Equivalent Circuit of a Power System by Mesh Method

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By

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The simple equivalence of a power system is obtained in the form of a Lagrangian tree, through the transformation of the circuit matrix where mesh currents are variables and some nodes or elements (load, etc.,) of the system are eliminated, then the equivalence is extremely simplified from the view point of network topology.

The application of this equivalence to the calculation of system power flow, and transmission loss is studied.

Preface

When one intends to solve a power system by means of a digital computer, it is convenient for purpose of computation to simplify the system and to form the equivalent circuit. For this purpose, Kron, Kirchmayer and others have devised a matrix for the transformation of a circuit. The transformation method proposed by M.B. Reed seems to be especially useful.

In general, if one applies an incident or circuit matrix to a computation for a power system problem, the operation to simplify the system and to solve the problem is mainly accomplished by multiplication of the matrices. Because a digital computer is originally suitable to matrix computation, the application of an incident or circuit matrix is useful for the digital approach.

In the present paper, the equivalent circuit is formed by the application of a circuit matrix, and the number of buses and elements in the system is reduced, and the various problems of the system are easily solved by use of the equivalent circuit.

For example, power flow and transmission loss are studied here, and the mesh method proposed in this paper is compared with the nodal method proposed by M.B. Reed.

1. Equivalent Circuit for a Power System

a. General

The equivalent circuit described in this paper is formed as follows: first, the

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original power system is represented by an oriented linear graph, the circuit matrix corresponding to the graph is obtained, and the transformation is executed by the combination of the impedance matrix of the system and the circuit matrix, and forms a palm-formed or fan-formed equivalent circuit as described later. Though this transformed circuit is quite equivalent to the original system electrically, it is reduced in number of buses and the elements, and is extremely simple from the viewpoint of geometrical topology.

Prior to the calculation, following the definition by M.B. Reed who proposed a transformation of a circuit by the incident matrix, the assortment and naming of the node, element and graph are carried out as follows:

D-element: element to be dispalced by formation of an equivalent circuit (transmission line, earth capacity, impedance load, etc.)

S-element: element to be retained in the equivalent circuit.

D-graph: any connected proper subdiagram containing only D-elements.

S-graph: any connected proper subdiagram containing only S-element (namely , the compliment of a D-graph)

J-vertex: the buses common to both the S-graph and the D-graph, the J-vertices are v_J in number.

N-graph: an auxiliary Lagrangian tree which contains only J-vertices, it has (v_J-1) elements and v_J nodes.

E-graph: a graph having the same vertices, oriented elements, node voltage and magnitude of current as the N-graph, but having the opposite direction of current.

One obtains an ND-graph from the union of a D-graph and its auxiliary N-graph, likewise, one obtains an ES-graph which is the equivalent circuit of the entire power system.

Further, symbols are defined as follows:

b=a circuit matrix of the power system with which we are concerned

b_{mn}=a partitioned submatrix of the B-matrix

b'_{mn} (or b_{mnt})=a transposed matrix of b

Z_D=an impedance matrix of a D-graph

Z_E=an impedance matrix of an E-graph

b. Forming an Equivalent Circuit

A circuit matrix expresses the topological relation between the nodes and elements in an oriented linear graph in a manner similar to that of an incident matrix, but the former is convenient for expressing Kirchhoff's 2nd law.

The steps in forming an equivalent circuit are indicated in Fig.s 2~6, which are concerned with the theoretical system shown in Fig. 1. For the sake of simplicity, the earth capacity of a transmission line is equivalently contained in generators or loads at the end of the line. The example system is expressed by the oriented linear graph shown in Fig. 2. In this example, if it is assumed that the lines are

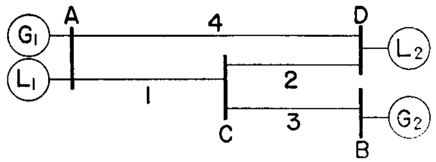


Fig. 1. Example of a theoretical system.

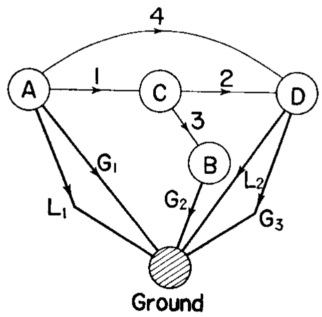


Fig. 2. Oriented linear graph corresponding to Fig. 1.

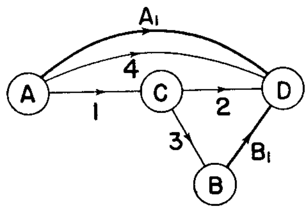


Fig. 3. ND-graph.

D-elements and the generators and loads are S-elements, then the nodes A, B and D are J-vertices by the definitions above. If node D is chosen as a reference point, the ND-graph is formed as in Fig. 3. In this case, A_1 and B_1 are N-elements and the N-graph forms a Lagrangian tree. In Fig. 3, each element of the graph is sorted according to the above difinition, and arranged as the elements of a circuit matrix, which is obtained as follows: (App. 1)

$$b = \begin{array}{l} \text{loop having N-elements as co-trees} \\ \hline \text{other loops} \end{array} \quad \begin{array}{cc} \text{N-element} & \text{D-element} \\ \left[\begin{matrix} u \\ 0 \end{matrix}\right. & \left.\begin{matrix} b_{12} \\ b_{22} \end{matrix}\right] \end{array} \qquad (1)$$

From Kirchhoff's 2nd law,

$$\begin{bmatrix} u & b_{12} \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} v_N \\ v_D \end{bmatrix} = 0 \qquad (2)$$

where:

v_N, v_D=respectively (the matrix of) the voltage of node pair at the N or D-element

The relation of the voltage to the current of a D-element is as follows:

$$v_D = Z_D \cdot i_D \tag{3}$$

where:

i_D=current of the D-element

By the mesh transformation,

$$\begin{bmatrix} i_N \\ i_D \end{bmatrix} = \begin{bmatrix} u & 0 \\ b'_{12} & b'_{22} \end{bmatrix} \begin{bmatrix} i_N \\ i_{ND} \end{bmatrix} \tag{4}$$

where:

i_N, i_D=current of the N-element, or D-element respectively

i_{ND}=mesh current of the circuit made up of only D-elements in the ND-graph

From Eqq. (1)∼(4)

$$i_{ND} = -(b_{22} Z_D b'_{22})^{-1} (b_{22} Z_D b'_{12}) i_N \tag{5}$$

$$i_D = \{b'_{12} - b'_{22}(b_{22} Z_D b'_{22})^{-1} b_{22} Z_D b'_{12}\} i_N \tag{6}$$

From the above equations, Eqq. (2) and (3) become

$$v_N = -b_{12} Z_D \{b'_{12} - b_{22}(b_{22} Z_D b'_{22})^{-1} b_{22} Z_D b'_{12}\} i_N \tag{7}$$

$$v_D = Z_D \{b'_{12} - b'_{22}(b_{22} Z_D b'_{22})^{-1} b_{22} Z_D b'_{12}\} i_N \tag{8}$$

In the case where there is no circuit made up of only D-element in the ND-graph, the above equations are simplified as follows:

$$i_D = b'_{12} i_N \tag{6'}$$

$$v_N = -b_{12} Z_D b'_{12} i_N \tag{7'}$$

$$v_D = Z_D b'_{12} i_N \tag{8'}$$

From the definition of the E-graph

$$i_E \equiv -i_N, \qquad v_E \equiv v_N \tag{9}$$

Then, Z_E, that is, the impedance matrix of an E-graph, is obtained as follows:

$$Z_E = b_{12} Z_D \{b'_{12} - b'_{22}(b_{22} Z_D b'_{22})^{-1} b_{22} Z_D b'_{12}\} \tag{10}$$

In the case where there is no circuit made up of only D-elements in the ND-graph:

$$Z_E = b_{12} Z_E b'_{12} \tag{10'}$$

From Eqq. (8) and (10),

$$v_E = Z_E i_E \tag{11}$$

Eq. (10) shows the impedance matrix corresponding to the E-graph shown in Fig. 4, where v_E is the bus voltage to the reference bus D, and i_E is the current flowing into the J-vertex (namely, the independent mesh current).

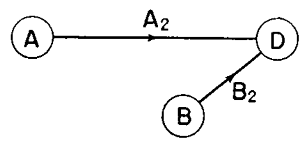


Fig. 4. E-graph.

In this equivalent circuit, the relation between v_J (J-vertex voltage to the ground reference) and v_E (J-vertex voltage to the reference bus) is as sfollows:

$$v_J = v_E + E_R \tag{12}$$

where:

E_R = the voltage of the reference bus to the gounud

The independent variable i_E in Eq. (11) is the algebraic sum of currents flowing into a J-vertex from S-elements, and can be direclty obtained from the graph, or formally obtained as follows:

By connection of the S-element to the E-graph, we obtain the ES-graph, that is, the equivalent circuit of the system, as shown in Fig. 5.

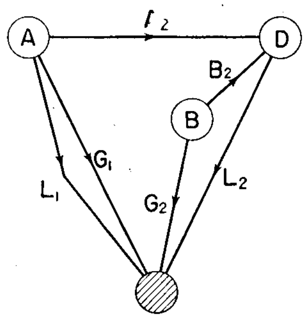


Fig. 5. ES-graph (D bus reference).

The circuit matirx (b^{ES}) corresponding to the ES-graph is then obtained, where all the E-elements and one of the S-elements are used as the trees of the graph (App. 2). Then:

$$b^{ES} = \begin{array}{c} \text{circuits having an S-element} \\ \text{as a cotree} \end{array} \quad \begin{array}{cc} \text{S-element} & \text{S- and E-element} \\ [u, & b_{12}^{ES}] \end{array} \tag{13}$$

By the transformation of mesh method:

$$\begin{bmatrix} i_{s-1} \\ i_{e+1} \end{bmatrix} = b_t^{ES}[i_{s-1}] = \begin{bmatrix} u \\ b_{12t}^{ES} \end{bmatrix} [i_{s-1}] \tag{14}$$

where suffix $(s-1)$ denotes the group of S-elements, one of which is omitted as a tree of the graph, suffix $(e+1)$ denotes all E-elements plus the one S-element which is used as an element of the tree, and t denotes a transposed matrix.

In the above description, the equivalent circuit takes a bus as the reference point, and it will be called a "fan-like form". When an impedance load of the system is regarded as a D-element, the equivalent circuit becomes a palm-like

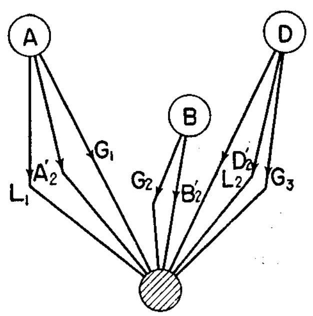


Fig. 6. ES-graph (ground reference).

form as shown in Fig. 6. The present paper is concerned mainly with the former.

When an E-graph is formed, the buses which are not J-vertices (C in this example) are eliminated, but the voltages and currents of the eliminated buses can be restored if necessary. That is, for the restoration of the voltage, Eq. (8) is rewritten,

$$\begin{aligned} v_D &= Z_D \tau_1 i_N \\ \tau_1 &= \{b_{12}' - b_{22}'(b_{22} Z_D b_{22}')^{-1} b_{22} Z_D b_{12}'\} \end{aligned} \tag{15}$$

When v_D in the above equation is added to the known voltage (v_E) of a bus, the required voltage can be obtained. Further, the following equations for mesh and branch currents are obtained from Eqs. (5) and (6):

$$\begin{aligned} i_{ND} &= \tau_2 i_N \\ \tau_2 &= -(b_{22} Z_D b_{22}')^{-1} (b_{22} Z_D b_{12}') \\ i_D &= \tau_1 i_N \end{aligned} \tag{16}$$

c. The Separation and Reunion of Systems

If a connected system is mathematically treated as one system, the number of circuits becomes large and the rank of circuit matrices increases: then the difficulty of computation will increase in proportion to the saquare of the matrix rank. In this section, a system is divided, and reunited after an equivalent circuit is formed for each division. This operation is repeated until finally a fan-formed equivalent circuit is obtained.

The theoretical model system is shown in Fig. 7 and each step is illustrated in Figs. 8~12. That is, the buses for system separation are properly chosen (buses C and F in the example), and each division is changed to an E-graph in fan form for which A_2, C_{21}, and D_2, E_2, C_{22} are obtained for E-elements as shown in Fig. 10.

After buses used for separation are reunited, the old E-elements are regarded as secondary D-elements, as shown in Fig. 11, and a new ND-graph is formed, from which the equivalent circuit and impedance matrix for the connected system are obtained.

The circuit matrix corresponding to the reunited ND-graph is obtained in the same method as in Eq. (1), as follows:

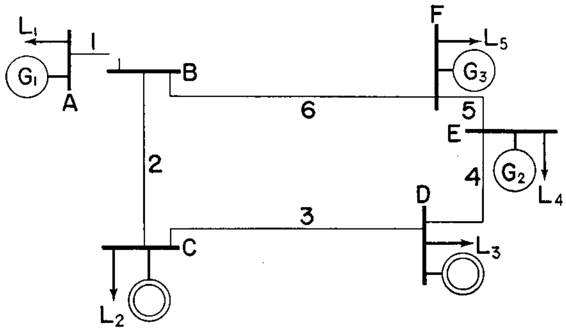


Fig. 7. Example of an interconnected theoretical system.

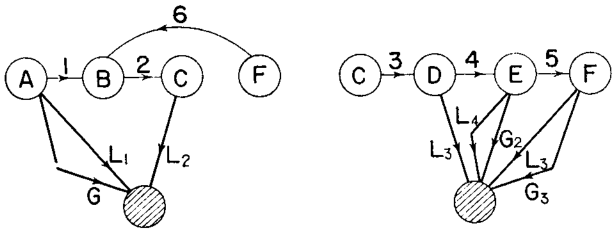


Fig. 8. Oriented linear graph of zoned systems.

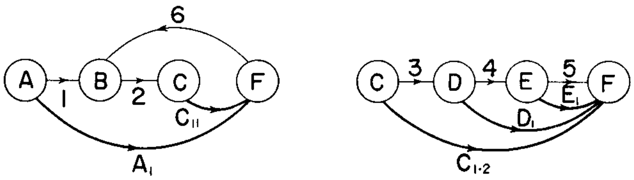


Fig. 9. ND-graphs corresponding to zoned systems.

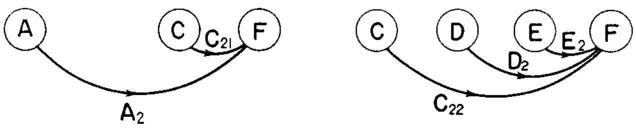


Fig. 10. E-graphs corresponding to zoned systems.

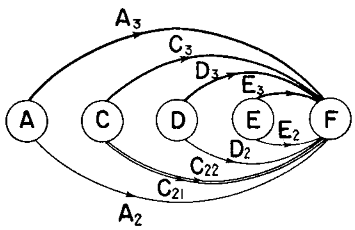


Fig. 11. ND-graph of an interconnected system.

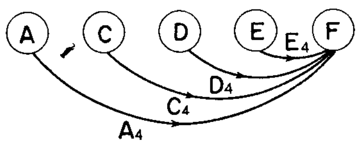


Fig. 12. E-graph of an interconnected system.

$$b=\begin{bmatrix} u & b_{12} \\ 0 & b_{22} \end{bmatrix} \tag{18}$$

When the system is divided into two section as in the example, then (App. 3)

$$\begin{aligned} b_{12}&=[b_{121}, \quad b_{122}] \\ b_{22}&=[b_{221}, \quad b_{222}] \end{aligned} \tag{19}$$

The equivalent impedance matrix for the entire system is obtained as follows:

$$\begin{aligned} Z_E= & \sum_{j=1}^{2} [b_{12j} Z_{Ej} b'_{12j}] \\ & -[\sum_{j=1}^{2} b_{12j} Z_{Ej} b'_{12j}][\sum_{j=1}^{2} b_{22j} Z_{Ej} b'_{22j}]^{-1}[\sum_{j=1}^{2} b_{22j} Z_{Ej} b'_{12j}] \end{aligned} \tag{20}$$

In dividing a system, the followiag points must be considered: (1) In a subdivision of the system, the number of circuits formed by lines is reduced, and (2) between divided systems, there must be no connection except separation buses.

d. Further Considerations

As described above, after a system has been simplified, and the E-graph obtained, the independent variables in the system equation are the currents in the S-elements, and the voltages and currents of which are eliminated in the simplification of the system can be restored using Eqq. (5)$\sim$(8).

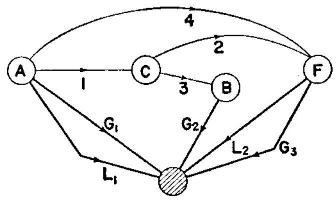
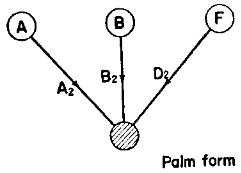
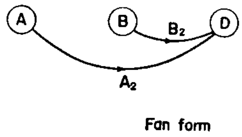
The equivalent circuit obtained by use of a circuit matrix is compared in Table 1 with one obtained by use of an incident matrix. In this table, both palm-formed and fan-formed equivalent circuits are illustrated.

The circuit or incident matrix is to be formed at the begining of these calculations. To form an incident matrix, one need only to know whether an element is incident to a node or not: but to form a circuit matrix, one must classify elements into trees and cotrees, and form successively independent circuits. It has been held that the difficulty of matrix formation from a branch numbering table is the same for either type of matrix, but in fact, the formation of the circuit matrix is more difficult than that of the incident matrix. This is why the nodal method is used much more frequently than the mesh method.

To aboid the difficulty of a circuit matrix formation, the method of changing an incident matrix to a circuit matrix is proposed. This method was experimentally applied to the example in this paper by the authors, and no difficulty was experienced in the program, the change of matrix was easily accomplished.

As described above, there are two kinds of equivalence of circuit, that is,

Table 1. The correspondence between mesh method and nodal method.

	Nodal ethod	Mesh method
Equation	i-explicit form $i_E = Y_E v_E$	v-explicit form $v_E = Z_E i_E$
Original system		
Equivalent circuit (E-graph)		
Expression	Admittance matrix $Y_E = a_{12} Y_D \{a_{12t} - a_{22t} [a_{22} y_D a_{22t}]^{-1} \times a_{22} y_D a_{12t}\}$	Impedance matrix $Z_E = b_{12} Z_D \{b_{12t} - b_{22t} [b_{22} Z_D b_{22t}]^{-1} \times b_{22} Z_D b_{12t}\}$

palm-formed or fan-formed one. In a system where any bus is not connected to the ground by a D-element such as impedance load, it is difficult to obtain the palm-formed equivalenee by means of a circuit matrix. There is no such restriction in the nodal method. In a system with a steady state of power, the performance of an element which connects the bus to the gound is ordinary expressed by its generating (or consuming) power, and when the fan-formed equivalence is employed, there is no difficulty in the treatment of system problems. The advantages of the palm-formed equivalence are about equal to those of the fan-formed one, and this paper is mainly concerned with the fan-formed one. In a system fault calculation, however, the ground is to be regarded as a bus.

The relation between an impedance matrix Z_E and an admittance one Y_E for a fan-formed equivalence is as follows:

$$[Z_E][Y_E] = [U] \tag{21}$$

Where any bus of the system is not connected to the ground by a D-element, the admittance matrix Y_E is singular, and connot be inverted. (App. 4)

2. Power Flow Solution

a. General

As is well known, for the analysis of various problems in a power system, it is necessary to know the power flow distribution for the initial state. Ward and Hale have proposed an iterative technique which employed a nodal method, and various methods for determining the power flow follow this technique. The method for power flow in this paper also follows it, except in the method of obtaining the impedance matrix of the system, in which the mathematical treatment is simplified.

b. Calculation of Power Flow in Equivalent Circuit

Because of the character of the equivalent circuit used in this paper, the power flow program employs the impedance matrix. For the fan-formed equivalence, taking a bus as the reference point,

$$v_K = \sum_{m=1}^{M} Z_{Km} I_m + E_R \tag{22}$$

where:

v_k =voltage of k-bus to ground
Z_{Km}=element of Z_E in Eq. (10)
M =number of independent circuits
E_R =voltage of the reference point to ground

In the calculation, ΔI_R (the correcting term of the current) is obtained from the prescribing conditions, and if the assumed initial values are suitable, ΔI_K is obtained as a solution of simultaneous linear equations. (App. 5)

c. Example of Calculation

The following calculation is carried out with the object of comparing the mesh method (namely impedance method) and the nodal method (admittance method).

For the forming of the equivalence and the calculation of the power flow, a serial program was used. The program for forming an equivalent circuit is the same in both the Z impedance-method and the Y admittance-method, but the data used for them are respectively different. (cf. Table 1) The **b**-matrix and the Z-matrix are put into computer with the Z-method, and similarly, the **a**-matrix Y-matrix with the Y-method.

For the determination of the power flow in the system, the program for computation by means of Ward and Hale's method (Y-method) was composed. Then it was changed partially and applied to the computation by means of Z-method.

In this computation, the power system shown in Fig. 1 and 16-buses system inserted in Reference 3 are used for the example. The time forming the equivalence of 16-buses system is about equal in the calculation by both Z- and Y-method, but longer a little (about 10 sec in 16-buses system) in Z-method calculation than in Y-method. Usually, the convergency in the numerical iteration for power flow determination is more quick in the process by Z-method than in one by Y-method. This advantage compensates the defect in the forming of the equivalence by the circuit method.

The initial estimated current in the S-element may be taken as $P+j\cdot O$ for convenience, where

$$E_E=1+j\cdot O \tag{23}$$

$$(P_K+jQ_K)=(E_R+\sum_{m=1}^{M} I_m Z_m)I_K^* \tag{24}$$

As is well known, with a reasonable estimate of bus currents, satisfactory convergence is obtained by solving the resulting system of linear equations for α_k and β_k, where α_k and β_k make a corrective set, namely $\Delta I_k=\alpha_k+\Delta\beta_k$. But, one must note that, if the initial estimation is drastically in error, a corrective set of current cannot be obtained by linear equations.

The bus currents are obtained by the iteration method, the currrent of the slack generator (a_0+jb_0) is finally obtained as follows:

$$a_0+jb_0=-\sum_{k=1}^{M}(a_k+jb_k) \tag{25}$$

The results in the calculation of power in the system shown in Fig. 1 are compliled as Table 2.

Table 2. Example of power flow.

case	P_1	P_2	P_3	Q_1	Q_2	Q_3	E_1	E_2	E_3	Loss
1	−1.5	0.8	0.813	0.159	−0.019	0.412	1.00	1.05	1.1	0.113
2	−1.6	0.8	1.024	0.518	−0.206	0.291	1.05	1100	1.1	0.124
3	−1.4	0.6	0.888	0.577	−0.166	0.018	1.10	1.05	1.1	0.088
4	−1.4	1.0	0.567	1.157	−0.487	0.156	1.15	0.09	1.1	0.167
5	−1.2	0.5	0.776	−0.209	0.034	0.546	0.95	1.05	1.1	0.076

Pi $(i=1, 2, 3)$: frowing into or our from a bus A, B or D

3. Transmission Loss

a. General

For the computation of transmission loss, the transformation of a circuit has been derived by Kron and Kirchmayer. In this paper, using a fan-formed equivalent circuit having the slack generator bus as the reference bus, the transmission loss and loss coefficient are obtained by a serial progran, following the power flow determination.

b. Loss and Loss Coefficient

From the fan-formed equivalence illustrated in Fig. 4 or 12, the loss formula is obtained as follows:

$$P_{\text{Loss}}=\Re(i^*_{Et}\cdot v_E)=\Re(i^*_{Et}\cdot Z_E\cdot i_E) \tag{26}$$

where i^*_E=conjugate matrix of i_E

v_E=matrix of bus voltage to reference bus

If the power flow is determined by calculation in the preceding section, the loss is obtained by Eq. (26). But the power flow calculation is to be made every time the system operation changes, and Eq. (26) is not pratical because power flow determination is troublesome.

Usually, in the transmission system with a constant volatege bus, the magnitude of v_E (in Eq. 26) is known, but its phase is unknown. To avoid this difficulty, the loss coefficient is defined as in Kirchmayer's method, and the loss can be then be obtained as follows:

$$P_{\text{Loss}}=\sum_m\sum_n P_m B_m P_n \tag{27}$$

B_{mn} is easily obtained by the transformation of Eq. (26) and by the following assumptions: (App. 6)

(1) The load current at any bus remains a constant complex fraction of the total load current.

(2) The generator bus voltage magnitudes remain constant.

(3) The phase angle in each generator bus is assumed to remain constant, which is equal to one in the base case.

(4) The source reactive power may be approximated by the sum of a component which varies with the system load and a component which varies with the source output.

In a fan-formed ES-graph where generators and loads are regarded as S-element, the currents of E-elements (i_E) are made by product of b^{ES} and i_S in Eq.

(14). Then,

$$i_E = b^{ES} i_S = [b^{ES}] \begin{bmatrix} i_G \\ i_{L-1} \end{bmatrix} \tag{28}$$

where : i_G=generator current

$i_{L-1}=(L-1)$ load currents where L is number of load

Generator current i_G is expressed by its effective power as floows:

$$i_G = C_1 P_G \tag{29}$$

where : P_G=effective power of generator

C_1=transformation matrix

The above assumption (1) is combined with Eq. (29), and the following equation is obtained: (App. 6)

$$\begin{bmatrix} i_G \\ i_{L-1} \end{bmatrix} = i_S = C_0 \cdot C_1 \cdot P_G \tag{30}$$

where : C_0=transformation matrix

From Eqq. (28)~(30).

$$i_E = b^{ES} \cdot C_0 \cdot C_1 \cdot P_G \tag{28$'$}$$

Comparing Eq. (26):

$$P_{\text{Loss}} = \Re\{P_{Gt} C_t^* \cdot Z \cdot C \cdot P_G\}$$

where

$$C \equiv C_0 \cdot C_1, \quad Z \equiv b_t^{ES} \cdot Z_E \cdot b^{ES}$$

Furthermore, Z and C are defined as follows:

$$Z \equiv R_E + jX_E, \quad C \equiv \alpha + j\beta$$

Then,

$$\begin{aligned} C_t^* &= \alpha' - j\beta' \\ B_{mn} &= \Re(\alpha' - j\beta')(R_E + jX_E)(\alpha + j\beta) \\ &= \alpha' \cdot R_E \cdot \alpha + \beta' \cdot R_E \cdot \beta - \beta' \cdot X_E \cdot \alpha + \alpha' \cdot X_E \cdot \beta \end{aligned}$$

Since X_E is a symmetric matrix,

$$-\beta' X_E \alpha + \alpha' X_E \beta = \alpha' X_E \beta - (\alpha' X_E \beta)_t \equiv S$$

S is twice the skew symmetric part of $\alpha' X_E \beta$, and has no effect on the value of $P_t B_{mn} P$, which has a quadratic form. Then,

$$B_{mn}=\Re[C_t^* Z_E C]=\alpha' R_E\alpha+\beta' R_E\beta$$

In this calculation, Z_E is precisely the open circuit impedance of the transmission system.

The advantages of this method are as follows:

(1) The fan-formed equivalence which was used for determination of the power flow is serially applied to the loss computation, and Z_E, which is obtained as the impedance of equivalence, is the same as the open circuit impedance of the system.

(2) The various transformations and operations in the system are mainly carried out by matrix multiplication, the programming of the instruction for the computation being simple.

(3) A skew symmetric part of B_{mn} does not exist. This fact can be used in checking the computation.

c. Example

As an example of loss computation, the equivalent circuit and power flow described above were used here. The results are shown in Table 3, and are consistent with the sum of loss in each line. That may be readily verified by comparing Table 2 with 3.

Table 3. Loss coefficient.

case 1

	P_1	P_2	P_3	
P_1	0.00130	−0.00246	−0.00664	
P_2	−0.00246	0.07436	0.01208	$\Sigma PB_{mn}P=0.11281$
P_3	−0.00664	0.01208	0.03476	

case 3

	P_1	P_2	P_3	
P_1	0.00157	−0.00202	−0.00618	
P_2	−0.00202	0.07961	0.01183	$\Sigma PB_{mn}P=0.08784$
P_3	−0.00618	0.01183	0.03069	

case 5

	P_1	P_2	P_3	
P_1	0.00131	−0.00257	−0.00597	
P_2	−0.00257	0.07479	0.01304	$\Sigma PB_{mn}P=0.07619$
P_3	−0.00597	0.01304	0.03793	

Conclusion

(1) The equivalent circuit obtained by means of the mesh method is derived from the combination of the loop matrix in the original system and the simple formula described in this paper, and it can be expressed by an impedance matrix.
(2) For the analysis of the system, only the least variable needs to be used in this equivalence.
(3) The voltage and current of buses or lines which are elemintated for the simplification of the system can be restored when needed.

For the applications of this equivalence,
(1) For the power flow determination, the number of buses and elements becomes less than in ordinary methods, and the covergence is rapid because this method has the features of an impedance method.
(2) For the transmission loss computation, this equivalence is especially useful, the loss and the loss coefficient being easily obtained following the power flow determination.

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Appendix

(1) Matrix b in Eq. (1) for the ND-graph shown in Fig. 3 is as follows:

$$b=\begin{array}{ll} & \\ \text{N-element}\left\{\begin{array}{l}A_1\\B_1\end{array}\right. \\ \text{D-element} & 4\end{array}\overset{\begin{array}{cc}\overbrace{A_1\ \ B_1}^{\text{N-element}} & \overbrace{4\quad 1\quad 2\quad 3}^{\text{D-element}}\end{array}}{\begin{pmatrix} 1 & 0 & \vdots & 0 & -1 & -1 & 0 \\ 0 & 1 & \vdots & 0 & 0 & -1 & 1 \\ \cdots & \cdots & & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 1 & -1 & -1 & 0 \end{pmatrix}}=\begin{bmatrix} u & b_{12} \\ 0 & b_{22} \end{bmatrix}$$

$$\underbrace{\quad\quad}_{\text{cotree}}\qquad\underbrace{\quad\quad}_{\text{ttee}}$$

i_{ND} in Eq. (4) is the current in element 4 in this example.

(2) Matrix b^{ES} in Eq. (13) for the ES-graph shown in Fig. 5 is as follows:

$$b^{ES}=\text{S-element}\begin{array}{c} G_1\\ G_2\\ G_3\\ L_1\end{array}\overset{\begin{array}{cc}\overbrace{G_1\ \ G_2\ \ G_3\ \ L_1\ \ \ L_2}^{\text{S-element}} & \overbrace{A_2\ \ B_2}^{\text{E-element}}\end{array}}{\begin{pmatrix} 1 & 0 & 0 & 0 & \vdots & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & \vdots & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & \vdots & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \vdots & -1 & -1 & 0 \end{pmatrix}}=[u,\ b_{12}^{ES}]$$

$$\underbrace{\quad\quad\quad}_{\text{cotree}}\qquad\underbrace{\quad\quad\quad}_{\text{tree}}$$

In the above equation, an S-element L_2 is chosen as a tree, and the remaining S-elements are chosen as cotrees. From Eq. (14),

$$[i_{e+1}]=\begin{pmatrix} i_{L2} \\ i_{A_2} \\ i_{B_2} \end{pmatrix}=\begin{pmatrix} -1 & -1 & -1 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} i_{G_1} \\ i_{G_2} \\ i_{G_3} \\ i_{L_1} \end{pmatrix}$$

(3) Mesh matrixes for each ND-graph in Fig. 9 are respectively as follows:

$$b'=\text{N-element}\left\{\begin{array}{l} C_{11} \\ A_{11} \end{array}\right.\overset{\begin{array}{cc}\overbrace{C_{11}\ \ A_2}^{\text{N-element}} & \overbrace{1\quad 2\quad 6}^{\text{D-element}}\end{array}}{\begin{bmatrix} 1 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 1 & \vdots & -1 & 0 & 1 \end{bmatrix}}=[u,\ b_{12}^1]$$

$$b^2=\begin{array}{c} C_{12} \\ D_1 \\ E_1 \end{array}\overset{\begin{array}{cccccc} C_{12} & D_1 & E_1 & 3 & 4 & 5 \end{array}}{\begin{pmatrix} 1 & 0 & 0 & \vdots & -1 & -1 & -1 \\ 0 & 1 & 0 & \vdots & 0 & -1 & -1 \\ 0 & 0 & 1 & \vdots & 0 & 0 & -1 \end{pmatrix}}=[u,\ b_{12}^2]$$

In this example, there is no circuit made only of D-elements, and Z_{Ei} for the circuits in Fig. 10 are as follows:

$$Z_{Ei}=b_{12}^{i}Z_{Et}b_{12t}^{i}, \quad i=1 \text{ or } 2$$

The circuit matrix for the circuit in Fig. 11 is as follows:

$$b=\begin{array}{c} \\ \\ A_D \\ C_i \\ D_3 \\ E_3 \\ C_{21} \end{array}\begin{array}{c} \begin{array}{cccccccccc} & & & & \overbrace{\hphantom{C_{21}\ A_2}}^{1} & \overbrace{\hphantom{C_{22}\ D_2\ E_2}}^{2} \end{array} \\ \begin{array}{ccccccccc} A_B & C_3 & D_3 & E_3 & C_{21} & A_2 & C_{22} & D_2 & E_2 \end{array} \\ \left(\begin{array}{cccc:cc:ccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ \hdashline 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \end{array}\right) \end{array}=\begin{bmatrix} u, & b_{121}, & b_{122} \\ 0, & b_{221}, & b_{222} \end{bmatrix}$$

(4) For a palm-formed equivalence, the rank of Y_E in Table 1 cannot be greater than that of $[a_{12}]$. If any bus in the system is not connedted to the ground by a D-element and the number of buses (exclusive of the gound) is ν, the rank of $[a_{12}]$ cannot be more than $(\nu-1)$. Furthermore, the order of $[y_E]$ is (ν, ν), finally, $[y_E]$ is signular.

(5) The impedance mehtod for a fan-formed equivalence is compared with the admittance method. The symbols are defined as follows:

$[\varDelta E]$ =bus voltage to the ground.

$[P]$ =effective power computed from the estimated value of the voltage or current.

$[P_S]$, $[E_S]$ =prescribed effective power and voltage of buses, the suffix denoting k-th bus.

$$[Z_E]=[R_{ij}+jX_{ij}], \quad [Y_E]=[G_{ij}+jB_{ij}]$$

(a) Admittance method

$$\text{initial estimated voltage: } [\varDelta E]=[e_k+jf_k] \tag{App. 1}$$

A corrective voltage $(\varepsilon_k+j\xi_k)$ is obtained as the solution of following sinultaneous equations, and the correction of the voltage is done by Eq. (App. 3), and the whole process $(k=1, M)$ is repeated over and over unitl the power correction appearing in the process becomes smaller than a predetermined value.

$$\left.\begin{array}{l} \varDelta P_k=P_{sk}-P_k=C_{p1k}\cdot\varepsilon_k+C_{p2k}\cdot\zeta_k \\ \varDelta|E_h|^2=|E_{sk}|^2-|E_k|^2=C_{e1k}\cdot\varepsilon_k+C_{e2k}\cdot\xi_k \end{array}\right\} \tag{App. 2}$$

where: $C_{p1k}=e_k G_{kk}+f_k B_{kk}+a_k$

$C_{p2k}=-e_k B_{kk}+f_k G_{kk}+b_k$

$C_{e1k}=2e_k \quad C_{e2k}=2f_k$

$$[\varDelta E]=[(e_k+\epsilon_k)+j(f_k+\xi_k)] \qquad \text{(App. 3)}$$

(b) Impedance method

initial estimated current: $[I_E]=[a_k+jb_k]$ (App. 1′)

A corrective current $(\alpha_k+j\beta_k)$ is obtained by solving Eq. (App. 3′).

$$\left.\begin{aligned}\varDelta P_k&=C'_{p1k}\cdot\alpha_k+C'_{p2k}\cdot\beta_k\\ \varDelta|E_k|^2&=C'_{e1k}\cdot\alpha_k+C'_{e2k}\cdot\beta_k\end{aligned}\right\} \qquad \text{(App. 2′)}$$

where: $C'_{p1k}=a_k R_{kk}+b_k X_{kk}+e_k$

$C'_{p2k}=-a_k X_{kk}+b_k R_{kk}+f_k$

$C'_{e1k}=2(e_k\cdot R_{kk}+f_k\cdot X_{kk})$

$C'_{e2k}=2(f_k\cdot R_{kk}-e_k X_{kk})$

$$[I_E]=[(a+\alpha_k)+j(b_k+\beta_k)] \qquad \text{(App. 3′)}$$

Thus the iterative process by means of admittance method is similar to one by means of impedance method and the program for a computer is almost the same for both methods, except that the current in the latter process replaces the voltage in the former.

(6) Let the ratio of each load-current (i_{Lj}) to total load-current (i_{L0}) be l_j,

$$i_{Lj}=l_j\cdot i_{L0}, \quad i_{L0}=-\sum i_{Gk}$$

then,

$$\begin{bmatrix} i_G \\ i_{L-1}\end{bmatrix}=\begin{bmatrix} u \\ -l\end{bmatrix}[i_G]=C_0\cdot i_G \qquad \text{(App. 4)}$$

where:

$$[l]=\begin{array}{c} \\ L_1 \\ L_m\end{array}\!\!\begin{array}{c} G_1 \cdots\cdots G_n \\ \begin{bmatrix} l_1 & \cdots\cdots & l_1 \\ l_m & \cdots\cdots & l_m\end{bmatrix}\end{array}$$

i_G can be represented by generator powers as follows:

$$i_G^* v_{GK}\equiv P_{GK}+jQ_{GK}\equiv(1-jS_{GK})P_{GK}$$

$$\therefore \quad i_G=\begin{pmatrix} \dfrac{1+jS_{G1}}{v_{G1}^*} & * \cdots\cdots\cdots & 0 \\ & & \vdots \\ 0 \cdots\cdots\cdots & 0 & \dfrac{1+jS_{Gn}}{v_{Gn}^*}\end{pmatrix}\begin{pmatrix} P_{Gn} \\ \vdots \\ \vdots \\ P_{G1}\end{pmatrix}=C_1\cdot P_G \qquad \text{(App. 5)}$$

Eq. (28′) is obtained by the combination Eqq. (28), (App. 4) and (App. 5).