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By

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This paper describes the results of experimental studies made on the effects of differential shrinkage of the composite beams having the flange or the upper half portion cast on the precast prestressed beam. The cast-in-place concrete was made with artificial lightweight aggregates.

In all thirty beam specimens were fabricated and the variables involved in the specimens were: shape of cross-section, amount of shear connecter, ratio of shear span to beam depth, and age at casting slab.

Differences of shrinkage and creep characteristics between the cast-in-place lightweight concrete and the precast normal concrete were measured by using the control specimens.

The differential shrinkage set up internal stresses in the composite construction and its effects on the cracking moment and on the ultimate strength of the composite beams were investigated and compared with the theoretical analysis.

It is concluded that the differential shrinkage has so profound an effect that it should be taken into consideration in design, especially when the slab concrete is placed at a later time.

I Introduction

Composite prestressed concrete girder which consists of precast prestressed concrete girder and cast-in-place slab is a favorite construction from the economical and technologycal points of view. Many more economical advantages could be expected by using for the cast-in-place slab the artificial lightweight aggregate conrete which has been widely used. This type of composite construction, however, has some problems to be solved which are listed below.

(1) Solid interaction between the cast-in-place slab and the precast beam. The interface between the slab and the girder is apt to become structurally weak and therefore the bond strength of the interface and the contribution of shear connector to the shear resistance should be clarified.

(2) Effects of internal stress due to the difference of shrinkage and creep chara-

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cteristics between the cast-in-place slab and the precast beam.

This differential shrinkage stress may in some cases affects greatly the warping and the cracking strength of the composite construction.

With the problems described above in mind, this study was carried out especially to clarify the effects of differential shrinkage on the composite prestressed concrete beam having a rectangular or T-shaped cross section, of which the flange or the upper portion was cast on the precast beam with the artificial lightweight aggregate concrete.

II Analytical Method on Differential Shrinkage

Stress over the cross section due to the differential shrinkage varies depending on the differences of mechanical properties between the slab and the girder concretes (modulus of elasticity, shrinkage and creep), the characteristics of cross section, the age of slab casting and so on. Generally, two analytical methods on differential shrinkage effects have been proposed, one is the so-called composite section method proposed by Mörsch and Birkeland, and the other is the separate section method proposed by Evans[4], Parker, Ozell[5], Branson[5] and one of the authors[1~3]. One of the authors analysed the differential shrinkage stresses considering the relaxation due to creep and the rotation of section of precast prestressed concrete girder as briefly discribed below.

Notation (suffixes 1, 2 indicate slab portion and precast prestressed concrete beam, respectively)

- M_1, M_2: internal moment induced by differential shrinkage
- N_1, N_2 : internal normal force induced by differential shrinkage
- S_1, S_2 : free shrinkage strain
- ΔS_2 : relative difference of shrinkage between the upper fiber and the lower fiber of precast girder induced by eccentrical prestressing
- φ_{1t}, φ_{2t} : creep factor at the age "t"
- φ_{1n}, φ_{2n} : final creep factor
- E_1, E_2 : modulus of elasticity
- A_1, A_2 : area
- I_1, I_2 : moment of inertia
- a : distance from the centroid of slab portion to that of precast concrete girder
- h : height of precast concrete girder

Normal force N_1 and moment M_1 in the slab portion and N_2, M_2 in the precast

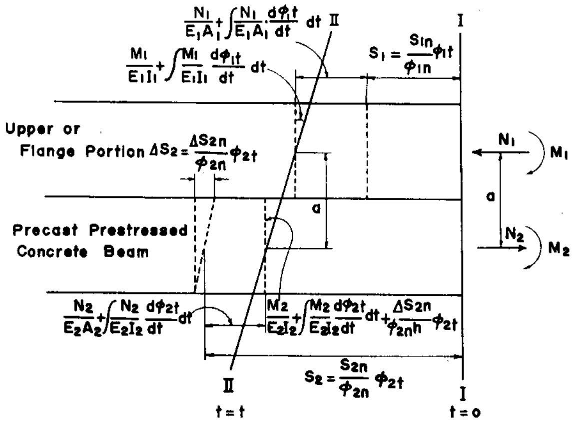

Fig. 1 Strains over Cross Section

girder are induced by differential shrinkage as shown in Fig. 1.

Assuming that shrinkage strains develop with time similarly to creep factor, shrinkage strains are given as follows.

$$S_1 = \frac{S_{1n}}{\varphi_{1n}} \cdot \varphi_{1t}, \quad S_2 = \frac{S_{2n}}{\varphi_{2n}} \cdot \varphi_{2t}, \quad S_2 = \frac{\Delta S_{2n}}{\varphi_{2n}} \cdot \varphi_{2t} \tag{1}$$

Equilibriums of normal forces and moments are held because no external force is applied, so we can obtain the following equations.

$$N_1 = N_2 \tag{2}$$

$$M_1 + M_2 = N_1 \cdot a \tag{3}$$

Strains induced over the cross section consist of shrinkage strains, elastic and creep strains due to normal forces and moments, which are shown in Fig.-1. Now, assuming that the plane of the cross section remains plane also after the interaction between the slab and the girder is completed, the next two equations are obtained. (see Fig. 1)

$$\frac{M_1}{E_1 I_1} + \int_0^t \frac{M_1}{E_1 I_1} \cdot \frac{d\varphi_{1t}}{dt} dt = \frac{M_2}{E_2 I_2} + \int_0^t \frac{M_2}{E_2 I_2} \frac{d\varphi_{2t}}{dt} dt + \frac{\Delta S_{2n}}{\varphi_{2n} h} \cdot \varphi_{2t} \tag{4}$$

$$\begin{aligned} \frac{S_{2n}}{\varphi_{2n}} \cdot \varphi_{2t} = {} & \frac{S_{1n}}{\varphi_{1n}} \cdot \varphi_{1t} + \frac{N_1}{E_1 A_1} + \int_0^t \frac{N_1}{E_1 A_1} \frac{d\varphi_{1t}}{dt} dt \\ & + a\left(\frac{M_1}{E_1 I_1} + \int_0^t \frac{M_1}{E_1 I_1} \frac{d\varphi_{1t}}{dt} dt\right) + \frac{N_2}{E_2 A_2} + \int_0^t \frac{N_2}{E_2 A_2} \frac{d\varphi_{2t}}{dt} dt \end{aligned} \tag{5}$$

Solutions of the above simultanious integral equations are too complex for practical use, so the following approximate equation on creep strain can be utilized.

$$X_t+\int_0^t X_t\frac{d\varphi_t}{dt}dt = X_t\left(1+\frac{1}{2}\varphi_t\right) \tag{6}$$

Where X_t is elastic strain varying under creep phenomenon and φ_t is creep factor.

By substituting eq. (6) into eqs. (4) and (5), the unknown normal forces and moments N_1, N_2, M_1 and M_2 are obtained as follows.

$$M_1 = \frac{\Delta_1}{\Delta},\ M_2 = \frac{\Delta_2}{\Delta},\ N_1 = N_2 = \frac{1}{a}(M_1+M_2) \tag{7}$$

where

$$\Delta = A_1B_2-A_2B_1,\quad \Delta_2 = F_1B_2-B_1F_2,\quad \Delta_1 = A_1F_2-F_1A_2$$

$$A_1 = \mu\left(1+\frac{1}{2}\varphi_{2t}\right)+\left(1+\frac{1}{2}\varphi_{1t}\right)$$

$$B_1 = \mu\left(1+\frac{1}{2}\varphi_{2t}\right)+\left(1+\frac{D_1a^2}{K_1}\right)\left(1+\frac{1}{2}\varphi_{1t}\right)$$

$$A_2 = \lambda\left(1+\frac{1}{2}\varphi_{2t}\right),\quad B_2 = -\left(1+\frac{1}{2}\varphi_{1t}\right)$$

$$F_1 = (S_2-S_1)aD_1,\quad F_2 = -\left(\frac{\Delta S_2}{h_2}\right)K_1$$

$$\mu = \frac{D_1}{D_2} = \frac{E_1A_1}{E_2A_2},\quad \lambda = \frac{K_1}{K_2} = \frac{E_1I_1}{E_2I_2}$$

Thus, the stresses over cross section due to differential shrinkage can be easily computed with the above N_1, N_2, M_1 and M_2.

The separate section methods proposed by the others are also based on a similar procedure, but the above method has characteristics of dealing with the rotation of section of precast prestressed concrete beam due to eccentric prestressing just as considered by Evans[4] the influence of reinforcement in cast-in-situ slab on differential shrinkage stress.

III Tests on the Composite Beams

(1) Materials used

Artificial lightweight aggregates were used in the concrete of upper or flange portion cast on the precast prestressed beam, and natural sand and gravel in the concrete of precast concrete beam. These artificial lightweight aggregates are of a pelletized type named "Lionite" for structural use. The physical properties of those aggregates are shown in Table 1. The ϕ14 mm and ϕ16 mm prestressing bars were used in the precast portions of the rectangular and T-shaped composite beams, respectively. The mechanical properties of those prestressing bars are shown in Table 2.

The $\phi 6$ mm round bars were used both as the stirrup and the shear connector as shown in Fig. 2.

Table 1 Physical Properties of Aggregates

Aggregates \ Properties		Specific Gravity*	Absorption (% by wt.)	Fineness Modulus	Bulk Density (kg/m³)
(I)	Sand	2.00	4.20	2.99	1120
	Gravel	1.41	2.55	6.46	910
(II)	Sand	2.61	0.98	2.97	1660
	Gravel	2.64	0.98	6.38	1690

(I); Lightweight Aggregate (II); River Aggregate
* Saturated Surface-dry Condition

Table 2 Mechanical Properties of Prestressing Bars

Diameter (mm)	Yield Strength (kg/mm²)	Tensile Strergth (kg/mm²)	Elongation (%)	Modulus of Elasticity (kg/mm²)	Note
14.86	131.7	140.9	7.3	20.20×10^3	for T-shaped Test Beams
12.87	137.0	144.0	8.0	20.00×10^3	for Rectangular Test Beams

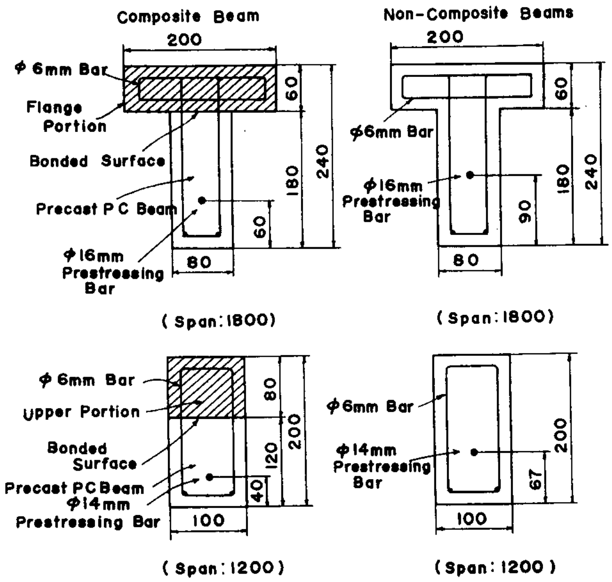

Fig. 2 Cross Section of Test Beams

Table 3 Mix Proportions

*	Slump (cm)	Cement (kg/m³)	Water (kg/m³)	Water-Cement Ratio (%)	Sand (kg/m³)	Gravel (kg/m³)	Max. size (mm)
(I)	7.5±1	300	175	66.3	575	616	15
(II)	5.0±1	450	160	36.0	649	1130	15

* (I): Lightweight Concrete in Flange Portion
(II) Normal Concrete in Precast PC Beam

Table 4 Mechanical Properties of Concretes (at the Age of Loading Test)

Kind of Concrete \ Properties	Compressive Strength (kg/cm²)	Splitting Strength (kg/cm²)	Modulus of Rupture (kg/cm²)	Modulus of Elasticity (kg/cm²)	Age of the Concrete (weeks)
Lightweight Concrete	211	14.6	28.2	14.1×10^4	4
	196	14.0	26.1	15.0×10^4	5
Normal Concrete	374	25.8	48.4	35.6×10^4	9
	402	24.8	47.8	37.0×10^4	19

(2) Mix proportions of concrete

The mix propotions of the above two concretes are shown in Table 3, and the mechanical properties of those concretes in Table 4.

(3) Test beams

Two kinds of beam specimens were made, one has rectangular cross section and the other T-shaped cross section, both as shown in Fig. 2.

Precast concrete beams were prestressed by the prestressing bars at the age of 3 weeks to about 100 kg/cm² at the lower fiber and zero at the upper fiber, and represtressed and grouted at the age of 4 weeks. Concrete of flange potion was cast on the precast prestressed beam at 5 or 14 weeks. These specimens were cured in the laboratory till they were tested statically at the age of 9 or 19 weeks.

Specimens used in this test have three variables, these are the treatment of the bonded surface, the ratio of shear span length to beam height and the age of flange casting. The former two variables were accommodated to investigate the strength of bonded surface. Details of the specimens are given in Table 5 and Fig. 2.

Control specimens were also made to measure the free shrinkage and the free creep strains of the concretes used. Ordinary prestressed concrete beams (non-composite beams) were also made for comparison.

To make clear the effect of differential shrinkage on the cracking strength, cracking load of the composite beam was measured by using wire strain gages at-

Table 5 Test Beams

		Kind of Beam	Treatment of* Bonded Surface	Age of Flange Casting (weeks)	Age of Test (weeks)	a/h Ratio	Notes
Rectangular Beams	Group I	R-HL-I-5-2.0	I	5	9	2.0	
		R-HL-II-5-2.0	II	5	9	2.0	
		R-HH-II-2.0	(II)		9	2.0	non-Compos.
		R-HL-II-5-2.5	II	5	9	2.5	
		R-HH-II-2.5	(II)		9	2.5	non-Compos.
	Group II	R-HL-II-14-2.0	II	14	19	2.0	
		R-HL-II-14-2.5	II	14	19	2.5	
T-shaped Beams	Group I	T-HL-I-5-2.0	I	5	9	2.0	
		T-HL-II-5-2.0	II	5	9	2.0	
		T-HL-III-5-2.0	III	5	9	2.0	
		T-HH-II-2.0	(II)		9	2.0	non-Compos.
		T-HL-II-5-3.0	II	5	9	3.0	
		T-HH-II-3.0	(II)		9	3.0	non-Compos.
	Group II	T-HL-II-14-2.0	II	14	19	2.0	
		T-HL-II-14-3.0	II	14	19	3.0	

* I: rough without shear connector
II: rough with shear connector (20 cm spacing)
III: rough with shear connector (10 cm spacing)

tached at the lower fiber of the beam.

(4) Test results and discussion

(a) Calculated differential shrinkage stresses in test beams

Differential shrinkage strains expected to occur in the test beams were estimated from the measured free shrinkage strains in the control specimens as shown in Fig. 3.

According to Fig. 3, the differential shrinkage strain at the time of loading test is about 17×10^{-5} for the composite beams of which the flange portions were cast at 5 weeks and about 28×10^{-5} for the beams of 14 weeks flange casting.

By substituting the above differential shrinkage strain and other measured data into the theoretical solutions, the differential shrinkage stress over the cross section of the test beam can be calculated.

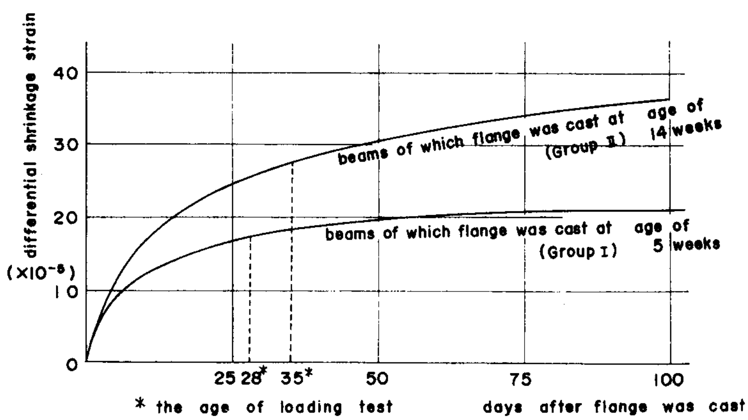

Fig. 3 Differential Shrinkage Strain of Test Beams

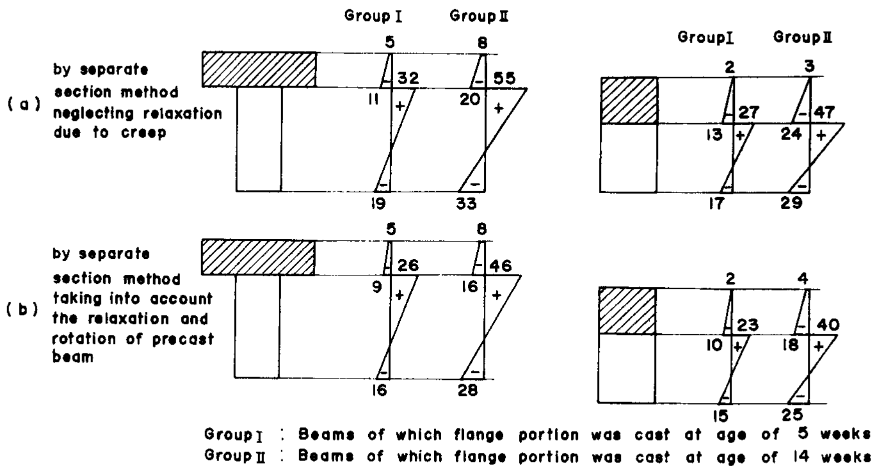

Fig. 4 Calculated Differential Shrinkage Stresses (kg/cm²)

Fig. 4 shows these differential shrinkage stresses calculated by the separate section method in two ways, that is, (i) neglecting the relaxation due to creep and (ii) taking into account the relaxation due to creep as well as the rotation of the section of precast beam.

It is seen from Fig. 4 that considerably large stresses set up due to differential shrinkage especially in group II beams in which the flanges were cast at 14 weeks.

(b) Cracking strength

In Table 6 the measured and calculated cracking moments (M_{cr}) of each beam are given. The cracking moment is computed by the following formula.

$$M_{cr} = W_c \cdot (\sigma_{cb} + \sigma_{ce}) \tag{8}$$

Table 6 Measured and Calcurated Cracking Moments of Test Beams

(unit : $t\cdot$cm)

		Kind of Beam	Measured Cracking Moment M_{cr}	Calculated Cracking Moment		
				M_1 (M_{cr}/M_1)	M_2 (M_{cr}/M_2)	M_3 (M_{cr}/M_3)
Rectangular Beams	Group I	RH-HL-I-5-2.0	85.0	80.4 (1.06)	71.1 (1.20)	72.3 (1.18)
			80.0	80.4 (1.00)	71.1 (1.13)	72.3 (1.11)
		R-HL-II-5-2.0	90.0	84.7 (1.06)	75.6 (1.19)	76.8 (1.17)
			85.0	84.7 (1.00)	75.6 (1.12)	76.8 (1.10)
		R-HH-II-2.0	85.0	98.6 (0.86)	—	—
			80.0	98.6 (0.81)	—	—
		R-HL-II-5-2.5	75.0	81.1 (0.92)	72.0 (1.04)	73.2 (1.02)
			68.0	81.1 (0.84)	72.0 (0.94)	73.2 (0.93)
		R-HH-II-2.5	93.8	98.1 (0.96)	—	—
			87.5	98.1 (0.89)	—	—
	Group II	R-HL-II-14-2.0	60.0	78.5 (0.76)	62.6 (0.96)	65.0 (0.92)
			55.0	78.5 (0.70)	62.6 (0.88)	65.0 (0.86)
		R-HL-II-14-2.5	62.5	78.5 (0.80)	62.6 (1.00)	65.0 (0.96)
			56.3	78.5 (0.71)	62.6 (0.90)	65.0 (0.87)
T-shaped Beams	Group I	T-HL-I-5-2.0	114.0	120.6 (0.95)	105.0 (1.09)	107.5 (1.06)
			108.0	120.6 (0.90)	105.0 (1.03)	107.5 (1.00)
		T-HL-II-5-2.0	108.0	127.5 (0.85)	112.7 (0.96)	114.4 (0.94)
			108.0	127.5 (0.85)	112.7 (0.96)	114.4 (0.94)
		T-HL-III-5-2.0	120.0	126.5 (0.95)	110.9 (1.08)	113.5 (1.06)
			114.0	126.5 (0.90)	110.9 (1.03)	113.5 (1.00)
		T-HH-II-2.0	132.0	142.6 (0.93)	—	—
			138.0	142.6 (0.97)	—	—
		T-HL-II-5-3.0	126.0	122.1 (1.03)	106.4 (1.18)	109.0 (1.15)
			108.0	122.1 (0.88)	106.4 (1.02)	109.0 (0.99)
		T-HH-II-3.0	117.0	141.8 (0.83)	—	—
			117.0	141.8 (0.83)	—	—
	Group II	T-HL-II-14-2.0	96.0	119.1 (0.81)	92.1 (1.04)	96.1 (1.00)
			84.0	119.1 (0.79)	92.1 (0.91)	96.1 (0.87)
		T-HL-II-14-3.0	99.0	119.1 (0.84)	92.1 (1.08)	96.1 (1.03)
			90.0	119.1 (0.76)	92.1 (0.98)	96.1 (0.94)

where

M_{cr}: cracking moment

W_c: transformed modulus of section of composite beam

σ_{cb} : flexural strength of precast concrete

σ_{ce} : effective prestress in lower fiber of the beam at the age of loading test

The effective prestress σ_{ce} in eq. (8) is estimated in the following three ways:

(i) (σ'_{ce}); Considering only the loss of prestress due to shrinkage and creep of the precast prestressed beam alone as usually done in ordinary prestressed beam, and neglecting the effects of differential shrinkage between the slab and girder concretes.

(ii) (σ''_{ce}, σ'''_{ce}); In addition to (σ'_{ce}), considering the differential shrinkage stresses in two ways as described before and corresponding to Fig. 4 (a) and (b), respectively.

Thus, three kinds of cracking moment M_1, M_2 and M_3 are calculated each corresponding to the effective prestress σ'_{ce}, σ''_{ce} and σ'''_{ce}.

Table 6 shows that the measured cracking moments M_{cr} of the beams of which the flange portions were cast at the age of 14 weeks, are about 0.7 times as large as those of the similar beams of 5 weeks flange casting. Comparison M_{cr} with M_1 gives that M_{cr} is much smaller than M_1 on the beams of 14 weeks flange casting. This fact shows that the differential shrinkage stresses, which are neglected in calculating M_1, have a large influence on the cracking moment of the composite beam, especially when the flange portion was cast later. The calculated moments M_2 and M_3 taking into account the differencial shrinkage show fairly good agreement with the measured moment M_{cr}.

(c) Ultimate strength

All the test beams failed in flexure except the non-composite T-shaped beams which were loaded with 2.0 a/h ratio. The ultimate flexural moments of the test beams are shown in Table 7 as well as the calculated ones. The calculated ultimate flexural moments of the composite beams are computed by the same method as is used for the ordinary prestressed concrete beams.

In Table 7, the measured ultimate moments show fairly good agreement with the calculated ones. Standing on another view point, this means that the ultimate flexural moments are little affected by the differential shrinkage stresses.

(d) Other results

Other results concerning the differential shrinkage are as follows.

(i) No harmful slip was found during the flexural test even in the beams which

Table 7 Measured and Calculated Ultimate Flexural Moments

(unit : t·cm)

		Kind of Beam	Measured Ultimate Moment M_u	Calculated Ultimate Moment M_{cal}	Calculated Ultimate Moment M_u/M_{cal}
Rectangular Beams	Group I	R-HL-I-5-2.0	170.0	164.0	1.04
			160.0	164.0	0.98
		R-HL-II-5-2.0	170.0	147.0	1.15
			168.0	147.0	1.14
		R-HH-II-2.0	173.0	180.2	0.95
			159.0	180.2	0.88
		R-HL-II-5-2.5	160.0	160.6	1.00
			150.0	160.6	0.94
		R-HH-II-2.5	166.3	174.3	0.95
			175.0	174.3	1.00
	Group II	R-HL-II-14-2.0	130.0	139.2	0.94
			135.0	139.2	0.97
		R-HL-II-14-2.5	161.3	156.2	1.03
			180.0	156.2	1.15
T-shaped Beams	Group I	T-HL-I-5-2.0	296.4	338.0	0.88
			300.0	338.0	0.89
		T-HL-II-5-2.0	164.0	309.8	0.85
			285.6	309.8	0.92
		T-HL-III-5-2.0	322.8	313.8	1.03
			336.0	313.8	1.07
		T-HH-II-2.0	333.6*	341.7	0.98
			350.4*	341.7	1.03
		T-HL-II-5-3.0	318.6	330.0	0.97
			324.0	330.0	0.98
		T-HH-II-3.0	324.0	318.0	1.02
			309.6	318.0	0.97
	Group II	T-HL-II-14-2.0	289.2	295.9	0.98
			278.4	295.9	0.94
		T-HL-II-14-3.0	345.6	324.8	1.06
			324.0	324.8	1.00

* failed in shear

have smaller *a*/*h* ratio and even no shear connector. And all the composite beams used in this test failed in flexure, neither in slip of bonded surface nor in shear.

(ii) As far as differential shrinkage was concerned, lightweight aggregate concrete appears to be more advantageous because of its small modulus of elasticity.

IV Conclusions

The results obtained from this test are summarized as follows.

(i) Differential shrinkage may considerably affect the cracking load and the warping of the composite prestressed concrete beams especially when the flange portion is cast at later age. The ultimate flexural strength, however, is little affected by differential shrinkage.

(ii) The cracking moment calculated by the method taking into account the differential shrinkage stress as described in this paper shows good agreement with the measurement.

(iii) It appears more favourable to use the lightweight conrete in flange portion of the composite beam in respect to reducing the differential shrinkage effects because of its small modulus of elasticity.

V References

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