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By

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This paper presents the experimental results for static and dynamic characteristics of a suspension bridge with inclined hanger system comparing those with ordinary vertical hanger system by use of three spans suspension bridge model (3,250+6,500+3,250). The discrepancy due to the inclination of hangers is studied by the model test on the natural frequencies, the frequency responses and the damping-amplitude relations for the above cases. The results indicate that the inclination of hanger system may increase the deflectional stiffness when the load acts asymmetrically and dynamically the damping effect in the free vibrations.

1. Introduction

Recently in Japan the possibility of construction of longer as well as high rise structures is considered from a number of national demands, such as the plan for the Akashi-Straits Bridge connecting the Honshu Island and the Shikoku Island. A typical feature of long spanned suspension bridge is associated with dynamic instability due to various time dependent external actions as wind and earthquake motions, the response of which mainly depends on its free vibrational characteristics. Recnet results of researches by C. Scruton[1], Leonhardt[2], etc., indicate the effective possibilities of increasement of dynamic stability by means of (i) increasement of flexural and torsional rigidities[3], (ii) increasement of structural damping, (iii) improved cross sectional shapes of stiffening girders. To satisfy the first and second effects one may consider the inclined hangers instead of the customary vertical hanging systems. The change of inclination of hanger system may result in stiffening the flexibility of structures and simultaneously increasing the structural damping. In order to clarify this effect of improvement a model of suspension bridge (3250+6500+3250) is made based on the first plan of the proposed Akashi-Strait Bridge with both vertical and inclined hangers. The static loading tests and the free vibrational tests for vertical flexural modes are performed

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and compared with two cases. Consequently, the suspension bridge with inclined hangers proves to be remarkably less flexible for asymmetric loads than the one with vertical hangers and dynamically the damping for inclined hanger system is also considered to be larger to compare with the case of vertical hangers.

2. Fundamental Differential Equations

According to the linear theory of structures the fundamental equations for a suspension bridge[4)] are obtained as follows:

$$(EI\eta'')''-H_w\eta'' = p(x)+H_p h''$$

$$\frac{H_p L_k}{E_c A_c} \pm \alpha_T \Delta T L_T + h\int_0^L \eta''(x)\,dx = 0$$

where $$L_k = \int_L \frac{dx}{\cos^3\varphi}, \quad L_T = \int_L \frac{dx}{\cos^2\varphi}$$

For free vibrations, neglecting the thermal elongation of structural elements, we have

$$\frac{w}{g}\frac{\partial^2\eta}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2\eta}{\partial x^2}\right)-H_w\frac{\partial^2\eta}{\partial x^2}+\frac{w}{H_w}H_p = 0$$

and

$$\frac{H_p L_k}{E_c A_c}-\frac{w}{H_w}\int\eta(x)\,dx = 0$$

for which the characteristic equation for natural frequencies is of the form

$$\frac{wlf}{\Phi^3(z^2-1)\,z}\left\{\frac{\Phi}{\sqrt{2}}z-\frac{z+1}{z-1}\tan\left(\frac{\Phi}{\sqrt{2}}\frac{\sqrt{z-1}}{2}\right)-\frac{z-1}{\sqrt{z+1}}\tanh\left(\frac{\Phi}{\sqrt{2}}\frac{\sqrt{z+1}}{2}\right)\right\}$$
$$+\frac{2w_1 f_1 l_1}{\Phi_1^3(z_1^2-1)z_1}\left\{\frac{\Phi_1}{\sqrt{2}}z_1-\frac{z_1+1}{z_1-1}\tan\left(\frac{\Phi_1}{\sqrt{2}}\frac{\sqrt{z_1-1}}{2}\right)-\frac{z_1-1}{\sqrt{z_1+1}}\tanh\left(\frac{\Phi_1}{\sqrt{2}}\frac{\sqrt{z_1+1}}{2}\right)\right\}$$
$$-\frac{L_k}{E_c A_c}\frac{H_w}{32\sqrt{2}} = 0$$

where $$\Phi = l\sqrt{\frac{H_w}{EI}}, \quad \Phi_1 = l_1\sqrt{\frac{H_w}{EI_1}}, \quad z=\sqrt{1+\frac{32f}{g\Phi^2}\omega^2}, \quad z_1=\sqrt{1+\frac{32f_1}{g\Phi_1^2}\omega^2}$$

A model of 1/200 of the 1st plan of the Akashi-Strait Bridge is so designed that the vertical flexural vibrational modes are similar to the prototype under the linear reduction factor $\lambda=1/200$ and the slicing factor $m=5.798$; which is thus featured as Table -1 (Fig. 1).

Numerical results of approximate natural frequencies for free vibrations of the model by Rayleigh Ritz method are given as the Table-2.

Table 1. Dimensions of a model.

Length of main span	$l=6500$ mm
Length of side span	$l_1=3250$ mm
Cross Sectional Area of Cable	$F_k=5.309$ mm^2/cable
2nd Moment of Inertia	
for Main Span	$I=3.770$ cm^4
for Side Span	$I=5.278$ cm^4
Sag for Main Span	$f=540$ mm
Sag for Side Span	$f_1=135$ mm
Dead Load	$w=0.2646$ kg/cm
Live Load	$p=0.1026$ kg/cm
Young's modulus for Cable & Tower	$E=2.1\times10^6$ kg/cm^2
Young's modulus for Stiffening Girder	$E=2.5\times10^4$ kg/cm^2
Panel Length of Stififfening Girder	62.5 mm
Type of Stiffening Girder	: Warren Type of Truss
Height of Stiffening Girder	75 mm
Spacing of Cables	150 mm

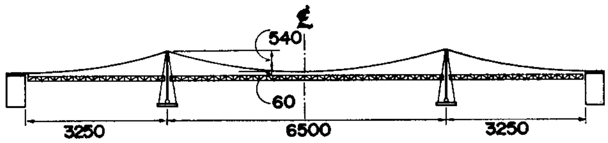


Fig. 1. General view of model (scale 1/200).

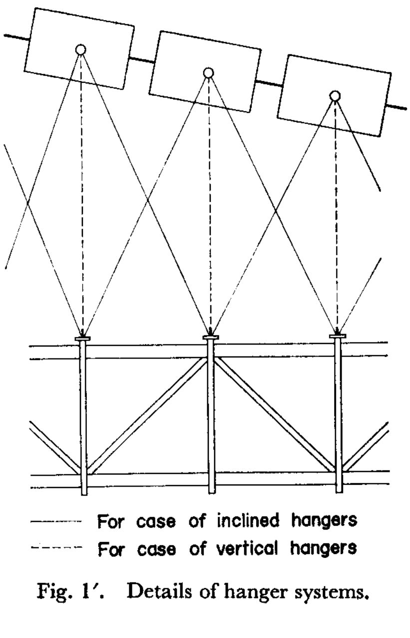


Fig. 1′. Details of hanger systems.

Table 2. Natural Frequencies and Modes (Calculated)

Symmetric Modes

* Inflexible Tower

$EI=1.35\times 10^5$ kg·cm²

n	ω	N (c/s)	T (sec)	Modes Main Span
1	13.69	2.18	0.459	$\sin\frac{\pi x}{l}-1.353\sin\frac{3\pi x}{l}$
2	20.68	3.29	0.304	$\sin\frac{\pi x}{l}+0.661\sin\frac{3\pi x}{l}$

* Flexible Tower

$EI=1.35\times 10^5$ kg·cm²

n	ω	N (c/s)	T (sec)	Modes main	side
1	7.36	1.17	0.86	$\sin\frac{\pi x}{l}-0.062\sin\frac{3\pi x}{l}$	$-0.790\sin\frac{\pi x_1}{l_1}$
2	14.50	2.31	0.43	$\sin\frac{\pi x}{l}-0.484\sin\frac{3\pi x}{l}$	$1.623\sin\frac{x\pi_1}{l_1}$

(Three ferms approximation)

$$\eta=\sin\frac{\pi x}{l}+a_3\sin\frac{3\pi x}{l}+a_5\sin\frac{5\pi x}{l}$$

$$\eta_1=\bar{a}\sin\frac{\pi x_1}{l_1}$$

n	ω (rad/sec)	N (c/s)	T (sec)	Modes		
				a_3	a_5	$\bar{a}_1$
1	7.535	1.171	0.854	−0.062	−0.009	−0.789
2	14.478	2.305	0.434	−4.440	−0.072	1.616
3	20.362	3.242	0.309	0.684	−0.248	1.222
4	27.39	4.362	0.229	0.461	7.406	1.109

3. Static Experiments and Results

In order to clarify the mechanical response of a suspension bridge with vertical and inclined hangers static loading tests and free vibrations tests are performed in this investigation; for the former test, deflections of stiffening girders are measured at the 18 different loading conditions as specified in Fig. 2 for the cases of (i) inflexible towers and (ii) flexible towers under the uniform load (namely 615.6 gr/one pannel (6.25 cm)/one side).

Static tests are performed under 20 kinds of loadings as indicated in Fig. 2; No. 1–8th loading conditions form symmetric loading acting on the main girder, while No. 9–15th conditions asymmetric on the main girder, and under No. 16–20th conditions main and side spans are fully or partly loaded as in the figure.

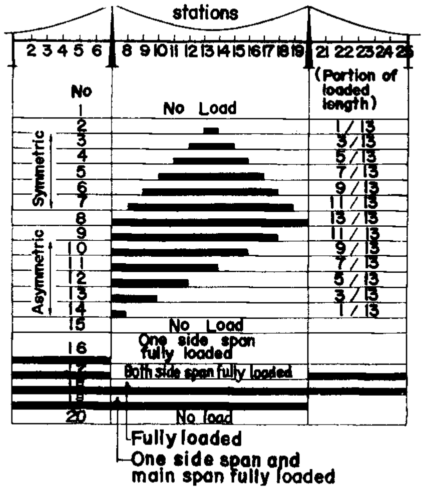


Fig. 2. Static loads.

Deflections of the stiffening girders under above distriubted loads are measured directly by use of the telescope and the results for both cases of vertical hangers

Table 3. Deflection of stiffening girders

Laod Conditions / Stations	4			8		
	Calculated	vertical	inclined	Calculated	vertical	inclined
2	−1.05	−0.92	−0.71	−1.90	−1.45	−1.23
3	−1.72	−1.53	−1.16	−3.13	−2.40	−1.95
4	−1.94	−1.69	−1.33	−3.71	−2.66	−2.28
5	−1.72	−1.47	−1.20	−3.13	−2.34	−2.08
6	−1.05	−0.86	−0.77	−1.90	−1.34	−1.29
8	0.18	0.11	0.17	1.60	1.22	1.05
9	0.74	0.58	0.65	2.97	2.26	1.95
10	1.77	1.44	1.36	4.03	3.06	2.71
11	2.99	2.53	2.29	4.77	3.68	3.40
12	3.96	3.33	3.00	5.23	4.03	3.82
13, 14	4.33	3.64	3.21	5.39	4.18	3.93
15	3.96	3.34	2.86	5.23	4.03	3.71
16	2.99	2.52	2.12	4.77	3.68	3.26
17	1.77	1.40	1.20	4.03	3.07	2.63
18	0.74	0.56	0.58	2.97	2.25	1.90
19	0.18	0.10	0.15	1.60	1.19	0.97
21	−1.05	−0.89	−0.72	−1.90	−1.40	−1.22
22	−1.72	−1.51	−1.20	−3.13	−2.40	−2.00
23	−1.94	−1.75	−1.41	−3.71	−2.78	−2.30
24	−1.72	−1.57	−1.25	−3.13	−2.48	−2.04
25	−1.05	−0.99	−0.79	−1.90	−1.55	−1.28

and inclined hangers for flexible and inflexible twoer systems are given in Table 3 and Fig's. 3–13 from which one may conclude the following characteristics.

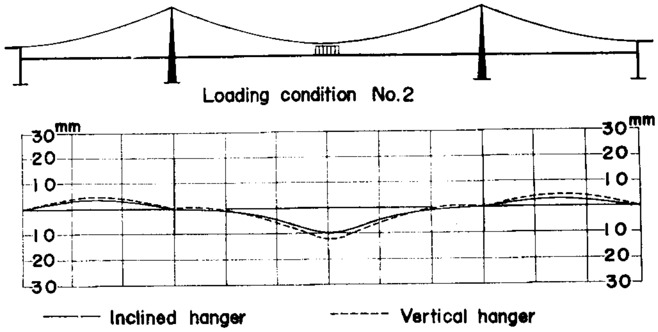


Fig. 3. Static deflectional curves (Loading condition No. 2).

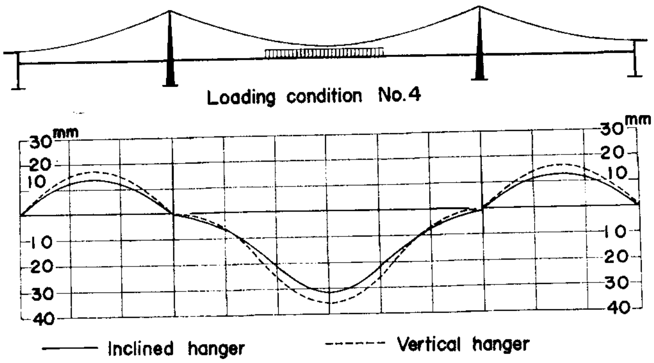


Fig. 4. Static deflectional cruves (Loading condition No. 4).

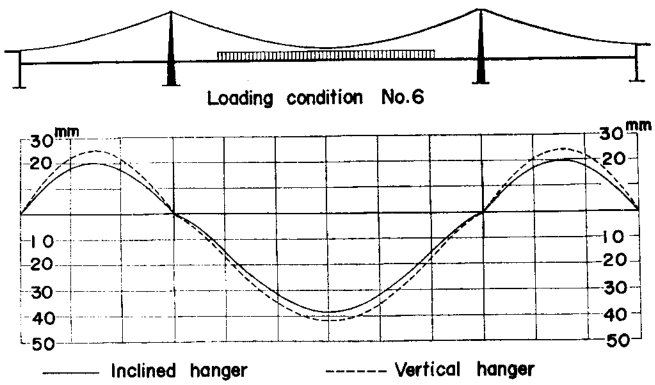


Fig. 5. Static deflectional curves (Loading condition No. 6).

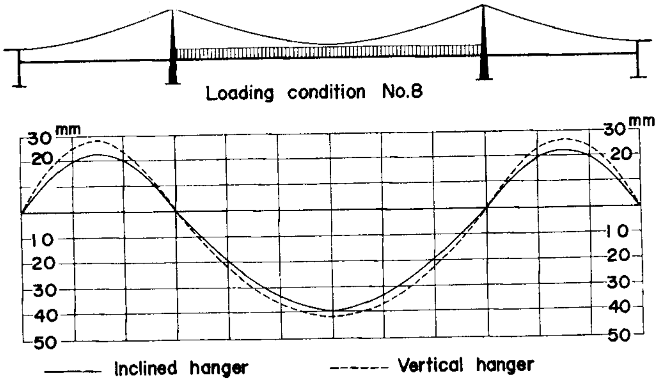


Fig. 6. Static deflectional curves (Loading condition No. 8).

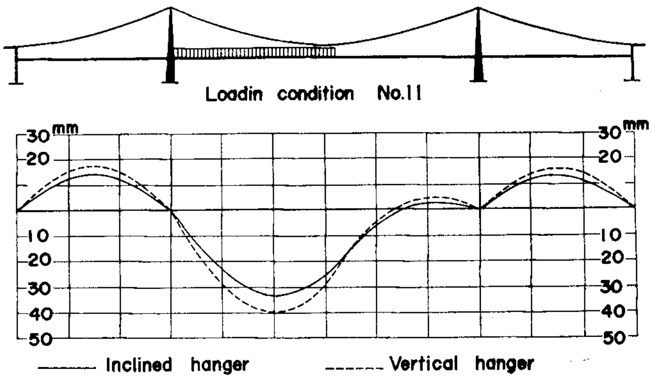


Fig. 7. Static deflectional curves (Loading condition No. 11).

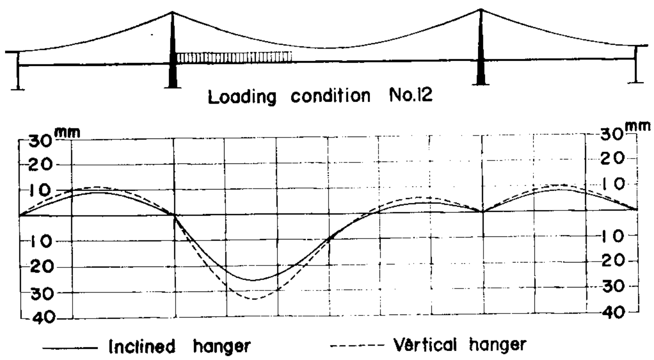


Fig. 8. Static deflectional curves (Loading condition No. 12).

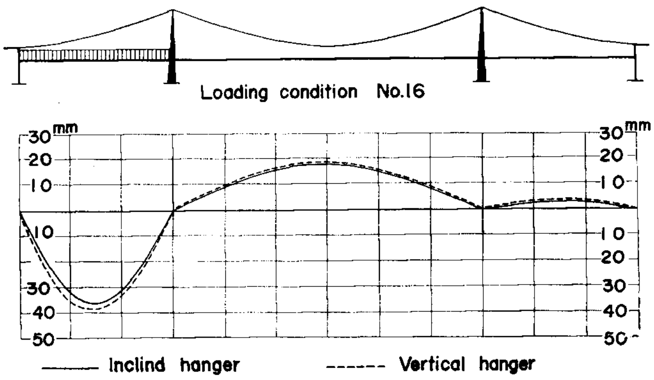


Fig. 9. Static deflectional curves (Loading condition No. 16).

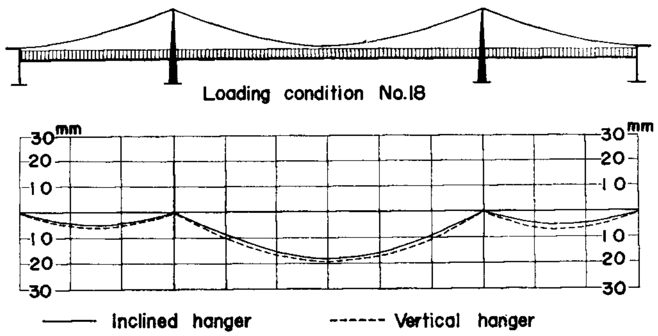


Fig. 10. Static deflectional curves (Loading condition No. 18).

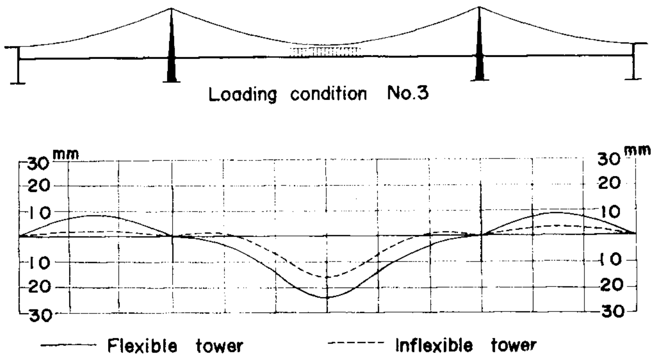


Fig. 11. Static deflectional curve (inclined hanger)
(Loading condition No. 3).

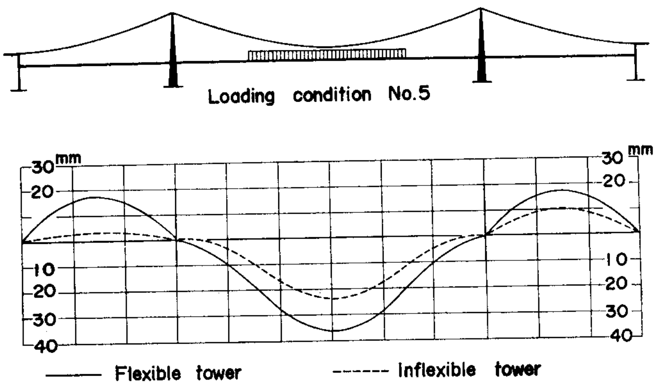


Fig. 13. Static deflectional curve (inclined hanger) (Loading condition No. 5).

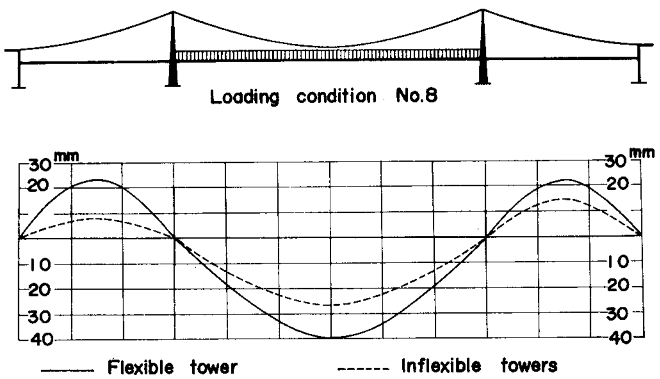


Fig. 13. Static deflectional curve (inclined hanger) (Loading condition No. 8).

(1) For the case of inclined hangers the maximum static deflections are obtained under the No. 8th loading conditions (Fig. 10), namely fully loaded on the main span, and measured as 3.930 cm which corresponds to 1/165 of span length; for the case of the vertical hangers the ratio of maximum deflection to span length reaches to 1/154 under the corresponding load conditions.

The model for inclined hangers is considered, as the result, to deflect approximately 10% less than for the vertical hangers. The decrease of deflections due to the different hanger system tend to become remarkable when loads act asymmetrically or concentrated.

(2) When the tops of towers are fixed so as to have inflexible towers the deflections of stiffening girders are about 35% less than the case of so called flexible towers.

Throughout all kinds of loading conditions slopes of deflectional curves as the supports are comparatively small for the case of inclined hangers because of less flexibility in comparison with the case of vertical hangers (Fig's. 11–13)

(3) Stresses in cables reach to the maximum value, when the loads act fully on both main and side spans. However the stresses in chord members of stiffening truss vary differently in upper and lower chords. The stresses in lower chords tend to reach to high stress level when the main span is loaded, while the stresses in upper chords tend to reach to the high stress level when side spans are loaded.

4. Dynamic Experiments and Results

Experimental investigation is performed mainly to disclose the free vibrational characteristics of a suspension bridge with such an inclined hanger system as shown in Photo 2. In this study the deflections of stiffening girders are obtained by use of the differential transformers, (type 150 BL, Shinkou Electric Co.) and the amplifier (type M1–6W–13 Shinkou Electric Co.) and the strains are measured by the wire strain gauges amplified by the DS6–MTH (Shinkou Communication Co.) and recorded by the electromagnetic oscillograph, type FR–101 (San-ei Sokki Co.). To have the steady state response the electrodynamic vibration exciter, type VS–VVE–3202 AS(IMV Co. Photo 3) is used, the vibrator of which varied from 1 to 100 cps in ferquency and up to 26 mm in amplitude, exerting the sinusoidal external forces up to 125 kg at the maximum.

In dynamic analysis of this investigation, the natural frequencies, the corresponding modes, and the logarithmic decrement are obtained by measuring deflections and strains of stiffening girders and cables in free vibrations and under the steady state excitations for models with vertical & inclined hangers (Fig's 17, 18 & 19). In order to estimate the effect on the deflection of stiffening girders due to flexibility of towers, the tests are performed when the tops of the towers move freely and when completely fixed by additional supports, as shown in photograph 2. The results are as follows, (Table 4). (1) With flexible towers the principal vibrational modes, as shown in Fig. 14, remain of almost similar form for vertical and inclined hangers, while the frequency for the former case is $N=1.23$ cps and the one for the latter case is $N=1.45$ cps, namely 18% larger than for the vertical hangers. In spite of the fact that the calculated shape of modes accords with the experimental result, the frequency is estimated as $N=1.17$ cps.

For the first asymmetric modes (Fig. 15) the frequencies are measured as 1.58 cps and calculated as 1.54 cps for vertical hangers, while the frequency is obtained as 2.00 cps for inclined hangers. Theoretically, in the first asymmetric

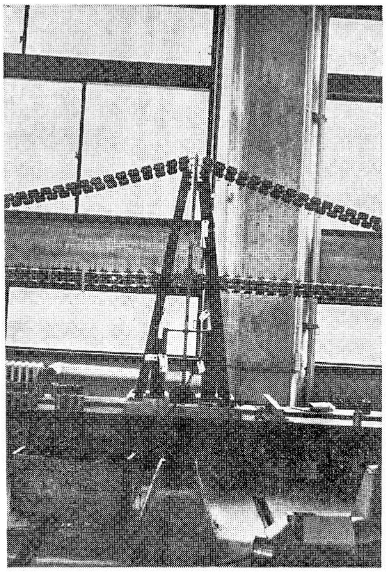


Photo. 1. Model for suspension bridge.

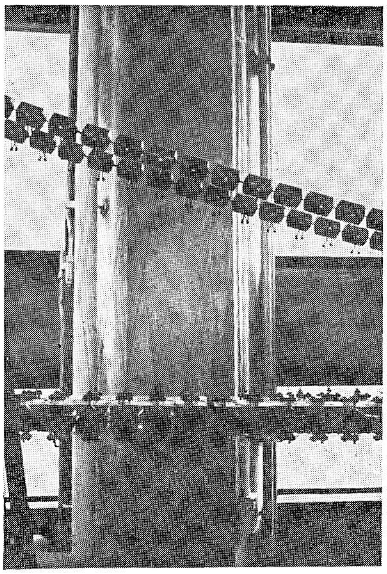


Photo. 2. Inclined hangers.

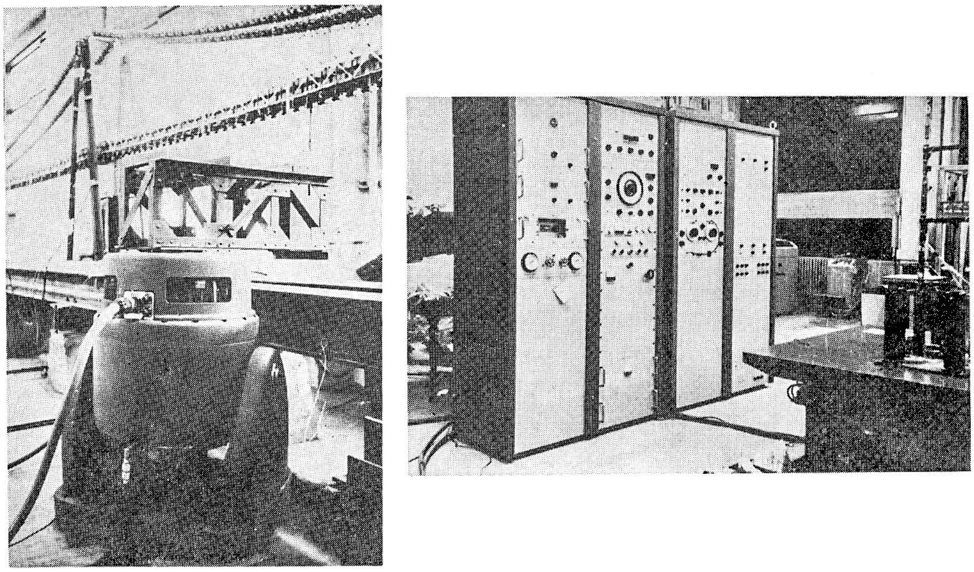
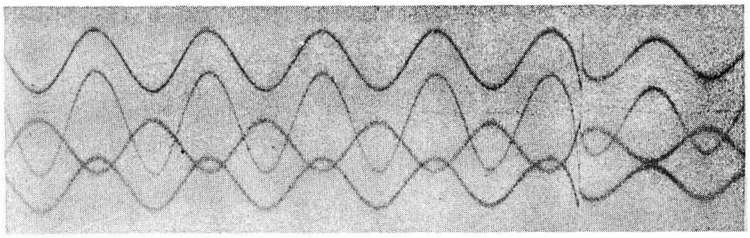
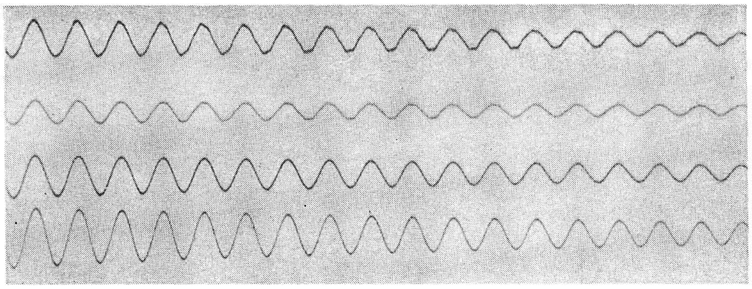


Photo. 3. Electro dynamic vibration excitor.

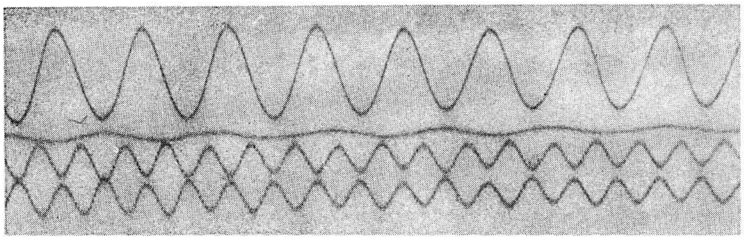


(a) Vertical hangers, flexible tower

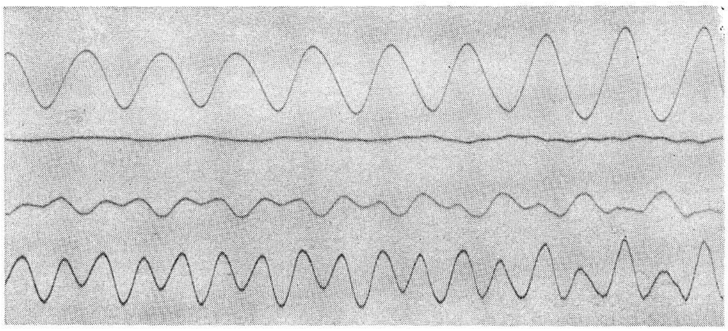


(b) Inclined hangers, flexible tower

Photo. 4. Deflectional & strain variations in 1st symmetric modes.



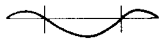
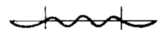
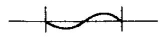
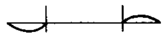
(a) Vertical hangers, flexible tower



(b) Inclined hangers, flexible tower

Photo. 5. Deflectional & Strain variations in 1st Anti-symmetric modes.

Table 4. Natural Frequencies (Measured)

	order	inclined hanger		inclined hanger	vibrational modes
		calculated	measured	measured	
symmetric	1st	1.17	1.28	1.45	
	2nd	2.30	2.38	3.05	
	3rd	3.33	—	4.00	
	4th	4.36	—	5.50	
asymmetric	1st	1.54	1.58	2.00	
	2nd	—	—	2.30	
	3rd	3.29	—	—	

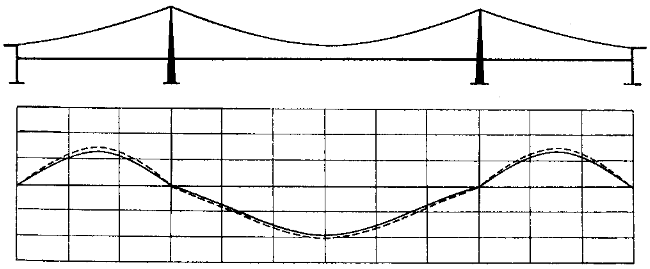


Fig. 14. Measured 1st symmetric modes of vertical vibrations.

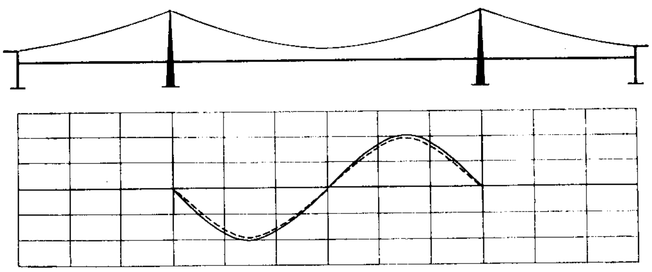


Fig. 15. Measured 1st asymmetric modes of vertical vibrations.

modes the motions of stiffening girders of main and side spans are independent to each other, but experimentally the side spans are exerted by motions of stiffening girders in main span. Comparing the mode of vertical hangers with the one of inclined hangers, the ratio of maximum amplitudes of defledtions of side spans to that of main span reamins 0.96 for vertical hangers and 0.63 for inclined hangers.

As shown in the Table 2, the frequencies for the model with inclined hangers are approximately 20–30% larger than those for vertical hangers; this fact may indicate to some extent the increase of deflectional stiffeness of a suspension bridge.

(2) The stresses in cables and the deflections of stiffening girders at quarter points of main span are obtained simultaneously as shown in the photographs 4. Particular attention should be placed on the fact that for the first asymmetric mode the frequency of stresses in cables is twice as much as the frequency of deflection of stiffening girders for both cases of vertical and inclined hangers. However, for the case of inclined hangers, strains in cables change less regularly through the course of time. From the frequency-response curve for strains it may be found that strain energy in cables of the model used in this investigation play a more important role in free vibrations of high degrees than generally considered. Thus, in connection with this fact, one may consider that such free vibrations as the case of inclined hangers are characterized by taking into an account of strain energy stored in various structural members. Though in this experimental study measurements are restricted to vertical deformation of stiffening girders, vertical deflections seem to exaggerate the lateral (horizontal) deflections of stiffening girders on an account of less flexibility in the vertical direction, which is thus considered to casue some disturbances on strains in various members.

(3) The logarithmic decrements for models with vertical and inclined hangers are obtained and compared in Fig. 16. The relation between decrements and amplitude can not be clearly found at the present, but the decrements tend to increase when the amplitudes decrease. And generally speaking, the damping effect for the

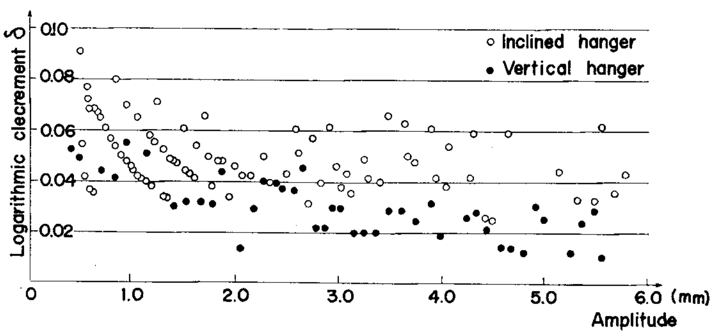


Fig. 16. Logarithmic decrements of 1st symmetric modes of vertical vibrations.

case of inclined hangers is considered to be larger than the case of vertical hangers.
(4) With fixed tops of towers the natural frequencies for the case of inclined hangers seem to increase about 10% to compare with those for the case of vertical hangers. In the first asymmetric modes the frequency of strains in cable accords with that of deflections of stiffening girders for both cases of vertical and inclined hangers.
(5) There appears to be no significant vibrational characters by the longitudinal horizontal settlement of bases of towers up to 3 mm in this scale of model.

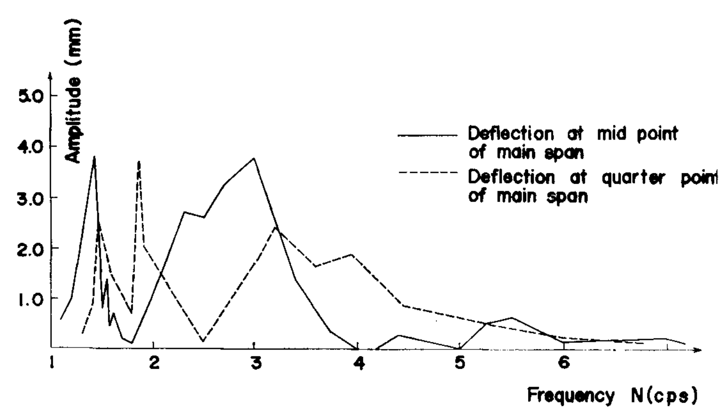


Fig. 17. Frequency response diagram for deflections (inclined hanger).

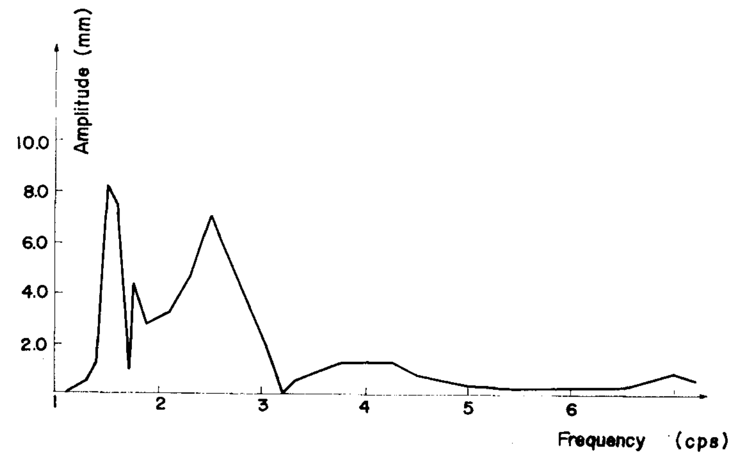


Fig. 18. Frequency response diagram for deflections at midpoint of side span (inclined hanger).

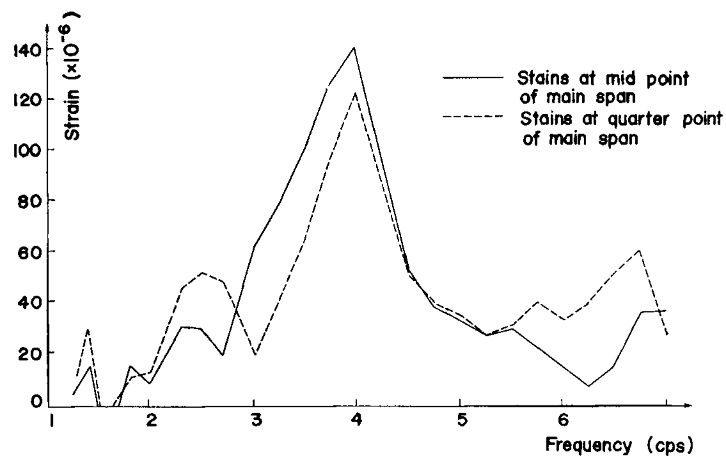


Fig. 19. Frequency response diagram for strains in cables.

Conclusions

In this experimental investigation the vibrational characteristics of a model of suspension bridge with inclined hangers are considered in comparison with case of vertical hangers. Statically, the inclined hanger system may cause less flexibility when loads act asymmetrically and partly distributed and, thus, the slope of stiffening girders at the ends can be comparatively small. Dynamically, the natural frequencies increase about 20% to compare with the case of vertical hangers, and also damping is considered to increase significantly. This investigation obviously calls for future thoroughful theoretical investigations which make the design of such improvement as adoption of inclined hangers technically possible.

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