in tree branch i and the positive orientation of current in *J*-edge j are in the same direction in the loop formed by the insertion of *J*-edge j into the tree
$T_{ij}=-\mathbf{1}$; if the positive orientation of current in tree branch i and the positive orientation of current in *J*-edge j are in opposite direction in the loop formed by the insertion of *J*-edge j into the tree
$T_{ij}=\mathbf{0}$; if the tree branch i does not lie in the loop formed by the insertion of *J*-edge j into the tree
where T_{ij} is a 3×3 submatrix, $\mathbf{1}$ is a 3×3 unit matrix, and $\mathbf{0}$ represents a null matrix of all orders

If a set of composite vectors, $\boldsymbol{V}_r$, $\boldsymbol{V}_l$, $\boldsymbol{V}_J$, $\boldsymbol{I}_r$, $\boldsymbol{I}_l$, $\boldsymbol{I}_J$ are introduced whose components are tree-branch-resistor, tree-branch-inductor and *J*-edge voltages and their currents respectively, the application of Kirchhoff's node and loop laws for current and voltage variables and partitioning the transformation matrix $\boldsymbol{T}$ gives:

$$\begin{bmatrix} \boldsymbol{I}_r \\ \boldsymbol{I}_l \\ \boldsymbol{V}_J \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{T}_1 \\ \mathbf{0} & \mathbf{0} & \boldsymbol{T}_2 \\ -\boldsymbol{T}_1^T & -\boldsymbol{T}_2^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_r \\ \boldsymbol{V}_l \\ \boldsymbol{I}_J \end{bmatrix} \tag{B-1}$$

Then, the relationships of edge-voltages to edge-currents give:

$$\boldsymbol{V}_r = \boldsymbol{R}\boldsymbol{T}_r \qquad \boldsymbol{V}_l = \boldsymbol{M}_p\boldsymbol{I}_l \tag{B-2}$$

where

$$\boldsymbol{R} = \begin{bmatrix} R_1 & \mathbf{0} & - \\ \mathbf{0} & R_2 & - \\ - & - & - \end{bmatrix} \qquad \boldsymbol{M} = \begin{bmatrix} L_1 & M_{12} & - \\ M_{21} & L_2 & - \\ - & - & - \end{bmatrix}$$

To obtain the relationship between $\boldsymbol{V}_J$ and $\boldsymbol{I}_J$ from eqns. (B–1) and (B–2), the vectors $\boldsymbol{V}_l$, $\boldsymbol{V}_r$, $\boldsymbol{I}_l$ and $\boldsymbol{I}_r$ are eliminated to give:

$$\boldsymbol{V} = (\boldsymbol{T}_1^T\boldsymbol{R}\boldsymbol{T}_1 + \boldsymbol{T}_2^T\boldsymbol{M}\boldsymbol{T}_2 p)(-\boldsymbol{I}_J) \tag{B-3}$$

Since the relationship of $\boldsymbol{V}_J$ to $\boldsymbol{I}_J$ is obtained so that the Kirchhoff's law of eqn. (B–1) may pertain to the *ND*-graph, the effect of the currents flowing in the *D*-graph is completely countered by the effect of $\boldsymbol{I}_J$, and the voltages across the *J*-buses are conserved in $\boldsymbol{V}_J$. Therefore, the relationship between the currents which counter $\boldsymbol{I}_J$, i.e. $(-\boldsymbol{I}_J)$ and $\boldsymbol{V}_J$ may represent the transmission network precisely.

In the case of the transmission network containing loops in its structure, there necessarily exist some cutsets among the fundamental system of cutsets that cut simultaneously a combination of the *J*-edges and the two edges contained in a loop. Consequently, it is impossible to express all the currents flowing in the transmission network with only the *J*-edge currents. It is necessary to divide the *J*-bus contained in a loop into two *J*-buses in order to get the intermediate tree-formed network. After constructing its equivalent circuit, the short circuit of the separated *J*-buses gives the equivalent circuit of the transmission network. This procedure may be applied also to the network, in which some loops do not contain any *J*-buses, by changing a bus contained in the loop to a *J*-bus. However, when the division of all *J*-buses is needed to form the intermediate tree formed network, this algorithm cannot be used. Even if this is the case, considerable simplification may be expected in the application of this algorithm to the tree-part of the transmission network having no mutual inductances between the tree-part and the remainder.