Calculating Method of Dynamic Stability in a Multi-machine System allowing for Margin

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By

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First, the equivalent circuit of a power system transformed into the form of Lagrangian tree is applied to obtain the basic equations required for criterion of system stability. On the next step, the characteristic equation can be obtained by transformation of the matrices which consist of the coefficients of basic equations. Furthermore, a new concept about the margin of system stability is proposed, and the assessment of system stability is carried out by obtainning the root-loci of a characteristic equation.

1. Introduction

On calculating dynamic stability in a power system, many excellent results[1] have been obtained about multi-machine problem dealt with by means of Hurwitz's criterion etc. owing to the progress of a digital computer with a large capacity of memory and quick processing ability. The computing method described in the present paper is also in accord with the recent tendencies as described above, and furthermore, it has the following features:

(1) An equivalent circuit[2] transformed into the form of Lagrangian tree from a system network using linear graph theory is applied to formulate the relation between voltages and currents of each synchronous machine connected to the system, and then the basic equation required for the criterion of multi-machine system stability can be introduced by simple routine work. By means of this method, the basic equations about the stability in a multi-machine system can be deduced easily as well as the two machine problem even if the system network is complicated.

(2) On the next step, after coefficients of basic equations are obtained, the matrices of which they consist are transformed by the same method as the one used in the one machine problem which is described in this paper. Since matrix-transformations are easily applicable for digital computer programming, and the troublesome part of analytical processing in the calculation decreases by the transformations, the characteristic equation can be easily obtained.

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(3) Considering the criterion of the stability of the power system, when all roots of characteristic equation lie on the left side of imaginary axis on the Gauss-plane, the system has been regarded stable. In this paper, however, a certain critical line is specified by two parameters α and μ on the left side of the imaginary axis on the Gauss-plane, where α and μ are related with attenuation and frequency of hunting in the system. α and μ are used and difined as the margin of the system stability, then it is investigated whether all characteristic roots are on the left side of this critical line or not, and the system is regarded stable if all roots are on the left side. As the examples of computation by this method, one-machine and three-machine problems are illustrated and the results are studied.

2. Deduction of Basic Equations and a Characteristic Equation in a One-machine System

In this section, we formulate the basic equations where one synchronous machine is connected to an infinite bus through a transmission line as shown in Fig. 1 and the authers apply matrix transformations to these basic equations. These steps will be applied to multi-machine system in the next section.

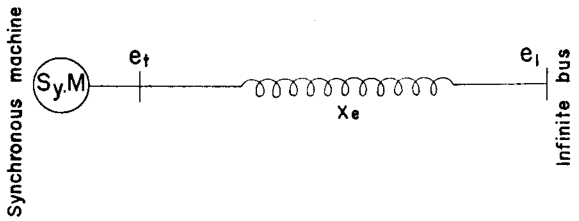

Fig. 1. Theoritical system of one machine problem.

2.1 Basic Equations[3]

For the synchronous machine in the system shown in Fig. 1, the vector diagram of the voltages and the currents is obtained as shown in Fig. 2, and furthermore the relations between these vectors are formulated as the following equations, where voltages, currents, fluxes etc. are represented with p.u. quantities, and resistances of armature windings and of transmission line are ignored.
(The notations of symbols are shown at the end of this paper).

$$e_d = p\psi_d - \psi_q \cdot p\theta \fallingdotseq -\psi_q \quad (1)$$

$$e_q = p\psi_q + \psi_d \cdot p\theta \fallingdotseq \psi_d \quad (2)$$

From the view point of the infinite bus,

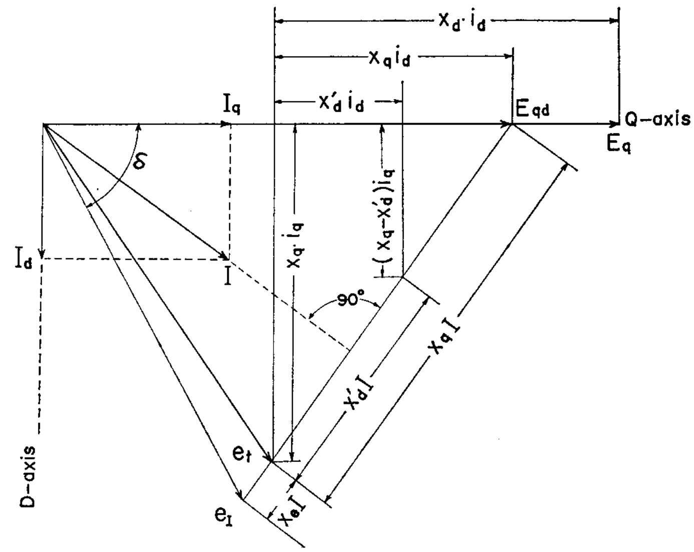

Fig. 2. Vector diagram of a synchronous machine with leading current.

$$e_d = e_I \sin\delta - x_e \cdot i_q \tag{3}$$

$$e_q = e_I \cos\delta + x_e \cdot i_d \tag{4}$$

In steady-state

$$\psi_d = E_q - x_d \cdot i_d \tag{5}$$

$$\psi_q = -x_q \cdot i_q \tag{6}$$

The voltages of a synchronous machine in a transient state are defined as follows:

$$\psi_{fd} = E_q - (x_d - x_d') \cdot i_d \tag{7}$$

Or

$$\psi_{fd} = e_q + x_d' \cdot i_d \tag*{(7)$'$}$$

$$\psi_{fq} = e_d - x_q' \cdot i_q \tag{8}$$

And

$$T_{d0}' \cdot p\psi_{fd} = E_{fd} - E_q \tag{9}$$

$$T_{q0}' \cdot p\psi_{fq} = (x_q - x_q') \cdot i_q - \psi_{fq} \tag{10}$$

On the terminal voltage

$$e_t^2 = e_d^2 + e_q^2 \tag{11}$$

The equation of the rotor motion in a synchronous machine is

$$M \cdot p^2\theta = T_m - \psi_d \cdot i_q + \psi_q \cdot i_d + P_d \cdot p\theta \tag{12}$$

Since the voltage of the infinite bus is assumed constant, then

$$e_I = e_{I0} \tag{13}$$

When an infinitesimal disturbance occures in the system, a small change of some variable quantity is represented as follows:

$$\psi_d = \psi_{d0} + \Delta\psi_d \tag{14}$$

where suffix 0 means the value in a steady state.

And the unit time in this paper is $2\pi f_0$.
The equations for small changes in variables of Eq. (3), (4) and (7)′ are arranged as follows:

$$[A_1]\begin{pmatrix}\Delta e_d \\ \Delta e_q \\ \Delta i_d \\ \Delta i_q\end{pmatrix} = [A_2]\begin{pmatrix}\Delta\delta \\ \Delta\psi_{fd} \\ \Delta\psi_{fq}\end{pmatrix} \tag{15}$$

where

$[A_1] =$

1			$-x_q'$
	1	$-x_e$	
	1	x_d'	
1			x_e

where a blank means a zero-element.

$[A_2] =$

		1
$-e_{I0}\cdot\sin\delta_0$		
	1	
$e_{I0}\cdot\cos\delta_0$		

and $[A_3]$, $[A_4]$ matrices are given as follows

$$[A_3] \equiv [A_1]^{-1}, \quad [A_4] \equiv [A_3][A_2]$$

and the column matrices which appear in both side of Eq. (15) are shown by the sign $[\ ]_t$ because the authors intend to save space, then Eq. (15) is rewritten as follows:

$$[\Delta e_d, \Delta e_q, \Delta i_d, \Delta i_q]_t = [A_4][\Delta\delta, \Delta\psi_{fd}, \Delta\psi_{fq}]_t \tag{15$'$}$$

From Eqs. (9)~(12), the four equations for variances of currents, voltages and fluxes of a machine can be obtained, and furthermore the following three equations which are concerned with angular velocity, effects of governor and A.V.R. are obtained,

$$p\Delta\delta = \Delta\omega \tag{16}$$

$$\Delta T_m = \frac{-\mu_m}{(1+\tau_1 \cdot p)}\Delta\omega \tag{17}$$

$$\Delta E_{fd} = \frac{-K}{1+pT_f}\cdot \Delta e_t \tag{18}$$

These equations are combined and arranged in the form of a matrix equation as follows:

$$\begin{aligned}&[p\Delta\delta,\ p\Delta\psi_{fd},\ p\Delta\psi_{fq},\ p\Delta E_{fd},\ p\Delta\omega,\ p\Delta T_m]_t \\ &= [A_5][\Delta e_d,\ \Delta e_q,\ \Delta i_d,\ \Delta i_q,\ \Delta\delta,\ \Delta\psi_{fd},\ \psi_{fq},\ \Delta E_{fd},\ \Delta\omega,\ \Delta T_m]_t\end{aligned} \tag{19}$$

where $[A_5]$ is as follows:

$[A_5] =$

								1	
		$-\frac{(x_d-x_{d'})}{T_{d0}'}$			$\frac{-1}{T_{d0}'}$		$\frac{1}{T_{d0}'}$		
			$\frac{x_q-x_q'}{T_{q0}'}$			$-\frac{1}{T_{q0}'}$			
$-\frac{K\cdot e_{d0}}{T_f\cdot e_{i0}}$	$-\frac{K\cdot e_{q0}}{T_f\cdot e_{t0}}$						$-\frac{1}{T_f}$		
$-\frac{i_{d0}}{M}$	$-\frac{i_{q0}}{M}$	$-\frac{e_{d0}}{M}$	$-\frac{e_{q0}}{M}$					$-\frac{P_d}{M}$	$\frac{1}{M}$
								$-\frac{\mu_m}{\tau}$	$-\frac{1}{\tau}$

In $[A_5]$ matrix, the column corresponding to $[\Delta\delta]$ is introduced to compose the following matrix $[A_6]$.

In the next step, in order to substitute Eq. (15) into Eq. (19) in the form of matrix multiplication, a compound matrix is formed as follows:

$$[A_6] \equiv \begin{bmatrix} [A_4] & [0] \\ & [U] \end{bmatrix} \tag{20}$$

where, [0] is zero-matrix of (4×3) order, $[U]$ is unit matrix of (6×6) order. Then, $[A_6]$ has order of (10×6) and the following equation is obtained,

$$[\Delta e_d, \Delta e_q, \Delta i_d, \Delta i_q, \Delta\delta, \Delta\psi_{fd}, \Delta\psi_{fq}, \Delta E_{fd}, \Delta\omega, \Delta T_m]_t$$
$$= [A_6][\Delta\delta, \Delta\psi_{fd}, \Delta\psi_{fq}, \Delta E_{fd}, \Delta\omega, \Delta T_m]_t \tag{21}$$

From Eqs. (19) and (21)

$$[p\Delta\delta, p\Delta\psi_{fd}, p\Delta\psi_{fq}, p\Delta E_{fd}, p\Delta\omega, p\Delta T_m]_t$$
$$= [A_5][A_6][\Delta\delta, \Delta\psi_{fd}, \Delta\psi_{fq}, \Delta E_{fd}, \Delta\omega, \Delta T_m]_t \tag{22}$$

The definition of matrix $[BH]$ is as follows:

$$[BH] \equiv [A_5][A_6]$$

Then, $[BH]$ has order of (6×6), and Eq. (22) is rewritten as follows:

$$[p[U]-[BH]][\Delta\delta, \Delta\psi_{fd}, \Delta\psi_{fq}, \Delta E_{fd}, \Delta\omega, \Delta T_m]_t = [0] \tag{23}$$

Thus, when the system under consideration is affected by the small disturbance, the system stability is investigated as follows: At first a secular equation is formed as follows.

$$\det. |p[U]-[BH]| = 0 \tag{24}$$

and from the above equation the polynomial of p is formed

$$C_0p^6 + \cdots + C_4p^2 + C_5p + C_6 = 0 \tag{25}$$

Hurwitz's criterion is applied to the coefficients of Eq. (25), and the stability of the system is investigated. In this manner of the computation, matrices $[A_1]$, $[A_2]$ and $[A_5]$ are first obtained, and inversion, multiplication and combination of these three matrices are carried out, then the required coefficients of the cahracteristic equation can be obtained, where very little analytical operation is needed, and the coding and programming of computations from Eq. (24) to Eq. (25) and for Hurwitz's criterion are prepared as subroutines in the calculation about automatic controll engineering, and these computations are executed with simple programs.

2.3 Margin of the Stability

By Hurwitz's criterion which is used for the assesment of dynamic stability of a power system, the system has been regarded stable, when all roots of characteristic equation lie on the left side of the imaginary axis of the Gauss-plane as shown in Fig. 3(a). But the state of system operation, in which a characteristic root is on the imaginary axis, or in the neighbourhood of it, is easy to become unstable, then in the practical operation it is desirable that the roots lie on the left

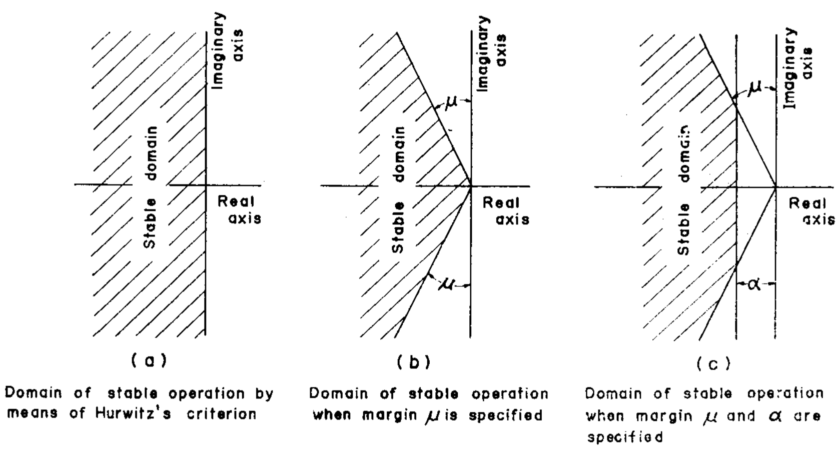

Fig. 3. Domains of stable operation defined by various criterions.

plane apart from the imaginary axis.

When the domain of the characteristic roots is restricted on the left side from the lines specified by α on the Gauss-plane (c.f. Fig. 3(c)) for the purpose of the stable operation of a system, in the critical state of stable operation, the root, corresponding to the oscillation with time constant $1/\alpha$, is located on this line. Then the other oscillations have a smaller time constant than $1/\alpha$, and the system has some margin for stable operation.

To obtain the critical operating condition under this restriction, $(p+\alpha)$ is substituted in stead of p into Eq. (25), and a new characteristic equation can be formed. Or a new secular determinant is formed from Eq. (24) as follows:

$$\text{det.}\,|\,p[U]+[\alpha]-[BH]\,|=0 \tag{26}$$

where $[\alpha]$=diagonal matrix having α as elements

Then Hurwitz's criterion is applied to the characteristic equation of the above.

Alternatively to restrict root positions, two lines are specified with angle μ against the imaginary axis as shown in Fig. 3(b), where both the frequency and the attenuation time of the hunting of the system is restricted.
For this purpose,

$$p = w \exp(-j\mu) \tag{27}$$

is substituted into Eq. (25), and Bilharz-Frank's criterion can be applied.

Furthermore both α and μ can be used together as shown in Fig. 3 (c). In this paper by means of numerical method, the authors get the root loci with parameters

P and Q etc. to obtain the critical condition for various α and μ (c.f. example of calculation). The margin of the system stability can be defined by many different ways, but in this paper α and μ are used.

3. Multi-machine System

3.1 Formation of the Vector Diagram of Synchronous Machines Using a Tree-formed Equivalent Circuit.

The analysis of each machine connected to a multi-machine system is same as the one-machine problem fundamentally, but it is different from the one-machine problem in that the phase relations between voltages and currents of the machines and the power flow among then have to be studied. When the network of the system is complicated, the investigation of the relation between quantities of the machines becomes very troublesome. In this paper, in order to avoid such difficulties, the authors use a tree-formed equivalent circuit of a power system by means of linear graph theory.[3] In this equivalent circuit as shown in Fig. 4, a bus of the system is adopted as the reference point and for simplicity the earth capacitances of lines are included equivalently in the impedance loads at the line terminals, or otherwise are ignored. In this paper, the latter is adopted.

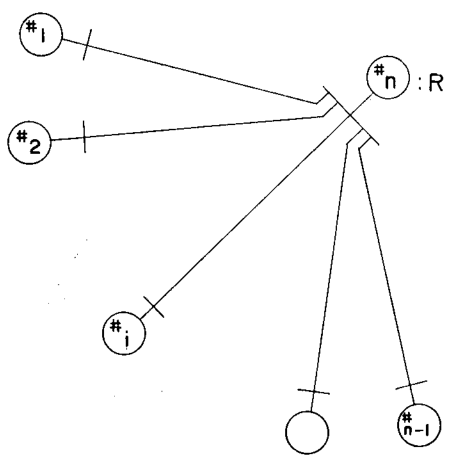

Fig. 4. Equivalent circuit having the form of Lagrangian tree in n-machines system.

Under the assumption described above, the realtion between voltages and currents in the system are given as follows:

$$[v_E] = [Z_E][i_E] \tag{28}$$

where $[v_E]$: Column matrix which consists of the bus voltage measured from the reference point .

$[i_E]$: Column matrix of the line currents corresponding to $[v_E]$

$[Z_E]$: Impedance matrix of $(n-1)$, $(n-1)$ order for an n-terminal system

From the equivalent circuit described above, the relations between the reference bus voltage and D-, Q- axis of each synchronous machine are shown in Fig. 5, and

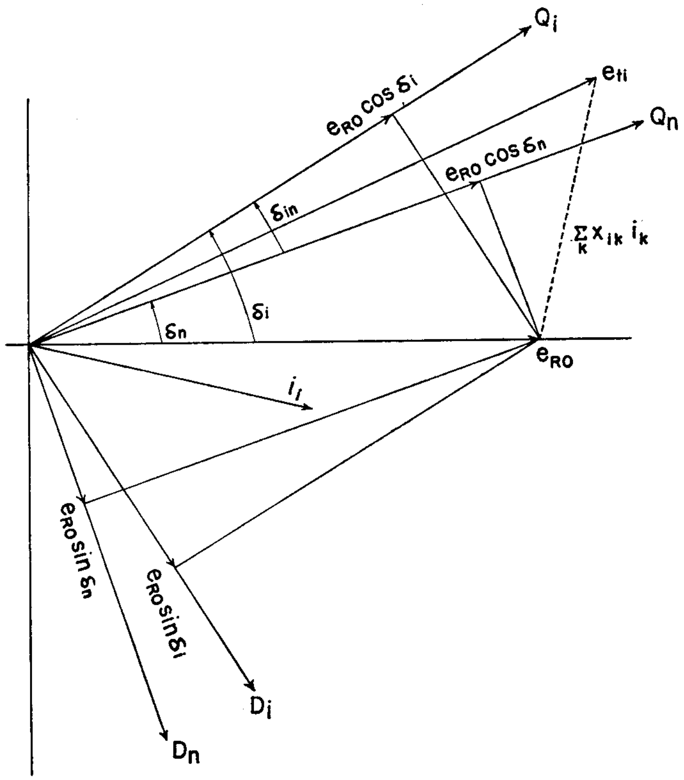

Fig. 5. Vector diagram of each component of terminal voltage in synchronous machine connected to multi-machine system.

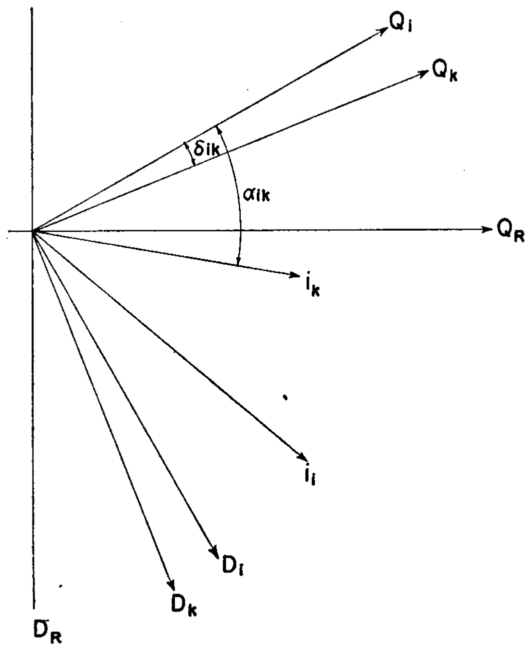

Fig. 6. Vector diagram of armature current of each synchronous machine in multi-machine system.

the current vectors are shown in Fig. 6. To obtain these relations, the bus voltages and currents have to be computed by power flow calculation in the power system beforehand.

3.2 Establishment of Fundamental Equations and the Deduction of a Characteristic Equation

According to the vector diagrams shown in Fig. 5, 6, the authors formulate the relation between the bus voltage and the current of i-th synchronous machine and ones of the reference in the case of $n \geqq 2$ as follows (c.f. Fig. 5, 6)

$$\begin{bmatrix} e_{R0}\cdot\sin\delta_i \\ e_{R0}\cdot\cos\delta_i \end{bmatrix} = \begin{bmatrix} \cos\delta_{in} & \sin\delta_{in} \\ -\sin\delta_{in} & \cos\delta_{in} \end{bmatrix} \begin{bmatrix} e_{R0}\sin\delta_n \\ e_{R0}\cos\delta_n \end{bmatrix} \tag{29}$$

where e_{R0}=voltage of the reference bus in a steady state

δ_i=angle between e_{R0} and Q_i-axis

$\delta_{in}=\delta_{in}-\delta_i$

n=number of reference bus

and the voltage relation is as follows[3].

$$\left.\begin{aligned} e_{R0}\sin\delta_i &= e_{di}+\sum_{k=1}^{n-1} x_{ik}\cdot i_k\cdot\cos\alpha_{ik} \\ e_{R0}\cos\delta_i &= e_{qi}-\sum_{k=1}^{n-1} x_{ik}\cdot i_k\cdot\sin\alpha_{ik} \end{aligned}\right\} \tag{30}$$

where x_{ni}, $x_{in}=0$ $(i=1\sim n)$ holds according to the definition of Z_E.

Then,

$$e_{R0}\cdot\sin\delta_n = e_{dn}\,,\quad e_{R0}\cdot\cos\delta_n = e_{qn}$$

Eq. (29) is rewritten as follows:

$$\begin{bmatrix} e_{R0}\cdot\sin\delta_i \\ e_{R0}\cdot\cos\delta_i \end{bmatrix} = \begin{bmatrix} \cos\delta_{in} & \sin\delta_{in} \\ -\sin\delta_{in} & \cos\delta_{in} \end{bmatrix} \begin{bmatrix} e_{dn} \\ e_{qn} \end{bmatrix} \tag{29$'$}$$

Since α_{ik} means the angle between i_k and Q_i-axis,

$$\alpha_{ik} = \delta_{ik}+\tan^{-1}(i_{dk}/i_{qk}) \tag{31}$$

where i, $k=1\sim(n-1)$

According to Eq. (29)$'$, (30)

$$e_{di}+\sum_{k=1}^{n-1} x_{ik}\cdot i_k\cdot\cos\alpha_{ik} = e_{dn}\cdot\cos\delta_{in}+e_{qn}\cdot\sin\delta_{in} \tag{32}$$

$$e_{qi}-\sum_{k=1}^{n-1} x_{ik}\cdot i_k\cdot\sin\alpha_{ik} = -e_{dn}\cdot\sin\delta_{in}+e_{qn}\cdot\cos\delta_{in} \tag{33}$$

And the sum of all current flowing into the network is zero, then,

$$\left.\begin{aligned}\sum_{k=1}^{n} i_k\cdot\cos\alpha_{ik} &= 0\\ \sum_{k=1}^{n} i_k\cdot\sin\alpha_{ik} &= 0\end{aligned}\right\} \tag{34}$$

where the saffix 'i' is selected arbitrarily within n. (c.f. Appendix)

The transient voltages of synchronous machines are formulated similarly to the one-machine problem as follows:
Eq. (7) becomes

$$\left.\begin{aligned}&\psi_{fdi} = E_{qi}-(x_{di}-x'_{di})i_{di} = e_{qi}+x'_{di}\cdot i_{di}\\ (8)\quad &\psi_{fqi} = e_{di}-x'_{qi}\cdot i_{qi}\end{aligned}\right\} \tag{35}$$

$$(9)\quad T'_{d0i}\cdot p\psi_{fdi} = E_{fdi}-\psi_{fdi}-(x_{di}-x'_{di})\cdot i_{di} = E_{fdi}-E_{qi}$$

$$(10)\quad T'_{q0i}\cdot p\psi_{fqi} = (x_{qi}-x'_{qi})i_{qi}-\psi_{fqj}$$

where $i=1\sim n$

If $T'_{q0j}=0$ *i.e.* $x_{qi}=x'_{qi}$ is assumed for simplicity, the equation corresponding to Eq. (10) is not needed. The above assumption ($T'_{q0i}=0$) is adpoted here for simplicity and for reduction of the storages for the computor programming.

$$(11)\quad e_{ti}^2 = e_{di}^2+e_{qi}^2 \tag{37}$$

$$(19)\quad \Delta E_{fdi} = -K_i\cdot\Delta e_{ti}/(1+pT_{fi}) \tag{38}$$

Relative angular acceleration is,

$$p\Delta\delta_{ij} = \Delta\omega_{ij} \tag{39}$$

The equation for the relative motion between i-th machine and n-th one is

$$\begin{aligned}(12)\quad p^2\theta_{in} = &-(P_{di}/M_i)p\theta_{in}\\ &+\{M_n(P_{Mi}-P_{Ei})-M_i(P_{Mn}-P_{En})\}/M_i\cdot M_n\end{aligned} \tag{40}$$

where, it is assumed for simplicity that

$(P_{di}/M_i)=$constant for all i.

In the above equation, the governer action is ignored, that is,

$$P_{Mi}=P_{M0i} \tag{41}$$

Thus, the mechnical input is constant even if $p\omega_i$ varies slightly.
For the electrical output of the i-th machine,

$$P_E = e_{di}\cdot i_{di}+e_{qi}\cdot i_{qi} \tag{42}$$

For the small distubance in the system, Eqs. (20)$\sim$(40) are linearized similarly to the one-machine problem, and the work for linearization becomes complicated when the number of machines connected to the system increases. (c.f. three-system machine in Appendix)

(15) $[A_1^n][\Delta e_{di}, \Delta e_{qi}, \Delta i_{di}, \Delta i_{qi}]_t$

$$= [A_2^n][\Delta\delta_{in}, \Delta\psi_{fdi}, \Delta\psi_{fqi}]_t \quad (43)$$

where $i=1\sim n$

$[A]_1^n$, $[A_2^n]$ have respectively order of $(4n, 4n)$ and $\{4n, (3n-1)\}$, but if $T'_{q0i}=0$ is assumed, $[A_2^n]$ has order of $\{4n, (2n-1)\}$

Eq. (15′) is rewritten as follows:

(15′) $[\Delta e_{di}, \Delta e_{qi}, \Delta i_{di}, \Delta i_{qi}]_t$

$$= [A_4^n][\Delta\delta_{in}, \Delta\psi_{fdi}, \Delta\psi_{fqi}]_t \quad (44)$$

where $[A_4^n]$ has order of $\{4n, (3n-1)\}$ but if $T'_{q0i}=0$ is assumed, $\{4n, (2n-1)\}$

Similarly to the one machine problem

(21) $[p\Delta\delta_{in}, p\Delta\psi_{fdi}, p\Delta\psi_{fqi}, p\Delta E_{fdi}, p\Delta\omega_{in}]_t$

$$= [A_5^n][\Delta e_{di}, \Delta e_{qi}, \Delta i_{di}, \Delta i_{qi}, \Delta\delta_{in}, \Delta\psi_{fdi}, \Delta\psi_{fqi}, \Delta E_{fdi}, \Delta\omega_{in}]_t \quad (45)$$

where, $[A_5^n]$ has order of $\{(5n-2), (9n-2)\}$, but if $T'_{q0i}=0$ is assumed, $\{(4n-2), (8n-2)\}$.

Because Eq. (45) does not have the column corresponding to $\Delta\delta_{ij}$ in Eq. (19), the constitution of $[A_6^n]$ is different from one-machine problem and has the form as follows:

$$[A_6^n] = \begin{bmatrix} [A_4^n] & [O_a^n] \\ [O_b^n] & [U^n] \end{bmatrix}$$

where $[O_a^n]$ and $[0_b^n]$ are zero-matrices and have order of $\{(4n), (2n-1)\}$, $\{(4n-1), (n-1)\}$ reaspectively and $[U^n]$ is a unit matrix and has order of $\{(4n-1), (4n-1)\}$. When $T'_{q0i}=0$ $(i=1\sim n)$ is assumed, $[O_a^n]$ and $[O_b^n]$ have order of $\{(4n), (2n-1)\}$ and $\{(3n-1), (n-1)\}$ respectively.

And then similarly to the one machine problem, $[BH^n]=[A_5^n][A_6^n]$ where $[BH^n]$ is a square matrix of order of $\{(5n-2), (5n-2)\}$, if $T'_{q0i}=0$ $(i=1\sim n)$ is assumed, the order is $\{(4n-2), (4n-2)\}$. Secular determinant for $[BH^n]$, the characteristic equation and criterion for system stability can be obtained similarly to the one machine problem.

4. Examples and Study of the Results

4.1 One Machine Problem

The system parameters in the theoretical system as shown in Fig. 1 are assumed as follows:

$x_d = x_q = 1.6$ (p.u)
$x'_d = 0.24 \quad x'_q = 0.9 \quad x_e = 0.4 \quad e_t = 1.0$
$T'_{d0} = 7.3$ (sec) $\quad T'_{q0} = 1.6$ (sec) $\quad M = 7.6$ (sec) $\quad \mu_m = 20$
$\tau = 1.0$ (sec)
where time is multiplied by $2\pi f_0 = 377$ as described above, then
T' (radian) $= 2\pi f_0$ (radian/sec) $\times T$ (sec)

is used.

Furthermore in this unit system, $P_d = 60$ (dimensionless) is assumed and K and T_f are shown in the figures as parameters.

For the initial value of the system, e_{I0} is set under the condition that $e_t = 1.0$ (constant) is held for various P, Q.

The root loci of Eq. (25) under the condition described above are shown on the Gauss-plane in Fig. 7 where $K = 15.0$, $T_f = 25$ sec and P, Q are parameters. Numerical analysis by Hitchcock-Baistow method, Newton Method e.t.c. are used here. In the figure, T_1, T_2 and T_3 mean inverses of $2\pi f_0\alpha_1$, $2\pi f_0\alpha_2$ and $2\pi f_0\alpha_3$. Since Eq. (25) is the 6th degree polynomial in p with real coefficients, it has 6 roots. But the authors pay attention to only one of the 6 roots that is the nearest to the

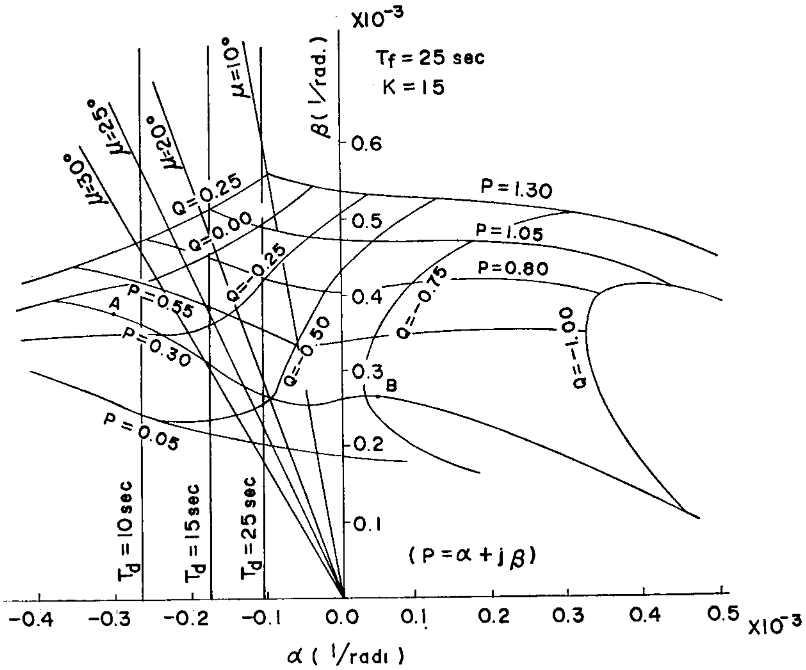

Fig. 7. Loci of complex roots of characteristic equation in one-machine system (parameter P, Q) (1st).

imaginary axis of the Gauss-plane. Only the group of characteristic roots considered is selected and shown in Fig. 7.

From the root loci, it becomes clear that the root (e.g. point A) traverses the critical line from the stable domain to the unstable domain in Fig. 7 as Q increases (*i. e.* consumption of leading reactive power increases) along the straight line AB in Fig. 8. If the point (P, Q) which corresponds to the root on the specified critical line in Fig. 7 is obtained, it is the point on the critical operating line in Fig. 8.

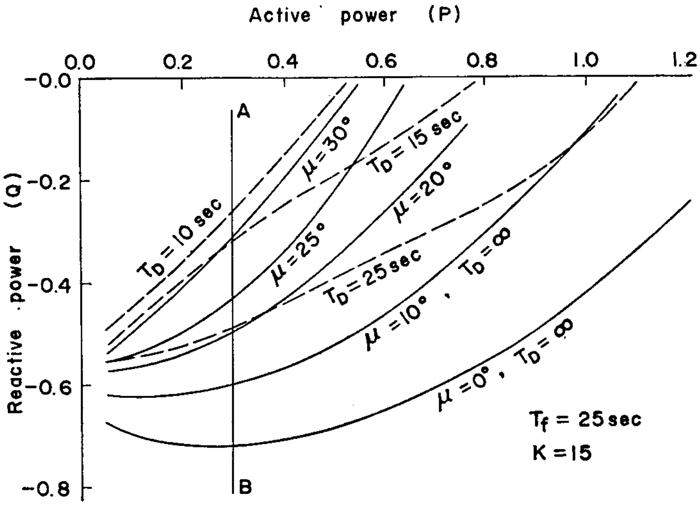

Fig. 8. Domain of stable operation in one-machine system allowing for margin (parameter α, μ on P–Q plane).

Thus under the conditions of ($\mu = 0$, $T_D = 10$, 15, 25 sec.) and ($T_D = \infty$, $\mu = 0°$, 10°, 20°, 25°, 30°), the critical operating states represented by P and Q are shown in Fig. 8. Furthermore, the roots shown in Fig. 9 as well as in Fig. 7 are complex ones, but they are far from the imaginary axis and out of the question. Each of the two groups of roots shown in Fig. 7 and 9 has its conjugate root group which lies symmertrically to the real axis. Since the critical lines for criterion are also symmertric to the real axis, only either one of the conjugate roots is disccused here.

The critical operating line on the P-Q plane obtained by the method described above coincides with one obtained by the method of Hurwitz or Bilharz-Frank criterion. The root-loci shown in Fig. 10, 11 are groups of real roots taking Q as a parameter, and all lie on the left side far from the ones shown in Fig. 7, then, they are out of the stability criterion.

Fig. 12 shows the root-loci with parameters Q and K under constant P. Using this figure, the critical operating line is obtained as shown in Fig. 13.

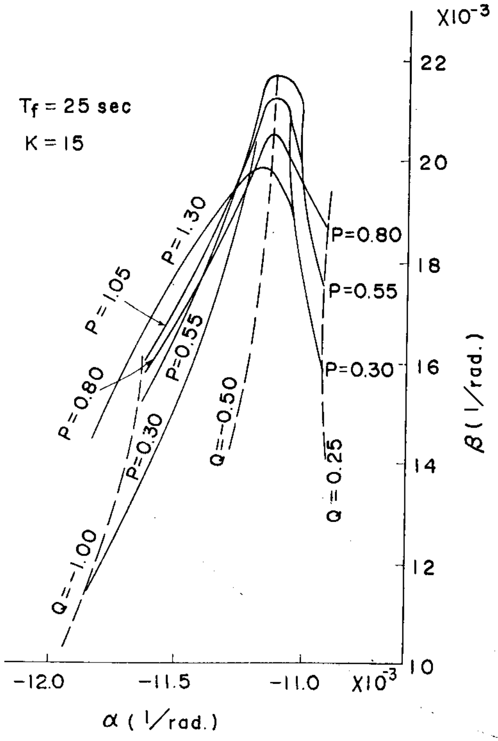

Fig. 9. Loci of complex roots of characteristic equation in one-machine system (prarameter P, Q) (2nd).

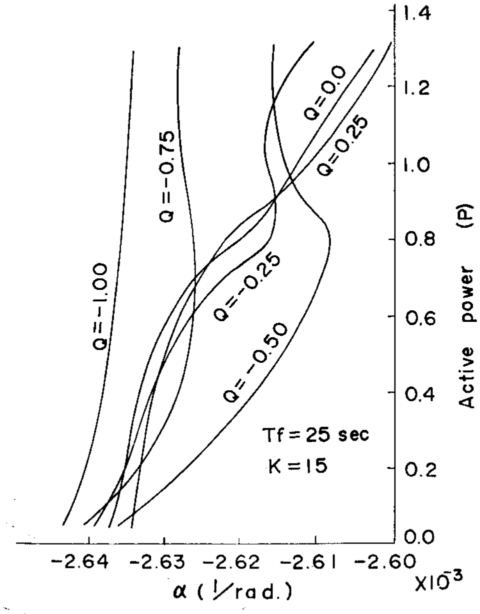

Fig. 10. Loci of real roots of characteristic equation in one-machine system (parameter Q on $\alpha-P$ plane) (1st).

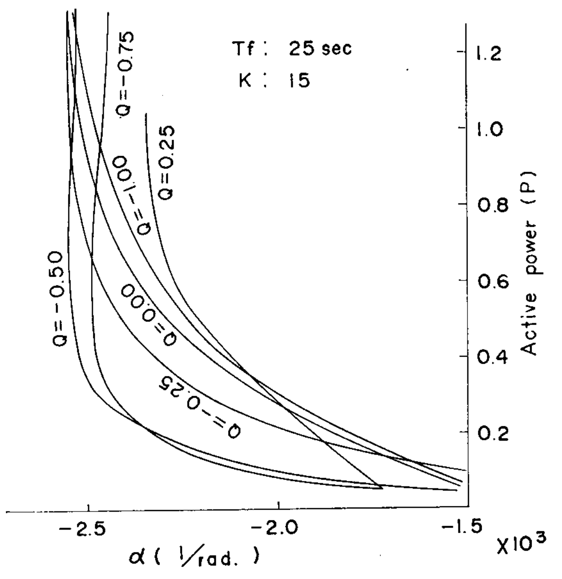

Fig. 11. Loci of real roots of characteristic equation in one-machine system (parameter Q on $\alpha-P$ plane) (2nd).

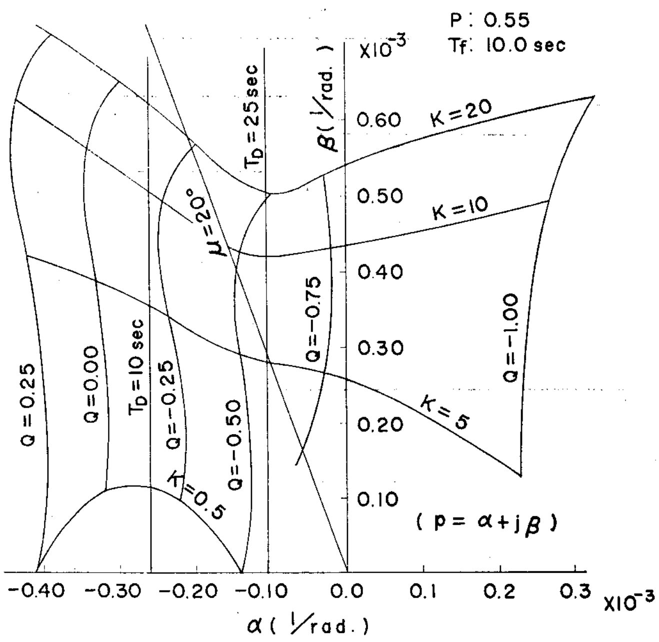

Fig. 12. Loci of complex roots of characteristic equation in one-machine system (parameter Q, K)

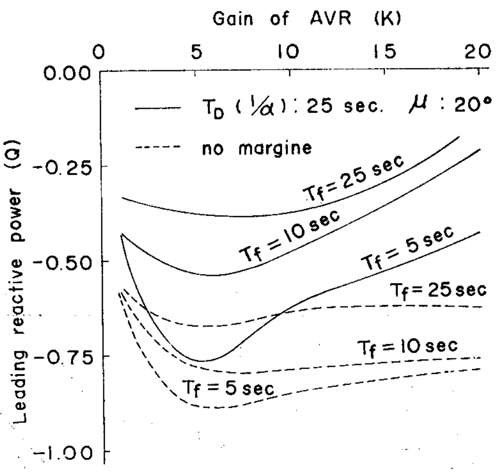

Fig. 13. Domain of stable operation in one-machine system allowing for margin (parameter T_f, on K-Q plane).

4.2 Three Marchine System

For the theoretical three machine system shown in Fig. 14, the tree-formed equivalent circuit is formed, where #3–bus is regarded as the reference bus. When

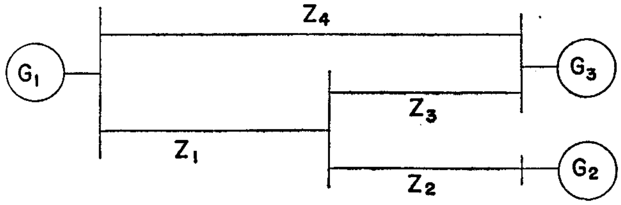

Fig. 14. Model system of 3-machine problem.

resistances and earth capacitances of lines are ignored, the impedance matrix for the equivalent circuit is obtained as follows:

$Z_E=$

	#1	#2
#1	$j0.251$	$j0.126$
#2	$j0.126$	$j0.439$

where $Z_1=Z_2=j0.250$, $Z_3=j0.252$ and $Z_4=j0.592$ are assumed in Fig. 14.

The constants of the synchronous machines are assumed as the following table.

The constants of synchronous machines

	#1-, #2-Machine	#3-Machine
x_d	1.6	0.8
x_q	1.6	0.8
x_d'	0.24	0.12
x_q'	1.6	0.8
T_{d0}'	7.3 sec	7.3 sec
M	7.6 sec	15.2 sec

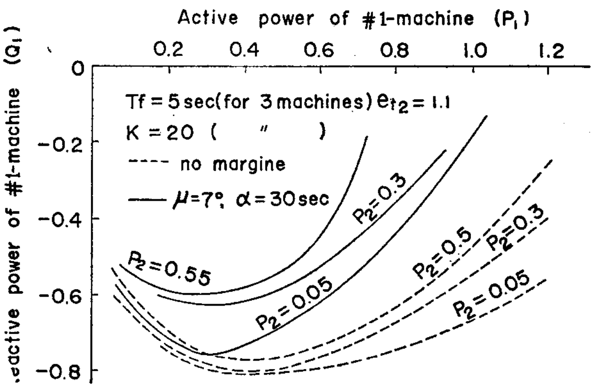

Fig. 15. Domain of stable operation in 3-machine system allowing for margin (parameter P_2 on P_1-Q_1 plane).

And $P_d = 60$ (dimensionless, c.f. preceding section) is assumed and the constants of AVR is shown in Fig. 15.

To determine the initial condition for the computation of dynamic stability, the calculation of the power flow of the system is necessary. In the computation, #1-bus is $P-Q$ node (*i.e.* P_1, Q_1 are specified and e_{t1} is calculated) and #2-bus is $P-V$ node (*i.e.* P_2, e_{t2} are specified and Q_2 is calculated).
In this paper, P_2 is taken as a parameter and shown in Fig. 15, and for #3-synchro-

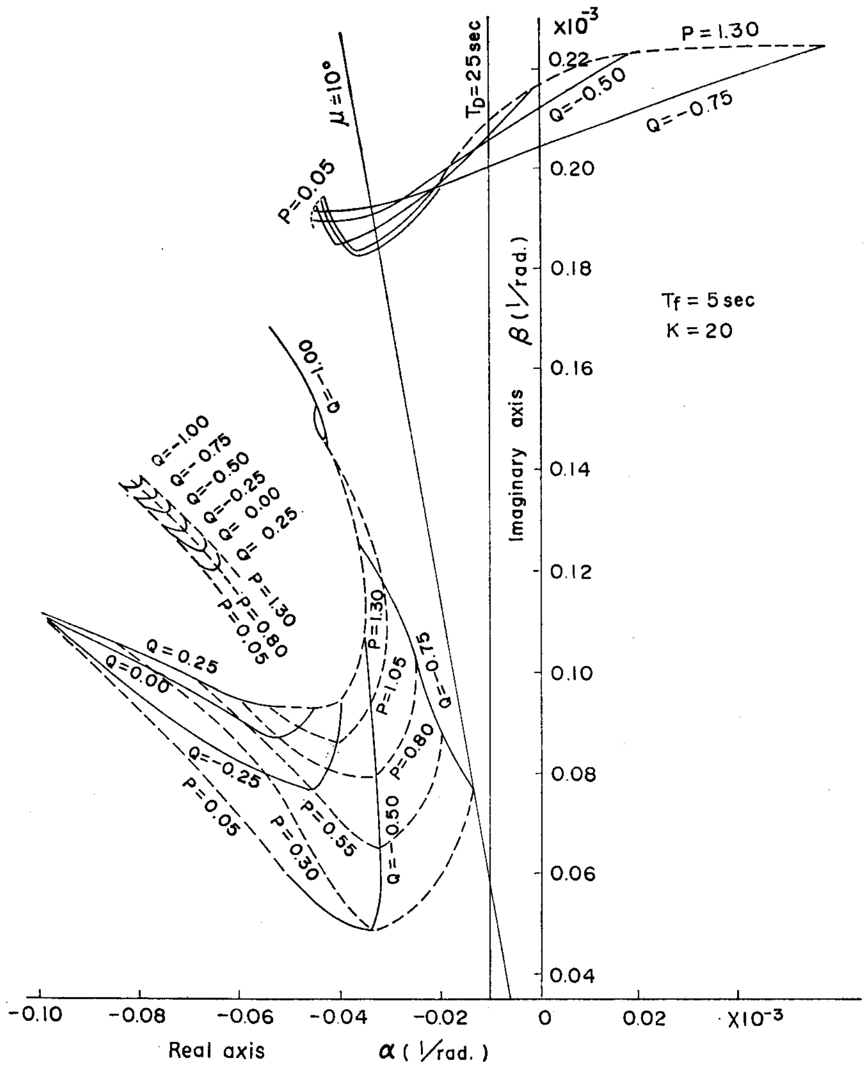

Fig. 16. Loci of complex roots of characteristic equation in 3-machine system (parameter P, Q).

nous machine (motor), $e_{t3}=1.1$ and P_3, Q_3 are obtained from the power demand in the system.

For the criterion of dynamic stability, attention is paid only to the group of the nearest root to the imaginary axis. The root-loci of the characteristic equation of the three-machine system are shown in Fig. 16 and 17. From the root distribu-

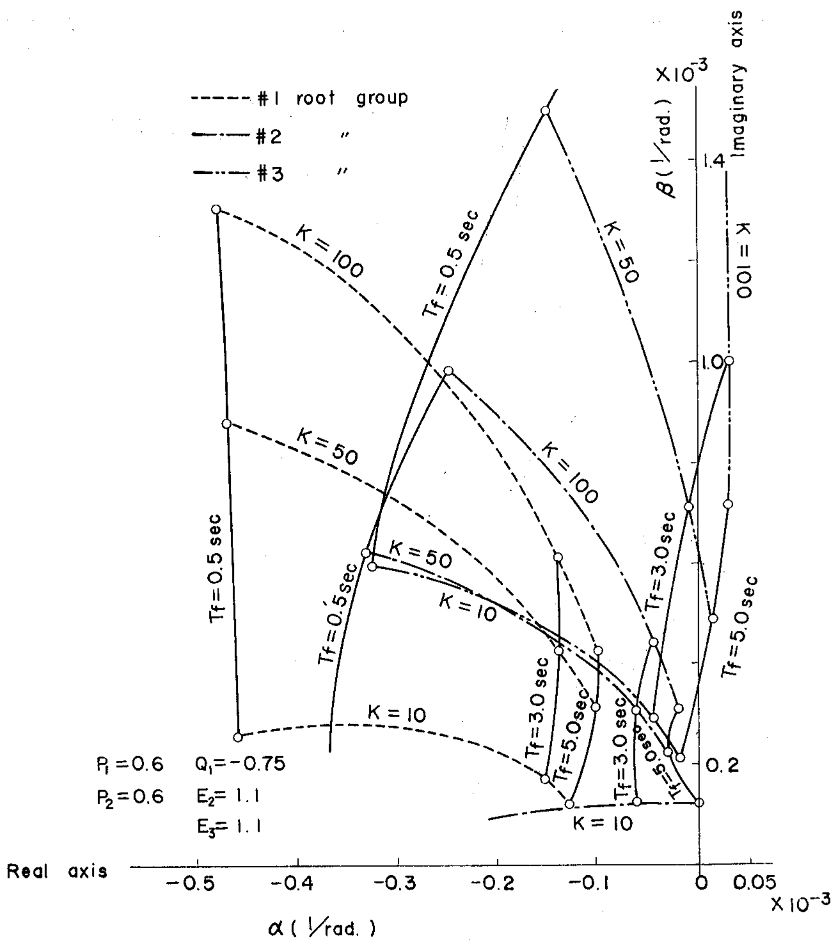

Fig. 17. Loci of complex roots of caaracteristic equation in 3-machine system (parameter K, T_f).

tion shown in Fig. 17, it is clear that the frequency of the oscillation in the system increases as K (the gain of AVR) increases and the root comes nearer the imaginary axis as T_f (the time constant of AVR) increases, where the time constants and gains of three machines are assumed to be the same in all.

4.3 Further Consideration on the Calculating Method for Stability Criterion

When Bilharz-Frank criterion is applied to dynamic stability in a power system specifying α and μ arbitrarily, the P, Q point of stable operation sometimes can not be found anywhere. It often occurs especially when the margin is large. And the following results are obtained from the numerical computation:

The n-th degree characteristic equation has n roots, which are classified as follows:

(1) Groups of high frequency and short attenuation time. In one-machine system positions of them do not change so much against changing P, Q. The movement of these roots scarcely affect the system stability.

(2) The group of real roots which are stable and do no affect the system stability as group (1).

(3) The groups which have the minimum absolute value and the behavior of which decide the system stability.

In this paper, attention is paid only to group (3) for system stability criterion as described above. When the values of α and μ are too large; group (3) does not move to the left side of the critical line, even if the P, Q point largely changes and stable operation of the system is impossible with the specified margin in such a case.

When Bilharz-Frank's criterion is used to get the critical curves of stable operation in the $P-Q$ plane against various values of α and μ, the criterion must be carried out at every point in the neighborbood of the cruves in order to search them. Because the procedure of this criterion is very troublesome and it must be repeated so often, the work increases exceedingly.

Once, however, the root-loci are drawn varying the values of P and Q as parameters, the critical curves of stable operation can be obtained easily against any values of α and μ as described here. They can be obtained only by making the critical line specified by α and μ across the root-loci to find the values of P and Q at the cross points. This procedure is not so troublesome.

The methods of Hurwitz and Bilharz-Frank seem to be sometimes used to avoid the impossibility or the difficulty of solving high order characteristic equations. But, recently, digital computers have come to possess high abilities and have made it easy to calculate roots of high order equations.

Therefore, there are some advantages now in calculating the roots of them directly.

5. Conclusion

The deduction of the characteristic equation of the dynamic stability in multi-machine system becomes easier as follows. (1) By the application of an equivalent circuit, the establishment of basic equations can be carried out as easily as in a two-machine system. (2) On the next step, the matrices which consist of the coefficients of the basic equations, are transformed by the same method as discribed for the one-machine system in this paper, and the necessary characteristic equation can be obtained. (3) This method is advantageous for a digital computer, and the part of analytical processing in the calculation decreases by means of these matrix-transformations, and therefore the programming for the computer becomes simpler.

Furthermore, it has been proposed that the parameters to restrict the attenuation and the frequency of the oscillation in the system should be introduced as the margin of stability. Along with this opinion, the method to get the critical condition for stable operation and the examples of the computation for one- and three-machine systems have been shown. The results of the examples have also been considered in this paper.

This method shows great advantages in studying the stability criterion allowing for the margin.

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Nomenclature

Nomenclatures are defined mainly according to reference 5

e_t = terminal voltage of synchronous machine
e_q = direct axis voltage
e_d = quadrature axis voltage
e_I = voltage of infinite bus
E_q = induced voltage at no load
ψ_d = direct axis flux linkage in armature winding
ψ_q = quadrature axis flux linkage in armature winding
ψ_{fd} = direct asis field flux linkage
ψ_{fq} = quadrature axis field flux linkage
x_d = direct axis synchronous reactance
x_q = quadrature axis synchronous reactance

x'_d = direct axis transient reactance
x'_q = quadrature axis transient reactance
i_d = direct axis current
i_q = quadrature axis current
T'_{d0} = open-circuit generator field time constant in direct axis
T'_{q0} = open-circuit generator field time constant in quadrature axis
M = $I\omega^2$ where I is per-unit inertia constant
T_m = prime mover torque
P = active power
Q = reactive power
δ = angle between quadrature axis of machines
θ = angular position of direct axis with respect to stator
K = gain of AVR
T_f = time constant of AVR
μ_m = gain of governor
τ = time constant of governer
P_d = damping coefficient of generator

The subscript zero denotes initial operating condition

Appendix

When $T'_{q0i}=0$ ($i=1\sim3$) is assumed in three-machine problem, the elements of matrices $[A_1{}^3]$, $[A_2{}^3]$ and $[A_5{}^3]$ are obtained as follows:

From eq. (35)

$$\Delta\psi_{fdi} = \Delta e_{qi}+x'_{di}\cdot\Delta i_{di} \qquad (i=1\sim3) \qquad \text{(App. 1)}$$

$$\Delta e_{di}-x'_{qi}\cdot\Delta i_{qi} = 0 \qquad (i=1\sim3) \qquad \text{(App. 2)}$$

$i=1$, $n=3$ are substituted in eq. (32), (33),

$$\begin{aligned}&\Delta e_{d1}-(\cos\delta_{130})\cdot\Delta e_{d3}-(\sin\delta_{130})\cdot\Delta e_{q3}+x_{11}\cdot\Delta i_{q1}\\&-(x_{12}\cdot\sin\delta_{120})\cdot\Delta i_{d2}+(x_{12}\cdot\cos\delta_{120})\cdot\Delta i_{q2}\\&=\{(e_{q30}\cdot\cos\delta_{130}-e_{d30}\cdot\sin\delta_{130})+(i_{d20}\cdot\cos\delta_{120}+i_{q20}\cdot\sin\delta_{120})\cdot x_{12}\}\Delta\delta_{13}\\&-(i_{d20}\cdot\cos\delta_{120}+i_{q20}\cdot\sin\delta_{120})\cdot x_{12}\cdot\Delta\delta_{23}\end{aligned} \qquad \text{(App. 3)}$$

$$\begin{aligned}&\Delta e_{q1}+(\sin\delta_{130})\cdot\Delta e_{d3}-(\cos\delta_{130})\cdot\Delta e_{q3}-x_{11}\cdot\Delta i_{d1}\\&-(x_{12}\cdot\cos\delta_{120})\cdot\Delta i_{d2}-(x_{12}\cdot\sin\delta_{120})\cdot\Delta i_{q2}\\&=\{-(e_{q30}\cdot\sin\delta_{130}+e_{d30}\cdot\cos\delta_{130})+x_{12}(i_{q20}\cdot\cos\delta_{120}-i_{d20}\cdot\sin\delta_{120})\}\Delta\delta_{13}\\&+(-i_{q20}\cdot\cos\delta_{120}+i_{d20}\cdot\sin\delta_{120})\cdot x_{12}\cdot\Delta\delta_{23}\end{aligned} \qquad \text{(App. 4)}$$

Similarly, $i=2$, $i=3$ are substituted in eq. (32), (33),

$$\Delta e_{d2}-(\cos\delta_{230})\cdot\Delta e_{d3}-(\sin\delta_{230})\cdot\Delta e_{q3}-(x_{21}\cdot\sin\delta_{210})\cdot\Delta i_{d1}$$

$$+(x_{21}\cos\delta_{210})\cdot\Delta i_{q1}+x_{22}\cdot\Delta i_{q2} = -(i_{d10}\cdot\cos\delta_{210}+i_{q10}\cdot\sin\delta_{210})x_{21}\cdot\Delta\delta_{13}$$
$$+\{(e_{q30}\cdot\cos\delta_{230}-e_{d30}\cdot\sin\delta_{230})+(i_{d10}\cdot\cos\delta_{210}+i_{q10}\cdot\sin\delta_{210})\cdot x_{21}\}\Delta\delta_{23} \qquad \text{(App. 5)}$$

where, $\Delta\delta_{12}$ is eliminated in the above equation by the realtion of $\delta_{12}=\delta_{13}-\delta_{23}$ (similarly in the following)

$$\Delta e_{q2}+(\sin\delta_{230})\cdot\Delta e_{d3}-(\cos\delta_{230})\cdot\Delta e_{q3}-(x_{21}\cdot\cos\delta_{21})\cdot\Delta i_{d1}$$
$$-(x_{21}\cdot\sin\delta_{210})\cdot\Delta i_{q1}-x_{22}\cdot\Delta i_{q2} = (-i_{q10}\cdot\cos\delta_{210}+i_{d10}\cdot\sin\delta_{210})\cdot x_{21}\cdot\Delta\delta_{13}$$
$$+\{-(e_{d30}\cdot\cos\delta_{230}+e_{q30}\cdot\sin\delta_{230})+(i_{q10}\cdot\cos\delta_{210}+i_{q10}\cdot\sin\delta_{210})\cdot x_{21}\}\Delta\delta_{23} \qquad \text{(App. 6)}$$

When $i=3$ in eq. (34),

$$(\sin\delta_{130})\cdot\Delta i_{d1}+(\cos\delta_{130})\cdot\Delta i_{q1}+(\sin\delta_{230})\cdot\Delta i_{d2}$$
$$+(\cos\delta_{230})\cdot\Delta i_{q2}+\Delta i_{q3} = (-i_{d10}\cdot\cos\delta_{130}+i_{q10}\cdot\sin\delta_{130})\cdot\Delta\delta_{13}$$
$$+(-i_{d20}\cdot\cos\delta_{230}+i_{q20}\cdot\sin\delta_{230})\cdot\Delta\delta_{23} \qquad \text{(App. 7)}$$
$$(\cos\delta_{130})\cdot\Delta i_{d1}-(\sin\delta_{130})\cdot\Delta i_{q1}+(\cos\delta_{230})\cdot\Delta i_{d2}-(\sin\delta_{230})\cdot\Delta i_{q2}+\Delta i_{d3}$$
$$= (i_{q10}\cdot\cos\delta_{130}+i_{d10}\cdot\sin\delta_{130})\cdot\Delta\delta_{13}+(i_{q20}\cdot\cos\delta_{230}+i_{d20}\cdot\sin\delta_{230})\cdot\Delta\delta_{23} \qquad \text{(App. 8)}$$

Eqs. (App. 1)$\sim$(App. 8) are arranged and rewritten similar to eq. (43), then matrices $[A_1^3]$, $[A_2^3]$ can be obtained. Furthermore,

$$p\Delta\delta_{13} = \Delta\omega_{13}\,,\quad p\Delta\delta_{23} = \Delta\omega_{23} \qquad \text{(App. 9)}$$

$i=1\sim3$ is substituted in eq. (36)

$$p\Delta\psi_{fdi} = \{(x'_{di}-x_{di})/T'_{d0i}\}\cdot\Delta i_{di}$$
$$-(1/T'_{d0i})\cdot\Delta\psi_{fdi}+(1/T'_{d0i})\cdot\Delta E_{fdi} \qquad \text{(App. 10)}$$

$i=1\sim3$ is substituted in eq. (38)

$$p\Delta E_{fdi} = (-K_i\cdot e_{di0}/T_{fi}\cdot e_{ti0})\cdot\Delta e_{di}-(1/T_{fi})\cdot\Delta E_{fdi}$$
$$-(K_i\cdot e_{qi}/T_{fi}\cdot e_{ti0})\cdot\Delta e_{qi} \qquad \text{(App. 11)}$$

$n=3$, $i=1\sim2$ are substituted in eq. (40),

$$p\Delta\omega_{13} = -(i_{d10}/M_1)\cdot\Delta e_{d1}-(i_{q10}/M_1)\cdot\Delta e_{q1}+(i_{d30}/M_3)\cdot\Delta e_{d3}$$
$$+(i_{q30}/M_3)\cdot\Delta e_{q3}-(e_{d10}/M_1)\cdot\Delta i_{d1}-(e_{q10}/M_1)\cdot\Delta i_{q1}$$
$$+(e_{q30}/M_3)\cdot\Delta i_{q3}+(e_{d30}/M_3)\cdot\Delta i_{d3}-(P_{d1}/M_1)\cdot\Delta\omega_{13} \qquad \text{(App. 12)}$$
$$p\Delta\omega_{23} = -(i_{d20}/M_2)\cdot\Delta e_{d2}-(i_{q20}/M_2)\cdot\Delta e_{q2}+(i_{d30}/M_3)\cdot\Delta e_{d3}$$
$$+(i_{q30}/M_3)\cdot\Delta e_{q3}-(e_{d20}/M_2)\cdot\Delta i_{d2}-(e_{q20}/M_2)\cdot\Delta i_{q2}$$
$$+(e_{d30}/M_3)\cdot\Delta i_{d3}+(e_{q30}/M_3)\cdot\Delta i_{q3}-(P_{d2}/M_2)\cdot\Delta\omega_{23} \qquad \text{(App. 13)}$$

where, $(P_{d1}/M_1)=(P_{d2}/M_2)$

Eqs. (App. 9)$\sim$(App. 13) are arranged and rewritten as eq. (45), then matrix $[A_5^3]$ can be obtained.

Matrices $[A_1^3]$, $[A_2^3]$, $[A_5^3]$ and $[A_6^3]$ are shown as follows:

$[A_1^3]$ Matrix

Δe_{d1}	Δe_{q1}	Δe_{d2}	Δe_{q2}	Δe_{d3}	Δe_{q3}	Δi_{d1}	Δi_{q1}	Δi_{d2}	Δi_{q2}	Δi_{d3}	Δi_{q3}
1.0							$-x_{q1}$				
	1.0					x_{d1}'					
		1.0							$-x_{q2}$		
			1.0					x_{d2}'			
				1.0							$-x_{q3}$
					1.0					x_{d3}'	
	1.0			$\sin\delta_{130}$	$-\cos\delta_{130}$	$-x_{11}$		$-x_{12}\cos\delta_{120}$	$-x_{12}\sin\delta_{120}$		
1.0				$-\cos\delta_{130}$	$-\sin\delta_{130}$		x_{11}	$-x_{12}\sin\delta_{120}$	$x_{12}\cos\delta_{120}$		
			1.0	$\sin\delta_{230}$	$-\cos\delta_{230}$	$-x_{21}\cos\delta_{210}$	$-x_{21}\sin\delta_{210}$	$-x_{22}$			
		1.0		$-\cos\delta_{230}$	$-\sin\delta_{230}$	$-x_{21}\sin\delta_{210}$	$x_{21}\cos\delta_{210}$		x_{22}		
						$\cos\delta_{130}$	$-\sin\delta_{130}$	$\cos\delta_{230}$	$-\sin\delta_{230}$	1.0	
						$\sin\delta_{130}$	$\cos\delta_{30}$	$\sin\delta_{230}$	$\cos\delta_{230}$		1.0

$[A_2^3]$ Matrix

$\Delta\delta_{13}$	$\Delta\delta_{23}$	$\Delta\psi_{fd1}$	$\Delta\psi_{fd2}$	$\Delta\psi_{fd3}$
		1.0		
			1.0	
				1.0
a_1	a_2			
a_3	a_4			
a_5	a_6			
a_7	a_8			
a_9	a_{10}			
a_{11}	a_{12}			

$a_1=-e_{q30}\sin\delta_{130}-e_{d30}\cos\delta_{130}+(i_{q20}\cos\delta_{120}-i_{d20}\sin\delta_{120})x_{12}$
$a_2=(-i_{q20}\cos\delta_{120}+i_{d20}\sin\delta_{120})x_{12}$
$a_3=e_{q30}\cos\delta_{130}-e_{d30}\sin\delta_{130}+(i_{d20}\cos\delta_{120}+i_{q20}\sin\delta_{120})x_{12}$
$a_4=(-i_{d20}\cos\delta_{12}-i_{q20}\sin\delta_{120})x_{12}$
$a_5=(-i_{q10}\cos\delta_{210}+i_{d10}\sin\delta_{210})x_{21}$
$a_6=-e_{d30}\cos\delta_{230}-e_{q30}\sin\delta_{230}+(i_{q30}\cos\delta_{210}+i_{d10}\sin\delta_{210})x_{21}$
$a_7=(-i_{d10}\cos\delta_{210}-i_{q10}\sin\delta_{210})x_{21}$
$a_8=e_{q30}\cos\delta_{230}-e_{d30}\sin\delta_{230}+(i_{d10}\cos\delta_{210}+i_{q10}\sin\delta_{210})x_{21}$
$a_9=i_{q10}\cos\delta_{130}+i_{d10}\sin\delta_{130}$
$a_{10}=i_{q20}\cos\delta_{230}+i_{d20}\sin\delta_{230}$
$a_{11}=-i_{d10}\cos\delta_{130}+i_{q10}\sin\delta_{130}$
$a_{12}=-i_{d20}\cos\delta_{230}+i_{q20}\sin\delta_{230}$

$[A_6^3]$ Matrix

<table>
<tr><td colspan="2">$[A_3^3]$
(12×5)</td><td>[0]
(12×5)</td></tr>
<tr><td>[0]
(10×2)</td><td colspan="2">$[U]$
(10×10)</td></tr>
</table>

$[A_5^3]$ Matrix

	Δe_{d1}	Δe_{q1}	Δe_{d2}	Δe_{q2}	Δe_{d3}	Δe_{q3}	Δi_{d1}	Δi_{q1}	Δi_{d2}	Δi_{q2}
$p\Delta\delta_{13}$										
$p\Delta\delta_{23}$										
$p\Delta\psi_{fd1}$							$\frac{x_{d1}'-x_{d1}}{T_{d01}'}$			
$p\Delta\psi_{fd2}$									$\frac{x_{d2}'-x_{d2}}{T'_{d02}}$	
$p\Delta\psi_{fd3}$										
$p\Delta E_{fd1}$	$-\frac{K_1 e_{d10}}{T_{f1} e_{t10}}$	$-\frac{K_1 e_{q10}}{T_{f1} e_{t10}}$								
$p\Delta E_{fd2}$			$-\frac{K_2 e_{d20}}{T_{f2} e_{t20}}$	$-\frac{K_2 e_{q20}}{T_{f2} e_{t20}}$						
$p\Delta E_{fd3}$					$-\frac{K_3 e_{d3}}{T_{f3} e_{t3}}$	$-\frac{K_3 e_{q3}}{T_{f3} e_{t3}}$				
$p\Delta\omega_{13}$	$-\frac{i_{d10}}{M_1}$	$-\frac{i_{q10}}{M_1}$			$\frac{i_{d30}}{M_3}$	$\frac{i_{q30}}{M_3}$	$-\frac{e_{d10}}{M_1}$	$-\frac{e_{q10}}{M_1}$		
$p\Delta\omega_{23}$			$-\frac{i_{d20}}{M_2}$	$-\frac{i_{q20}}{M_2}$	$\frac{i_{d30}}{M_3}$	$\frac{i_{q30}}{M_3}$			$-\frac{e_{d20}}{M_2}$	$-\frac{e_{q20}}{M_2}$

	Δi_{d3}	Δi_{q3}	$\Delta\psi_{fd1}$	$\Delta\psi_{fd2}$	$\Delta\psi_{fd3}$	ΔE_{fd1}	ΔE_{fd2}	ΔE_{fd3}	$\Delta\omega_{13}$	$\Delta\omega_{23}$
$p\Delta\delta_{13}$									1.0	
$p\Delta\delta_{23}$										1.0
$p\Delta\psi_{fd1}$			$-\frac{1.0}{T_{d01}'}$			$\frac{1.0}{T_{d01}'}$				
$p\Delta\psi_{fd2}$				$-\frac{1.0}{T_{d02}'}$			$\frac{1.0}{T_{d02}'}$			
$p\Delta\psi_{fd3}$	$\frac{x_{d2}'-x_{d3}}{T_{d02}'}$				$-\frac{1.0}{T_{d03}'}$			$\frac{1.0}{T_{d03}'}$		
$p\Delta E_{fd1}$						$-\frac{1.0}{T_{f1}}$				
$p\Delta E_{fd2}$							$-\frac{1.0}{T_{f2}}$			
$p\Delta E_{fd3}$								$-\frac{1.0}{T_{f3}}$		
$p\Delta\omega_{13}$	$\frac{e_{d30}}{M_3}$	$\frac{e_{q30}}{M_3}$							$-\frac{P_{d1}}{M_1}$	
$p\Delta\omega_{23}$	$\frac{e_{d30}}{M_3}$	$\frac{e_{q30}}{M_3}$								$-\frac{P_{d2}}{M_2}$

where, $\frac{P_{d1}}{M_1}=\frac{P_{d2}}{M_2}$