Estimation of Dispersion Coefficients on Free Surface by Means of Particle Simulation Method

AUTHOR(S):

IWASA, Yoshiaki; IMAMOTO, Hirotake

CITATION:

IWASA, Yoshiaki ...[et al]. Estimation of Dispersion Coefficients on Free Surface by Means of Particle Simulation Method. Memoirs of the Faculty of Engineering, Kyoto University 1968, 30(1): 15-25

ISSUE DATE:

1968-03-15

URL:

http://hdl.handle.net/2433/280717

Estimation of Dispersion Coefficients on Free Surface by Means of Particle Simulation Method

By

Yoshiaki IWASA* and Hirotake IMAMOTO*

(Received September 30, 1967)

The present paper deals with a theoretical treatment of the size effect of solid particles on the estimation of dispersion coefficients at the free surface of flow by means of the tracer simulation method and experimental evidences made at a test flume in the laboratory and a navigation canal. The results obtained through experimentations indicate that the fluctuating velocites of solid particles on the free surface are of outstanding periodicity and therefore the Lagrangian correlation coefficient is not simply expressed in terms of an exponential function as done by Taylor, which must be further investigated by more extensive research programs.

1. General Scope of Present Problem

The diffusion process in a turbulent field will be simulated by the use of either deterministic or stochastic model. The deterministic simulation obtained by the analogous relation between momentum and mass transports describes well the dynamic behaviour of a suspended load in a flow under equilibrium state[1]. However, this type of simulation is sustained by a phenomenological motive and therefore general comprehension of the problem will remain unsolved.

The stochastic model is quite effective, if the underlying dynamic behaviour concerning the dispersion of turbulent flow is not subjected to a mathematical simulation. The theory of Taylor[2] is then the main contribution of the present problem, though it concerns the basic knowledge of a mixing process in the homogeneous isotropic field in turbulence. Actual estimations of eddy diffusivity given by the dispersion coefficient are made by flow visualization techniques, in which the movement of discrete particles of finite size and specified weight will be traced. The physical and geometric effects of discrete particles as simulating tracers must be then disclosed. The densimetric effect of particles on the longitudinal dispersion was experimentally made by Binnie and Phillips[3] with the use of slightly buoyant

* Department of Civil Engineering

and heavy particles in pipe flows. The result obtained indicates no significance in the difference of density between fluid and simulating tracers used.

The present paper concerns the size effect of discrete particles as a measure of geometric parameter to the eddy diffusivity of the free surface shear flow.

2. Mathematical Theory of Dispersion

The longitudinal dispersion in the direction of flow will be considered. The stochastic model for the continuous movement of fluid particles due to Taylor[2] gives the longitudinal variance of

$$\overline{X^2(t)} = 2\overline{u'^2}\int_0^t\int_0^{t_1} R_L(\xi)\,d\xi\,dt_1 = 2\overline{u'^2}\int_0^t (t-\xi)\,R_L(\xi)\,d\xi \tag{1}$$

where

X : longitudinal deviation from mean value

R_L : Lagrangian correlation coefficient, which is

$$R_L(\xi) = \frac{\overline{u'(t)\,u'(t+\xi)}}{\overline{u'^2}} \tag{2}$$

The dispersion coefficient is then

$$K(t) = \frac{1}{2}\frac{d\overline{X^2}}{dt} = \overline{u'^2}\int_0^t R_L(\xi)\,d\xi \tag{3}$$

Between the Lagrangian correlation coefficient $R_L(\xi)$ and the Lagrangian spectrum function $F_L(n)$,

$$F_L(n) = 4\int_0^\infty R_L(\xi)\cos(2\pi n\xi)\,d\xi \tag{4}$$

or inversely

$$R_L(\xi) = \int_0^\infty F_L(n)\cos(2\pi n\xi)\,dn \tag{5}$$

Inserting Eq. (5) into Eqs. (1) and (3), the variance of $X(t)$ and the dispersion coefficient $K(t)$ expressed in terms of the Lagrangian spectrum function are

$$\overline{X^2(t)} = \overline{u'^2}\int_0^\infty F_L(n)\left[\frac{1-\cos(2\pi nt)}{2(\pi n)^2}\right]dn = t^2\overline{u'^2}\int_0^\infty F_L(n)\frac{\sin^2(\pi nt)}{(\pi nt)^2}\,dn \tag{6}$$

$$K(t) = t\,\overline{u'^2}\int_0^\infty F_L(n)\frac{\sin(2\pi nt)}{2\pi nt}\,dn \tag{7}$$

Next will be described a similar statistical treatment for the continuous movement of discrete particles of finite size and specified weight by means of the correlation coefficient of Lagrange, because the spectrum function method deals with the periodic variation. An exact simulation to the fluid flow through the movement of discrete particles is impossible, so that the treatment will be developed under the following assumptions that

(1) the instantaneous velocity of a discrete particle is given by a volumetric mean value of fluid velocities, and furthermore

(2) the volumetric averaging can be replaced by the one-dimensional treatment using an averaging-time, in which the fluid passes the diameter of particle.

Then, the fluctuation velocity of particle is approximated by

$$u_p'(t) = \frac{1}{V}\int_V u'(t)\,dV = \frac{1}{p}\int_{t-(p/2)}^{t+(p/2)} u'(t_1)\,dt_1 \tag{8}$$

The correlation of $u_p'(t)$ is[4)]

$$\overline{u_p'(t_0)\,u_p'(t_0+\xi)} = \frac{\overline{u'^2}}{p^2}\left[\int_{\xi-p}^{\xi}(\eta-\xi+p)\,R_L(\eta)\,d\eta + \int_{\xi}^{\xi+p}(-\eta+\xi+p)\,R_L(\eta)\,d\eta\right] \tag{9}$$

where

$\overline{u'^2}$: intensity of fluid turbulence

Putting $\xi=0$ in Eq. (9) makes

$$\overline{u_p'^2} = \frac{2\overline{u'^2}}{p^2}\int_0^p (p-\eta)\,R_L(\eta)\,d\eta \tag{10}$$

and integrating Eq. (9) with respect to ξ from 0 to ∞

$$\int_0^\infty \overline{u_p'(t_0)\,u_p'(t_0+\xi)}\,d\xi = \int_0^\infty \overline{u'(t_0)\,u'(t_0+\xi)}\,d\xi \tag{11}$$

which indicates the integral is independent of p. The right side term of Eq. (11) is equivalent to $\overline{u'^2}t_*$, so that

$$\overline{u_p'^2}\cdot t_{*p} = \overline{u'^2}\cdot t_* \tag{12}$$

where

t_* : Lagrangian integral time scale for fluid

t_{*p} : Lagrangian integral time scale for discrete particle

A numerical illustration will be made by using

$$R_L(\xi) = \exp(-\xi/t_*) \tag{13}$$

as done by Taylor. The ratio of turbulent intensities between the fluid and the discrete particle is then

$$\frac{\overline{u_p'^2}}{\overline{u'^2}}=\frac{t_*}{t_{*p}}=\frac{2}{(p/t_*)^2}[(p/t_*)+\exp(-p/t_*)-1] \tag{14}$$

A curve in Fig. 1 is the numerical solution of Eq. (14). With the increase of p for a constant t_*, $\overline{u_p'^2}/\overline{u'^2}$ decreases and inversely t_{*p}/t_* increases.

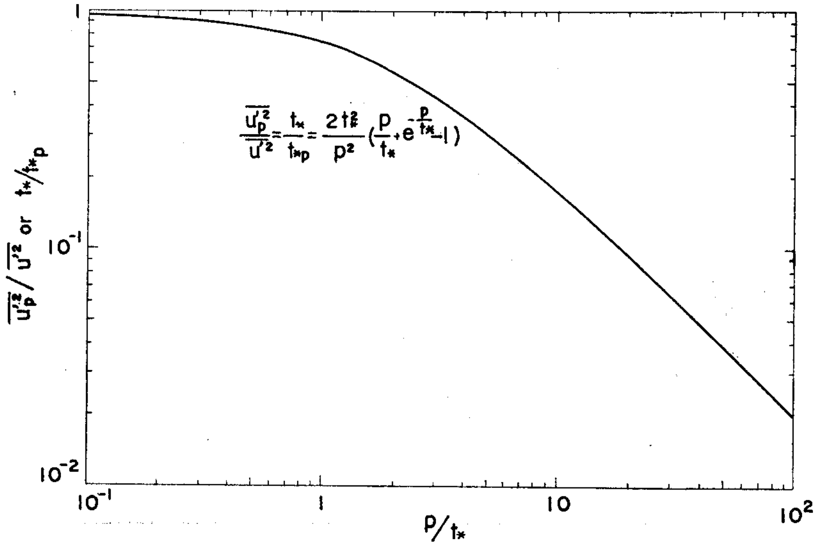

Fig. 1 Numerical Plot of Eq. (14)

The variation of the Lagrangian correlation coefficient with the change of p is obtained by inserting Eq. (13) into Eq. (9) and shown in Fig. 2.

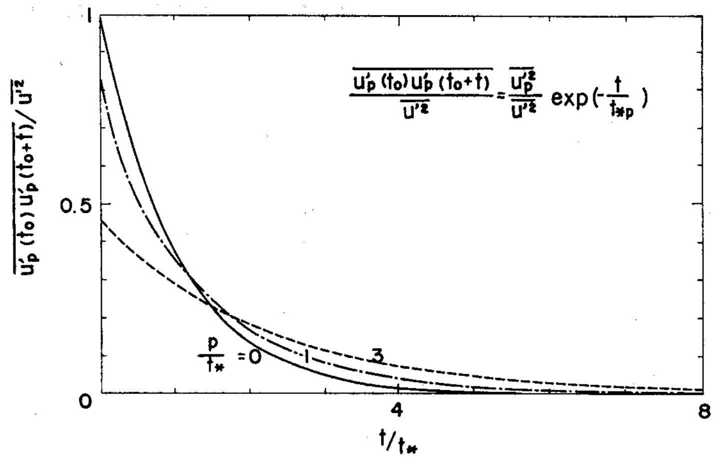

Fig. 2 Relation between Lagrangian Correlation Coefficient and Dimensionless Time

The Lagrangian correlation coefficient for discrete particles $R_{Lp}(\xi)$ is approximated by

$$R_{Lp}(\xi) = \exp(-\xi/t_{*p}) \tag{15}$$

The longitudinal dispersion and the dispersion coefficient for discrete particles are then

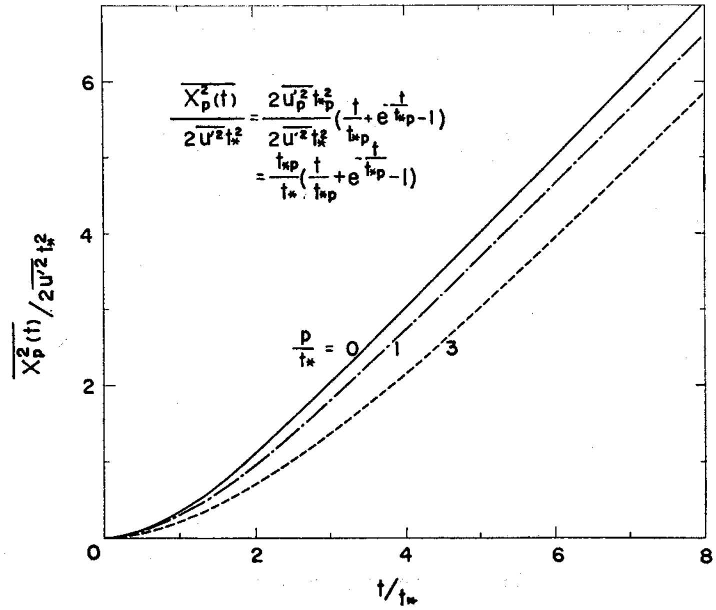

Fig. 3 Numerical Plot of Dimensionless Variance of $\overline{X_p^2}$

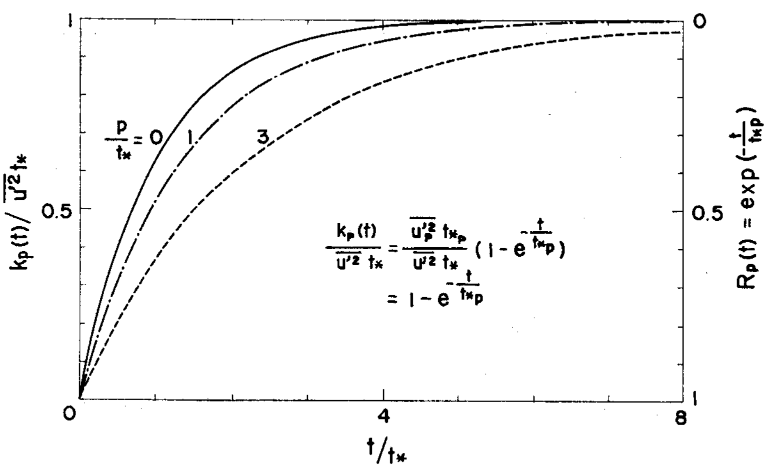

Fig. 4 Numerical Plot of Dispersion Coefficient of Discrete Particles

$$\overline{X_p^2(t)} = 2\overline{u_p'^2}t_{*p}^2[(t/t_{*p})+\exp(-t/t_{*p})-1] \tag{16}$$

$$K_p(t) = \overline{u_p'^2}t_{*p}[1-\exp(-t/t_{*p})] \tag{17}$$

which make a significant evidence of the size effect of simulating tracers to the dispersion process in a turbulent field. Figs. 3 and 4 are numerical plots of Eqs. (16) and (17)

3. Verification through Laboratory Experimentation and Field Observation

The laboratory experimentation was made in a straight lucite flume of 16 m in length, 25 cm in width, and 35 cm in depth at the Hydraulics Laboratory, Department of Civil Engineering. The bottom slope was set at 1/500. Discrete particles used in test runs are foamed polystyrene disks, of which the thickness is 0.3 cm and the diameter 0.5, 1.0, 2.0 and 4.0 cm, respectively. The movement of each particle flowing down on the free surface was taken in pictures at every 0.05 sec. Each test run is made by 100 observations of particles, and hydraulic conditions given at all test runs are shown in Table 1.

Table 1 Hydraulic Condition for Laboratory Experiment

Run	Discharge (l/sec)	Flow Depth (cm)	Temperature (°C)	Reynolds No.	Froude No.
L–1	15.90	9.52	14.4	5.49×10^4	0.692
L–2	9.17	6.73	23.9	3.99	0.671
L–3	8.07	9.52	14.4	2.79	0.351
L–4	4.08	4.00	14.4	1.41	0.767

The field observation was made at a straight reach of 200 m of the municipal canal, which has a trapezoidal section of 5.6 m in top width and 4.8 m in bottom width. The particles used for this observation were a polyethylene disk of 0.3 cm in thickness, 1.0 cm in diameter and 0.935 in specific weight, and two plywood disks of 0.8 in specific weight, 0.5 cm in thickness, 10 and 100 cm in diameter, respectively. The reason why such a large disk was used for the observation of longitudinal dispersion is to check the limit of the present particle simulation method. The measurements of times required to pass 5, 10, 20, 40, and 80 m in distance along the canal for each particle were made. Each test run is also constituted by 100 measurements.

The correlation coefficient and the spectrum function of Lagrange were evaluated by the following relations of

$$R_L(j\tau) = \sum_{i=0}^{N-j} (u_i' u_{i+j}') / [\sum_{i=0}^{N-j} u_i'^2 \sum_{i=0}^{N-j} u_{i+j}'^2]^{1/2} \tag{18}$$

and

$$S_L(f) = \left[\frac{1}{N}\sum_{i=1}^{N} u_i' \cos(2\pi f i)\right]^2 + \left[\frac{1}{N}\sum_{i=1}^{N} u_i' \sin(2\pi f i)\right]^2 \tag{19}$$

where

u' : fluctuating velocity from mean value of particle velocity at every 0.05 sec.

After making the systematic measurements of dispersion by means of the discrete particle simulation method in a laboratory flume and a municipal canal, the following results are obtained. The relative intensities of turbulence measured for various particles are shown in Fig. 5. The longitudinal variances $\overline{X_p^2(t)}$ are also shown in Figs. 6-a and -b. The laboratory experiments, as seen in Fig. 6-a, can not give any verification to the mathematical theory deduced in the foregoing. The cause will be the unfitness of exponential law for the Lagrangian correlation coefficient. For instance, Fig. 7 shows the Lagrangian correlation coefficient measured for two test runs of *L*–1 and *L*–4.

A considerable periodicity in the variation of u' will be seen, though the reliability of computation is not enough because of shortage in duration of observation (1.5 sec for *L*–1 and 2.5 sec for *L*–4). The normalized Lagrangian spectrum functions measured for both test runs of a particular size (0.5 cm) are shown in Fig. 8-a. The figure indicates there is a peak in $R_{L_p}(t)$ at $f=3$ cycles for *L*–1 whereas no peak for *L*–4, which describes the variation of Lagrangian spectrum function with the change of external hydraulic conditions. Fig. 8-b indicates the normalized Lagrangian spectrum functions measured for various particle sizes in the test run *L*–1 and no distinct trend concerning the size effect to the spectrum function through the present discrete particle simulation method will be seen.

4. Conclusive Remarks

In this paper, a statistical theory to estimate the dispersion coefficient on the free surface by means of a particle simulation method was concerned. However, the laboratory experimentation and the field observation can not give a real verification to the theory deduced. The main cause in the unsuccess of the research program will be due to the experimental techniques in laboratory and field works. Nevertheless, all the results treated in this paper will contribute to the further extensive research program.

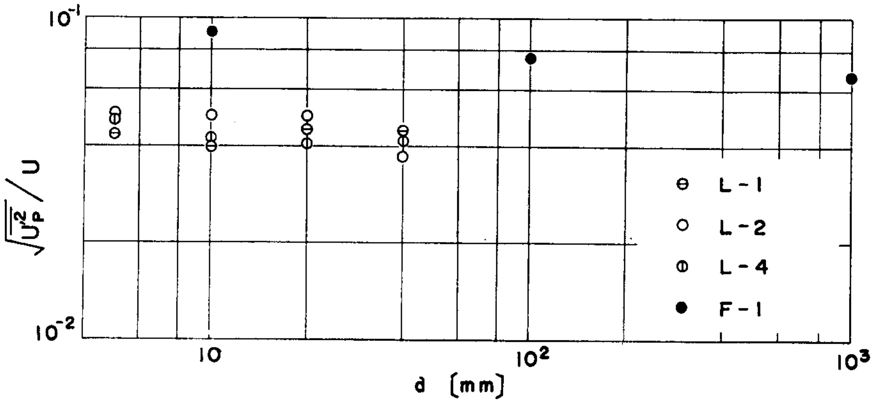

Fig. 5 Relative Intensity of Turbulence in Terms of Particle Diameters Measured by Simulation Method

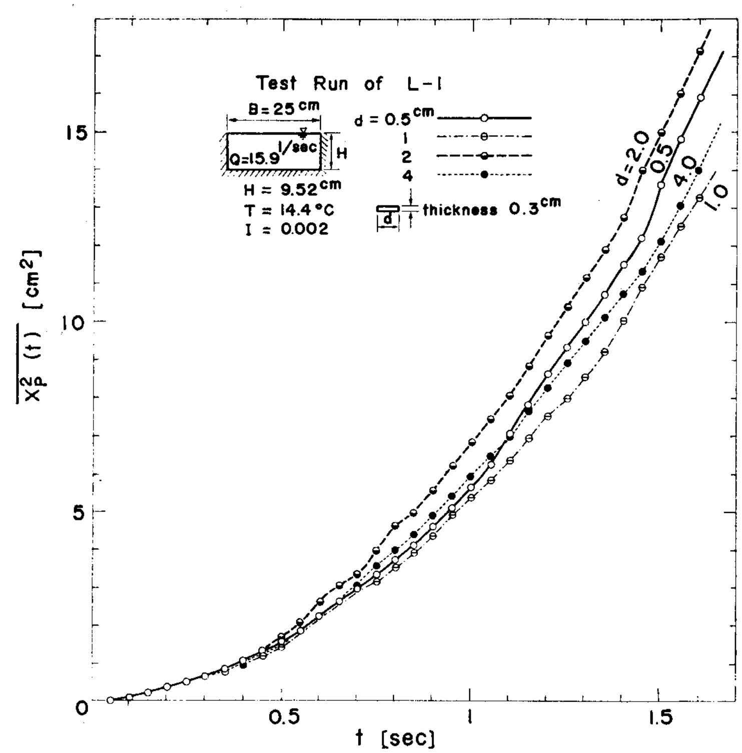

Fig. 6-a Longitudinal Dispersions Measured in Test Flume at Test Run, *L*-1

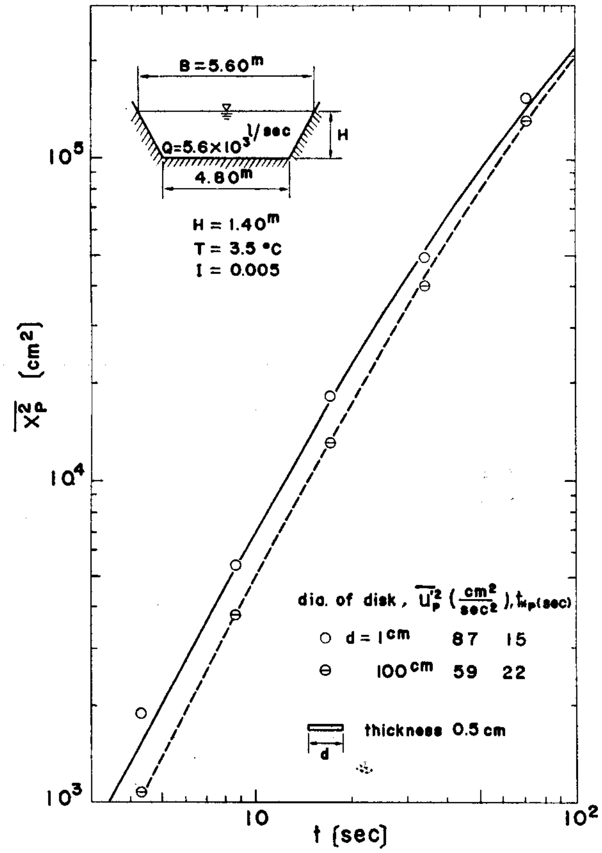

Fig. 6-b Longitudinal Dispersions Measured in Canal

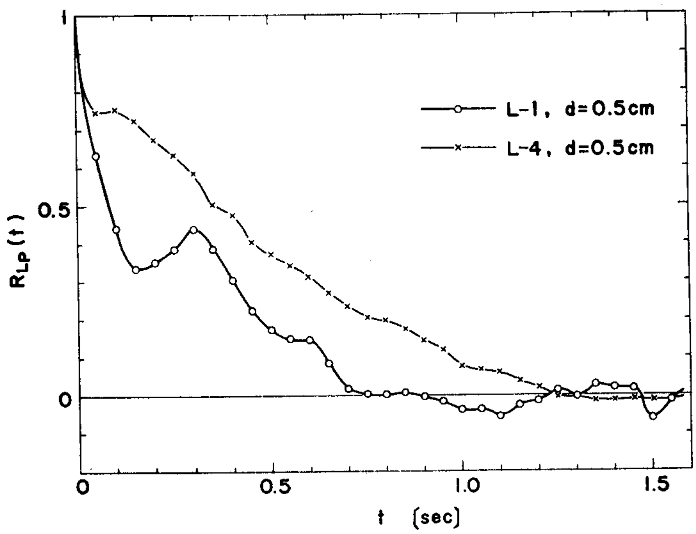

Fig. 7 Lagrangian Correlation Coefficients for Particle Size of 0.5 cm at Test Runs, *L*-1 and *L*-4

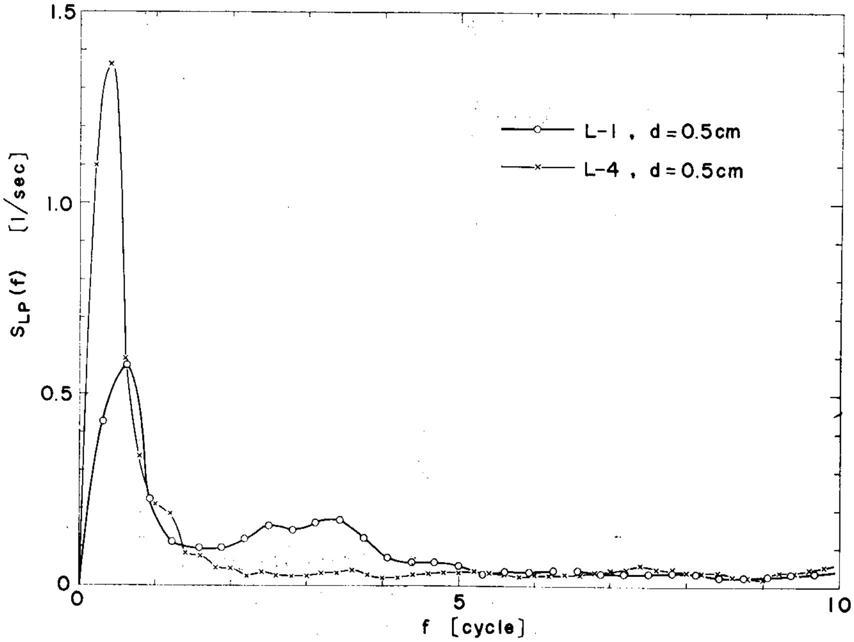

Fig. 8-a Lagrangian Spectrum Functions for Particle Size of 0.5 cm at Test Run, *L*–1 and *L*–4

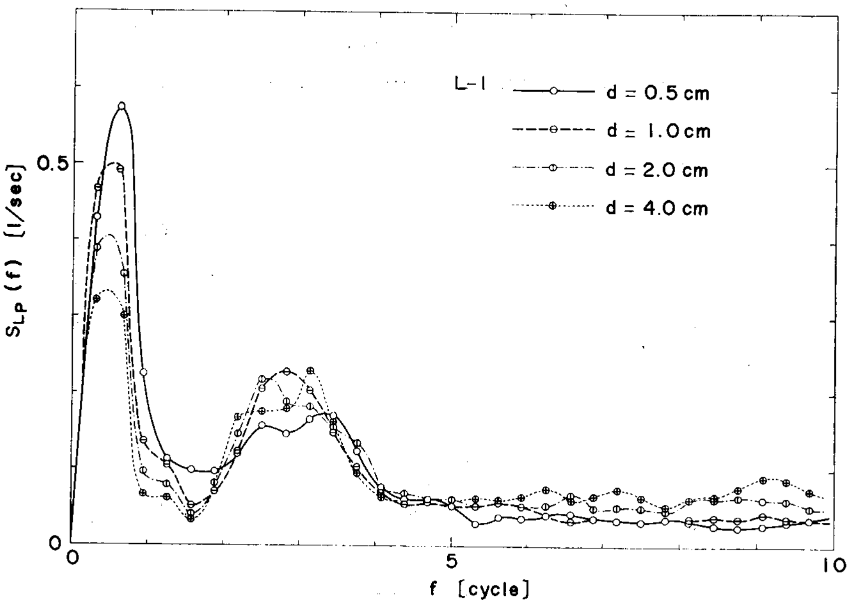

Fig. 8-b Lagrangian Spectrum Functions for Various Particle Sizes at Test Run, *L*–1

References

1) Rouse, H.: Transportation of Sediment, Chatpter XIV of Fluid Mechanics for Hydraulic Engineers, McGraw-Hill, New York, 1938.
2) Taylor, G.I.: Proc. Roy. Soc. A, **223,** 1954.
3) Binnie, A.M. and O.M. Phillips: Jour. Fluid Mech., **4,** 1958.
4) Iwasa, Y. and H. Imamoto: Proc. 12th Congress of IAHR, Fort Collins, Colo., U.S.A., **4,** 1967.