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On the Condition required for Discharge and Dielectric Breakdown

By

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The condition required for the electric breakdown of gas, liquid or solid against impulse voltage is tentatively assumed. The calculated value by the equation which is derived from this condition is compared with the experimental data about the breakdown of air, heptan and chloroprene, and the validity of this theory is investigated.

1. Preface

The phenomena of discharge and dielectric breakdown have been studied for many years, and experimental results under various conditions have been obtained and the theories for these results have been given. However, it seems that these theories are inadequate to explain and describe the phenomena over a wide range of conditions.

In the present paper, referring to many contributions on the dielectric breakdown of gases, liquids or solids, the authors assume that the condition required for discharge on dielectric breakdown is satisfied when the electron density of the electron avalanche becomes larger than a certain value. It is verified in the following manner that the assumption proposed by the authors is applicable to discharge phenomena in various conditions, and that the computations necessary to establish this value are simple.

First the radial width of the head of the electron avalanche grown in gas subjected to electric stress is calculated against three kinds of domain of the gap space, and are then combined with the critical condition described above and calculated values are obtained and compared to the experimental values. Then the same theory is applied to the dielectric breakdown of an organic liqued or solid, and the validity of the theory is investigated.

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2. Discharge in gases

2-1. Electron ionization coefficient

The energy distribution of electrons in gas subjected to an electric field is described separately in two cases, where (1) the electrons do not exchange their energy with each other, or where (2) electrons exchange their energy with each other, and significant mutual actions are observed. When an electron avalanche grows in a gas, the electron avalanche at first takes the former distribution. But, just before breakdown occurs, the number of electrons becomes more than 10^{10}, and they may act as a group, and exchange their energy mutually, in which case the distribution becomes close to the latter.

In this paper, for the sake of simplicity of mathematical treatment, it is assumed that the following distribution holds in the range where the discharge is investigated. (App. 1)

$$dN = A\exp\left(-\frac{\varepsilon}{kT_e}\right)d\varepsilon \tag{1}$$

where;

dN=number electrons with energy between e and $(e+de)$
A=constant for normalization
k=Boltzmann constant
T=electron temperature

This energy distribution is related to the electric field in the following equation, in which the electron temperature in the gas is defined.

$$kT_e = aeE\lambda \tag{2}$$

where;

a=constant for the gas
e=electric charge of electron
E=applied electric field
λ=mean free path of electron in the gas before the electric field is applied

Ionization efficiency (S), the value of which does not quite reach the maximum value, is described by the following equation induced from experimental data,

$$S = Cp(\varepsilon-\varepsilon_i)\ \text{(number/cm)} \qquad \varepsilon_i<\varepsilon<\varepsilon_p \tag{3}$$

where;

C=constant for the gas
p=gas pressure (mmHg)
ε_i=minimum energy necessary for ionization
ε_p=electron energy corresponding to the maximum value of S

The value of C has been obtained by many researchers as shown in Table 1, which shows the number of pair of ions produced by one primary electron per 1 mmHg per 1 eV.

As far as α, it is sufficient to consider Eq. (3) only over the domain of ε more than ε_i (minimum ionization energy). The following equation can be derived from Eqq. (1)~(3).

Table 1. Inclination of ionization efficiency in the neighbourhood of the original point.

Kind of gas	O_2	N_2	Air
C	24×10^{-2}	26×10^{-2}	26×10^{-2}

$$\alpha=\frac{1}{N}\int_N SdN=\int_{\varepsilon_i}^{\varepsilon_p}\frac{1}{kT_e}Cp(\varepsilon-\varepsilon_i)\exp\left(-\frac{\varepsilon}{kT_e}\right)d\varepsilon \tag{4}$$

For the sake of ease of caluculation, constants A and B are defined by the following equations, and Eq. (5) is derived:

$$\varepsilon_p=A\varepsilon_i,\qquad \varepsilon_i=BkT_e$$
$$\alpha=CpkT_e\{\exp(-B)-(AB+1-B)\exp(-AB)\} \tag{5}$$

For the gas whose insulation is investigated, B=2~3, and $A\fallingdotseq5$ are obtained experimentally, and $\exp(-B)\gg\exp(-AB)$ holds. Then, on the right side of Eq. (5), the second term is negligible compared to the first, and the equation is simplified as follows:

$$\alpha\fallingdotseq CpkT_e\exp\left(-\frac{\varepsilon_i}{kT_e}\right) \tag{6}$$

When the applied field is very large, B is very small and this approximation is inadequate. In this paper, however, only the critical breakdown field is studied, and the phenemenon where B is extraordinary small is disregarded.

The mean free path of electrons is as follows:

$$\lambda=\frac{L}{p} \tag{7}$$

where;

L=mean free path of electron at 1 mmHg

From Eqq. (2), (6) and (7)

$$\frac{\alpha}{p} = CaeL\left(\frac{E}{p}\right)\exp\left(-\frac{p}{E}\frac{\varepsilon_i}{aeL}\right) \tag{8}$$

On the other hand, the following is obtained as experimental, semitheoretical equation,

$$\frac{\alpha}{p} = g\exp\left(-\frac{p}{E}h\right)$$

where;

g, h=constants for gas

The above equation is different from the form of Eq. (8), but the values calculated by the two equations nearly coincides.

The experimental values obtained by Harrison, Geballe and Brose are used for the constants in Eq. (8) and constants used in the calculation of α/p are compiled as shown in Table 2. The constants for air are obtained

Table 2. Constants used for the calculation of α/p.

Constant / Kind of gas	a	L (cm)	ε_i (eV)
O_2	3.33	3.6×10^{-2}	12.2
N_2	3.24	2.7×10^{-2}	15.6
Air	3.15	2.9×10^{-2}	14.0

L (cm), ε_i (eV)

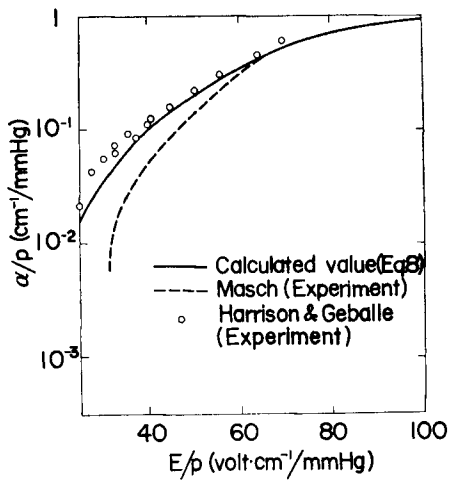

Fig. 1. Calculated values of α/p for oxigen compared with experimental values.

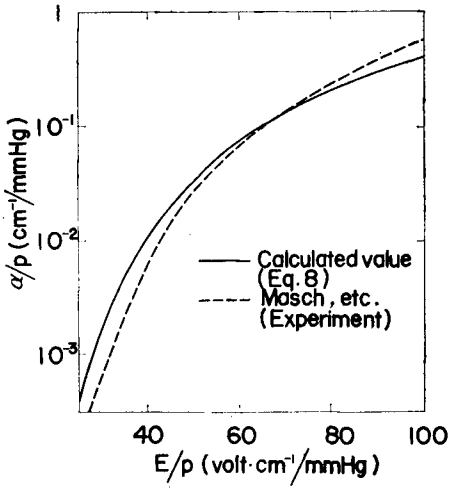

Fig. 2. Calculated values of α/p for nitrogen compared with experimental values.

from the weighted mean value of the constants for nitrogen and oxygen, where the ratio of nitrogen to oxygen is 4. In Figs. (1)~(3), the values calculated by Eq. (8) are compared with the experimental data found in the references. From these results, it is deduced that Eq. (8) can describe the relation between E/p and α in the range where the electric discharge in a gas is investigated.

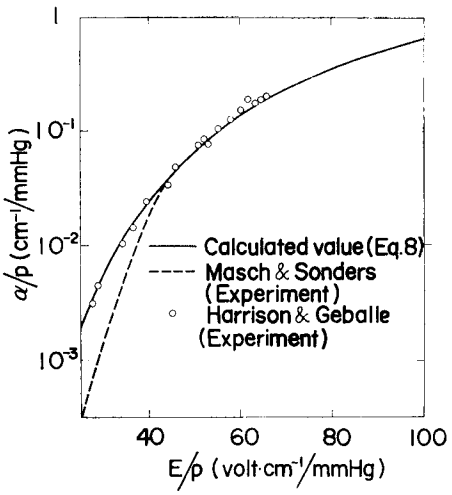

Fig. 3. Calculated values of α/p for air compared with experimental value.

On the other hand, attachments of electrons to gas molecules are to be considered, but these phenomena are neglected in the present paper since they occur at much lower energy level than ionization and the probability of their occurence is very small.

2-2. Condition required for discharge

When the electrons are accelerated in an electric field and the space charge of the positive ions is neglected, according to Townsend's theory, the number of electrons (n) grows as $n=n_0 \exp(z)$ by ionizing collision, and the electron avalanche appears.

However, according to the experimental result, at a pressure of 760 mmHg and a gap distance of 5 cm, αd equals about 40 under critical breakdown field in air, then n would increase to 10^{17}.

If there are 10^{17} electrons in the electron swarm, the width of the electron swarm must become much larger than the experimental results which show the width to be smaller than about 5 cm. It is unlikely that, 10^{20}~10^{40} electrons are contained in this electron swarm, or avalanche. Then, it can be assumed that, after the electron density exceeds certain value, the density does not increase further and the electron avalanche changes to a streamer and spark discharge (or dielectric breakdown) occurs. This assumption is proposed also for a breakdown in a solid.

After the electron density becomes saturated, electrons act as a group, plasma is formed in the back of the electron avalanche, and the spark bridges over the gap. That is, in the case where the gap distance is relatively large, the group of electrons spends the energy obtained from the electric field to increase the mean energy of the group, and energy distribution may

change to Maxwell-Boltzmann distribution. At that time, new electrons are formed from ionization by collision at the head of the swarm, and the electrons in excess of a certain density are left at the tail of the swarm. The electrons which leave the group, are mixed with floating positive ions and form the plasma. The result is that the electric field around the swarm head is enhanced, and this phenomena is repeated again and again until the bridgeover occurs at the gap space. Thus, the spark channel should be established very soon after the electron density of the avalanche becomes larger than the threshold value.

On the other hand, where the gap space is relatively small, the electron density of the avalanch which is about to reach the anode becomes the threshold value described above, and the plasma formed in front of and behind the abalanche enhances the applied field, following which the spark channel should be formed.

2-3. Calculation of breakdown field

First, the value calculated by Raether and Meek are compared with experimental results in Table 3, where Meek brought his values close to experimental ones by using some parameter.

Table 3. Calculated values of discharging voltage in air compared with experimental values.

Discharging voltage (kV) / Gap space (cm)	Experimental value	Raether's calculated value	Meek's calculated value
1.0	31.6	32.4	32.2
2.5	73	71.3	70.5
5.0	138	132	132
10	265	249	249
15	386	378	363

Note : 760 mmHg, 20°C.

It is deduced from this table that the calculated values deviate further from the experimental ones in the wide gap. On the other hand, Fletcher's theory originally proposed for time lag in spark discharge, is suitable for the time lag, but not adequate for the critical breakdown field.

As well known, in the study of discharge domains, not all discharge are shown in one equation, but are divided among various domains of gap space or p·d, and the condition required for discharge varies for each domain, the boundary of which is related to the width of the electron

avalanche. The authors follow this opinion also in this paper.

In the early period when the number of the electrons increases as exp (αz), the diffusing width (r_D) of the electron avalanche studied in 3-dimensional space in air is as follows:

$$r_D = \left(0.400 \frac{z}{p}\right)^{1/2} \tag{9}$$

In the above equation the constants for air are adopted.

Then, in the case when the gap space is small, the diffusing radius can be calculated from the extension caused by thermal diffusion, and the electron density (n_{01}) is as follows:

$$n_{01} = \frac{N_0 \varepsilon^{\alpha d}}{\frac{4}{3}\pi r_0^3} = \frac{N_0 \varepsilon^{\alpha d}}{\frac{4}{3}\pi \left(0.400 \frac{d}{p}\right)^{3/2}} \tag{10}$$

where:

N=number of electron released from the cathode

When the avalanche grows large, the effect of the positive ions becomes small, the avalanche extends itself by electro-static repulsion, and its width becomes as follows:

$$r_e = \left(1.44 \times 10^{-7} \frac{3}{\alpha E}\right)^{1/3} \cdot \exp\left(\frac{\alpha z}{3}\right) \tag{11}$$

where:

E=V/cm

z=cm

α=1/cm

ε^*=1(for air)

But, when the number of electrons becomes more than above described, the electron density approaches the threshold value, and the width (r_e') of the avalanche becomes as follows (App. 2):

$$r_e' = r_0 \exp\left(1.44 \times 10^{-7} \frac{4\pi}{3} \cdot \frac{n_0 z}{E}\right) \tag{12}$$

where:

r_0=the value of r_e' when r_e shifts to r_e'

n_0=the electron density

The above equation holds in the case where the gap space is large, and when n_{03} is defined as the threshold value of the electron density in that case, the following equation is obtained:

$$n_{03}=\frac{N_0\varepsilon^{\alpha d}}{\frac{4}{3}\pi r_e'^3}=\frac{N_0\varepsilon^{\alpha d}}{\frac{4}{3}\pi r_0^3\exp\left(1.44\times10^{-7}\cdot4\pi n_{03}\frac{d}{E}\right)} \tag{13}$$

Because n_{03} is contained in both sides of this equation, n_{03} is implicitly expressed.

Here, it is inconsistent with the assumption described above that the total number of electrons is calculated by $N_0=\exp(\alpha d)$, but it is formal only. That is, it is assumed that the condition required for shifting from avalanche to streamer would be satisfied at the point z, and the avalanche width increases in proportion to the number of electrons in such a way that the electron density remains constant. Therefore r_e' becomes much greater than the width in real phenomena, and does not exist in fact.

In the case where gap space is between the domains of Eq. (12) and Eq. (13), the radius of the electron alalanche is expressed by the sum of thermal diffusion and electrostatic repulsion: then

$$r=r_D+r_e \tag{14}$$

The threshold value of electron density (n_{02}) in this case is as follows:

$$n_{02}=\frac{N_0\varepsilon^{\alpha d}}{\frac{4}{3}\pi r^3}=\frac{N_0\varepsilon^{\alpha d}}{\frac{4}{3}\pi(r_D+r_e)^3} \tag{15}$$

The equations (12), (13) and (15) are rewritten as follows:

$$\alpha d=\ln\left\{\frac{4}{3}\pi\frac{n_{01}}{N_0}\left(0.400\frac{d}{p}\right)^{3/2}\right\}+\frac{3}{2}\ln\frac{d}{p} \tag{16}$$

$$\alpha d=\ln\left(\frac{4}{3}\pi\frac{n_{02}}{N_0}\right)+3\ln\left\{\left(0.400\frac{d}{p}\right)^{1/2}+\left(\frac{3}{\alpha E}1.44\times10^{-7}\right)^{1/3}\cdot\varepsilon^{\alpha d/3}\right. \tag{17}$$

$$\alpha d=\ln\left(\frac{4}{3}\pi\frac{n_{03}}{N_0}\right)+1.44\times10^{-7}\cdot4\pi n_{03}\frac{d}{E}+3\ln r_0 \tag{18}$$

Note that αd is contained in both sides of Eq. (17).

For the sake of ascertaining the domains of Eqq. (13)~(15), Schumann's experimental results are plotted on Fig. 4, where the ordinate is αd and the abscissa is $\ln(d/p)$, according to Eq. (16). From this graph, it is deduced that Eq. (16) holds for a space gap of less than 0.1 cm, where the plotted data are linear. ($n_{01}=5.5\times10^9\,N_0/\mathrm{cm}^3$) is obtained from experimental values and Eq. (16), but N_0 remains unknown here.

The same data are plotted in Fig. 5, where the ordinate is αd, and the abscissa is d/E, according to Eq. (18). From Fig. 5, it is deduced that Eq. (18) holds for a space gap of more than 2.4 cm, where the plotted data are

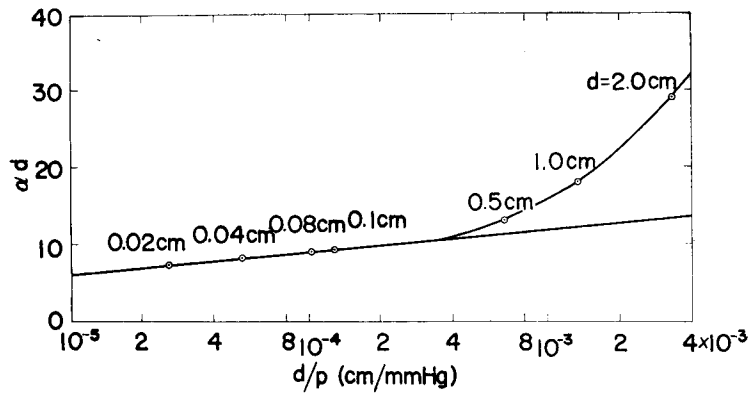

Fig. 4. Experimental values of αd against d/E for air.

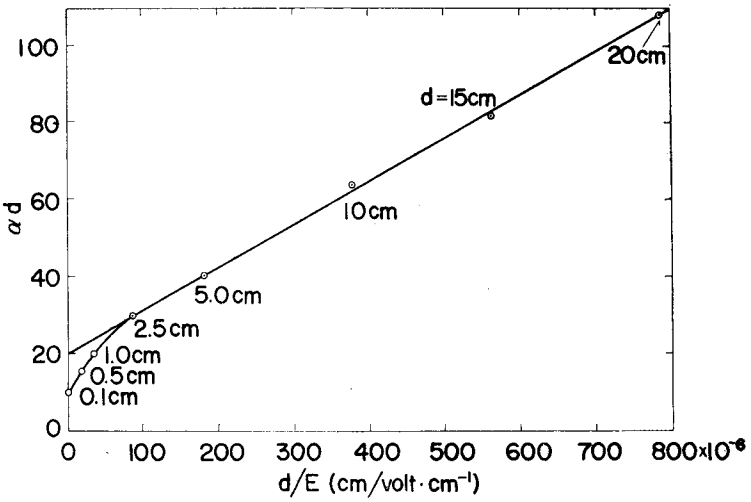

Fig. 5. Experimental values of αd against $\ln d/p$ for air.

linear. ($n_{03}=6.1\times10^{10}/\text{cm}^3$) is obtained from experimental data and Eq. (18).

For the intermediate range of space gap ($0.1<d<2.5$ cm), Eq. (17) is applied to the data, with αd(in Eq. 16) equal to αd(in Eq. 17) in their boundary, and ($n_{02}=5.6\times10^9 N_0/\text{cm}^3$) is obtained. These calculated values are gathered and plotted together in Fig. 6, compared with experimental results for all ranges of space gap. If ($N_0=10$) is assumed, ($n_{0i}=6\times10^{10}$, $i=1, 2, 3$) is obtained, and the threshold value for electron density is constant for all ranges of space gap. It is permissible for n_{0i} to take a different value in each range, on the other hand, N_0 is originally stastical, and can not be precisely ascertained. It may be appropriate to regard N_0 as a parameter which is varied to hold n_{0i} constant.

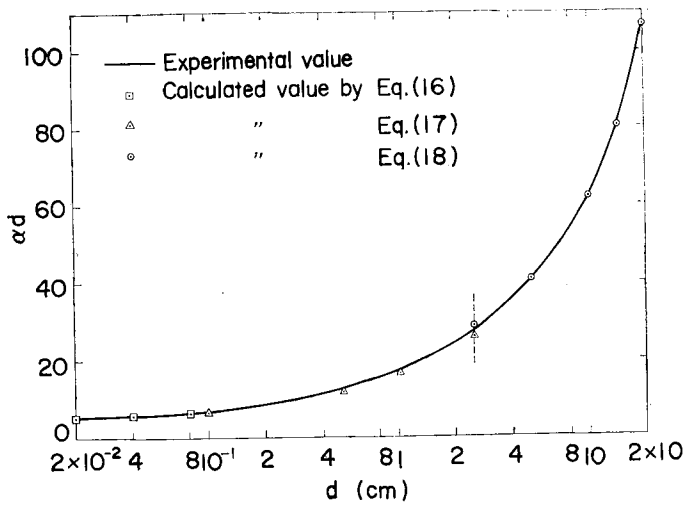

Fig. 6. Calculated values of αd against d for air compared with experimental values.

Table 4. Calculated values related to discharge in air compared with experimental values.

Gap space d(cm)	Experimental value*				Calculated value					
	r(V)	E (V/cm)	α (1/cm)	αd	αd	r_D (cm)	r_e (cm)	r_D+r_e (cm)	r_e' (cm)	n_0 (1/cm^3)
0.02	1520	76000	399	7.98	7.5	3.25×10^{-3}	3.50×10^{-4}	3.60×10^{-3}	1.25×10^{-2}	5.5×10^{9} N_0
0.04	2320	58000	205	8.20	8.0	4.60×10^{-3}	4.70×10^{-4}	5.07×10^{-3}	1.27×10^{-2}	
0.08	3760	47000	101	8.78	8.9	6.51×10^{-3}	6.57×10^{-4}	7.19×10^{-3}	1.32×10^{-2}	
0.10	4500	45000	87.4	8.84	9.1	7.27×10^{-3}	8.83×10^{-4}	9.15×10^{-3}	1.34×10^{-2}	
0.50	17300	34600	27.5	13.8	12.8	1.63×10^{-2}	7.48×10^{-3}	2.38×10^{-2}	2.10×10^{-2}	5.6×10^{9} N_0
1.0	31600	31600	17.7	17.7	15.3	2.30×10^{-2}	3.20×10^{-2}	5.54×10^{-2}	3.91×10^{-2}	
2.5	73000	29300	7.50	28.7	$\frac{25.4}{29}$	3.64×10^{-2}	1.55	1.59	2.84×10^{-1}	
5.0	138000	27600	8.06	40.3	41	5.15×10^{-2}	⋮	⋮	5.5	6.1×10^{10}
10	265000	26500	6.43	64.3	62	7.21×10^{-2}	⋮	⋮	⋮	
15	386000	25700	5.28	79.2	82	8.90×10^{-2}	⋮	⋮	⋮	
20	510000	25500	5.24	107	107	1.03×10^{-1}	⋮	⋮	⋮	

Note : * 1 atm, 20°C, by Schumann

⋮ omitted because r' becomes much grater than real phenomenon (c.f. §2-3)

The calculated values which appear in each stage of the calculation are compiled as Table 4. It becomes clear from the table that the calculated value of αd is different from the experimental one in the range ($1.0<d<2.5$ cm). This may be due to the fact that the range where r_e shifts to r_e' is not defined, and that the shift is slow and the effect of r_e' appears in Eq. 17 in the range ($d<2.5$ cm).

2-4. Comparison of the present assumption with previous ones

The present assumption about the condition required for discharge or breakdown will now be compared with previously published theories.

a) Raether proposed that the electron avalanche turns into a streamer at the point where the distance from the cathode satisfies the following equation:

$$\alpha Z = \ln 2\times 10^8 + \ln Z \tag{19}$$

This equation is like Eq. (16), but the coefficient of $\ln Z$ is different. The different coefficient has a greater effect at a longer distance; consequently, his calculation is considerably in error for a large space gap.

b) Meek proposed that the streamer grows at the point where the following equation is satisfied:

$$n_+(Z) = n_{+a} \tag{20}$$

where;

$n_+(Z)$=positive ion density at the distance z from the cathode

n_{+a}=the threshold value of positive ion density

His theory relates to positive ion density rather than to electron density, with which the present paper is concerned. In the early period of electron avalanche, or in the case of small gap space, electron density (n_-) is approximately equal to positive ion density (n_+), and his assumption is the same the one in this paper, and $1\times 10^{11}/\text{cm}^3$ is obtained for the value of n_{+a}, which is about 10 times value of n_0 in this paper. This difference may be due to the fact that time required for discharge is different in the two theories, and $n_a \fallingdotseq n_-$ does not hold there. The value calculated by Meek is smaller than the experimental value in the case where the gap space is large. This result may be due to the fact that the width of the electron or positive ion swarm is larger than his calculated result where the gap space is large, and $n_+>n_a$ is in fact not yet quite satisfied.

c) Loeb proposed that a discharge occurs when the number of electron at a certain moment equals the number of electrons formed at the next moment when the avalanche propagates at some distance; in other words (electron

density at a certain moment) equals (electron number in the next generation) /(space volume occupied by the electron swarm at the next moment). If it can be considered that the right side of the above equation remains constant, his proposal is equivalent to the present assumption.

d) Fletcher proposed that a streamer can be generated in the space between the electron swarm and the positive ion swarm, when the resultant field (derived from the impressed field and the space charge) becomes zero. It is difficult to relate his assumption to the present one. However, he proposed that the condition required for discharge would change at a certain space gap (l_c), and obtained 2.3 cm as l_c, which approximately equals the boundary (d=2.4 cm) of Eq. (17) and (18) in this paper.

e) From the radius of electron swarm head derived from research on discharge domain, 0.64 cm is obtained as the gap space of the boundary where the extension of the swarm by thermal diffusion eqvals that by electrostatic repulsion, which is in the neighbourhood of 0.1 cm obtained in the present paper. This result shows that the method of determining domain in the present paper is approximately right.

As above described, the assumption about discharge condition in this paper has something in common with the theories of pioneer reserchers.

3. Breakdown of Liquid and Solid

3-1. Consideration of breakdown of liquid and solid

For liquid breakdown, Sharbough described the characteristic derived from a simple theoretical equation, and obtained a fine result, but some imperfections are found. (App. 3)

For solid breakdown, many theories are proposed by Bloch, Hippel and others; each theory is different in respect to the mechanism by which electrons increase in the solid but can explain the experimental results, in which the main object of study is crystaline material.

But practical insulation materials are so complex in construction and so irregular in molecular arrangement that these thories are not applicable. In this chapter, the theory studied in the preceding chapter is applied to the exprimental results obtained by Sharbaugh for Heptan, and by the authors for Neoprene rubber, and the mechanism of its breakdown is studied. Furthermore, an equation expressed for breakdown characteristic is obtained.

First, for liquids and solids in general, the following are assumed:

(1) The ionization energy of dielectrics is ε_i, and when an electron with energy greater than ε_i collides against a molecule, one new electron grows.

(2) ε_d is defined as the disassociation energy and when an electron with energy between ε_d and $\varepsilon_i(\varepsilon_d<\varepsilon<\varepsilon_i)$ collides against a molecute, it is disassociated.

(3) An electron with energy less than ε_d collides elastically with molecules, but the electron is not decelerated by collision, because the electron mass is much smaller than the molecule mass.

(4) The molecules of the dielectrics are arranged uniformly and at random.

(5) The electrons move in the dielectrics without restriction until they disassociate or ionize the molecule.

(6) The electrons form the avalanche by multiplying themselves and the avalanche extends itself mainly by electrostatic repulsion.

When the electron density of an avalanche exceeds a certain value, a streamer grows behind the avalanche, and breakdown occurs.

The energy distribution of electrons is assumed to be the same as in the preceding chapter; then

$$dN = \frac{N}{kT_e}\exp\left(\frac{-\varepsilon}{kT_e}\right)d\varepsilon \tag{21}$$

From the above assumption and the difinition of λ, the ionization coeficient α is calculated as follows,

$$\alpha\lambda = \int_{\varepsilon_i}^{\infty}\frac{1}{kT_e}\exp\left(\frac{-\varepsilon}{kT_e}\right)d\varepsilon$$

$$\alpha = \frac{1}{\lambda}\exp\left(\frac{-\varepsilon_i}{kT_e}\right) \tag{22}$$

Since the width of the electron avalanche is caused by electrostatic repulsion according to author's assumption, Eq. (13) and (22) are combined and the relation between E and d is obtained as follows:

$$\exp\left(\frac{-\varepsilon_i}{aeE\lambda}\right) = \frac{\lambda}{d}\ln\frac{4\pi n_0 r_0^3}{3N_0} + 1.44\times10^{-7}\frac{4\pi n_0\lambda}{\varepsilon^* E} \tag{23}$$

Likewise as Eq. (2)

$$kT_e = aeE\lambda \tag{24}$$

where;

λ=apparent mean free pass of electron

Since it is different from the case of gas, α of dielectrics has not been measured, and the value of α must be obtained by a method different from that of the preceding chapter. The existing ratio of the electrons which participate in the ionization or disassociation of molecules are marked by

n_i or n_d respectively.

$$n_i = \frac{1}{kT_e}\int_{\varepsilon_i}^{\infty} \exp\left(-\frac{\varepsilon}{kT_e}\right)d\varepsilon \tag{25}$$

$$n_d = \frac{1}{kT_e}\int_{\varepsilon_d}^{\varepsilon_i} \exp\left(-\frac{\varepsilon}{kT_e}\right)d\varepsilon \tag{26}$$

In this paper, 12 eV is chosen as ε_i for carbon, hydrogen and chlorine and 4 eV as ε_d for $-c-c-$, $-c-$, in the lump.

The mean velocity of electrons which participate in the ionization or disassociation of molecules is marked by $\bar{v}_i$ or $\bar{v}_d$ respectively, and obtained as follows:

$$\begin{aligned}\bar{v}_i &= \frac{1}{n_i kT_e}\int_{\varepsilon_i}^{\infty} v \exp\left(-\frac{mv^2}{2kT_e}\right)d\varepsilon \\ \bar{v}_d &= \frac{1}{n_d kT_e}\int_{\varepsilon_d}^{\varepsilon_i} \bar{v} \exp\left(-\frac{mv^2}{2kT_e}\right)d\varepsilon\end{aligned} \tag{27}$$

Electrons repeat elastic or inelastic collision with molecules, the energy which the electrons lose before ionization being consumed mainly in disassociation, and the ratio of disassociation energy to all electron energy is marked as b, which is obtained as follows:

$$b = \frac{n_d\bar{v}_d\Sigma_d\varepsilon_d}{n_i\bar{v}_i\Sigma_i\varepsilon_i + n_d\bar{v}_d\Sigma_d\varepsilon_d} \tag{28}$$

where;

Σ_i, Σ_d: ionization or disassociation cross area of dielectrics against electron

Then, constant a in Eq. (23) is obtained as follows:

$$a = 1-b = \frac{n_i\bar{v}_i\Sigma_i\varepsilon_i}{n_i\bar{v}_i\Sigma_i\varepsilon_i + n_d\bar{v}_d\Sigma_d\varepsilon_d} \tag{29}$$

3-2. Comparison of theoretical and experimental results

From the experimental results, the physical constant in Eq. (24) is obtained and studied in this section.

a) Heptan The experimental results as shown in Fig. 7 are the relation of breakdown field (E: V/cm) to space gap (d: cm) in Heptan, and are substituted in Eq. (23), then

$$\left.\begin{aligned} k_1 &= \frac{\varepsilon_i}{ae\lambda} = 1.06\times10^7 \text{ (V/cm)} \\ k_2 &= \lambda \ln\frac{\frac{4}{3}\pi n_0 r_0^3}{N_0} = 8.53\times10^{-6} \text{ (cm)} \\ k_3 &= 1.44\times10^{-7}\frac{4\pi n_0\lambda}{\varepsilon^*} = 3.84\times10^2 \text{ (V/cm)} \end{aligned}\right\} \tag{30}$$

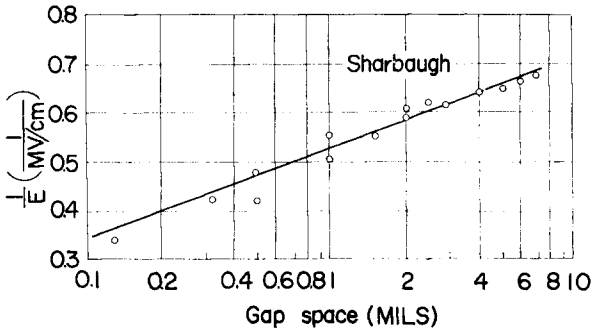

Fig. 7. Calculated value of dielectric breakdown strength of heptan compared with calculated values.

Now, as Σ_i and Σ_d are not measured, constant a cannot be obtained by Eq. (29), but for simplicity it is assumed here that Σ_i approximately equals Σ_d, the value of E being chosen as 1.8×10^6 V/cm from the experimental data, and ε^* being equal to 1.889, then;

$$\lambda = 1.26\times10^{-5}\,\text{cm}$$

and

$$n_0 = 3.24\times10^{13}/\text{cm}^3$$

On the other hand, the maximum collision cross area of gas molecules against electrons is about $10^{-5}\,\text{cm}^2$ and this value is used here, and the density of Heptan is $0.6836\,\text{g/cm}^3$, then;

$$\lambda_0 = 5\times10^{-6}\sim1\times10^{-7}\,\text{cm}$$

where;

λ_0=mean free pass of electron in Heptan

The value of λ (apparent mean free pass) obtained from the breakdown results being compared with λ_0, it is deduced that the energy of the electrons reaches ε_i after 50~100 collisions with molecules. This is consistent with the result obtained by the earlier theories. The values calculated from Eq. (23) using $k_1\sim k_3$ obtained from the experimental data correspond approximately to the straight line in Fig. 7. (App. 4)

b) Chloroprene The breakdown field (E: V/cm) of chloroprene sheet of thickness (d: cm) for impulse voltage ($0.1\times400\mu$S) is measured, and the results are shown in Fig. 8. Since the sheet varies in thickness, the thickness is measured at a breakdown point after breakdown occurs, and the impressed field is then calculated. Thus, the one point plotted on the figure is obtained from about 50 measured values. The form and arrangement of electrodes is designed such that the measuring condition is constant.

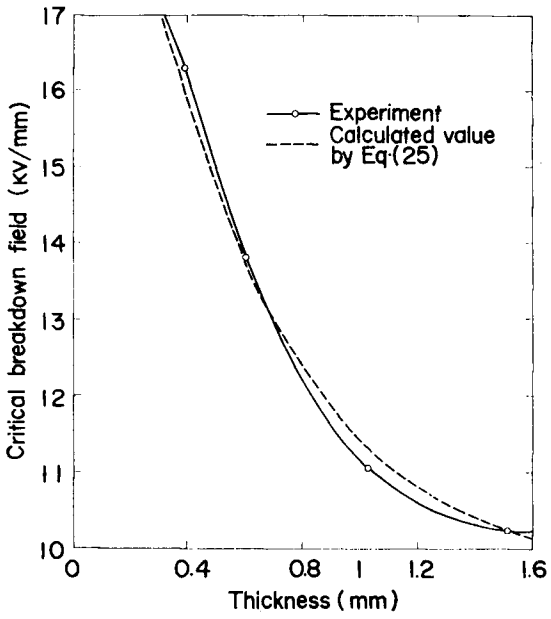

Fig. 8. Dielectric breakdown strength to thickness of chroloprene sheet.

As with the preceding term, the measured E (V/cm) and d (cm) are substituted in Eq. (23), and constants are obtained as follows:

$$k_1 = 26.7\times10^4 \text{ (V/cm)}$$
$$k_2 = 6.92\times10^{-3} \text{ (cm)}$$
$$k_3 = 3.19\times10^3 \text{ (V/cm)}$$

Now, $E=1.1\times10^5$ V/cm, $\varepsilon_i=12$ eV, $\varepsilon_d=4$ eV, $\varepsilon^*=80$ and $\Sigma_i \fallingdotseq \Sigma_d$ are assumed: then

$$a = 0.610$$
$$\lambda = 7.35\times10^{-5} \text{ cm}$$
$$n_0 = 1.92\times10^{14}/\text{cm}^3$$

It is deduced from these results that the electron density required for breakdown of cholroprene is several times that for Heptan, and the apparent mean free pass of electrons is about 10 times that for Heptan. If λ_0 in chloroprene were of the same order as that for Heptan, electrons would be obtained ε_i after 500~1000 collisions. The value of breakdown field is calculated from Eq. (23) using these constants, and the dotted line in Fig. 8 is obtained and its characteristic differs only slightly from the experimental values.

4. Conclusion

For the required condition for breakdown of gas, liquid and solid, the derived equation and calculated value for the relation of breakdown field to gap space (or thickness) have been compared with the experimental results, and it is deduced that the assumption in the present paper is applicable to breakdown phenomena over a wide range. In this paper, however, only the relation between gap space and breakdown field is studied, and the time required for spark formation and statistical time lag for discharge is not studied. From this study, the following conclusions are derived:

(1) The equation calculating α for gases holds over a wide range of d and p.

(2) The width of electron avalanche for 3 domains of gap-space is calculated, electron density and breakdown field are obtained and compared with the experimental results, and found to be consistent with them.

(3) The theory in the present paper seems to be applicable to breakdown phenomena in gas, liquids and solid.

(4) The theory in this paper is compared with earlier theories and found to share common ground with them.

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App. 1

In the space and time with wihch we are concerned in this paper, the exchange of energy between electrons, and between electrons and molecules, is not sufficient, and phenomenon is transient, that is, perfect Maxwell-Boltzmann distribution is not achieved, and the distribution may also be difierent from the Druyvestem one.

When electrons are accelerated by an applied field in a gas, collide repeatedly with the gas molecules, and are in a stationary state, the energy distribution of the electrons is as follows:

$$dN = A_{11}\sqrt{\varepsilon}\exp\left(-\frac{\varepsilon}{kT_e}\right)d\varepsilon \tag{31}$$

$$3k(T_e - T) = m_1\langle v_z\rangle^2 \tag{32}$$

where;

T_e=electron tempeature
T=gas temperature
m_1=electron mass
$\langle v_z\rangle$=mean moving velocity of electrons in the direction of the impressed field

As the mathematical treatment of the above equation is complex, for the sake of simplicity in the present paper the distribution is assumed to be as in Eq. (1). It is similar to the distribution proposed by Sharbaugh, but is different from the coeficient of kT_e as shown in Eq. (2). When ε_m is defined as the mean energy of the electrons, the following equation is obtained from the distribution shown in Eq. (1).

$$\varepsilon_m = A\int_0^\infty \varepsilon\exp\left(-\frac{\varepsilon}{kT_e}\right)d\varepsilon = kT_e \tag{33}$$

Furthermore, the kinetic energy gained by electrons traveling along the mean free path is calculated as follows:

$$\varepsilon_m = aeE\lambda \tag{34}$$

Eqq. (28) and (29) lead to Eq. (2).

App. 2

Radius r_e in Eq. (11) has been induced by Raether, who assumed the electron velocity in the radial and axial direction of the avalanche to be propotional to the electric field, then

$$\frac{dr_e}{dt} = v_r = kE_-, \quad \frac{dz}{dt} = \nu_- = kE \tag{35}$$

$$E_{-} = \frac{e\exp(dz)}{Kr_e^2} \tag{36}$$

These two equations lead to Eq. (11). In Eqq. (30) and (31), E_{-} expresses the field caused by all the electrons in the gap space, but after the electron density reaches and exceeds the threshold value, and some electrons begin to leave from the avalanch, the electric field caused the space charge must be varied, then the electric field (E'_{-}) in this instance is obtained as follows:

$$E'_{-} = \frac{4e\pi n_0 r_0^3}{3K} \tag{37}$$

Eqq. (33) and (35) lead to Eq. (12).

App. 3

Sharbough obtained the following equation by the same method as Eq. (22),

$$\alpha = \frac{1}{\lambda}\exp\left(-\frac{\varepsilon}{eE\lambda}\right) \tag{38}$$

and for the condition required for the discharge, he assumed the following equation, which is similar to Townsend's method,

$$\alpha d = n \tag{39}$$

where:

n=constant

From these two equations, the following equation is obtained for critical breakdown field:

$$E = \frac{\varepsilon}{e\lambda}(\ln d/n\lambda)^{-1} \tag{40}$$

Then, the relation between $1/E$ and d becomes linear according to his theory, and his experimental result satisfieds Eq. (40). But physical constants such as ε_i and λ which are calculated from Eq. (40) and his experimental results, are inadequate. Then these equations do not seem generally suitable for the breakdown of gases, liquids and solids.

App. 4

From Eq. (25),

$$\left.\begin{aligned} n_i &= \frac{1}{kT_e}\int_{\varepsilon_i}^{\infty}\exp\left(-\frac{\varepsilon}{kT_e}\right)d\varepsilon = \exp\left(-\frac{\varepsilon_i}{kT_e}\right) \\ n_d &= \exp\left(-\frac{\varepsilon_d}{kT_e}\right) - \exp\left(-\frac{\varepsilon_i}{kT_e}\right) \end{aligned}\right\} \tag{41}$$

From Eqq. (24) and (30)

$$\exp\left(-\frac{\varepsilon_i}{kT_e}\right)=\exp\left(-\frac{k_1}{E}\right) \tag{42}$$

And, if $\varepsilon_i/\varepsilon_d$ equals 3, then

$$\exp\left(-\frac{\varepsilon_d}{kT_e}\right)=\exp\left(-\frac{k_1}{3E}\right) \tag{43}$$

$\bar{v}_i$ is obtained from Eq. (27) as follows:

$$\bar{v}_i=\frac{1}{n_i kT_e}\int_{\varepsilon_i}^{\infty} v\exp\left(-\frac{mv^2}{2kT_e}\right)d\varepsilon$$

And if

$$kT_e\equiv\frac{1}{2}mC_m^2,\quad \alpha\equiv\frac{v}{C_m},\quad \alpha_i\equiv\frac{v_i}{C_m}$$

are deformed, then

$$\bar{v}_i=\frac{m}{n_i kT_e}C_m^3\int_{\alpha_i}^{\infty}\alpha^2\varepsilon^{-\alpha^2}d\alpha=\frac{m}{n_i kT_e}C_m^3\frac{\sqrt{\pi}}{4}\left\{\frac{2}{\sqrt{\pi}}\alpha_i\varepsilon^{-\alpha}+1-erf(\alpha_i)\right\} \tag{44}$$

In the case where, $\alpha_i>1.5$, erf (α_i) about equals 1.0 as is well known, then, Eq. (44) is expressed approximately as follows:

$$\bar{v}_i\fallingdotseq\frac{1}{n_i}\sqrt{\frac{2}{m}}\sqrt{\varepsilon_i}\exp(-\alpha_i^2) \tag{45}$$

Likewise, when $\alpha_i>1.5$, $\bar{v}_d$ is expressed as follow:

$$\bar{v}_d=\frac{1}{n_d}\sqrt{\frac{2}{m}}\{\sqrt{\varepsilon_d}\exp(-\alpha_d^2)-\sqrt{\varepsilon_i}\exp(-\alpha_i^2)\} \tag{46}$$

where;

$$\alpha_d=\frac{v_d}{C_m},\quad \varepsilon_d=\frac{1}{2}mv_d^2$$

From its definition, α_i is

$$\alpha_i^2=\frac{\varepsilon_i}{ae\lambda E}=\frac{k_1}{E} \tag{47}$$

Using α_i, Eqq. (41)~(43) lead to following equations,

$$\begin{aligned} n_i &= \exp(-\alpha_i^2)\\ n_d &= \exp\left(-\frac{1}{3}\alpha_i^2\right)-\exp(-\alpha_i^2)\end{aligned} \tag{48}$$

For the calculation of α_i, the value of E is chosen from the experimental values, the value of k_1 obtained in section (3.2) and Eq. (47) are used, and the values of α_i used in this paper are shown in Table 5.

Table 5. Values of α_i obtained from experimental values.

	k_1 (V/cm)	E (V/cm)	α_i
Heptan	1.06×10^{7}	1.8×10^{6}	2.43
Chloroprene	2.67×10^{5}	1.1×10^{5}	1.61

App. 5

Since the relation of $1/E$ and $\ln d$ is nonlinerar, the value calculated by Scharbaugh's equation does not consist with the one by Eq. (23). The difference in the both equation is clarified here.

First, using the constant defined in Section 3.2., Eq. (23) is rewritten as follows,

$$\exp\left(-\frac{k_1}{E}\right)=\frac{k_2}{E}+\frac{k_3}{E} \tag{49}$$

Table 6. Relation between E and d calculated by Eq. (23) compared with one by Scharbaugh's equation.

$1/E$ (cm/kV)*	$1/d$ (1/cm)*	d (MIL)*	d (MIL)**
3.20×10^{-4}	0.3938×10^{4}	0.100	0.100
5.32×10^{-4}	0.388×10^{3}	1.014	1.000
7.03×10^{-4}	0.364×10^{2}	10.810	10.000

* values calculated by Eq. (23)
** " by Scharbaugh's equation

The value of d to $1/E$ is calculated by the above equation, and compiled in Table 6. It is deduced from these results that the difference between Schabaugh's equation and Eq. (49) is too small to be indicated in a figure. Then, the calculated value by Eq. (49) is not plotted in Fig. 7.