Turbulent Heat Transfer in Non-Newtonian Fluids

By

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The heat transfer coefficients of the turbulent flow of not only pseudoplastic, but dilatant and also Bingham fluids in a rectangular duct and in a circular tube were measured experimentally, and correlated by the same characteristic idea with a modified Chilton-Colburn analogy. The equations of heat transfer coefficients thus derived were shown to represent the data well. Moreover, the temperature and velocity profiles in the rectangular duct were measured to compute the eddy diffusivities for momentum and for heat in Newtonian and non-Newtonian fluids, and the correlations of the eddy diffusivities were presented.

Nomencalture

a	$=\tau_0/\tau_w$	[—]
C_P	=Heat capacity, constant pressure	[cal/gr degree C]
D	=Diameter or equivalent diameter	[cm]
f	=Fanning's friction factor	[—]
k	=Thermal conductivity of fluid	[cal/cm^2 sec degree C/cm]
m	=Constant in power law model, equation (1)	[gr sec^{n-2}/cm]
n	=Power in power law model, equation (1)	[—]
q	=Heat flux	[cal/cm^2 sec]
r_0	=Half of distance between upper and lower plate of rectangular duct	[cm]
t	=Temperature	[°C]
u	=Axial velocity at a point in fluid	[cm/sec]
u_m	=Mean axial velocity of fluid in duct or tube	[cm/sec]
y	=Distance from the wall	[cm]
α	=Temperature conductivity	[cm^2/sec]
ε_M	=Eddy diffusivity for momentum	[cm^2/sec]
ε_H	=Eddy diffusivity for heat	[cm^2/sec]

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η	=Bingham viscosity in Bingham model, equation (2)	[gr/cm sec]
μ_{aw}	=Apparent viscosity at the wall	[gr/cm sec]
ν	=Kinematic viscosity	[cm^2/sec]
ρ	=Density of fluid	[gr/cm^3]
τ	=Shear stress	[gr/cm sec^2]
τ_0	=Yield stress in Bingham model, equation (2)	[gr/cm sec^2]
Nu	=Nusselt number	[—]
Re	=Reynolds number	[—]
Re_B	=Reynolds number for Bingham plastics, equation (21)	[—]
Re_P	=Reynolds number for pseudoplastic and dilatant fluids, equation (8)	[—]
Re'	$=Re_p\bigg/\left(\frac{3n+1}{4n}\right)^n 8^{n-1}$	[—]
St	=Stanton number	[—]
Pr	=Prandtl number	[—]
Pr_B	=Prandtl number for Bingham plastics, equation (22)	[—]
Pr_P	=Prandtl number for pseudoplastic and dilatant fluids, equation (9)	[—]

Subscript w refers to wall.

Introduction

Various investigators have published so many papers on fluid dynamics and heat transfer of turbulent flow of Newtonian fluids over a wide range of Prandtl number that there is enough knowledge about those fluids. On the other hand, though it is becoming more and more frequent to handle non-Newtonian fluids, only few experimental data and theoretical analysis are available on fluid dynamics and on heat transfer of the turbulent flow of those fluids. Especially lacking is the experimental data of dilatant fluids and also those of the eddy diffusivities in non-Newtonian fluids.

This paper reports the result of an experimental and analytical investigation of fluid dynamics and heat transfer of turbulent flow of Newtonian and non-Newtonian fluids and presents the correlation of the data of heat transfer coefficients for both kinds of fluids with a single equation. Four fluids were caused to flow turbulently in a rectangular duct and a circular tube. These four fluids were: (1)

aqueous solutions of glycerol as Newtonian fluids, (2) aqueous solutions of carboxy methyl-cellulose as pseudoplastic fluids, (3) corn starch dispersed in water as dilatant fluids, and (4) cement slurry in water as Bingham plastic fluids. The friction factors and heat transfer coefficients were found for these flow systems and correlated.

The characteristic idea for correlating the heat transfer coefficient is: "Since almost the all resistance for momentum and heat transfer in the turbulent flow of high viscous fluids as non-Newtonian fluids exists in the laminer film, the apparent viscosity evaluated at wall shearing stress may be the characteristic property. Therefore, if the viscosity term of Reynolds and Prandtl numbers in the equations for Newtonian fluids is replaced by the apparent viscosity at the wall, those equations may be applied to non-Newtonian fluids". This idea which is almost the same as that presented previously by Metzner and Friend [1] will be shown to be valid for the experimental range of this paper.

In addition the velocity and temperature profiles in the turbulent flows of Newtonian and non-Newtonian fluids in the rectangular duct were measured and the eddy diffusivities for momentum and heat were calculated. And an attempt was made to correlate the data of Newtonian and non-Newtonian fluids with a single relation.

Experimental Apparatus and Procedure

The flow diagram of the experiment is shown in Fig. 1. The liquid was caused to flow by a pump from a reservoir tank to the test section through a magnetic flow meter, and then to go back to the reservoir through a heat exchanger. The test sections are a horizontal rectangular duct and a circular tube which were placed parallel and switched with valves.

The rectangular duct made of 18 Cr–8 Ni stainless steel had a 24 *mm* high and 150 *mm* wide cross section, the hydraulic diameter of which was 41.4 *mm* and was equipped with a heating jacket along the upper side and a cooling jacket along the lower side, thus the heat transfered downward crossing the stream. The length of the duct was 5000 *mm* and the measuring point was at 4700 *mm* down stream from the entrance. The length of the foreflow was 113 times as big as hydraulic diameter.

The circular tube made of steel was of 50 *mm* in diameter with 10 *mm* wall thickness and of 5500 *mm* in length. The heating water was pumped in the annular jacket made of double tubes. The measuring point was at 5000 *mm* down stream from the entrance. Thus the ratio of the length of foreflow to diameter was 100.

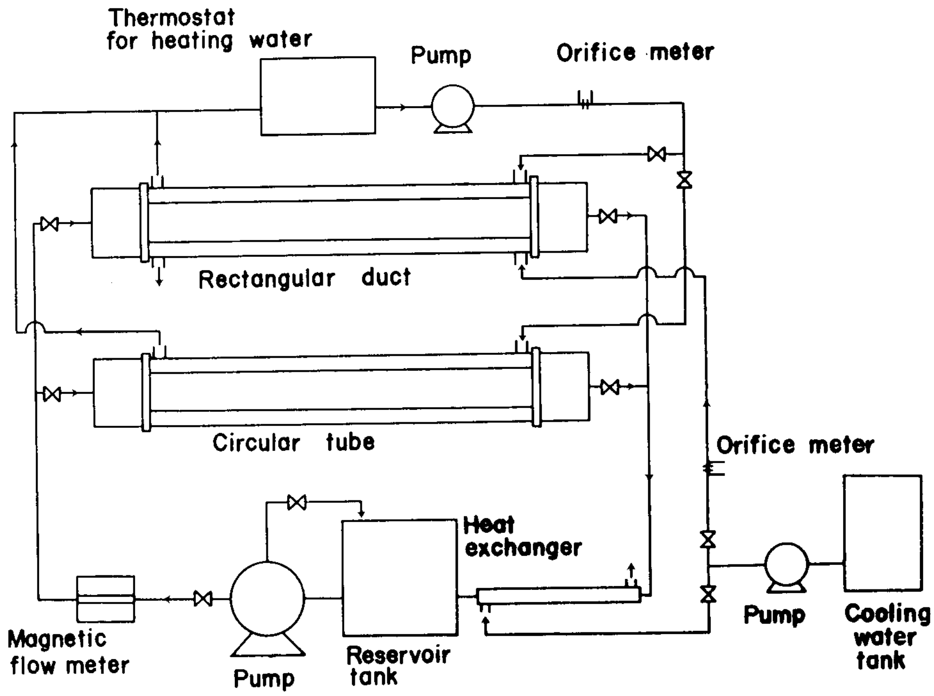

Fig. 1. Flow Diagram.

The temperatures of heating and cooling water were about 60°C and 30°C respectively and controlled within ±0.05 degree C. They were pumped to flow in the jackets countercurrently to the test fluids. The temperature rise of cooling water and descent of heating water in the jackets were less than 0.5 degree C.

The pressure drops in the test section were measured with inclined U tube mercury manometers. The distance of pressure taps were 600 *mm* in the rectangular duct and 1000 *mm* in the circular tube.

The heat fluxes at the walls were computed from the measurement of the temperature difference of the copper-constantan thermocouples fixed separately in the walls at the measuring point. The distance of those two thermocouples was about 11 *mm*, and the temperature difference ranged 4.4∼9.5 degree C in the rectangular duct, and the former was 3 *mm* and the latter were 0.4∼1.4 degree C in the circular tube. This measuring device was calibrated by comparing the thermal conductivity with the stagnant water filled in the duct for the rectangular duct and by the preliminary runs with water for the circular tube. Since the thermocouples can measure the temperature to 0.02 degree C, the error in measuring the heat flux is within $\pm1\%$ for the rectangular duct and $\pm10\%$ for the circular tube. The wall temperature at the measuring point was obtained by the extrapolation of the readings of the two thermocouples fixed in the wall. The average temperature of fluids at that point was calculated from the measured temperature and velocity profile at the point.

A Pitot tube for velocity profile and a copper-constantan thermocouple for temperature profile were traversed vertically in the rectangular duct and in the radial direction in the circular tube at the point of measuring.

The rheological properties of pseudoplastic and dilatant fluids will be represented by the power law model as

$$\tau = m\left(\frac{du}{dy}\right)^n \tag{1}$$

and those of Bingham plastics by the Bingham model as

$$\tau = \tau_0 + \eta\left(\frac{du}{dy}\right) \tag{2}$$

The constants of these equations were changed by varying the concentrations of materials of the solutions. And they were measured with Shimazu Universal Rheometer UR-1M (coaxial cylinders type).

The ranges of experimental conditions are tabulated in Table 1.

The anomal behavior of carboxy methyl-cellulose solution as significant as that reported by Dodge and Metzner [3] was not found, probably because the con-

Table 1. Ranges of experimental conditions

	Concentration [wt %]	Viscosity [c.p.]	Power n [—]	Yield Stress τ_0 [g/cm sec^2]	Reynolds Number Re, Re' or Re_B [—]	Prandtl Number Pr, Pr_P or Pr_B [—]
Glycerol in Water (Newtonian)	58~90	3.6~64	—	—	3.79×10^3~ 7.56×10^4 (Rect. Duct)	25~467
C.M.C in Water (Pseudoplastic)	0~1.3	—	0.66~1.00	—	2.5×10^3~ 5.23×10^4 (Rect. Duct) 2.5×10^3~ 5.79×10^4 (Circ. Tube)	5.8~846
Corn Starch in Water (Dilatant)	45~49	—	1.25~1.62	—	2.5×10^3~ 6.60×10^3 (Rect. Duct) 2.5×10^3~ 9.35×10^3 (Circ. Tube)	53~102
Cement Slurry in Water (Bingham Plastics)	35~72	—	—	27~169	2.70×10^4~ 7.18×10^4 (Rect. Duct) 5.00×10^4~ 1.53×10^5 (Circ. Tube)	14.7~18.2

centration was not so high. However, the rheological properties m and n changed with time by the effect of heating so much that it was necessary to measure those properties during every run.

Experimental Results

1. Friction factors and heat transfer coefficients

a. Pseudoplastic and Dilatant Fluids Applying equation (1) to the shear stress at the wall gives

$$\tau_w = m\left(\frac{du}{dy}\right)_w^n \tag{3}$$

Therefore, the apparent viscosity at the wall is

$$\mu_{aw} = \tau_w \Big/ \left(\frac{du}{dy}\right)_w = m^{1/n}\,\tau_w^{1-(1/n)} \tag{4}$$

The definition of Fanning's friction factor is

$$\tau_w = \frac{1}{2} f \rho u_m^2 \tag{5}$$

Following the idea mentioned in the introduction Re_w and Pr_w are difined as

$$Re_w = D u_m \rho / \mu_{aw} \tag{6}$$

$$Pr_w = C_p \mu_{aw} / k \tag{7}$$

On the other hand, the following Reynolds and Prandtl number are obtained from the dimensional analysis.

$$Re_P = D u_m \rho \Big/ \left[m\left(\frac{u_m}{D}\right)^{n-1}\right] \tag{8}$$

$$Pr_P = \frac{C_P m}{k}\left(\frac{u_m}{D}\right)^{n-1} \tag{9}$$

From equations (4) (5) (6) (7) (8) and (9), one obtains

$$Re_w = Re_P^{1/n}\left(\frac{2}{f}\right)^{1-(1/n)} \tag{10}$$

$$Pr_w = Pr_P Re_P^{1-(1/n)}\left(\frac{f}{2}\right)^{1-(1/n)} \tag{11}$$

The simplest equation for the friction factor of Newtonian fluids is that of Koo (2) as

$$f = 0.046\, Re^{-1/5} \tag{12}$$

Replacing Re in equation (12) with Re_w, the following equation is obtained.

$$f = 0.046\, Re_w^{-1/5} \tag{13}$$

As shown in Fig. 2 the experimental data of the friction factors of Newtonian as well as pseudoplastic and dilatant fluids are represented approximately by equation (13).

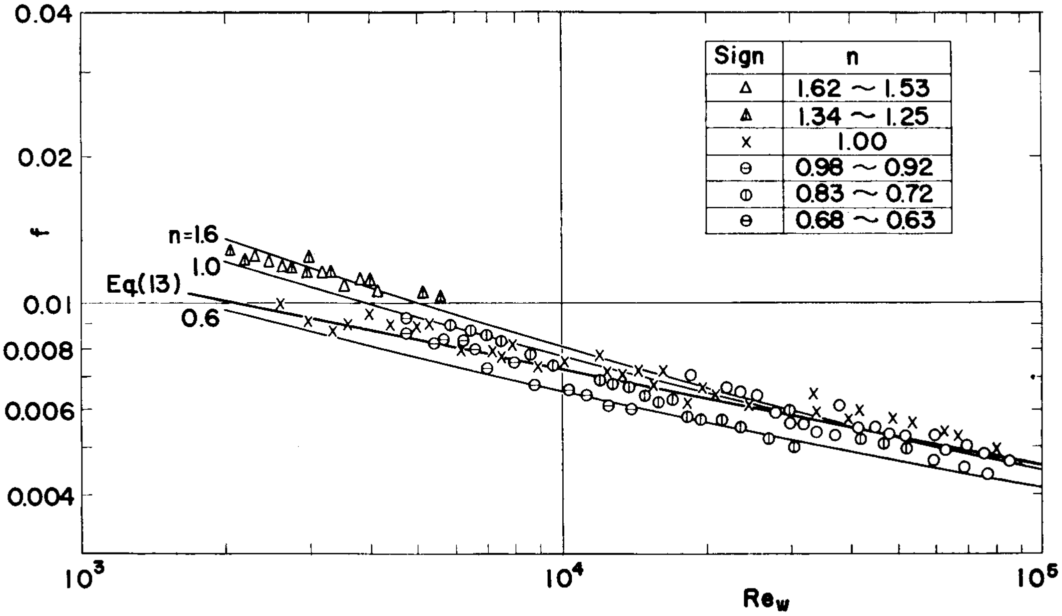

Fig. 2. Friction Factor of Pseudoplastic and Dilatant Fluids.

Dodge and Metzner [3] presented an equation of friction factor for power law fluids as

$$\sqrt{\frac{1}{f}} = \frac{4.0}{n^{0.75}} \log \left[Re' f^{(2-n)/2}\right] - \frac{0.4}{n^{1.2}} \tag{14}$$

Equation (14) is plotted in Fig. 2, which shows that equation (14) represents the authors' data well and that a family of curves, not just a single line are needed in f vs. Re_w plot as Bogue and Metzner [4] indicated. However, as the turbulent flow may occur practically for the range of $n = 0.6 \sim 1.6$, and taking into account the experimental error, the authors assume that the single line correlation is a good approximation.

Substituting equation (10) into equation (13), one obtains

$$\frac{f}{2}=(0.023)^{5n/(4n+1)}Re_P^{-1/(4n+1)} \tag{13a}$$

Relating to the heat transfer coefficient, Chilton-Colburn's [5] analogy is assumed to be valid for non-Newtonian fluids, and the Prandtl number in the equation is replaced by Pr_w. Thus one obtains

$$St=\frac{f/2}{Pr_w^{2/3}} \tag{15}$$

Substituting equations (11) and (13a) into equation (15), the following equation for the turbulent heat transfer coefficient is obtained.

$$Nu=(0.023)^{5(n+2)/[3(4n+1)]}Re_P^{4(n+2)/[3(4n+1)]}Pr_P^{1/3} \tag{16}$$

or approximately

$$Nu=0.023\,n^2 Re_P^{4(n+2)/[3(4n+1)]}Pr_P^{1/3} \tag{16a}$$

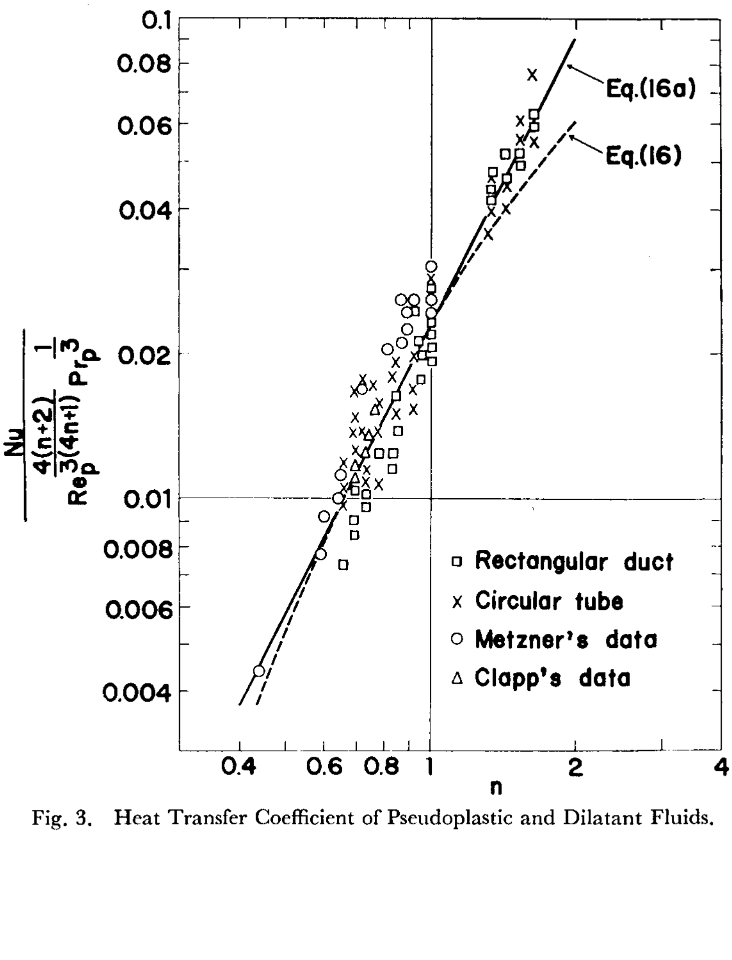

Fig. 3. Heat Transfer Coefficient of Pseudoplastic and Dilatant Fluids.

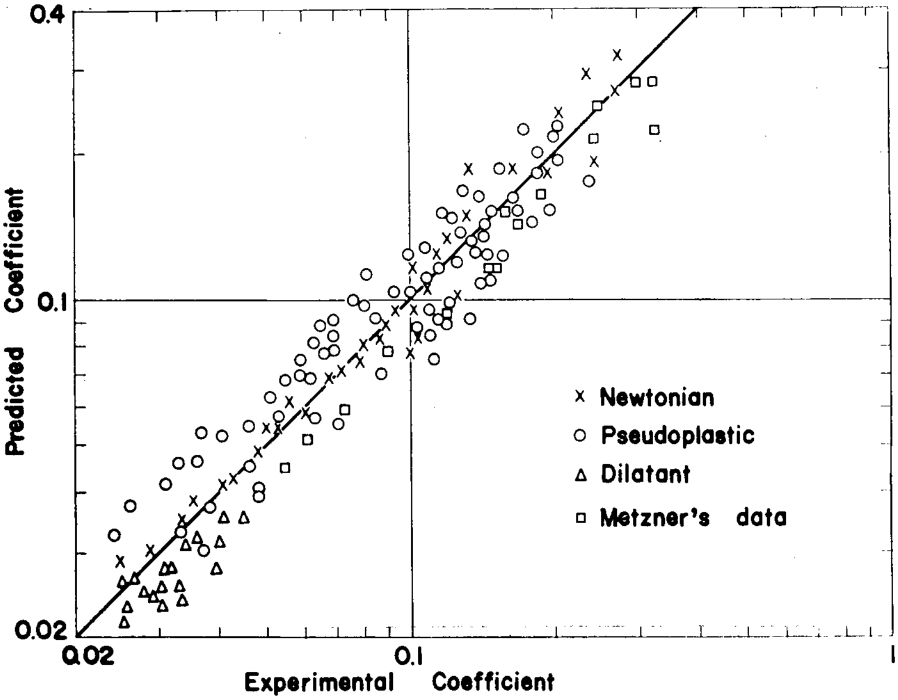

Fig. 4. Agreement between Experimental and Predicted Coefficients by Eq. (16).

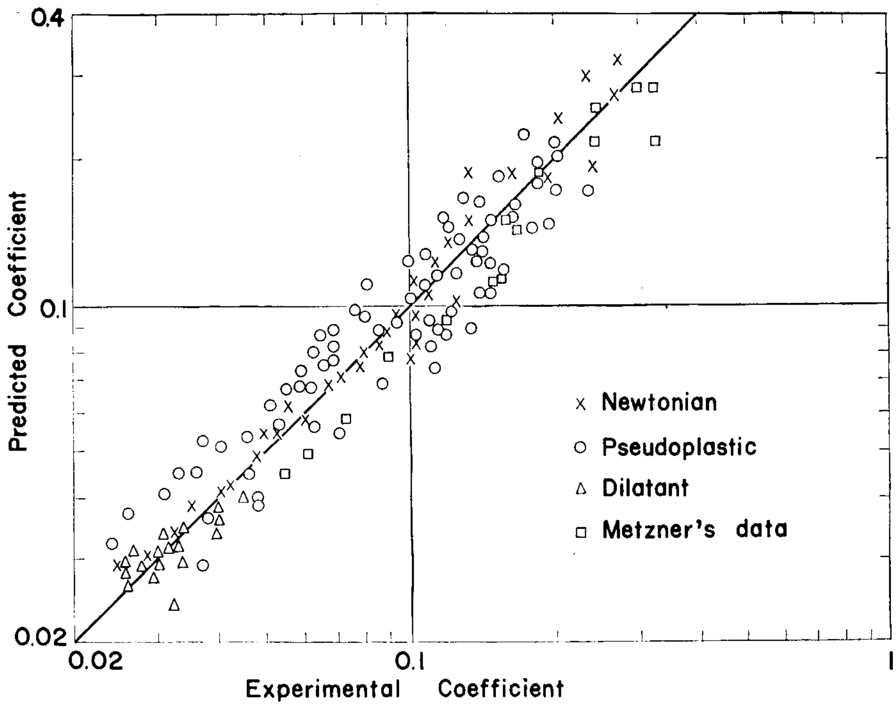

Fig. 5. Agreement between Experimental and Predicted Coefficients by Eq. (16a).

As shown in Fig. 3, the effect of n on Nu of the experimental data of Newtonian, pseudoplastic and dilatant fluids are well represented by equation (16) or (16a). Agreements between experimental and the predicted coefficients are shown in Fig. 4 and Fig. 5. In Fig. 3, 4 and 5 the data of Metzner and Friend [1], and in Fig. 3 those of Clapp [6] are also plotted.

Taking into account the small eddy in the laminar sublayer, Mizushina [7] made corrections in the analogy equations. Friend and Metzner [8] derived a similar equation and Metzner and Friend [1] used that to correlate non-Newtonian data by replacing the term of Pr in the equation with Pr_w. Though that method can predict the author's data too, they chose to use equation (16) in this paper, partly because the simpler equation is prefered and partly because the error of assuming an effective viscosity as μ_{aw} may be $(\mu_{ac}/\mu_{aw})^{2/3}$ for that method, while it may be $(\mu_{ac}/\mu_{aw})^{7/15}$ for equation (16), where μ_{ac} is the correct effective viscosity if there is any. In Fig. 6, the experimental values of heat transfer coefficients are compared with those predicted by Metzner and Friend's method.

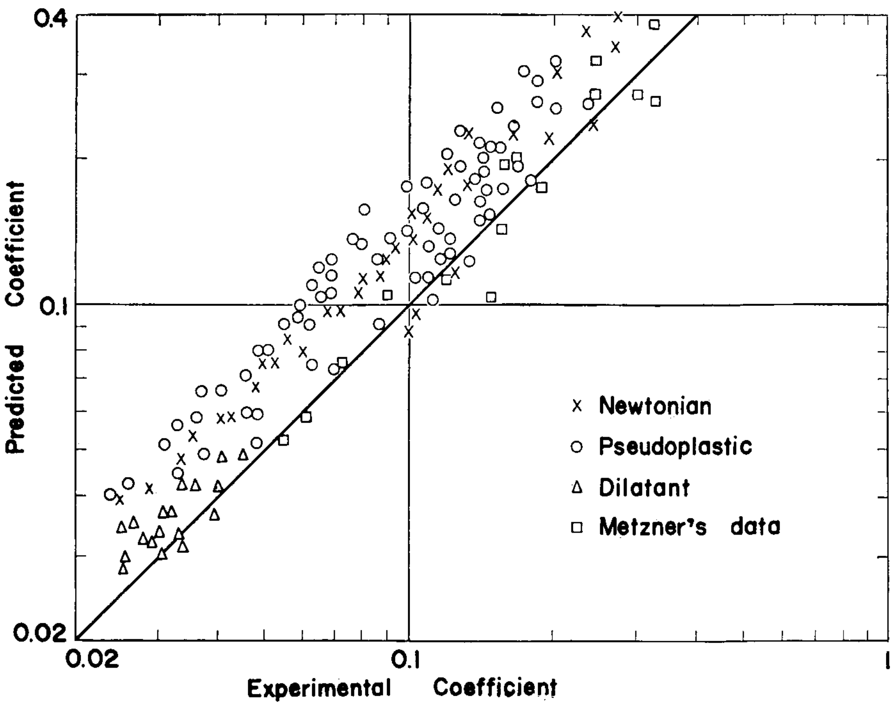

Fig. 6. Agreement between Experimental and Predicted Coefficients by Metzner and Friend's Analogy.

b. Bingham Plastics Applying equation (2) to the shear stress at the wall

$$\tau_w = \tau_0 + \eta\left(\frac{du}{dy}\right)_w \tag{17}$$

Therefore, the apparent viscosity at the wall is

$$\mu_{aw} = \tau_w \Big/ \left(\frac{du}{dy}\right)_w = \frac{\eta}{1-a} \tag{18}$$

where $\quad a = \tau_0 / \tau_w$

$$\therefore \quad Re_w = Du_m \rho / \mu_{aw} = Re_B(1-a) \tag{19}$$

$$Pr_w = C_P \mu_{aw} / k = Pr_B / (1-a) \tag{20}$$

where $\quad Re_B = Du_m \rho / \eta \quad (21)$

$$Pr_B = C_P \eta / k \tag{22}$$

Since the upper limit of the value of Hedström number in this experiment was 1.40×10^6, the highest value of the critical Reynolds number is predicted as about 2.7×10^4 with a diagram in the paper of Hedström [9]. Therefore, the experimental data of the friction factor of the slurry at $Re_w > 2.7 \times 10^4$ are plotted against Re_w computed from equation (19) in Fig. 7, which shows that equation (13) represents the experimental data in the range of the turbulent flow.

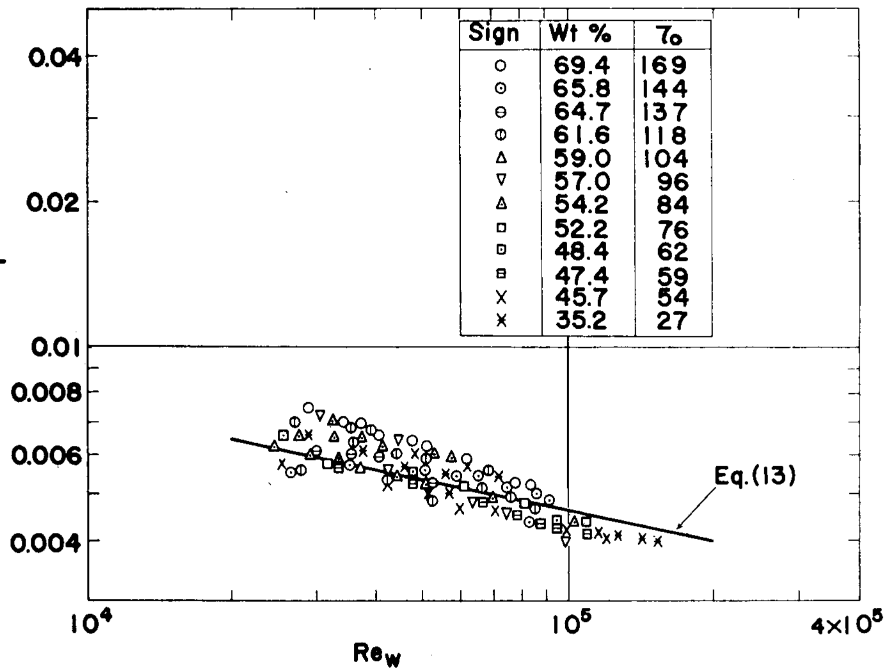

Fig. 7. Friction Factor of Bingham Plastics.

Thomas [10] published his extensive work on Bingham plastics. Though his equations may be more accurate to predict the friction factor, the simple single line correlation is assumed to be a good approximation at this moment.

Substituting equation (19) into equation (13) gives

$$\frac{f}{2}=0.023\,(1-a)^{-(1/5)}Re_B^{-(1/5)} \tag{13b}$$

From equations (15) (20) and (13b) the heat transfer equation is derived as

$$Nu=0.023\,(1-a)^{7/15}Re_B^{4/5}Pr_B^{1/3} \tag{23}$$

In Fig. 8, the experimental data of heat transfer coefficients of the slurry are plotted. Equation (23) represents the data well.

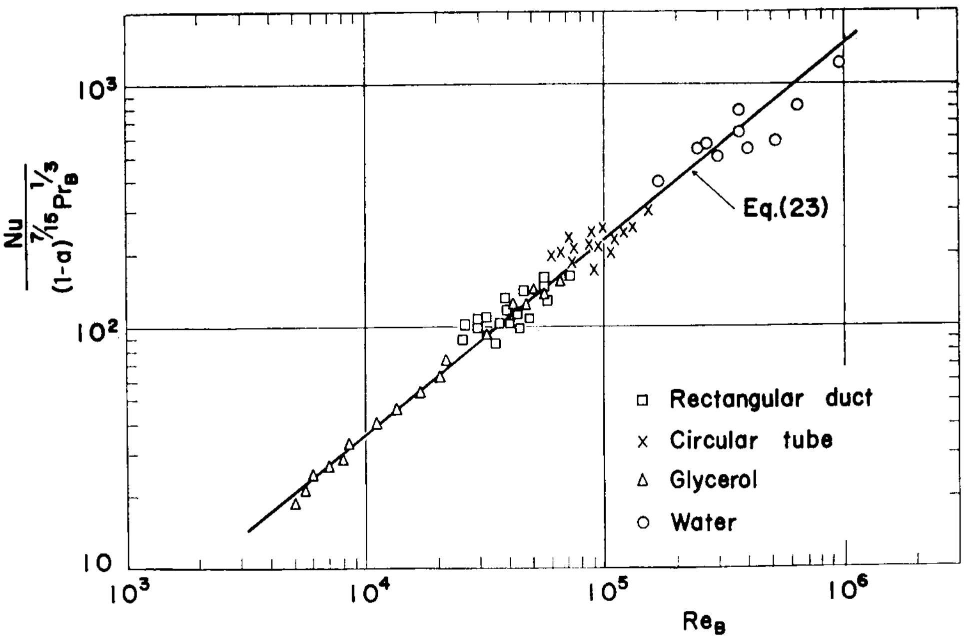

Fig. 8. Heat Transfer Coefficient of Bingham Plastics

For practical use, however, equation (23) is not convenient because it contains a, the value of which is to be computed by a trial and error method.

Fortunately, in the range of the experimental condition, the value of $(1-a)^{7/15}$ is $1\sim0.85$, and may be approximated to be unity. When the experimental error is taken into account, this may be a good approximation in any practical case. Thus,

$$Nu = 0.023 Re_B^{0.8} Pr_B^{1/3} \tag{24}$$

This equation coincides with the equation obtained by the previous investigators [10].

The experimental results are plotted again in Fig. 9, which shows equation (24) also predicts the data well.

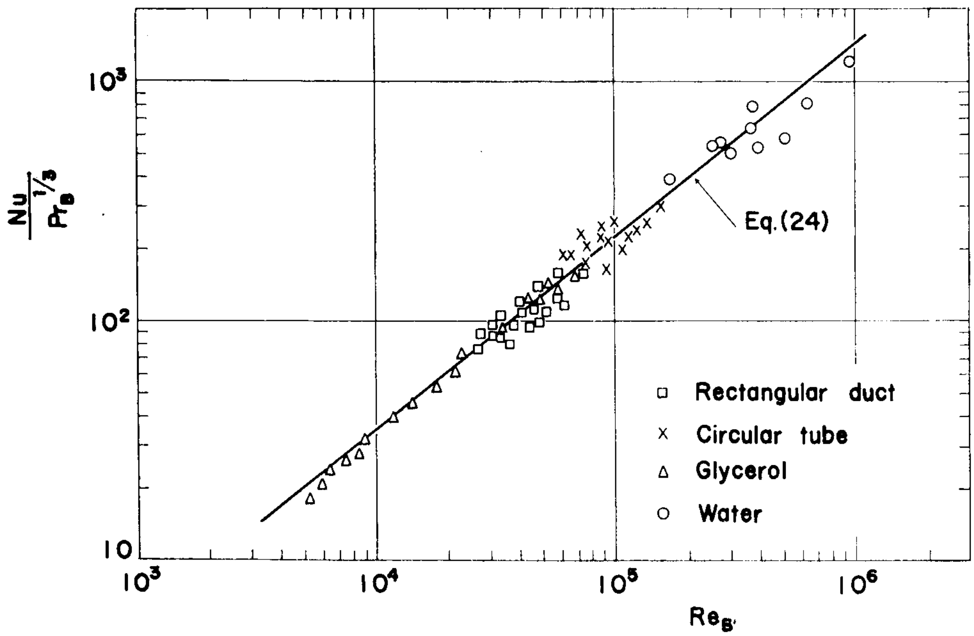

Fig. 9. Heat Transfer Coefficient of Bingham Plastics.

2. Eddy diffusivities for momentum and heat

Eddy diffusivities are calculated from the experimental data of the rectangular duct using the following equations.

$$\tau_w(1 - y/r_0)/\rho = (\nu + \varepsilon_M)\, du/dy \tag{25}$$

$$q_w/(C_P \rho) = (\alpha + \varepsilon_H)\, dt/dy \tag{26}$$

As ν in equation (25), the average kinematic viscosity calculated from $\nu = m(u_m/D)^{n-1}/\rho$ for the pseudoplastic and dilatant fluids and η/ρ for Bingham plastics are used. This approximation is rather conventional and may not be correct, but gives the most unscattered plots of data. As the heat flows from the upper jacket to the lower jacket crossing the fluid stream vertically the value of the heat flux q_w in equation (26) is constant through the stream at the measuring point.

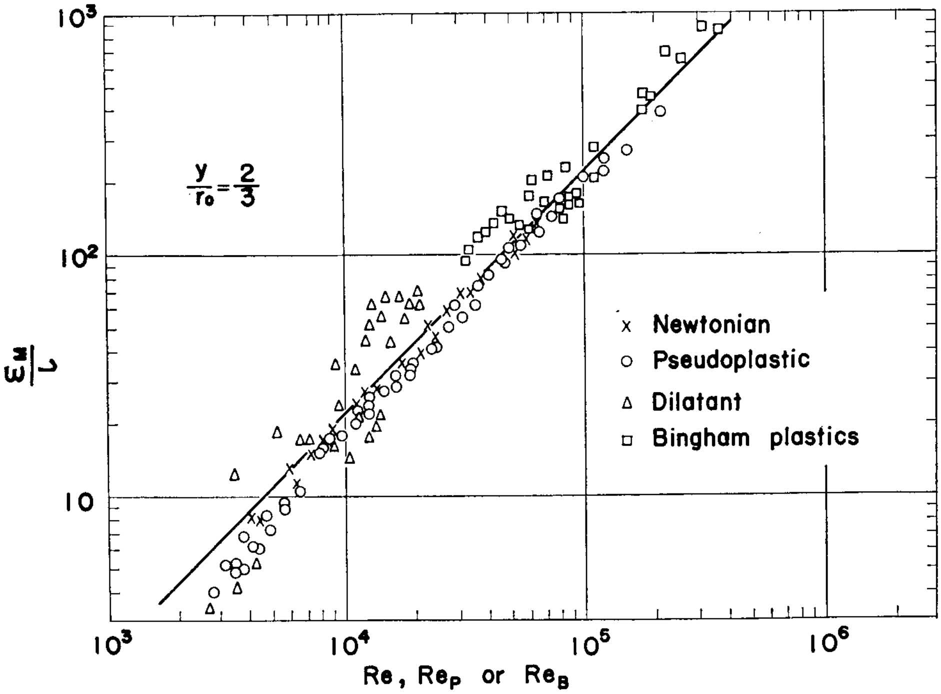

Fig. 10. ε_M vs. Re.

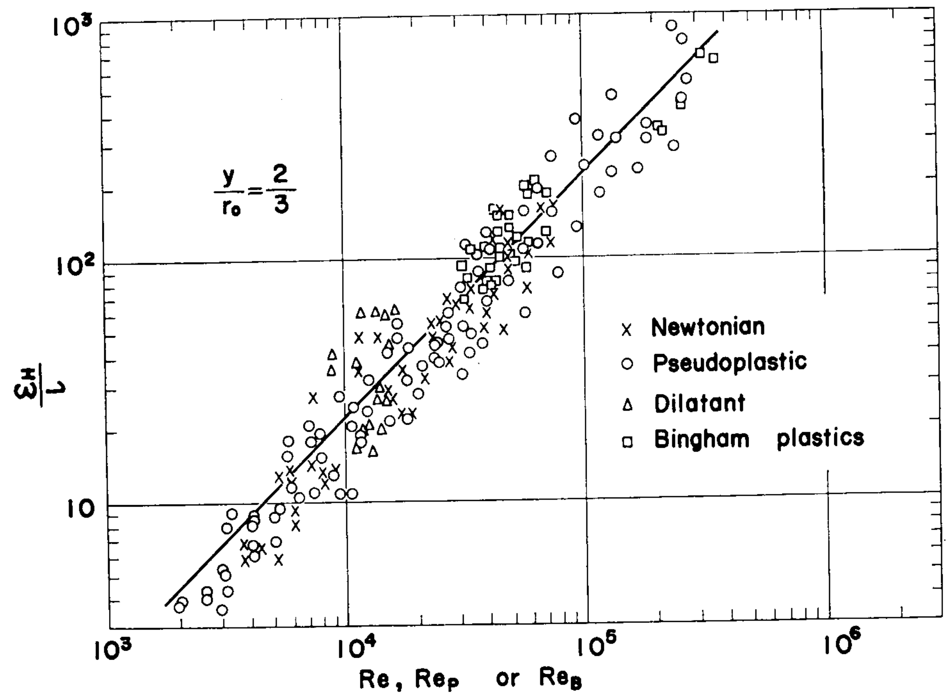

Fig. 11. ε_H vs. Re.

The experimental values of ε_M and ε_H of Newtonian, pseudoplastic or dilatant fluids and Bingham plastics at various values of y are plotted against Re, Re_P or Re_B respectively as shown in Fig. 10 and 11. As ν in ordinate is defined as $m(u_m/D)^{n-1}/\rho$ for the power law fluids and η/ρ for Bingham plastics, Re_P for the former and Re_B for the latter are used in abscissa. From these plots, it was found that the eddy diffusivities at any value of y are proportional to Reynolds numbers in the range of the experimental condition, and the following equations were obtained.

$$\varepsilon_M/\nu = 0.01\, Re fn_1(y/r_0) \tag{27}$$

$$\varepsilon_H/\nu = 0.01\, Re fn_2(y/r_0) \tag{28}$$

Therefore, the profile of ε_M and ε_H plotted in a semi-log paper as in Fig. 12 has the same shape respectively regardless of the difference of Reynolds number in the range of experimental condition. However, it should be noted that the level of the profile curve depends on Re. The values of ε_M and ε_H are maximum at $y/r_0 \doteqdot 1/2$ and seem to coincide with each other in the central part. This result differs from that of the previous investigators [11] which shows $\varepsilon_H/\varepsilon_M \doteqdot 1.5$ in the central part of the duct. In Fig. 12 the profile of eddy diffusivity for momentum

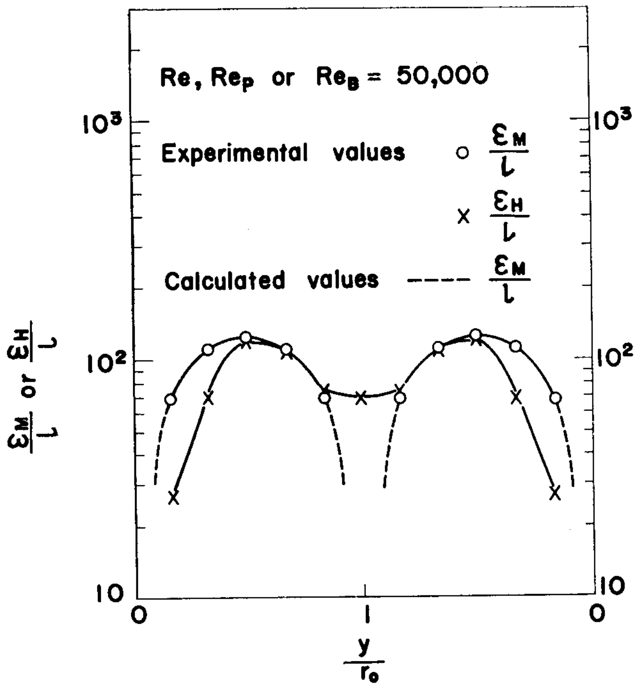

Fig. 12. Eddy Diffusivities Profile.

calculated from Deissler's [12] logarithmic velocity plofile for the turbulent flow of Newtonian fluids is plotted too. This is in good agreement with the experimental result.

Conclusions

1. The heat transfer coefficients of the turbulent flow of non-Newtonian fluids can be represented by the equations for Newtonian fluids, if the viscosity terms of those are replaced by the apparent viscosity at the wall shear stress.

2. The profiles of the eddy diffusivities for momentum and heat in non-Newtonian fluids are just as same as those in Newtonian fluids, if Reynolds numbers (Re, Re_P and Re_B) are the same.

Acknowledgments

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