Dynamic Characteristics of a Pneumatic Conveying Pipe Line

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By

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Measurement and control of the solid-gas mixture ratio in two phase flow were reported in the previous papers using a solid-gas two phase flowmeter[1,2]. However, dynamic characteristics of a pneumatic conveying pipe line could not be discussed because of large time-lags of primary means, manipulating device, etc.

In this paper, therefore, pressure measuring elements having small time-lags are used and pressure changes are taken as photographs by use of a synchroscope. Then, dynamic responses of pressure in a pneumatic conveying pipe line are discussed for a step change of flow rates of air and solids respectively.

It is concluded that time-lags of the pressure responses can be neglected for change of air flow rate alone. There is a relation between pressure P and volumetric flow rate of air V as follows;

$$P = ZV^2 = (Z^a + Z^s)V^2$$

where Z^a and Z^s are impedances of a pipe line for air and solid flow respectively. The experiment shows that Z^a is a constant irrespective of air flow rate, and that Z^s is proportional to the hold-up of solids in a pipe line.

1. Introduction

Automatic control techniques have been widely used in process industries, and process dynamics become important for process control system design. However, the techniques do not seem to be applied very much to the field of solid particle handlings, because no adequate device is available to measure and to manipulate the flow rate of solid particles. Flow rate measurement and control have been performed in the previous reports[1,2] for the solid-gas two phase flow. As one of a series of these works, this paper treats the dynamic characteristics of a pneumatic conveying pipe line for flow of solid-gas mixtures.

2. Experimental Apparatus

Experiments are conducted using a pneumatic conveying system of suction type as shown in **Fig. 1** and pressure responses are measured at the two

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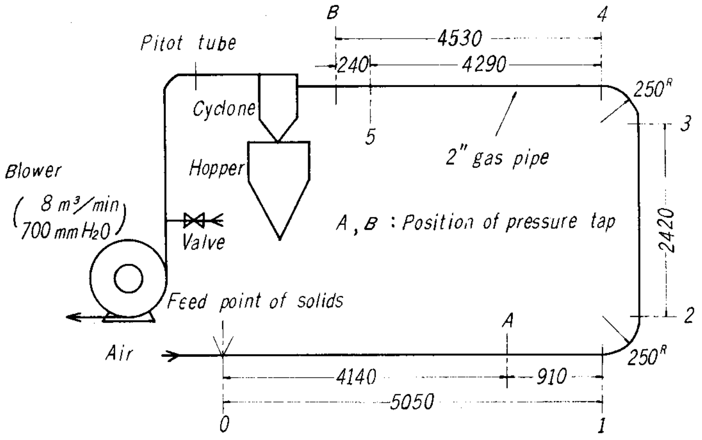

Fig. 1. Experimental apparatus.

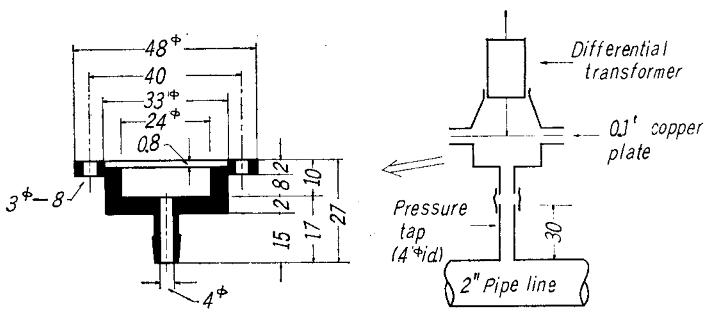

Fig. 2. Pressure measuring element used in the experiment.

points of A and B. **Fig. 2** shows a pressure measuring element used in the experiment. This element consists of a thin copper plate and a differential transformer. Deflections of the copper plate caused by pressure changes are measured by the differential transformer and pressure responses are taken as photographs by use of a synchroscope.

Dynamic characteristics of the pipe line are taken as pressure responses

Table 1. Solid particles used in the experiment.

Solid particles	50 wt. % median diameter [mm]	50 wt. % mean settling velocity [m/s]	n^* [−]	True density [g/cm³]
Millet	1.88	5.41	—	1.41
Aluminium No. 1	0.98	7.78	3.3	2.56
Aluminium No. 2	0.70	6.07	7.0	2.56
Silica sand No. 2	0.37	3.10	4.5	2.74

Note : $*$=Slope of the line written on a RRS size distribution diagram.

caused by a step change of flow rates of air and solids. In this paper, the step changes are given manually so that dynamic characteristics of the pipe line can be measured without any time-lags of manipulating devices such as an electromagnetic vibrating feeder. Properties of solid particles used are shown in **Table 1** and their motion belongs to the region of Newton's law.

3. Velocity and Residence-time of Solid Particles

Velocity and residence-time of solid particles are calculated in this section, which are required for dynamic characteristics study of the pneumatic conveying pipe line.

a) Lower horizontal pipe line

Motion of solid particles can be represented by the following equation for a horizontal pneumatic pipe line and for the region of Newton's law[3].

$$\frac{1}{g}\frac{dv}{dt}=\left(\frac{u-v}{v_t}\right)^2-\frac{\lambda_s v^2}{2gD}$$

Using initial condition of $v=0$ at $t=0$, the above equation can be solved as follows;

$$v=\frac{Au}{B-A}\cdot\frac{e^{2ABut}-1}{1+\frac{B+A}{B-A}e^{2ABut}} \tag{1}$$

where

$$A\equiv\sqrt{g/v_t^2},\quad B\equiv\sqrt{\lambda_s/(2D)},\qquad (B=\sqrt{\zeta}A)$$

Transported distance of solid particles along a pipe line is given by the following equation.

$$l=\int_0^t v\,dt \tag{2}$$

Then, the integration can be performed putting Eq. (1) into the above equation as follows;

$$(B+A)Au\left[t+\frac{(B-A)l}{Au}\right]=\ln\frac{1+\frac{B+A}{B-A}e^{2ABut}}{1+\frac{B+A}{B-A}} \tag{3}$$

Therefore, with the given values of A, B, and u, residence-time t of solid particles can be calculated from Eq. (3) for a given value of l, and then velocity of solid particles is obtained by Eq. (1).

b) Lewer curved pipe

For a curved pipe from horizontal to vertical direction, velocity of solid particles is given by the following equation[3].

$$v = e^{-\beta\theta}\sqrt{v_i^2+\frac{2R_c g}{4\beta^2+1}\left[(2\beta^2-1)-e^{2\beta\theta}\{(2\beta^2-1)\cos\theta+3\beta\sin\theta\}\right]} \tag{4}$$

Solid velocity at the inlet of the curved pipe, v_i, can be obtained by Eq. (1). With known values of β and R_c, therefore, solid velocity in a curved pipe can be calculated from Eq. (4) for a given angle θ of direction change.

c) Vertical pipe line

Solid velocity in a vertical pipe line can be calculated by the following equation for the region of Newton's law[4].

$$s = \left[\frac{1}{2(1-\zeta)}+\frac{1}{2(1-\zeta)\sqrt{1-(1-\zeta)(1-c^2)}}\right]\ln\left|\frac{(1-\zeta)\phi-1-\sqrt{1-(1-\zeta)(1-c^2)}}{(1-\zeta)\phi_i-1-\sqrt{1-(1-\zeta)(1-c^2)}}\right|$$
$$+\left[\frac{1}{2(1-\zeta)}-\frac{1}{2(1-\zeta)\sqrt{1-(1-\zeta)(1-c^2)}}\right]\ln\left|\frac{(1-\zeta)\phi-1+\sqrt{1-(1-\zeta)(1-c^2)}}{(1-\zeta)\phi_i-1+\sqrt{1-(1-\zeta)(1-c^2)}}\right| \tag{5}$$

Solid-air velocity ratio ϕ_i at the inlet of vertical pipe can be obtained from the solid velocity given by Eq. (4). Therefore, with known values of ζ and c, the velocity ratio can be calculated by the above equation for a given value of s. Then, the relation between v and l is obtained.

d) Upper curved pipe

For a curved pipe from vertical to horizontal direction, solid velocity is given by the following equation[3].

$$v = e^{-\beta\theta}\sqrt{v_i^2+\frac{2R_c g}{4\beta^2+1}\left[3\beta+e^{2\beta\theta}\{(2\beta^2-1)\sin\theta-3\beta\cos\theta\}\right]} \tag{6}$$

Solid velocity at the inlet of curved pipe, v_i, can be obtained from Eq. (5). Therefore, with known values of R_c and β, solid velocity in the curved pipe can be calculated from Eq. (6) for a given angle θ of direction change.

e) Upper horizontal pipe line

Solid velocity in a horizontal pneumatic pipe line can be calculated by the following equation for a given initial condition of $\phi=\phi_i$ at $s=0$ and for the region of Newton's law[4].

$$s+s_* = \frac{1}{2\sqrt{\zeta}}\left[\frac{\ln|1-(1-\sqrt{\zeta})\phi|}{1-\sqrt{\zeta}}-\frac{\ln|1-(1+\sqrt{\zeta})\phi|}{1+\sqrt{\zeta}}\right] \tag{7}$$

where

$$s_* \equiv \frac{1}{2\sqrt{\zeta}}\left[\frac{\ln|1-(1-\sqrt{\zeta})\phi_i|}{1-\sqrt{\zeta}}-\frac{\ln|1-(1+\sqrt{\zeta})\phi_i|}{1+\sqrt{\zeta}}\right]$$

Velocity ratio ϕ_i at the inlet of the pipe line can be obtained from solid velocity at the outlet of curved pipe given by Eq. (6). Therefore, $s\ (=gl/v_i^2)$

can be calculated for several given values of ϕ ($=v/u$), and the relation v and l is obtained.

According to the above procedure, solid velocity is calculated and shown in **Fig. 3** for several points of pneumatic pipe line given in Fig. 1. Empirical values used in the calculation are shown in **Table 2.**

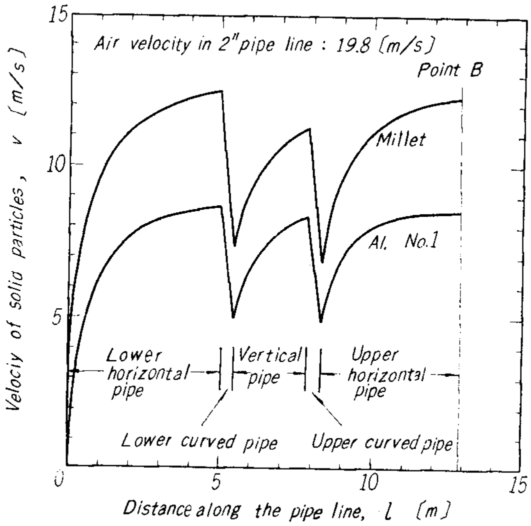

Fig. 3. Calculated particle velocity in the pneumatic conveying pipe line (c.f. Fig. 1).

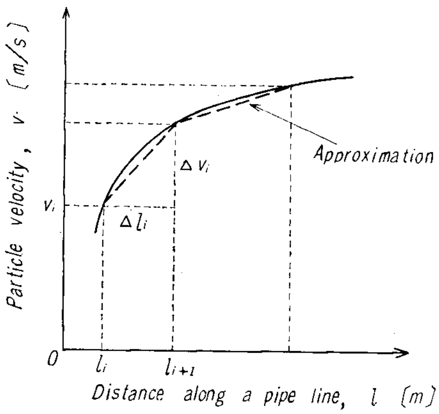

Fig. 4. Approximate calculation method for residence-time of solid particles.

Table 2. Empirical values used in the calculation of particle velocity.

Solid particles	Terminal settling velocity v_t [m/s]	$\zeta=\lambda_s v_t^2/(2gD)$ [−] Horizontal pipe	$\zeta=\lambda_s v_t^2/(2gD)$ [−] Vertical pipe	β [−]
Millet	5.41	0.336	0.21	0.336
Aluminium No. 1	7.78	1.71	0.77	0.33
Aluminium No. 2	6.07	0.845	0.065	0.33
Silica sand No. 2	3.10	0.794	0.23	0.61

Note : 1) $u=19.8$ [m/s], $R_c=0.277$ [m]
2) v_t : Empirical value obtained by air elutriation of particles

Residence-time of solid particles can be obtained by Eq. (3) for the lower horizontal pipe line, but it can not be calculated analytically for the other pipe lines. Therefore, the following approximate calculation method is used for this purpose. As is shown in **Fig. 4,** the solid velocity curve is approximated by small segments as follows;

$$v=\frac{dl}{dt}=\frac{\Delta v_i}{\Delta l_i}(l-l_i)+v_i$$

Integration of this equation over the range of $l_i \sim l_{i+1}$ gives residence-time of solid particles for the section of Δl_i as follows;

$$\Delta t_i = \frac{\Delta l_i}{\Delta v_i} \ln \left| 1 + \frac{\Delta v_i}{v_i} \right| \tag{8}$$

Therefore, summation of Δt_i gives the approximate residence-time of solid particles in a pneumatic conveying pipe line as follows;

$$t \fallingdotseq \sum_i \Delta t_i \tag{9}$$

Eqs. (3) and (9) are used in the calculation of residence-times for the lower horizontal pipe line and for the others respectively. The calculated results are shown in **Table 3.**

Table 3. Residence-time of solid particles for the sections of O–A and O–B (calculated results).

Time	Millet	Al. No. 1	Al. No. 2	Silica No. 2
t_A [sec]	0.44	0.60	0.50	0.42
t_B [sec]	1.27	1.69	1.43	1.29

Note : Air velocity in the 2″ pipe line is 19.8 [m/s].

4. Experimental Results and Discussions of the Dynamic Characteristics

Fig. 5 shows a step response of pressure measuring element used in the experiment. As is seen from the figure, the element has approximately a small time constant of about 0.11 sec. Step changes of air flow rate are given manually and pressure responses are shown in Fig. 5. There is only a little difference between the responses obtained at the two positions of A and B. Then, it is concluded that time-lags of the pressure responses are negligibly small for change of air flow rate alone. Time-lags of 0.2~0.3 sec

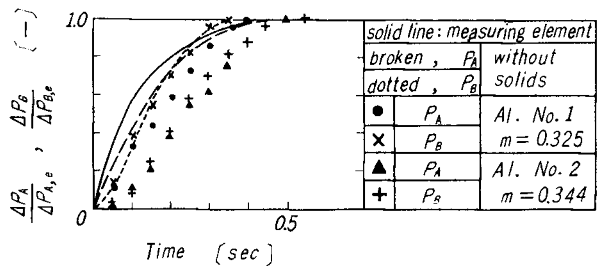

Fig. 5. Pressure responses for step change of air flow rate (air velocity: u=19.2~23.2 [m/s]; m: mixture ratio at u=20 [m/s])

appear in Fig. 5 because of incorrect step changes of air flow rate caused by manual operation and of dynamic characteristics of extended elements such as a blower. In this paper, dynamic characteristics of a pneumatic conveying pipe line are discussed neglecting time-lags for change of air flow rate alone.

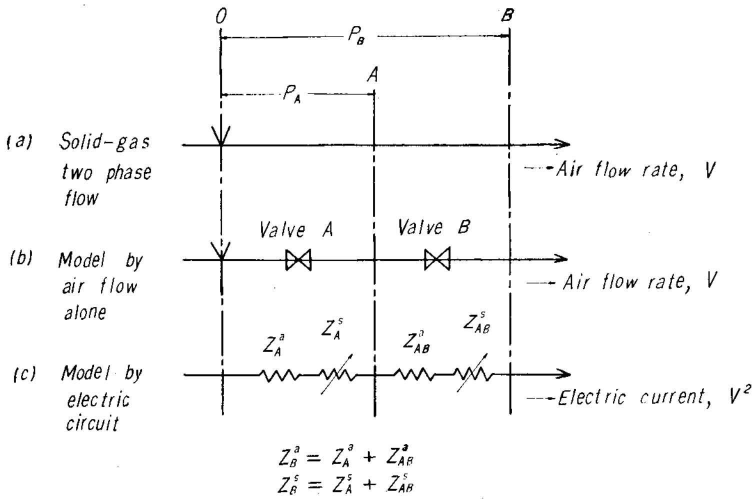

Fig. 6. Dynamic characteristics models of a pipe line flowing solid-gas mixtures.

Fig. 6 shows dynamic characteristics models of the pipe line. (*a*) is a pipe line flowing solid-gas mixtures. Generally, the following relation exists between pressure of a pipe line and volumetric flow rate of air.

$$P = ZV^2 \tag{10}$$

where Z is a pipe resistance, of which value is assumed to be sum of Z^a and Z^s for flows of air and solids respectively. Then, the following equations are obtained for the sections of 0–A and 0–B.

$$P_A = Z_A V^2 = (Z_A^a + Z_A^s) V^2 \tag{11}$$

$$P_B = Z_B V^2 = (Z_B^a + Z_B^s) V^2 \tag{12}$$

where Z_A^a and Z_B^a are constants irrespective of air flow rate. The value of Z_A^s and Z_B^s increases with solid flow rate, that gives rise to decrease the volumetric flow rate of air. Therefore, dynamic characteristics of the pipe line can be represented as a model by air flow alone as shown in Fig. 6 (b), where valve resistances are equivalent to the values of Z_A^s and Z_{AB}^s. Fig. 6 (c) is an electric circuit model simulated to Eqs. (11) and (12), regarding V^2 as electric current.

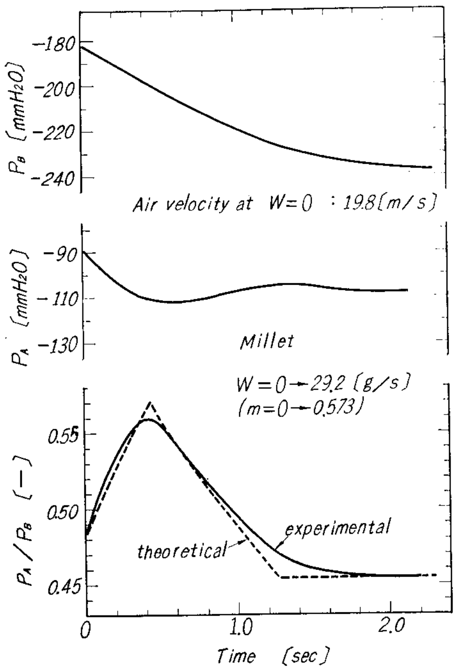

Fig. 7. Pressure response for a step feed of solids (millet).

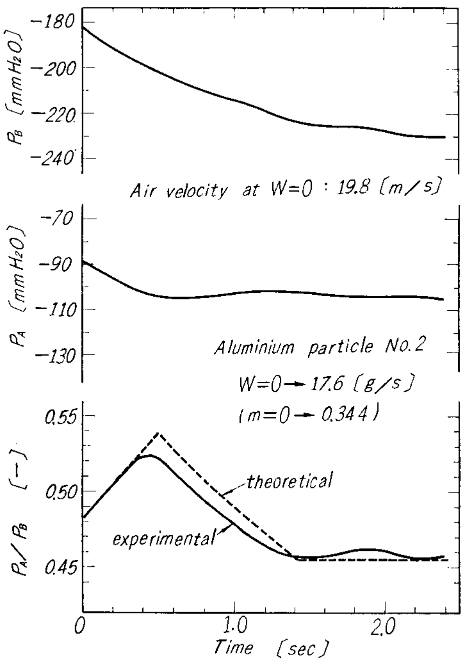

Fig. 8. Pressure response for a step feed of solids (Aluminium particle No. 2).

Pressure responses at the points of A and B are shown in **Figs. 7** and 8 for a step feed of solid particles. As is seen from these figures, pressure responses at the point of A have overshoots, which can be easily explained by use of Fig. 6 (b). For a step feed of solids, the resistance Z_A^s increases for the time interval of $0 \sim t_A$. This is equivalent of throttling the valve A, and then P_A decreases. After the time t_A is passed, the opening of valve A remains constant and pipe resistance beyond the point A increases. This is also equivalent to throttling the valve B and P_A increases. Therefore, pressure response at the point A, which is relatively close to the feed point, shows large overshoot. As mentioned above, since pressure responses caused by flow rate, change of solids are affected by the change of air flow rate; Eq. (11) is divided by Eq. (12) in order to eliminate the effect of air flow rate, and the pressure ratio is used in the following discussions.

$$\frac{P_A}{P_B} = \frac{Z_A^a + Z_A^s}{Z_B^a + Z_B^s} \tag{13}$$

Pipe resistances caused by solid flow can be assumed as variables which are directly proportional to the hold-up of solid particles. Then, the following equations are obtained for a step feed of solid particles into the pipe line,

$$\begin{aligned} Z_A^s &= k_A W t \ : \ 0 \leq t \leq t_A \\ &= k_A W t_A : \ t \geq t_A \end{aligned} \Bigg\} \tag{14}$$

$$\begin{aligned} Z_B^s &= k_B W t \ : \ 0 \leq t \leq t_B \\ &= k_B W t_B : \ t \geq t_B \end{aligned} \Bigg\} \tag{15}$$

where k_A and k_B are constants, and t_A and t_B are residence-times of solid particles for the sections of 0–A and 0–B respectively. Substituting the above two equations into Eq. (13), the pressure ratio becomes as follows;

$$\left.\begin{aligned} \frac{P_A}{P_B} &= \frac{Z_A^a + k_A W t}{Z_B^a + k_B W t} \ : \ 0 \leq t \leq t_A \\ &= \frac{Z_A^a + k_A W t_A}{Z_B^a + k_B W t} \ : \ t_A \leq t \leq t_B \\ &= \frac{Z_A^a + k_A W t_A}{Z_B^a + k_B W t_B} \ : \ t \geq t_B \end{aligned}\right\} \tag{16}$$

Empirical values of pipe resistances are shown in **Table 4.** With these values in hand, step responses of P_A/P_B are calculated from Eq. (16) and shown in

Table 4. Empirical values of pipe resistance.

Solid particles	Pipe resistance by hold-up of solids [mmH$_2$O/(m^3/min)^2]		Pipe resistance by unit hold-up of solids [mmH$_2$O/(m^3/min)^2 g-solid]	
	Z_A^s	Z_B^s	k_A	k_B
Millet	4.46	11.4	0.347	0.307
Aluminium No. 1	2.81	8.2	0.283	0.293
Aluminium No. 2	2.94	8.0	0.335	0.319
Silica sand No. 2	4.24	13.0	0.613	0.612

Note : $Z_A{}^a = 12.16$, $Z_B{}^a = 25.2$ [mmH$_2$O/(m^3/min)^2].

good agreements with experimental results as is given in Figs. 7 and 8. Alternatively, the pressure ratio becomes a step change of solid flow rate from a certain value to zero as follows;

$$\left.\begin{aligned} \frac{P_A}{P_B} &= \frac{Z_A^a + k_A W (t_A - t)}{Z_B^a + k_B W (t_B - t)} \ : \ 0 \leq t \leq t_A \\ &= \frac{Z_A^a}{Z_B^a + k_B W (t_B - t)} \ : \ t_A \leq t \leq t_B \\ &= \frac{Z_A^a}{Z_B^a} \ : \ t \geq t_B \end{aligned}\right\} \tag{17}$$

Calculated results of Eq. (17) are shown in **Figs. 9** and **10.** As is seen from these figures, the calculated results show good agreements with experimental ones. Therefore, it is justified that dynamic characterictics of a pneumatic conveying pipe line can be represented by the electric circuit model having

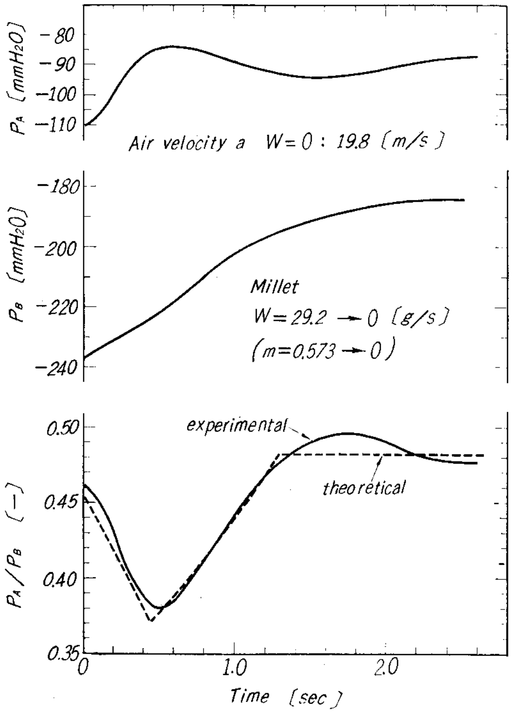

Fig. 9. Pressure response for a step change of solid flow rate (millet).

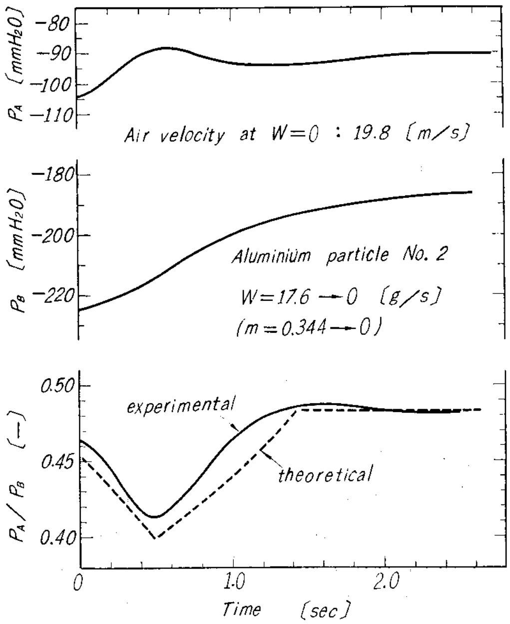

Fig. 10. Pressure response for a step change of solid flow rate (Aluminium particle No. 2).

variable resistances as shown in Fig. 6. At the neighborhood of peak and equilibrium points of the pressure ratio curves, however, the calculated results show little differences from experimental ones because of size distribution of solid particles and time-lags of the pressure measuring elements.

A pressure ratio response caused by an impulsive feed of solid particles into the pipe line is shown in **Fig. 11.** This response consists of two curves, because two photographs taken by synchroscope are connected with each other shifting their time scales, but a smooth curve can not be obtained. For an impulsive feed of solid particles, the pressure ratio becomes as follows;

$$\left.\begin{aligned} \frac{p_A}{p_B} &= \frac{Z_A^a}{Z_B^a} &&: \ t<0 \\ &= \frac{Z_A^a+kH}{Z_B^a+kH} &&: \ 0\leq t\leq t_A \\ &= \frac{Z_A^a}{Z_B^a+kH} &&: \ t_A\leq t\leq t_B \\ &= \frac{Z_A^a}{Z_B^a} &&: \ t\geq t_B \end{aligned}\right\} \qquad (18)$$

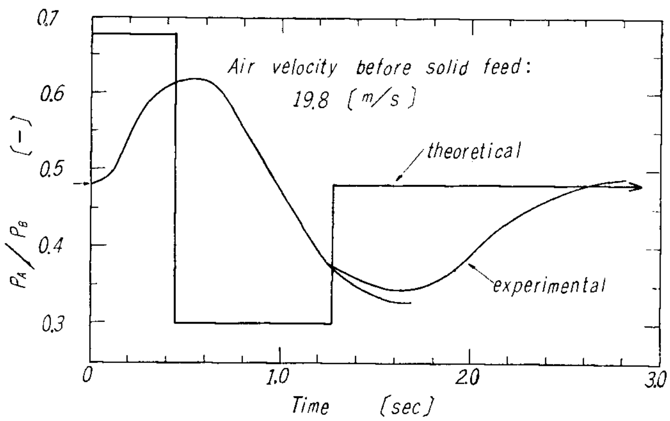

Fig. 11. Pressure response for 50 g impulsive feed of millet into the pipe line.

where k is a constant and H is weight of solid particles which are fed into the pipe line. For $k=k_B$ and $H=50$ g, Eq. (18) is calculated using empirical values in Table 4 and shown in Fig. 11. As is seen from the figure, the calculated results show a similar tendency to the experimental one, but there is large time difference between two curves. This reason seems to be that the amount of impulsive feed is too much to get a strict impulse response because of accumulation of solid particles on the bottom of the feed point. In case of the step response given in Fig. 7, hold-up of solids is $H=Wt_B=37.1$ g for the section of 0–B. Therefore, impulsive feed of 50 g into the pipe line seems to be too much for this system. On the other hand, the pressure response can not be measured accurately in case of a small amount of the impulsive feed.

5. Conclusion

Dynamic characteristics of a pneumatic conveying pipe line are discussed by use of pressure measuring elements having small time-lags. It is concluded that time-lags of the pressure response can be neglected for change of air flow rate alone. Pressure responses caused by change of solid flow rate depend on the change of pipe resistances which are proportional to the hold-up of solid particles. Therefore, the dynamic characteristics of a pneumatic conveying pipe line can be represented by an electric circuit model as shown in Fig. 6.

Notations

$c=v_t/u$		[−]
D	: diameter of a pipe	[m]
g	: acceleration of gravity	[m/s^2]

H	:	hold-up of solid particles	[g]
k	:	constant	[$mmH_2O/(m^3/min)^2$ g-solid]
l	:	distance along a pipe line	[m]
m	:	solid-air mixture ratio (=weight flow rate of solids/ weight flow rate of air)	[−]
P	:	pressure	[mmH_2O]
ΔP	:	deviation of pressure from its steady state value	[mmH_2O]
R_c	:	radius of curvature of a curved pipe	[m]
s	:	equivalent length ($=gl/v_t^2$)	[−]
t	:	time	[sec]
t_A, t_B	:	residence-time of solid particles for the sections of 0–A and 0–B respectively	[sec]
u	:	air velocity in a pipe line	[m/s]
V	:	volumetric flow rate of air	[m^3/min]
v	:	velocity of solid particles	[m/s]
v_t	:	mean settling velocity of solid particles	[m/s]
W	:	flow rate of solid particles	[g/s]
Z^a, Z^s	:	pipe resistance caused by air and solid flow respectively	[$mmH_2O/(m^3/min)^2$]
β	:	friction factor of solids flowing on a smooth plane (defined by equation of motion)	[−]
$\zeta = \lambda_s v_t^2/(2gD)$			[−]
θ	:	angle of direction change of a curved pipe	[deg.]
λ_s	:	friction factor of solid flow through a pipe (defined by pressure drops)	[−]
ϕ	:	solid-air velocity ratio ($=v/u$)	[−]

Subscript

A, B	:	for sections of 0–A and 0–B respectively
a	:	air flow
e	:	final value
s	:	solid flow

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