On the Iteration Method for the Optimal Transmission System Planning

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By

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The optimal transmission system design problem can be formulated by an integer linear program when the construction cost characteristics are expressed by a staircase function.

In this paper, we deal with the iteration method to obtain more accurate approximate solution of integer linear program and the procedure to solve effectively the large linear program by use of a decomposition principle and a network flow theory. Two examples on this problem are presented.

1. Introduction

The development of mathematical theory and digital computer has enabled more difficult transmission system planning considering load growth and topological situations. The method for solving a transmission system planning by means of integer linear programming with zero-one variables has already been presented in the previous publication,[1] where the cost characteristics for a construction of system elements are expressed by a staircase function. The integer linear program obtained in this fashion has an enormous number of constrained equations and to find its solution may be rather laborious even if we use a high speed electronic computer for a calculation.

This paper deals with the iteration method to obtain an approximate solution of integer linear program and the procedure to solve effectively the large linear program by use of a decomposition principle and a network flow theory.

2. Formulation of Power System Planning as Linear Programs[1]

The transmission system which consists of power plants, substations, transmission lines and loads can be represented by the directed network in which trans-

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mission lines correspond to arcs and power plants, substations and loads correspond to nodes. We can suppose that each are and some nodes have capacities; it may be thought of intutively as representing the maximal amount of transmitted electric power. In this network, there are multiple sources and multiple sinks, and several nodes have capacities.

However, by the addition of two nodes and several arcs, the network can be reduced to the extended network with a single source and a single sink. If the given transmission system is that of Fig. 1, the extended network is shown in Fig. 2, where $g(x)$, $d(x)$, $l(x)$ and $t(x)$ are respectively capacities of power plants, substations, loads and transmission lines.

In order to investigate whether the demand can be fulfilled from the supplies in the transmission system, calculate the maximal flow from source s_0 to sink t_0 on the extended network shown in Fig. 2. Let F_m be the maximal power flow obtained

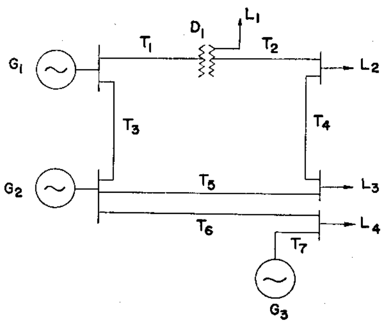

Fig. 1. Transmission system.

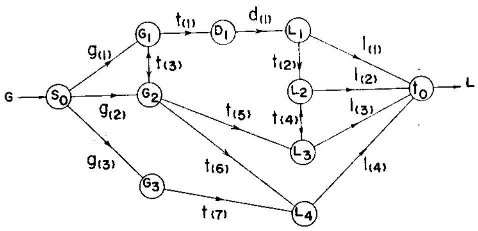

Fig. 2. Extended network for the transmission system shown in Fig. 1.

from the calculation and if we put the overall supply power and load power as G and L, these are written as

$$\left.\begin{aligned} G &= \sum_{x\in S} g(x) \\ L &= \sum_{x\in T} l(x) \end{aligned}\right\} \qquad (1)$$

where S and T are sets of all sources and all sinks, respectively. If $F_m = L \leq G$, the demands can be filfilled from supplies.

When we intend to design the transmission system to satisfy the increasing demand, the extended network corresponding to the transmission system involved plants, substations and transmission lines to be constructed in the future must satisfy the condition

$$F_m = L \leq G \qquad (2)$$

If the cost characteristics are given by a staircase function, we must study a special type of integer linear programming problem in which the integer variable has to be either 0 or 1, depending on whether or not some increase of the system element is used.

At present, several methods are available for solving integer linear programs. (2),(3),(4) Particuraly, B. Ealas' algorithm represents a combinatorial approach to the problem of solving integer linear programs with zero-one variables and it seems to work very efficiently.

3. Iteration Method

As mentioned in the previous section, a transmission system planning could be formulated as integer linear programs in the case where cost characteristics are given by a staircase function. However, if we assume that the construction cost is proportional to the capacity for an increase as shown in Fig. 3, the optimal (minimal cost) expansion planning can be reduced to the problem of constructing network flows that minimize cost as follows:

Minimize the linear function

$$\sum_{(i,j)\in A} a_{ij} f_{ij} \tag{3}$$

subject to the linear constraints

$$\left.\begin{array}{ll} \sum_{(i,j)\in A} (f_{ij}-f_{ji}) = \begin{cases} F & (i=s_0) \\ 0 & (i \neq s_0,\ t_0) \\ -F & (i=t_0) \end{cases} & \\ f_{ij} \leqq C_{ij} = C_{ij}^{(0)} + \Delta C_{ij} & (i,j)\in A \\ f_{ij} \geqq 0 & (i,j)\in A \end{array}\right\} \tag{4}$$

where f_{ij} : power flow from node i to node j
$C_{ij}^{(0)}$: capacity of the system element (i, j) being already constructed
ΔC_{ij}: capacity of the system element (i, j) for an increase
F : total demand, i.e. flow value from source s to sink t
a_{ij} : unit construction cost of the system element (i, j)
A : set of all arcs

In the network flow theory, this problem is called the minimal cost flow problem which was first studied by Ford and Fulkerson.[5] They introduced the primal dual algorithm for solving effectively the minimal cost flow problem.

If we approximate the cost characteristic by a linear function as shown in Fig. 4, a solution of the minimal cost flow problem can be considered as a first

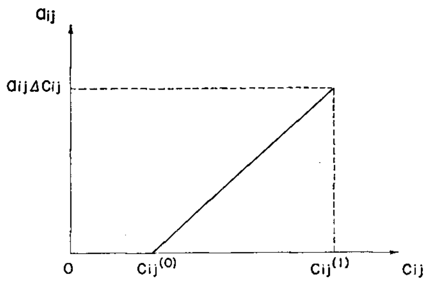

Fig. 3. Construction cost characteristic.

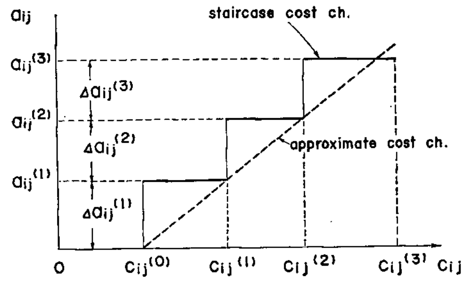

Fig. 4. Approximation of cost characteristic

approximate solution of a corresponding transmission system planning which may be described in an integer linear programming problem.

Using the results of the minimal cost flow problem, we may determine the capacity of each system element in the optimal transmission system as follows. If the power flow f_{ij} of arc (i, j) is greater than the arc capacity $C_{ij}^{(k-1)}$ and less than $C_{ij}^{(k)}$, in an approximate solution (i.e. solution of the minimal cost flow problem), the arc capacity is taken as the value $C_{ij}^{(k)}$ and if f_{ij} is equal to or less than $C_{ij}^{(0)}$, the arc capacity is taken as the value $C_{ij}^{(0)}$, namely,

$$C_{ij} = \begin{cases} C_{ij}^{(k)}, & \text{if } C_{ij}^{(k)} < f_{ij} \leqq C_{ij}^{(k)} \\ C_{ij}^{(0)}, & \text{if } f_{ij} \leqq C_{ij}^{(0)} \end{cases} \tag{5}$$

Now, to improve the accuracy of the approximation, we consider the following procedures. First find the power flow pattern $\{f_{ij}\}$ associated with the flow value F in the network corresponding to the transmission system constructed by the above mentioned procedure. When the arc capacity is taken as the value $C_{ij}^{(k)}$, we introduce the utilization factor given by the following equation

$$\lambda_{ij} = \frac{f_{ij} - C_{ij}^{(k-1)}}{C_{ij}^{(k)} - C_{ij}^{(k-1)}} \tag{6}$$

and calculate utilization factors for all arcs.

Let us group all arcs into the following two sets, one contains the arcs where

$$0 < \lambda_{ij} \leqq \beta \tag{7}$$

another contains the arcs where

$$\lambda_{ij} > \beta \quad \text{or} \quad \lambda_{ij} \leqq 0 \tag{8}$$

where β is a predetermined constant. We modify the unit construction cost a_{ij} by the following equations

$$\left.\begin{array}{ll} a'_{ij} = a_{ij}\dfrac{C_{ij}^{(k)}-C_{ij}^{(0)}}{C_{ij}^{(k-1)}-C_{ij}^{(0)}+\lambda_{ij}(C_{ij}^{(k)}-C_{ij}^{(k-1)})} & (0<\lambda_{ij}\leqq\beta) \\ a'_{ij} = a_{ij} & (\lambda_{ij}>\beta \quad \text{or} \quad \lambda_{ij}\leqq 0) \end{array}\right\} \tag{9}$$

Substitute the new unit construction cost given by Eq. (9) into Eq. (3), we solve again the minimal cost flow problem. Then, another approximate solution will be obtained.

In like manner, we can obtain successivelly many approximate solutions, but, unfortunately, we can not always expect a more accurate solution by the iteration. Therefore, after several iterations we intend to adopt the minimal cost flow solution as the approximate solution of integer linear programs.

Next consider the long term design of electrical transmission system. If the period over which the optimal design sequence is required is be divided into n intervals, an electrical transmission system must satisfy n constraints imposed by load demands in each interval. The optimal long term planning for load growth should have the lowest cost of accumulation of annual requirements. Therefore, the planning problem is to find the economically optimal sequence and if the cost characteristics are expressed by a staircase function, we can formulate this problem as a large scale integer linear programming problem. The linear program obtained in this fashion has an enormous number of constrained equation and this method does not suit practical use.

In order to find an approximate solution for the problem with staircase cost characteristics, we assume that the construction cost is proportional to the capacity for an increase. Then the planning problem will be exprestied as the minimal cost flow problem which is written as follows:

Minimize the linear function

$$z = \sum_{k=1}^{n} \sum_{(i,j)\in A} a_{ij}^{(k)} u_{ij}^{(k)} \tag{10}$$

subject to the linear constraints

$$\sum_{(i,j)\in A} (f_{ij}^{(k)}-f_{ji}^{(k)}) = \begin{cases} F^{(k)} & (i=s_0) \\ 0 & (i\neq s_0,\ t_0),\ (k=1,\ 2,\cdots,\ n) \\ -F^{(k)} & (i=t_0) \end{cases} \tag{11}$$

$$f_{ij}^{(k)} - \sum_{s=1}^{k} C_{ij}^{(s)} \leqq C_{ij}^{(0)} \qquad (i,\ j)\in A,\ (k=1,\ 2,\cdots,\ n) \tag{12}$$

$$u_{ij}^{(k)} \leqq v_{ij}^{(k)} \qquad (i,\ j)\in A,\ (k=1,\ 2,\cdots,\ n) \tag{13}$$

$$f_{ij}^{(k)} \geqq 0 \qquad (i,\ j)\in A,\ (k=1,\ 2,\cdots,\ n) \tag{14}$$

$$u_{ij}^{(k)} \geqq 0 \qquad (i, j) \in A, \ (k = 1, 2, \cdots, n) \tag{15}$$

where $a_{ij}^{(k)}$: unit construction cost of the system element (i, j) in kth interval

$f_{ij}^{(k)}$: power flow from node i to node j in kth interval

$u_{ij}^{(k)}$: capacity of the system element (i, j) to be constructed in kth interval

$F^{(k)}$: total demands in kth interval

$C_{ij}^{(k)}$: capacity of the system element (i, j) being already constructed

$v_{ij}^{(k)}$: maximal capacity of the system element (i, j) which is allowed to be constructed in kth interval

Using the above-mentioned iteration procedure, we have many approximate solutions of the problem with staircase cost characteristics and can adopt a more accurate solution as a solution of a large scale integer linear programming problem.

3. Procedure to Solve a Large Linear Program

In this section we deal with the procedure to solve effectively the large linear program described by Eqs. (10)–(15). The given problem is a minimal cost flow problem with multi-commodities. At present, we can not see the refined mathematical technique applied to this problem such as a primal dual algorithm for the minimal cost flow problem with one commodity flow.

In the minimal cost flow problem given by Eqs. (10)–(15), when $u_{ij}^{(k)}$ is not restricted, that is, $v_{ij}^{(k)}$ is equal to infinity, it is shown that this problem may be effectively solved by use of a decomposition principle and a network flow theory.(6) In this paper, we will follow this method.

The dual problem of the primal problem given by Eqs. (10)–(15) is written as follows:

Maximize the linear function

$$z = \sum_{k=1}^{n} \{F^{(k)}(\alpha_s^{(k)} - \alpha_t^{(k)}) - \sum_{(i,j)\in A} C_{ij}\alpha_{ij}^{(k)}\} \tag{16}$$

subject to the linear constraints

$$\alpha_i^{(k)} - \alpha_j^{(k)} - r_{ij}^{(k)} \leqq 0 \qquad (i, j) \in A, \ (k = 1, 2, \cdots, n) \tag{17}$$

$$\sum_{s=k}^{n} r_{ij}^{(s)} - \sigma_{ij}^{(k)} \leqq a_{ij}^{(k)} \qquad (i, j) \in A, \ (k = 1, 2, \cdots, n) \tag{18}$$

$$r_{ij}^{(k)} \geqq 0 \qquad (i, j) \in A, \ (k = 1, 2, \cdots, n) \tag{19}$$

$$\sigma_{ij}^{(k)} \geqq 0 \qquad (i, j) \in A, \ (k = 1, 2, \cdots, n) \tag{20}$$

where $\alpha_i^{(k)}$, $\gamma_{ij}^{(k)}$ and $\sigma_{ij}^{(k)}$ are dual variables corresponding to Eqs. (11), (12) and (13), respectively.

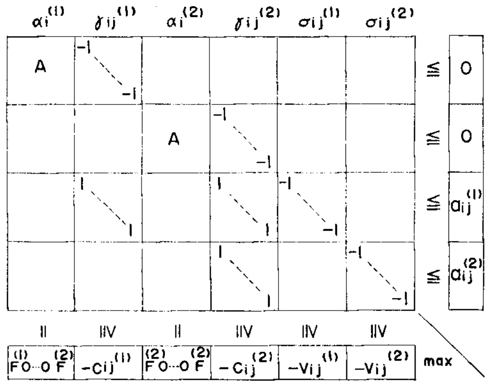

Fig. 5. Coefficient matrix of dual problem (k=2).

The coefficient matrix of the dual problem is shown in Fig. 5. For this problem, the constraints consist of independent sets of equations which refer to the same time interval and these subsets are tied together by a small set of equations. We can apply a decomposition principle for this dual problem. Namely, the dual problem will be divided into the following n decomposed subprograms of maximizing

$$z_k = F^{(k)}(\alpha_s^{(k)} - \alpha_t^{(k)}) - \sum_{(i,j)\in A} C_{ij}\gamma_{ij}^{(k)} \tag{21}$$

subject to

$$\begin{aligned} &\alpha_i^{(k)} - \alpha_j^{(k)} - \gamma_{ij}^{(k)} \leqq 0 \\ &\gamma_{ij}^{(k)} \geqq 0 \end{aligned} \qquad (i, j)\in A \tag{22}$$

and one special subprogram of maximizing

$$-\sum_{k=1}^{n} \sum_{(i,j)\in A} v_{ij}^{(k)}\sigma_{ij}^{(k)} \tag{23}$$

subject to

$$\sigma_{ij}^{(k)} \geqq 0 \tag{24}$$

We note that the variable $\sigma_{ij}^{(k)}$ in the last subprogram is only restricted by Eq. (24), then, the solution of the last decomposed subprogram may be infinity.

Now considering the dual problem for the decomposed subprogram given by Eqs. (21) and (22), we have the following problem of minimizing

$$z_k \tag{25}$$

subject to

$$\sum_j (f_{ij}^{(k)} - f_{ij}^{(k)}) + z_k = F^{(k)} \tag{26}$$

$$\sum_{(i,j)\in A} (f_{ij}^{(k)} - f_{ji}^{(k)}) = 0 \qquad (i \neq s,\, t) \tag{27}$$

$$0 \leqq f_{ij}^{(k)} \leqq C_{ij} \qquad (i, j) \in A \tag{28}$$

This is a maximal flow problem and we can solve easily by the labelling method.[7] Hence we may solve the dual problem given by Eqs. (25)–(28) instead of the primal problem given by Eqs. (21) and (22).

4. Examples

Let us consider the transmission system shown in Fig. 6. G, T and L are power plant, transmission line and load, respectively and the number appearing on the elements is a capacity being already constructed of each element. The number in a bracket on the bus is a node number when a transmission system is represented by the directed network. Now we assume that only the system element indicated by asterisk can be allowed to be constructed in the future. Figure 7 shows a unit capacity and a construction cost of system elements.

Making use of the iteration procedure described in the preceeding section, we get approximate solutions where we select $\beta = 0.6$. Figure 8 shows these results and we shall adopt third result as the optimal plan. In this plan generator G_3 and transmission lines T_4 and T_8 will be constructed and a total construction cost is $15{,}676 \times 10^6$ yen. From a combinatorial method we can prove that this result is a optimal (minimal cost) plan[8].

Next consider a long term design of the transmission system shown in Fig. 9. We adopt two years as the period considered for a planning and estimate the demand of each load at the end of each year as shown in Table 1. Unit capacity and construction cost of a system element (i, j) which may be constructed in each year are shown in Fig. 10, where dotted lines show approximate cost characteristics and numbers appearing on them are unit construction costs of system elements.

We assume that the cost characteristic is linear and by the iteration method in the preceeding section we calculate approximate solutions, where β is 0.6 and a number of iteration is two. Table 2 shows calculated results. Then we shall adopt the first result as the optimal plan.

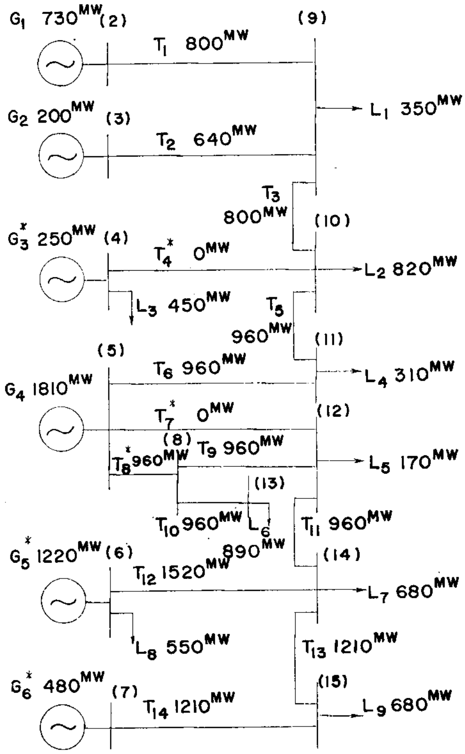

Fig. 6. Model transmission system (1).

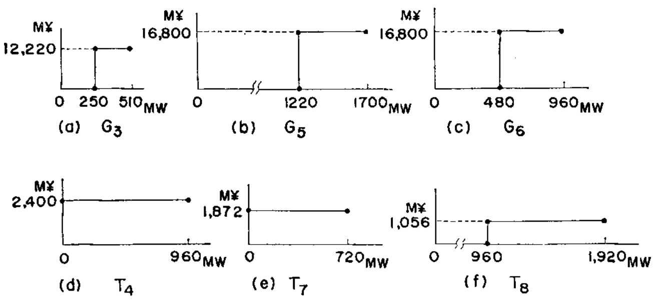

Fig. 7. Unit capacity and construction cost of system elements.

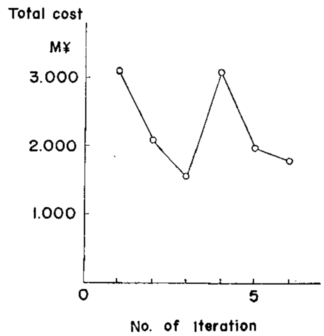

Fig. 8. Total construction cost.

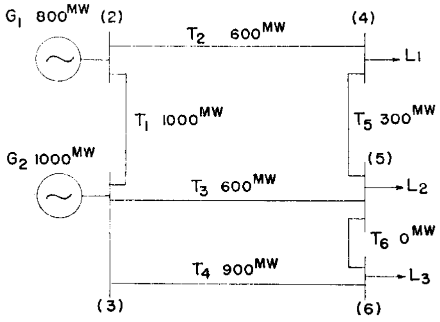

Fig. 9. Model transmission system (2).

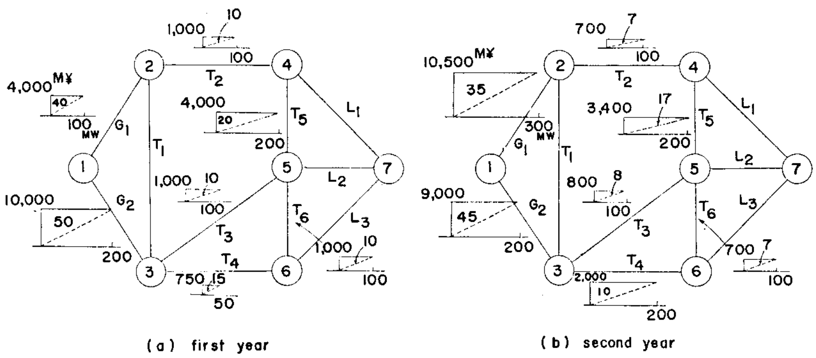

Fig. 10. Unit capacity and construction cost of system elements.

Table 1. Estimation of demand power.

	L_1 (MW)	L_2 (MW)	L_3 (MW)
first year	500	550	1,000
second year	600	600	1,200

Table 2. Calculated results.

No. of Iteration	System elements to be constructed		Total cost (M¥)
	first year	second year	
1	(1,2) (1,3) (3,6) (5,6)	(1,2) (1,3) (2,4) (3,6)	37,950
2	(1,2) (1,3) (3,6) (5,6)	(1,2) (1,3) (3,5) (3,6)	37,960

5. Conclusions

The optimal transmission system design problem can be formulated by an integer linear program, if we assume that the cost characteristics for a construction of system elements are expressed by a staircase function.

In this paper, we describe the iteration method to obtain more accurate approximate solution of integer linear program. Assuming that the construction cost is proportional to the capacity for an increase and solving the minimal cost flow problem, we obtain an approximate solution. Next calculating utilization factors we modify the unit construction costs by Eq. (9). We solve again the minimal cost flow problem and we have another approximate solution. In like manner, we can obtain successivelly many approximate solutions. After several iterations, we shall adopt the minimal cost flow solution as the approximate solution of integer linear program.

Using a decomposition principle and a network flow theory we show an effective method to solve a large scale linear program. Since we can formulate the long term design of transmission system as a large linear program, this method can be applied to the long term design.

From the results of the two examples, we are assured that the iteration method is a useful procedure to obtain more accurate approximate solution of integer linear program.

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