Particle Size Estimation from Pressure Drops of a Pneumatic Conveying Pipe Line

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By

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Size measurement of solid particles has been generally performed for a test sample. In such a conventional measurement, however, it is time-consumable and not easy to get test samples from a process line. A method of particle size estimation from pressure drops of a pneumatic conveying pipe line is herein discussed for the flow of Newton's region.

According to the previous report[1], pressure drops of a pneumatic conveying pipe line have been able to be calculated theoretically using empirical values of λ_s and v_t, i.e. friction factor of solid flow through a pipe and mean settling velocity of solid particles respectively. In this paper, λ_s and v_t are obtained from the relations between solid-gas mixture ratio and pressure drops of an accelerating and a constant velocity sections, where the accelerating section has to start at the position of $v=0$ such as the feed point of solid particles. Then the mean particle size equivalent to a sphere is calculated from v_t and shows good agreement with experimental results.

1. Introduction

Using notations of A, B, and C respectively for a physical property of solid particles (i.e. particle size, friction factor of solid flow through a pipe, etc.), the each flow rate of solid-gas mixtures, and the pressure drop of a pipe line, a method of theoretical calculation for the pressure drops of a pneumatic conveying pipe line can be represented symbolically by the following equation:

$$A,\ B \longrightarrow C$$

This equation shows that "C" is obtained from the values of "A" and "B". Similarly, a theoretical method of the flow rate measurement of solid-gas mixtures can be represented as follows;

$$A,\ C \longrightarrow B$$

As is seen from the above equations, there is an alternate relation between

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A, B, and C as follows;

$$B, C \longrightarrow A$$

This equation suggests a method which gives the physical property of solid particles from the values of flow rate and pressure drops.

According to this method, a particle size estimation can be performed in a pneumatic conveying pipe line.

2. Particle Size Estimation from Pressure Drops

Pressure drop, caused by solid flow, of a horizontal pneumatic pipe line can be obtained from the following equation for a constant velocity section and for the region of Newton's law[1].

$$\Delta P'_s \Big/ \left(\frac{\gamma_a u^2}{g}\right) = Km \tag{1}$$

where

$$K \equiv \frac{\zeta}{1+\sqrt{\zeta}} \cdot s \tag{2}$$

Therefore, experimental data relating $\Delta P'_s$ and m give the value of K in Eq. (2). In this case, the value of air velocity u should be measured by other method.

Alternatively, pressure drop, caused by solid flow, of a horizontal pipe line can be calculated from the following equation for an accelerating section of solid particles and for the region of Newton's law[1].

$$\Delta P_s = 2m(\Phi' + \zeta \phi_i s) \frac{\gamma_a u^2}{2g} \tag{3}$$

Taking the feed point of solid particles as the inlet of the section, $\phi_i = 0$ and the above equation becomes as follows;

$$\Delta P_s \Big/ \left(\frac{\gamma_a u^2}{g}\right) = \Phi m \tag{4}$$

where

$$\Phi \equiv \frac{\sqrt{\zeta}}{2(1-\sqrt{\zeta})^2} \ln|1-(1-\sqrt{\zeta})\phi| - \frac{\sqrt{\zeta}}{2(1+\sqrt{\zeta})^2} \ln|1-(1+\sqrt{\zeta})\phi| + \frac{\phi}{1-\zeta} \tag{5}$$

According to Eq. (4), experimental data relating ΔP_s and m give the value of Φ in Eq. (5).

For the motion of solid particles in a horizontal pipe line, the following equation is given by Weidner[2] in the region of Newton's law;

$$s = \frac{1}{2\sqrt{\zeta}} \left[\frac{\ln|1-(1-\sqrt{\zeta})\phi|}{1-\sqrt{\zeta}} - \frac{\ln|1-(1+\sqrt{\zeta})\phi|}{1+\sqrt{\zeta}} \right] \tag{6}$$

With the experimental values of K and Φ in hand, the three unknown variables (i.e. s, ϕ, and ζ) are determined from Eqs. (2), (5), and (6). However, it is difficult to solve these equations analytically and a graphical method is used below.

(**a**)—The graphical solution of Eq. (6) is shown by the broken lines of Fig. 1 which show the relation between ϕ and ζ for several values of the parameter s.

(**b**)—Similarly, the graphical solution of Eq. (5) is shown by the solid lines of

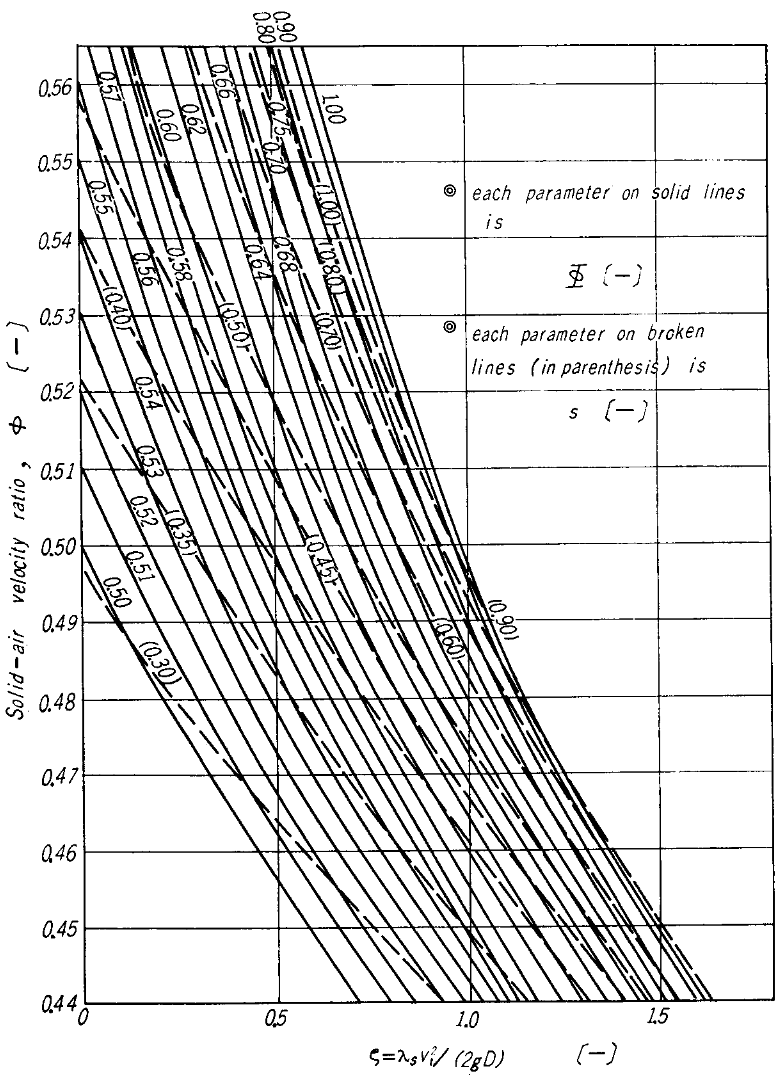

Fig. 1. Calculated results of the relation between ϕ and ζ.

Fig. 1 which show also the relation between ϕ and ζ for several values of the parameter Φ.

(c)—As is seen from Fig. 1, a solid line of Φ=const. gives several intersecting points with the broken lines, and a set of the values of s and ζ corresponds to the each point. Therefore, a relation between s and ζ is obtained as follows;

$$f(s, \zeta) = 0 \tag{7}$$

where the notation of "f" shows a function.

(d)—Alternatively, Eq. (2) gives another relation between s and ζ for an empirical value of K. Then Eqs. (7) and (2) are plotted on the graph at the same time and an intersection of the two curves gives the values of s and ζ as follows;

$$s = s^* = gl/v_t^2 \tag{8}$$

$$\zeta = \zeta^* = \lambda_s v_t^2/(2gD) \tag{9}$$

The two unknowns of λ_s and v_t can be obtained analytically from these equations. Therefore, the particle diameter, equivalent to a sphere, can be calculated from the value of v_t.

In this paper, Φ should be the value for the accelerating section which starts at the point of $v=0$ such as the feed point of solid particles. As is seen from Eq. (8), the variable s appeared in Eqs. (2)-(7) includes the pipe length l. Therefore, the values of K and Φ should be obtained under the condition that the length of the accelerating section is equal to that of the constant velocity section. Fortunately, the pressure drop of a constant velocity section proportionates directly to the pipe length and the data obtained from the section can be easily modified into that for the section which has the same length as an accelerating section. The velocity ratio converges to the value of $\phi=1/(1+\sqrt{\zeta})$ with increasing distance along a pipe line, and if the accelerating section is selected too long, the term of $|1-(1+\sqrt{\zeta})\phi|$ appeared in Eqs. (5) and (6) becomes too small for its effective value to be calculated. Therefore, large error comes out in the calculation. In such a case, pressure drop for an approximately constant velocity section is subtracted from the total pressure drop and the residue is treated as data for a new accelerating section. The detail will be shown later.

3. Experimental Results and Discussions

Experiments are conducted by use of a pneumatic conveying system of suction type as shown in Fig. 2, and the pressure drops are measured at the

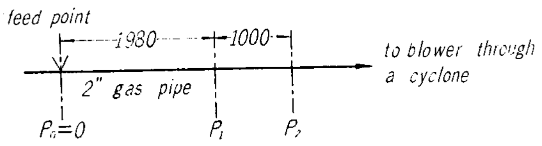

Fig. 2. Schematic diagram of the pipe line.

sections of P_1-P_2 and P_0-P_1. Experimental results are approximated by the lines which pass through the origin as shown in Fig. 3. Then $K'=0.48$

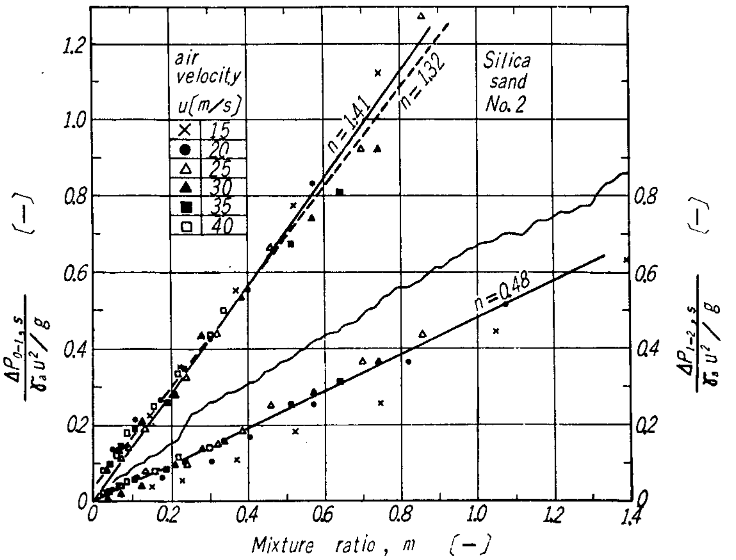

Fig. 3. Pressure drops caused by solid flow through an accelerating and a constant velocity sections.

and $\Phi'=1.41$ are obtained from the slope of the lines. As is seen from Fig. 2, however, the length of the section P_1-P_2 is not equal to that of the section P_0-P_1. Therefore, the values of K' and Φ' can not be used in the calculation of particle size estimation. In such a case, $K'=0.48$ obtained from the section P_1-P_2 of $l=1$ m is modified into the value for $l=1.98$ m which is the same length as the section P_0-P_1, and $K=0.48\times1.98=0.95$ and $\Phi=\Phi'=1.41$ should be used in the calculation. In this paper, however, an alternate following method is used to obtain the values of K and Φ in order to minimize the error of calculation.

Fig. 4 shows a calculated result of the relation between distance along a pipe line and solid-air velocity ratio. As is seen from this figure, the curve includes the relatively long length which can be treated approximately as

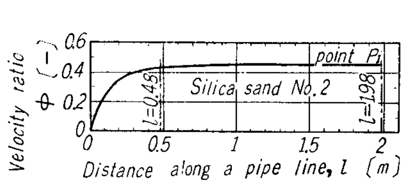

Fig. 4. Velocity of solid particles in a pipe line.

a constant velocity section and the curve is devided into two sections at the point of $l=0.48$ m. The right side can be regarded as a constant velocity section within the accuracy of 6%. Then, the left side section is treated as a new accelerating section. Hence, K' and Φ' obtained from the sections of P_1-P_2 and P_0-P_1 should be corrected to the values for $l=0.48$ m as follows;

$$K = 0.48K' = 0.48\times 0.48 = 0.23$$
$$\Phi = \Phi' - (1.98-0.48)K'$$
$$= 1.41 - 1.5\times 0.48 = 0.69$$

For the solid line of $\Phi=0.69$, Fig. 1 gives the relation between s and ζ as shown in a solid line of Fig. 5. Alternatively, Eq. (2) gives a broken line of Fig. 5 for the value of $K=0.23$. Then $s^*=0.66$ and $\zeta^*=0.63$ are obtained from the intersection of the two curves. Substituting these values into Eqs. (8) and (9), $v_t=2.62$ m/s and $\lambda_s=0.0953$ are obtained as shown in Table 1. This table also shows other results obtained for several particles used in the experiment. As is seen from Fig. 3, the experimental points obtained from the section P_0-P_1 do not exactly show

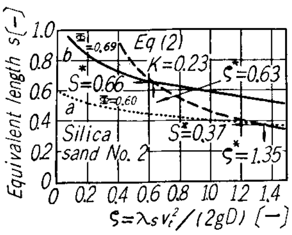

Fig. 5. Graphical solution of s and ζ.

Table 1. Experimental results.

Solid particles	approximate lines to determine Φ (c.f. Fig. 3)	calculated results obtained from pressure drops: mean settling velocity v_t [m/s]	calculated results obtained from pressure drops: mean particle diameter D_p [mm]	calculated results obtained from pressure drops: friction factor of solid flow λ_s [−]	empirical values obtained by air elutriation method: mean settling velocity v_t [m/s]	empirical values obtained by air elutriation method: mean particle diameter D_p [mm]	empirical values obtained by air elutriation method: friction factor of solid flow λ_s [−]
Aluminium No. 1	*a*	7.45	0.930	0.0318	7.78	0.983	0.0326
	b	4.61	0.576	0.0238			
Aluminium No. 2	*a*	5.46	0.683	0.0256	6.07	0.760	0.0272
	b	3.94	0.493	0.0214			
Silica sand No. 2	*a*	3.56	0.426	0.110	3.55	0.425	0.110
	b	2.62	0.314	0.0953			
Silica sand No. 4	*a*	1.28	0.153	0.0344	1.33	0.159	0.0328
	b	2.60	0.311	0.0434			

Note: The line "*a*" does not pass through the origin.
The line "*b*" passes through the origin.

good agreement with the solid line which passes through the origin, because of the end effect of feed point. Such a phenomenon appears also in case of the other particles. Therefore, the experimental points are tried to be approximated by the broken line of Fig. 3 which does not pass through the origin, and the value of $\Phi=0.60$ is obtained from the slope of the line. Then the dotted curve "a" of Fig. 5 is obtained in place of the solid curve "b". By use of the similar calculating method to stated before, λ_s and v_t are obtained again and show good agreement with the experimental results as given in Table 1. Particle sizes in the table are calculated from the alternate values of v_t as spheres. Experimental values of v_t are obtained by air elutriation method of particles. Empirical values of λ_s are calculated from those of v_t and the pressure drop caused by the solid flow through the section P_1-P_2.

Experimental points obtained under the condition of $u=10$ m/s show large deviation from the approximated line and they are omitted in Fig. 3. Therefore, air velocity of more than 15 m/s should be required for the particle size estimation. Using the value of ϕ for a constant velocity section, Reynolds numbers, $Re=D_p(u-v)/\nu_a=D_p u(1-\phi)/\nu_a$, are calculated and shown in Table 2.

Table 2. Reynolds numbers for solid flow
$Re=D_p(u-v)/\nu_a$

Solid particles	air velocity in a 2″ pipe line		$Re_t=\dfrac{D_p v_t}{\nu_a}$
	10 [m/s]	40 [m/s]	
Aluminium No. 1	369	1475	510
Aluminium No. 2	208	835	308
Silica sand No. 2	146	585	100.5
Silica sand No. 4	17.2	68.6	14.1

As is seen from the table, the flows of Silica sand No. 2 and No. 4 belong to the region of Allen's law, to which the particle size estimation method stated here can not be applied. But they are treated as flows of Newton's region in this paper and the calculated results shown in the column "a" are in good agreement with the experimental ones even for the case of Silica sand No. 2 and No. 4. The incorrect assumption of Newton's law to Allen's region may be compensated by the empirical values of K and Φ.

Particle size estimation is made possible from the pressure drops caused by solid-gas flow through a constant velocity and an acceleration sections of particles without sampling out of process line, where the accelerating section should be selected from the position of $v=0$. In this paper, therefore, the inlet is selected at the feed point of solid particles. A straight pipe section just behind of a curved pipe may be able to be treated as an accelerating section, but in this case there will arise a problem of segregation in solid flow just behind of the curved pipe.

4. Conclusion

A method of particle size estimation from the pressure drops is discussed in a horizontal pneumatic conveying pipe line, and the mean particle sizes are calculated from the pressure drops of a constant velocity and an accelerating sections of particles without sampling out of a process line. In this case, air velocity should be kept more than $u=15$ m/s. This is an indirect method of particle size measurement through the pressure drops and gives a contribution to automatic measurement in an industrial process.

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Notations

D	:	inside diameter of a pipe	[m]
D_p	:	50 wt. % median diameter of sold particles	[m]
g	:	acceleration of gravity	[m/s^2]
l	:	distance along a pipe line	[m]
m	:	solid-air mixture ratio	[−]
ΔP_s	:	pressure drop caused by solid flow	[mmH_2O]
s	:	equivalent length $(=gl/v_t^2)$	[−]
u	:	mean air velocity	[m/s]
v	:	mean velocity of solid particles	[m/s]
v_t	:	mean settling velocity of solid particles	[m/s]
γ_a	:	specific weight of air	[kg/m^3]
ζ	=	$\lambda_s v_t^2/(2gD)$	[−]
λ_s	:	friction factor of solid flow through a pipe	[−]
ν_a	:	kinematic viscosity of air	[m^2/s]
ϕ	:	solid-air velocity ratio $(=v/u)$	[−]

Subscript

a	:	air flow
s	:	solid flow
i-j	:	section i-j $(i, j=0, 1, 2)$

References

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