The Steady-State and Transient Characteristics of the Chopper-Modulated Circuit Having Four Circuit Modes

By

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The problem considered in this paper is one, in general, of making clear the characteristic of chopper-modulated circuits from the theoretical point of view.

The chopper-modulated circuit under consideration is assumed to have four circuit modes. In a previous investigation, we have already discussed a chopper-modulated circuit having common two circuit modes, and also have developed the valuable analytical method for a periodically interrupted electric circuit having two circuit modes excited by a complex sinusoidal input.

Here we shall generally extend the foregoing analytical method for a periodically interrupted electric circuit having m circuit modes driven by a complex sine wave input and basing on this method, we shall discuss a chopper-modulated circuit having four circuit modes. Only a type of practical interest is treated, that is, the transformer coupled chopper-modulated circuit which is frequently used with conventional chopper amplifiers, and other types which are not considered here. The method considered in this paper is an available technique explained to make clear the steady-state and transient performance of these types.

1. Introduction

In conventional chopper amplifiers, the chopper-modulated circuit commonly contains a mechanical contact modulator, or a chopper as an element of its circuit. This circuit has an important mean of converting an extremely low d-c voltage or a very low frequency alternating voltage with small amplitude into an alternating voltage being able to be amplified by a stable audio-frequency amplifier.

The usage of the modulating device of such a chopper-modulated circuit generating an alternating voltage has several advantages, that is, a very low internal impedance of the circuit, almost negligible internal noises, and no producing drift through the chopper amplifier.

The chopper may be supposed to take only two operations of "make" and "break" synchronously according to a driving force of the chopper, which is one of the types of the chopper-modulated circuits.

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However, other types of operation of the chopper in practice may be considered, that is, the chopper "makes before break" and "breaks before make" synchronously at an extrmely low voltage given as an input to the chopper amplifier. Here we shall discuss in detail the latter type of operation of the chopper. This type is seen to have four circuit modes per one cycle of a driving force to the chopper as shown later in this paper.

The circuit of such a chopper-modulated circuit converting d-c or a-c voltages into an alternating voltage may be considered as a periodically interrupted electric circuit of the first genus[1]. This circuit having two circuit modes driven by a complex sinusoidal input has been already discussed, we have developed the analytical method of such a circuit and, by the use of this method, we have clarified the performance of one type of the chopper-modulated circuit[2].

First, the remainder of this paper is devoted to the development of the foregoing method, which is expanded to be applicable to the circuits having m different modes[3].

Next, the analysis of the chopper-modulated circuit having four circuit modes is worked out in detail to illustrate the application of this expanded theory, and to make clear the circuit characteristics[4][5].

2. Analytical Method of a Periodically Interrupted Electric Circuit having m Circuit Modes

The general form of this interrupted circuit contains one chopper and several passive elements of networks with l storage elements $L.C$ as shown in Fig. 1. It may be supposed in general that this circuit may have the different operations of the chopper in each circuit modes, and also have the variable $L.C.R$ passive elements in different circit modes. That is, it may be considered one of the networks containing variable elements.

Now let us suppose that the interrupted circuit under discussion has m

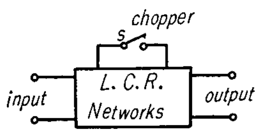

Fig. 1. Periodically interrupted electric circuit.

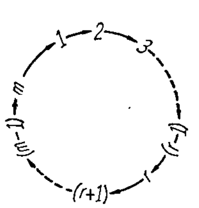

Fig. 2. Cyclic diagram illustrating the modes of a periodically interrupted electric circuit per one stage.

different circuit modes 1, 2, $\cdots$, r, $(r+1)$, $\cdots$, $(m-1)$, m, 1, 2, $\cdots$ altermately as illustrated in Fig. 2.

Hence the input signal is assumed to be the voltage signal of the form as a function of the frequency ω

$$E(t) = \varepsilon^{j\omega t} \tag{2.1}$$

which is connected with a loop containing the r-th element of the l storage elements L, C when the operation of the interrupted circuit exists in the r-th circuit mode, while which is connected with a loop containing the k-th element of the l strorage elements L, C when the operation of the circuit exists in the $(r+1)$-th circuit mode, and so on.

Furthermore, assume that the interval t_r of the r-th circuit mode is independent of the number of the stage, where $r=1, 2, \cdots, m$.

Then the differential equations can be written in the matrix notation for the r-th circuit mode

$$\begin{pmatrix} z_{r11} & z_{r12} & \cdots\cdots & z_{r1l} \\ z_{r21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ z_{rl1} & \cdots\cdots\cdots & & z_{rll} \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_r(t) \\ \vdots \\ y_l(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \varepsilon^{j\omega t} \\ \vdots \\ 0 \end{pmatrix} \tag{2.2}$$

where $y_1(t), \cdots, y_l(t)$ represent the unknown currents or voltages, which are denoted in such a way that if the elemet is a inductance L, $y(t)$'s is a current flowing through the inductance L, and if the element is a capacitacce C, $y(t)$'s is a voltage at terminals of the capacitance C, and z's are given by

$$\begin{aligned} z_{rij} &= Ld/dt + R \\ &= Cd/dt + G, \end{aligned}$$

or

$$\begin{aligned} z_{rij} &= 0 \\ &= \pm 1 \quad (r = 1, 2, \cdots, m. \quad i, j = 1, 2, \cdots, l). \end{aligned}$$

For simplicity, Eq. (2.2) is expressed of the abbreviate form

$$[z_r(D)][y(t)] = [w_r(t)] \tag{2.3}$$

where $D=d/dt$, $r=1,2, \cdots, m$ and the origin of the time is chosen at the initial instant of the r-th circuit mode.

By the use of the Laplace transformation*, Eq. (2.3)

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* $F(p) = p\int_0^\infty f(t)\varepsilon^{-pt}\,dt$

$$[Z_r(p)][Y(p)] = [W_r(p)] + p[z_r^0][y^{-0}], \tag{2.4}$$

then

$$[Y(p)] = [Z_r(p)]^{-1}[W(p)] + p[Z_r(p)]^{-1}[z_r^0][y^{-0}] \tag{2.5}$$

where the elements of the matrix $[y^{-0}]$ generally denote the initial values of the first kind corresponding to the elements of the matrix $[y(t)]$, which are identically equal to the final values of the element of the current- and voltage-matrices $[y(t)]$ in the preceding $(r-1)$-th circuit mode, and $[z_r^0]$ denote the initial matrix corresponding to the matrix $[z_r(D)]$, but in this paper, for simplicity of the analysis with no loss of generality, we shall treat the special case of that the matrix $[z_r]$ is uniform at $t=0$, or in other words, provided that $[z_r^{-0}] = [z_r^{+0}] \equiv [z_r^0]$, therefore the initial values of elements of the matrix $[y^{-0}]$ are uniform at $t=0$.

The operational solutions of Eq. (2.5) are transformed into the time function by the use of the inverse Laplace transformation*, then we have

$$[y(t)] = [\varphi_r(t)] + [x_r(t)][y^{-0}] \tag{2.6}$$

where

$$\left.\begin{aligned}[\varphi_r(t)] &= \mathfrak{H}[Z_r(p)]^{-1}[W_r(p)] \\ [x_r(t)] &= \mathfrak{H}p[Z_r(p)]^{-1}[z_r^0].\end{aligned}\right\} \tag{2.7}$$

In this equation, the origin of time is at the initial instant of the r-th circuit mode. Here for generality, we choose the origin of the time at a time when the 1st circuit mode on the 1st stage starts, and we suppose the input voltage $E(t)$ to be

$$E(t) = E\varepsilon^{j(\omega t+\theta)}. \tag{2.8}$$

By changing the origin of time, the input $E(t)$ is rewritten of the form

$$E(t) = E\exp\{j(\omega(nT+\sum_{i=1}^{r-1}t_i)+\theta)\}\cdot\exp\{j\omega(t-nT-\sum_{i=1}^{r-1}t_i)\} \tag{2.9}$$

which is seen to be applied at time $t = nT+\sum_{i=1}^{r-1}t_i$, that is, the input in the r-th circuit mode on the n-th stage, where $T=\sum_{r=1}^{m}t_r$ is one period of the operations of the chopper or a duration of one stage of the interrupted circuit, and $n=0, 1, 2, \cdots$.

Therefore, Eq. (2.6) may be rewritten as follows, for the input $E(t)$ of the form shown in Eq. (2.9) during the r-th circuit mode on the n-th stage.

$$\begin{aligned}[y(t)] = {} & E\exp\{j(\omega(nT+\sum_{i=1}^{r-1}t_i)+\theta)\}[\varphi_r(t-nT-\sum_{i=1}^{r-1}t_i)] \\ & +[x_r(t-nT-\sum_{i=1}^{r-1}t_i)][y(nT+\sum_{i=1}^{r-1}t_i)]. \quad nT+\sum_{i=1}^{r-1}t_i \leq t \leq nT+\sum_{i=1}^{r}t_i\end{aligned} \tag{2.10}$$

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* $f(t) = \mathfrak{H}F(p) = \frac{1}{2\pi i}\lim_{\beta\to\infty}\int_{r-i\beta}^{r+i\beta}\frac{F(p)}{p}\varepsilon^{pt}dp.$

The initial value matrix $[y(nT+\sum_{i=1}^{r-1} t_i)]$ on the right-hand side of Eq. (2.10) is represented by the following recurrence formula as reduced from Eq. (2.10)

$$[y(nT+\sum_{i=1}^{r-1} t_i)] = E\exp\{j(\omega(nT+\sum_{i=1}^{r-2} t_i)+\theta)\}[\varphi_{r-1}(t_{r-1})] + [x_{r-1}(t_{r-1})][y(nT+\sum_{i=1}^{r-2} t_i)]. \quad (2.11)$$

For abbreviation, using a symbol τ_{r-1}^n such as $\tau_{r-1}^n = nT+\sum_{i=1}^{r-1} t_i$, then we have

$$\begin{aligned}[y(\tau_{r-1}^n)] &= E\exp\{j(\omega\tau_{r-2}^n+\theta)\}[\varphi_{r-1}(t_{r-1})] + [x_{r-1}(t_{r-1})][y(\tau_{r-2}^n)] \\ &= E\exp\{j(\omega\tau_{r-2}^n+\theta)\}[\varphi_{r-1}(t_{r-1})] + E\exp\{j(\omega\tau_{r-3}^n+\theta)\}[x_{r-1}(t_{r-1})][\varphi_{r-2}(t_{r-2})] \\ &\quad + E\exp\{j(\omega\tau_{r-4}^n+\theta)\}[x_{r-1}(t_{r-1})][x_{r-2}(t_{r-2})][\varphi_{r-3}(t_{r-3})] + \cdots \\ &\quad + E\exp\{j(\omega\tau_0^n+\theta)\}[x_{r-1}(t_{r-1})][x_{r-2}(t_{r-2})]\cdots[x_2(t_2)][\varphi_1(t_1)] \\ &\quad + [x_{r-1}(t_{r-1})][x_{r-2}(t_{r-2})]\cdots[x_1(t_1)][y(\tau_0^n)]. \end{aligned} \quad (2.12)$$

This recurrence formula means that the initial value matrix $[y(nT+\sum_{i=1}^{r-1} t_i)] \equiv [y(\tau_{r-1}^n)]$ in the r-th circuit mode on the n-th stage is given by the initial value matrix $[y(nT)] \equiv [y(\tau_0^n)]$ which could be generally determined by the given initial value matrix $[y(0)]$ in the 1st circuit mode on the 1st stage as shown later.

Now substituting Eq. (2.12) into Eq. (2.10), we have

$$\begin{aligned}[y(t)] &= E\exp\{j(\omega\tau_{r-1}^n+\theta)\}[\varphi_r(t-\tau_{r-1}^n)] \\ &\quad + E\exp\{j(\omega nT+\theta)\}[x_r(t-\tau_{r-1}^n)]\sum_{u=1}^{r-1}\{\exp(j\omega\sum_{s=1}^{r-u-1} t_s)(\prod_{s=1}^{u-1}[x_{r-s}(t_{r-s})])[\varphi_{r-u}(t_{r-u})]\} \\ &\quad + [x_r(t-\tau_{r-1}^n)](\prod_{s=1}^{r-1}[x_{r-s}(t_{r-s})])[y(nT)]. \qquad \tau_{r-1}^n \leq t \leq \tau_r^n \end{aligned} \quad (2.13)$$

Setting $t=nT+T$ and $r=m$ in Eq. (2.13), we obtain the important relation of the form

$$[y(nT+T)] = E\exp\{j(\omega nT+\theta)\}[H] + [B][y(nT)] \quad (2.14)$$

where

$$\left.\begin{aligned}[H] &= \sum_{u=0}^{m-1}\{\exp(j\omega\sum_{s=1}^{m-u-1} t_s)(\prod_{s=0}^{u-1}[x_{m-s}(t_{m-s})])[\varphi_{m-u}(t_{m-u})]\} \\ [B] &= \prod_{s=0}^{m-1}[x_{m-s}(t_{m-s})]. \end{aligned}\right\} \quad (2.15)$$

Since it is evident that Eq. (2.14) is the difference equation having the difference T with respect to nT, the solution $[y(nT)]$ of this equation is easily determined by the general method solving the difference equation which was described in detail in Appendix 1 of References (2).

The result becomes

$$[y(nT)] = [B]^n[I] + E\exp\{j(\omega nT+\theta)\}\{\exp(j\omega T)[U]-[B]\}^{-1}[H] \quad (2.16)$$

where $[U]$ is a unit matrix and the initial value matrix $[I]$ is given by

$$[I] = [y(0)] - E\exp(j\theta)\{\exp(j\omega T)[U]-[B]\}^{-1}[H]. \quad (2.17)$$

The matrix $[B]^n$ is calculable by the use of the Sylvester expansion theorem (see Appendix 2 in References (2)) of the form

$$[B]^n = \sum_{r=1}^{l} \alpha_r^n \frac{\prod\limits_{\substack{s=1,2,\cdots,l \\ r\neq s}} \{\alpha_s[U]-[B]\}}{\prod\limits_{\substack{s=1,2,\cdots,l \\ r\neq s}} (\alpha_s-\alpha_r)} \tag{2.18}$$

where $\alpha_1, \cdots, \alpha_l$ are the distinct latent roots of $[B]$.

Substituting Eq. (2.16) into Eq. (2.13), the solution $[y(t)]$ can be determined, that is, the current and voltage of the interrupted circuit in any circuit mode on any stage are obtained in general with given initial values.

Now, clearly as was mentioned in References (2), the periodically interrupted circuit in question is stable, provided that the matrix $[B]$ has distinct latent roots and the absolute value of every root is less than unity.

If we consider only the case of the steady-state phenomena for sufficiently great value of n when the circuit is stable, then we have, from Eq. (2.18)

$$\lim_{n\to\infty} [B]^n = 0 \tag{2.19}$$

Therefore the intial value matrix $[y(nT)]$ becomes more simple forms as

$$[y(nT)] = E\exp\{j(\omega nT+\theta)\}\{\exp(j\omega T)[U]-[B]\}^{-1}[H] \tag{2.20}$$

and substituting Eq. (2.20) into Eq. (2.13) results in the steady-state solution.

3. Steady-State Characteristics of the Chopper-Modulated Circuit

Now let us consider a transformer coupled chopper-modulated circuit shown in Fig. 3, having four circuit modes which is a type of "break before make" as illustrated in Fig. 4.

Here we replace this modulated circuit by an equivalent circuit shown in Fig. 5, whose circuit constants are reduced to those on the primary side of an input transformer, and the chopper on s_1 and on s_2 are equivalent to the

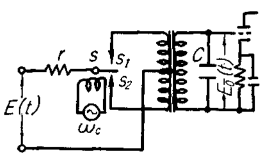

Fig. 3. Transformer coupled chopper-modulated circuit.

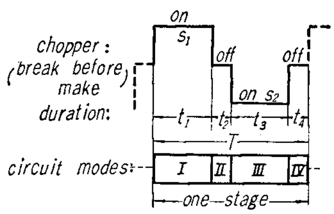

Fig. 4. Chopper operation and circuit modes.

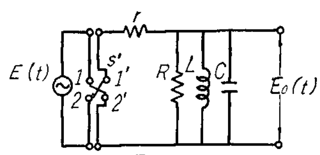

Fig. 5. Equivalent circuit corresponding to Fig. 3 (s on s_1 : 1–1′ & 2–2′, s on s_2 : 1–2′ & 1′–2).

switch s' on 1-1′ & 2-2′ and on 1-2′ & 2-1′ in Fig. 5 respectively.

By the application of the method in above section to the problem in this case, we can easily obtain the steady-state solution as follows.

The elements of the matrices in Eq. (2.13) in this case are given by

$$[y(t)] = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{3.1}$$

where $y_1(t)$ is the current flowing through the inductance L and $y_2(t) = E_0(t)$ is the terminal voltage of the capacitance C.

$$[\varphi_1(t)] = -[\varphi_3(t)] = \begin{bmatrix} \varphi_{11}(t) \\ \varphi_{12}(t) \end{bmatrix} \tag{3.2}$$

and

$$[\varphi_2(t)] = [\varphi_4(t)] = 0 \tag{3.3}$$

where

$$\varphi_{11}(t) = \mathfrak{H} p/\{\Delta_1(p-j\omega)\}, \quad \varphi_{12}(t) = \mathfrak{H} Lp^2/\{\Delta_1(p-j\omega)\}\ .$$

$$[x_1(t)] = [x_3(t)] = \begin{bmatrix} x_{111}(t), & x_{112}(t) \\ x_{121}(t), & x_{122}(t) \end{bmatrix} \tag{3.4}$$

where

$$x_{111}(t) = \mathfrak{H}\{LCrp^2 + L(1+r/R)p\}/\Delta_1, \quad x_{112}(t) = \mathfrak{H}(LCp)/\Delta_1,$$
$$x_{121}(t) = \mathfrak{H}-(rLp)/\Delta_1, \quad x_{122}(t) = \mathfrak{H}(LCrp^2)/\Delta_1.$$

$$[x_2(t)] = [x_4(t)] = \begin{bmatrix} x_{211}(t), & x_{212}(t) \\ x_{221}(t), & x_{222}(t) \end{bmatrix} \tag{3.5}$$

where

$$x_{211}(t) = \mathfrak{H}\{LCp^2 + (L/R)p\}/\Delta_2, \quad x_{212}(t) = \mathfrak{H} Cp/\Delta_2, \quad x_{221}(t) = \mathfrak{H} - Lp/\Delta_2,$$
$$x_{222}(t) = \mathfrak{H} LCp^2/\Delta_2$$

and

$$\Delta_1 = LCrp^2 + L(1+r/R)p + r, \quad \Delta_2 = LCp^2 + (L/R)p + 1\ .$$

By introducing the steady-state solutions in each circuit mode from Eq. (2.13), the solutions are expressed of the form, when n is sufficiently great value

1) for the 1st circuit mode

$$[y(t)] = E\exp\{j(\omega nT+\theta)\}[\varphi_1(t-nT)] + [x_1(t-nT)][y(nT)]$$
$$nT \leq t \leq nT + t_1 \tag{3.6}$$

2) for the 2nd circuit mode

$$[y(t)] = E\exp\{j(\omega nT+\theta)\}[x_2(t-nT-t_1)][\varphi_1(t_1)] + [x_2(t-nT-t_1)][x_1(t_1)][y(nT)]$$
$$nT + t_1 \leq t \leq nT + t_1 + t_2 \tag{3.7}$$

3) for the 3rd circuit mode

$$[y(t)] = E\exp\{j(\omega(nT+t_1+t_2)+\theta)\}[\varphi_3(t-nT-t_1-t_2)]$$
$$+E\exp\{j(\omega nT+\theta)\}[x_3(t-nT-t_1-t_2)][x_2(t_2)][\varphi_1(t_1)]$$
$$+[x_3(t-nT-t_1-t_2)][x_2(t_2)][x_1(t_1)][y(nT)]$$
$$nT+t_1+t_2 \leq t \leq nT+t_1+t_2+t_3 \tag{3.8}$$

4) for the 4th circuit mode

$$[y(t)] = E\exp\{j(\omega(nT+t_1+t_2)+\theta)\}[x_4(t-nT-t_1-t_2-t_3)][\varphi_3(t)]$$
$$+E\exp\{j(\omega nT+\theta)\}[x_4(t-nT-t_1-t_2-t_3)][x_3(t_3)][x_2(t_2)][\varphi_1(t_1)]$$
$$+[x_4(t-nT-t_1-t_2-t_3)][x_3(t_3)][x_2(t_2)][x_1(t_1)][y(nT)]$$
$$nT+t_1+t_2+t_3 \leq t \leq nT+T \tag{3.9}$$

where

$$[y(nT)] = E\exp\{j(\omega nT+\theta)\}\{\exp(j\omega T)[U]-[B]\}^{-1}[H]. \tag{3.10}$$

Substituting Eqs. (3.1), (3.2), (3.3), (3.4) and (3.5) into Eqs. (3.6), (3.7) (3.8) and (3.9), we can determine the steady-state solutions in this case.

Now to illustrate the present technique, some numerical results are presented as follows, by making use of the Digital Computer (KDC-1).

The circuit constants are assumed to be

$$r = 5\times10^2\,\Omega, \quad R = 10^3\,\Omega, \quad C=(5/\pi)\times10^{-6}\,F, \quad L = 1/20\pi\,H$$

and the chopper is driven synchronously at $\omega_c=1000$ cycle per second and, that is, $t_1=t_3=4\times10^{-4}$ sec, $t_2=t_4=1\times10^{-4}$ sec and $T=10^{-3}$ sec, and $n=10^{\alpha}$ is a sufficient great value, where α is integer.

Moreover the input signal to the chopper-modulated circuit under discussion is supposed to be $\sin\omega t$, putting $E=1$ and $\theta=0$

$$E(t) = E\varepsilon^{j(\omega t+\theta)} = \mathcal{I}m(e^{j\omega t}).$$

First we examine the stability of the circuit in question whether it is stable or not, by checking the matrix $[B]$ of the form

$$[B] = \begin{bmatrix} 3.981870\times10^{-1}, & 2.000955\times10^{-5} \\ 6.219725, & 4.895580\times10^{-1} \end{bmatrix} \tag{3.11}$$

Solving the following characteristic function of the matrix $[B]$

$$\delta\{\lambda[U]-[B]\} = 0, \tag{3.12}$$

the latent roots λ of the matrix $[B]$ are

$$\lambda_1 = 4.881754\times10^{-1}$$

and

$$\lambda_2 = 3.995696\times10^{-1},$$

whose volues are less than unity, then the circuit is stable.

From Eqs. (3.6), (3.7), (3.8) and (3.9), the wave forms of the modulated output $E_0(t)$ are obtained as illustrated in Figs. 6, 7, 8 and 9 with the sine wave

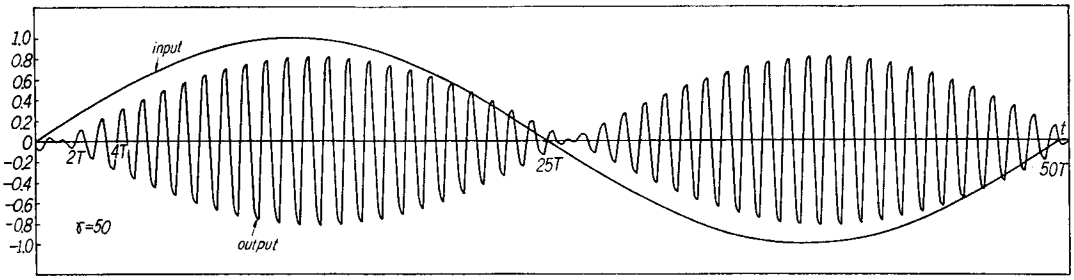

Fig. 6. Wave form of the modulated output voltage in steady-state where $\gamma=\omega_c/\omega=50$.

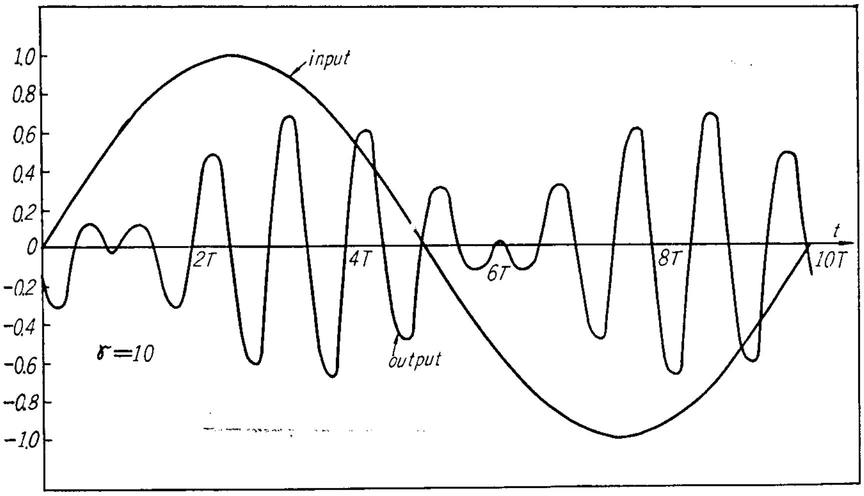

Fig. 7. Wave form of the modulated output voltuge in steady-state where $\gamma=10$.

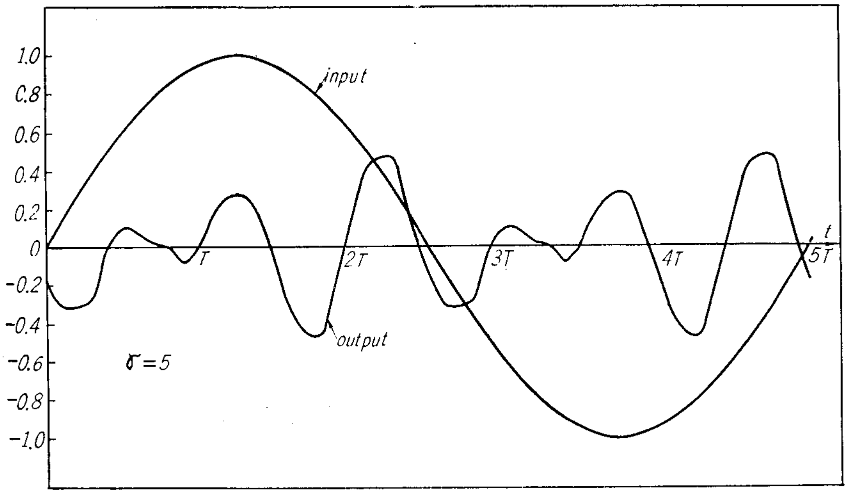

Fig. 8. Wave form of the modulated output voltage in steady-state where $\gamma=5$.

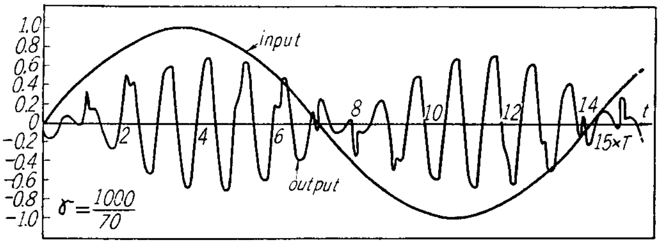

Fig. 9. Wave form of the modulated output voltage in steady-state where $\gamma=1000/70$ is irrational.

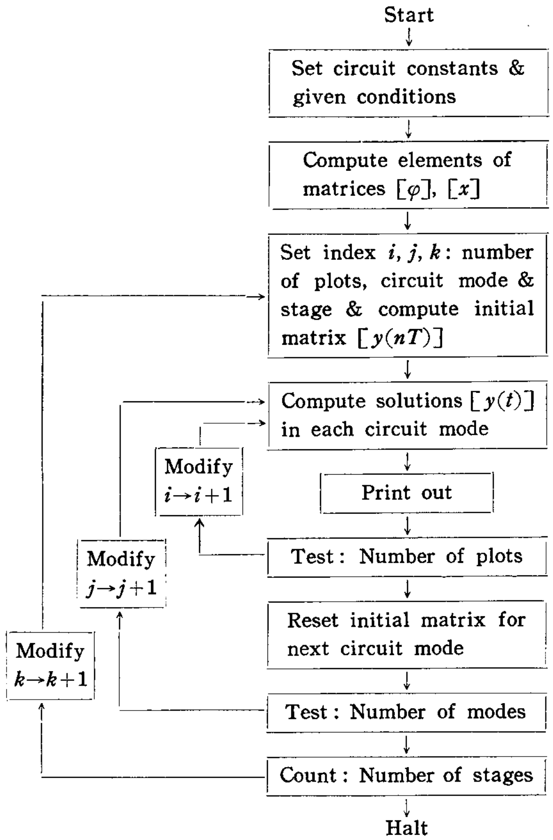

Fig. 10. Digital computer flowchart for calculating the modulated output voltage in steady-state.

input, whose calculations have been done in such a way as shown in Fig. 10.

By virtue of Figs. 6 and 7, the wave forms are considered sufficiently good results when $\gamma=\omega_c/\omega$ is a large value to some extent, but, when γ is small, the wave form is slightly distorted and its magnitude is highly decreased too, as shown in Fig. 8, When γ is irrational, the wave form is remarkably distorted as indicated in Fig. 9, whose phenomena seems physically to exist in the circuit considering the experimental point of view.

From many numerical calculations of the wave forms of the modulated outputs, Fig. 11 illustrates the frequency response of the transformer coupled chopper-modulated circuit, that is, the amplitue and phase characteristics as a function of the ratio ω/ω_c.

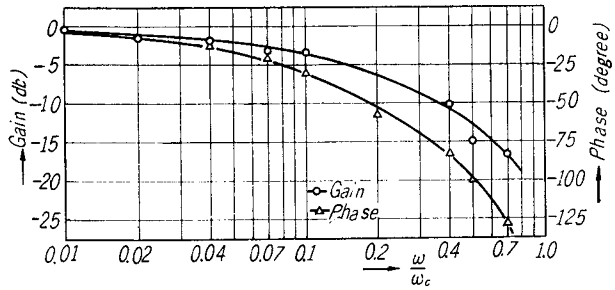

Fig. 11. Frequency response of the transformer coupled chopper-modulated circuit.

4. Transient Characseristics of the Chopper-Modulated Circuit

In this section we first consider the transient response of the same circuit of the type of the "break before make" treated in the 3rd section to a sinusoidal input. Next the transient response to a step input is discussed for two types of the "break before make" and "make before break" of the transformer coulped chopper-modulated circuit.

4.1. The Transient Response to the Sinusoidal Input

Setting the constants to be the same one as one in above section and other conditions are the same one too, the results can be obtained by using Eq. (2.13) where we take first $n=0$ and next $n=1$ and so on according to step by step calculations, and assume $[y(0)]=0$.

By means of the Digital Computer (KDC-1), the transient response of the transformer coupled chopper-modulated circuit of the type of the "break before make" is illustrated in Figs. 12 and 13 for the sine wave input with a frequency of 40 c/s and with a frequency of 100 c/s respectively, when the frequency of the chopper is 1000 c/s.

Accordingly it is concluded that in such a case where the frequency of input signal is 40 c/s or 100 c/s with respect to the chopper frequency 1000 c/s, the transient phenomena may be almost diminished in one cycle of the frequency of the input signal.

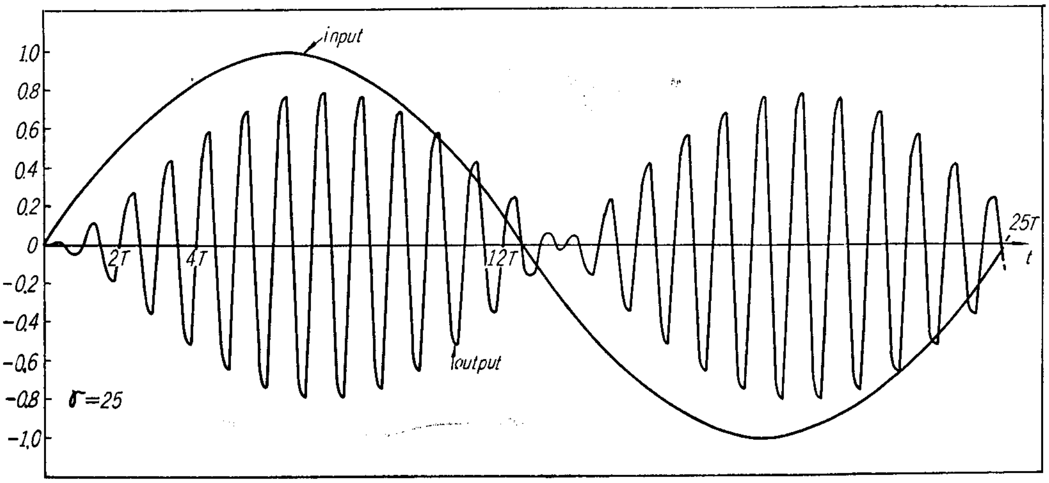

Fig. 12. Transient response of the modulated output voltage to the sinusoidal input where $\gamma=25$.

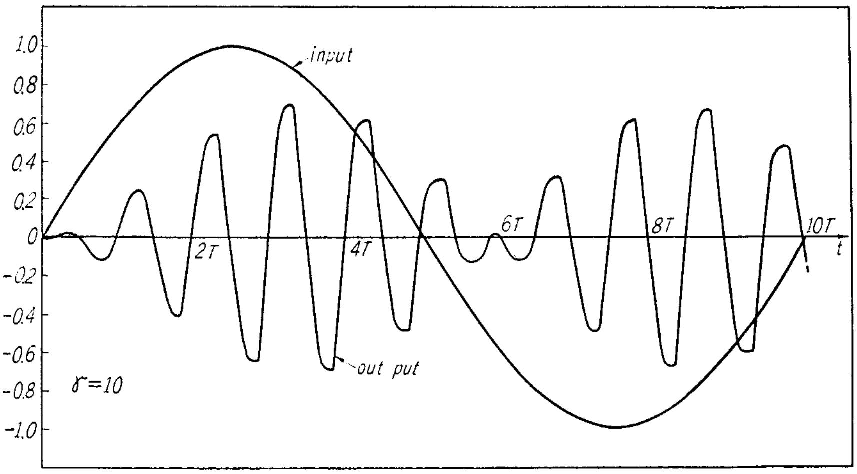

Fig. 13. Transient response of the modulated output voltage to the sinusoidal input where $\gamma=10$.

4.2. The Transient Response to the Step Input

For comparison, we consider the performances of the "break before make" and "make before break" types of the circuit shown in Fig. 3.

In the case of the latter, the chopper s operates of the form as shown in Fig. 14, therefore, for simplicity of the analysis, we may take the equivalent circuit shown in Fig. 15 corresponding to Fig. 3, in which the operation of the of the chopper s is replaced by the switch s' and s'' synchronously operating of the form as illustrated in Fig. 16 and an added resistance R' in Fig 15 is very small such as a contact one of the switch.

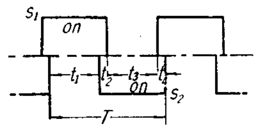

Fig. 14. Chopper operation of the "make before break" type.

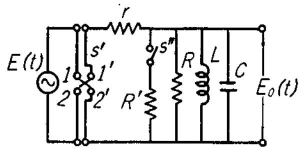

Fig. 15. Equivalent circuit when the chopper operates of the form as shown in Fig. 14.

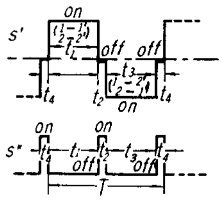

Fig. 16. Operations of the switch s' and s'' corresponding to the chopper s.

The operations of the switch s' and s'' are considered in such a way that when the chopper s is on s_1, the switch s' is off and s'' is on during t_4 and t_2, and during t_1 the switch s' is on (1-1′) & (2-2′) and s' is off, and that when the chopper s is on s_2, the swich s' is off and s'' is on during t_2 and t_4, and during t_3 the switch s' is on (1-2′) & (2-1′) and s'' is off.

The solutions in two cases under consideration become of the simple forms described in no complex domain, since the input signal is a unit step one.

Putting $n=0$, $\theta=0$ and $[y(0)]=0$ in Eqs. (3.6), (3.7), (3.8) and (3.9), the transient solutions to a unit step input are written of the forms, only on the 1st stage

1) for the 1st circuit mode

$$[y(t)] = [\varphi_1(t)] \qquad 0 \leq t \leq t_1 \tag{4.1}$$

2) for the 2nd circuit mode

$$[y(t)] = [\varphi_2(t-t_1)] + [x_2(t-t_1)][\varphi_1(t_1)] \qquad t_1 \leq t \leq t_2 \tag{4.2}$$

3) for the 3rd circuit mode

$$[y(t)] = [\varphi_3(t-t_1-t_2)] + [x_3(t-t_1-t_2)][\varphi_2(t_2)] + [x_3(t-t_1-t_2)][x_2(t_2)][\varphi_1(t_1)] \qquad t_2 \leq t \leq t_3 \tag{4.3}$$

4) for the 4th circuit mode

$$\begin{aligned}[y(t)] = {} & [\varphi_4(t-t_1-t_2-t_3)] + [x_4(t-t_1-t_2-t_3)][\varphi_3(t_3)] \\ & + [x_4(t-t_1-t_2-t_3)][x_3(t_3)][\varphi_2(t_2)] \\ & + [x_4(t-t_1-t_2-t_3)][x_3(t_3)][x_2(t_2)][\varphi_1(t_1)] \qquad t_3 \leq t \leq t_4\end{aligned} \tag{4.4}$$

where the elements of the matrices are given by Eqs. (3.1), (3.2), (3.3), (3.4) and (3.5), but in the case of the "make before break" type, the resistance R in Eqs. (3.2), (3.4) and (3.5) must be replaced by $R_0 = RR'/(R+R')$.

The transient solutions in any circuit mode on other stage are determined by step by step calculations basing on Eqs. (4.1), (4.2), (4.3) and (4.4).

The transient response to the unit step input is plotted as indicated in

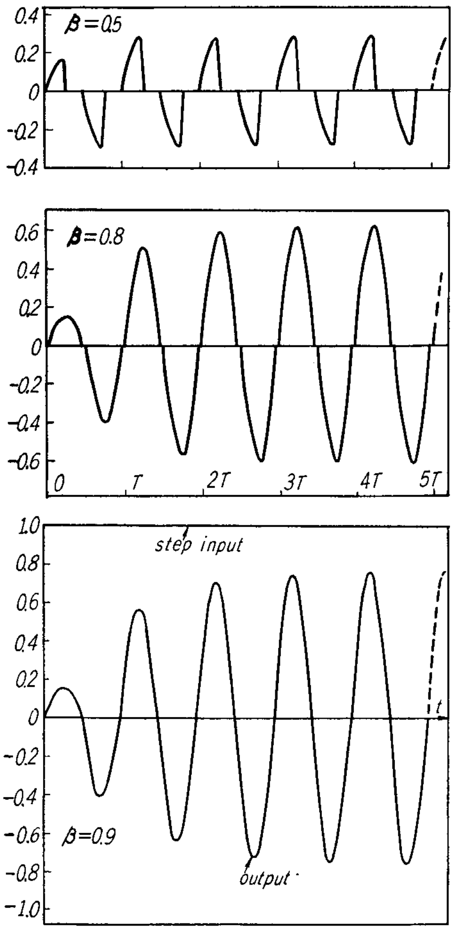

Fig. 17. Transient response to the unit step in the "make before break" type of the chopper where $\beta=0.5$, 0.8 and 0.9 respectively.

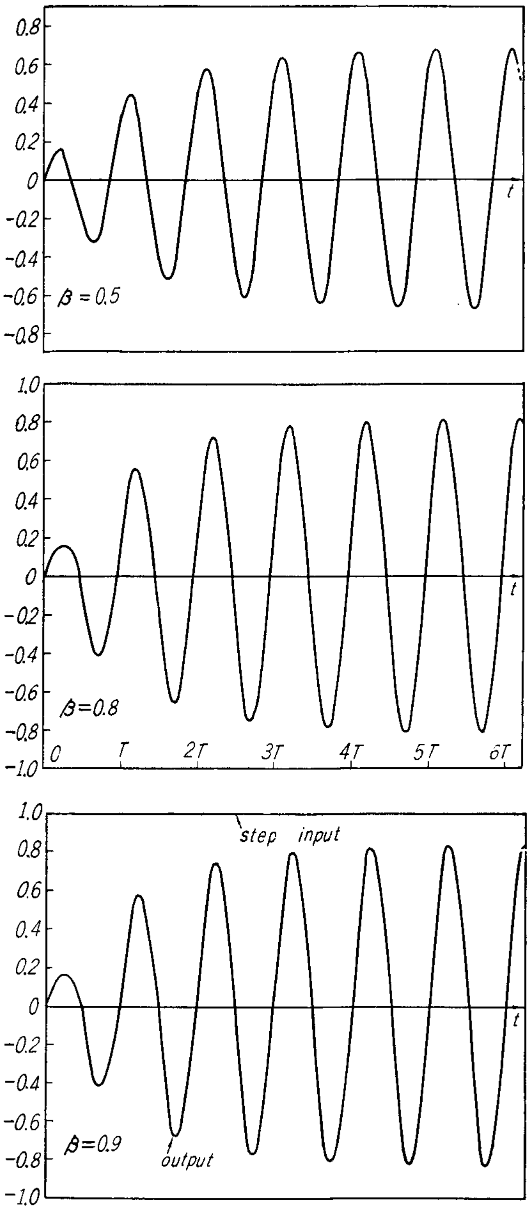

Fig. 18. Transient response to the unit step in the "break before make" type of the chopper where $\beta=0.5$, 0.8 and 0.9 respectively.

Figs. 17 and 18 by means of the Digital Computer (KDC-1), assuming that the constants and conditions are the same one as the previous case, but that in the case of the "make before break" type, the resistance R' is $R'=0.1\Omega$.

Figs. 17 and 18 illustrate the transient responses at three cases of $\beta=0.5$, 0.8 and 0.9 in the "make before break" (M.b.B) and "break before make" (B.b.M) types of the chopper respectively, where $\beta=t_1/(t_1+t_2)$.

From many numerical results of these transient responses, the transient upper envelopes of the modulated outputs to the unit step input are plotted

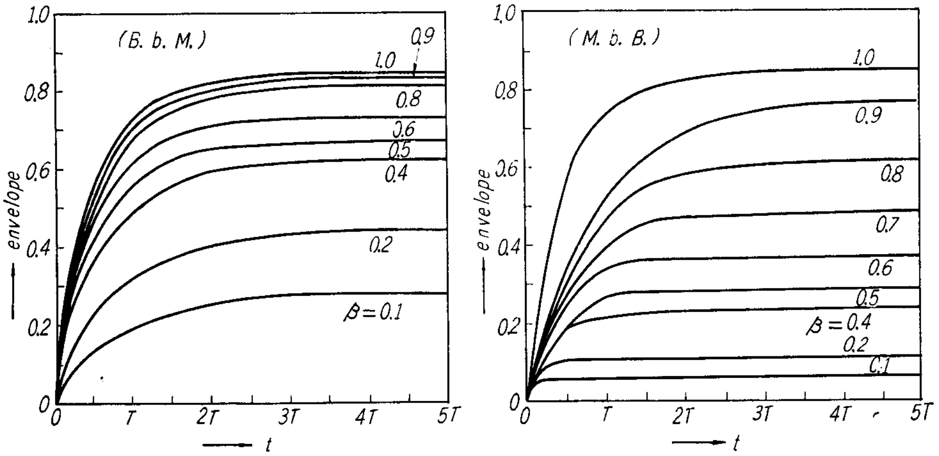

Fig. 19. Transient upper envelopes of the modulated output voltage to the unit step input when the operation of the chopper is (B.b.M) and (M.b.B) types respectively.

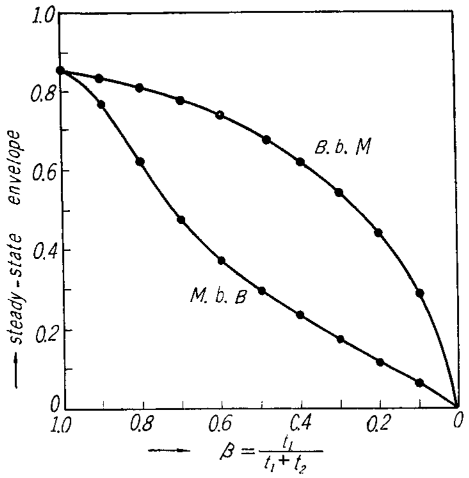

Fig. 20. Relations between the steady-state envelope and duty-ratio β for the *d.c* input signal.

with respect to the parameter of a duty-ratio β as shown in Fig 19 comparing (B.b.M) with (M.b.B) types.

Consequently it is apparent that when the response of envelope of the output becomes the steady-state value, we can obtain the relation between the envelope of the modulated output and duty-ratio β for the d.c input signal to the chopper-modulated circuit as illustrated in Fig. 20, which shows the great difference between (B.b.M) and (M.b.B) types.

From these results, it is concluded that the (B.b.M) type is superior to the (M.b.B) with respect to the performance of the chopper-modulated circuit under discussion.

5. Conclusion

It has been the purpose of this paper to examine the transient and steady-state response of the chopper-modulated circuit to sine wave and step inputs.

First we have introduced in general the analytical method of the periodically interrupted electric circuits of first genus having m circuit modes in complex domain.

On the basis of this new techniques, the transmission characteristics of the chopper-modulated circuits of two types have been clarified by observing the numerical results by means of the Digital Computer (KDC-1).

As the steady-state response of the output, the frequency response of the output, that is, the amplitude and phase characteristics have been obtained clearly only in the case of the "break before make" type of the transformer coupled chopper-modulated circuit, but in another case, not considered here, those performances could be obtained by the similar way of the previous case.

It is found that the transient behaviour of the output to the sinusoidal input is almost diminished in one cycle of the frequency of the input.

While for the unit step input, the transient response of the output have been made clear in both cases of the "break before make" and "make before break" types of the chopper-modulated circuit, and these results have shown the great difference of the characteristics between these two types.

It is hoped that the results given in this paper may serve as a reference to the circuit designer as well as other fields and that this analytical method will be widely available in other fields.

Acknowledgments

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