Temperature Distribution of the Bed during the Second Falling Rate Period of Drying

By

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(Received April 18, 1964)

The temperature distribution of the bed of the granular materials at drying during the second falling rate period was calculated by the analytical solution of the equation of heat conduction. The equation of heat conduction during the second falling rate period became the differential equations with moving boundary. The heat-balance integral method was applied to solve this problem which consists of a variable heat flux at surface. The calculated results by the analytical solution were displayed and compared with the numerical solution and also with the experimental ones.

1. Introduction

The drying mechanism for the second falling rate period of drying of the bed of the granular materials was investigated in our previous report[1]. The drying bed is separated into the dried-up zone and the wetted zone during the second falling rate period. The temperature of the boundary plane between these two zones remains nearly constant but does increase slightly; therefore this boundary plane retreats into the bed with increasing time. The temperature of the dried-up zone approaches rapidly to the air temperature as shown in **Fig. 1.**

This plane is considered as the evaporating plane and so evaporation may occur mainly at this plane. The asymptotic value of the temperature of the boundary plane was defined and analysed[1]. This temperature was named the asymptotic temperature, t_p.

It was also observed from the previous experimental moisture distribution curves during the second falling rate period that the evaporating plane retreats into the bed, keeping a simple geometric pattern of moisture distribution as shown schematically in **Figs. 2** (a), (b), (c).

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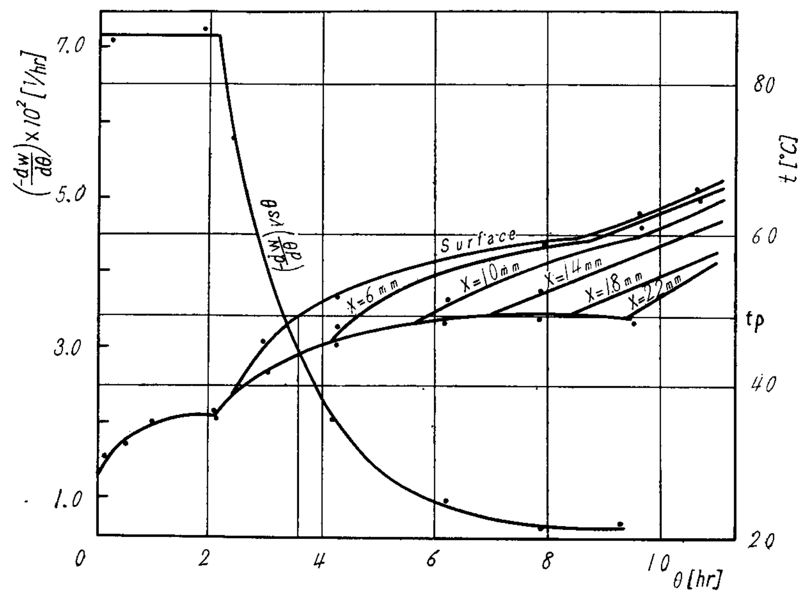

Fig. 1. Experimental temperature distribution curves (Run No. Ac-7, Acrican (42~60#), t_a=71.0°C, H_a=0.0113, V_a=1.8 m/sec, L=0.03 m).

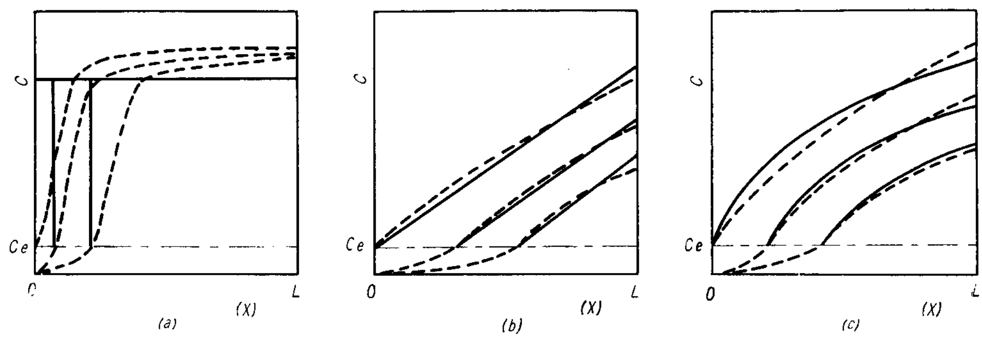

Fig. 2. Schematic model of the moisture distribution curves during the second falling rate period.

The behavior of the temperature change of the drying bed during the second falling rate period can be obtained by solving the equations of heat conduction with the moving boundary plane through the bed such as the melting of finite slabs.

A general analytical method which provides an exact solution to such a heat conduction problem has not been found. The "heat-balance integral" method by Goodman[2,3] is an unique one to get an approximate analytical solution which has been applied successfully to certain such problems. The application of this method is restricted to cases of constant heat flux at surface and could not be applied to the drying problem.

The heat-balance integral method is applied to the drying problem in cases of variable heat flux at surface, and the temperature distribution in the drying bed during the second falling rate period can be calculated by the authors.

The calculation procedures are presented as following and the calculated results are displayed and compared with the experimental ones.

2. Statement of the Basic Equations

A drying bed bounded by the planes $x=0$ and $x=L$ as shown in **Fig. 3** is considered. The interface between the dried-up zone and the wetted zone is the evaporating plane and is specified by function $\delta(\theta)$.

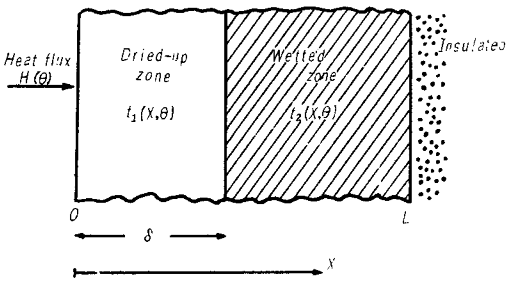

Fig. 3. Schematic representation of the drying bed during the second falling rate period.

The following heat conduction equations must be satisfied for dried-up zone;

$$\frac{\partial t_1}{\partial \theta} = \kappa_1 \frac{\partial^2 t_1}{\partial x^2} \quad \text{at} \quad 0 < x < \delta,\ \theta > 0 \tag{1}$$

for wetted zone;

$$\frac{\partial t_2}{\partial \theta} = \kappa_2 \frac{\partial^2 t_2}{\partial x^2} \quad \text{at} \quad \delta < x < L,\ \theta > 0 \tag{2}$$

where

$$\kappa_1 = \frac{\lambda_1}{\rho_1' c_{p_1}}, \quad \kappa_2 = \frac{\lambda_2}{\rho_2' c_{p_2}}$$

with the boundary conditions;

$$\lambda_1 \frac{\partial t_1}{\partial x} = -H(\theta) = -h_p(t_a - t_{1\,x=0}) \quad \text{at} \quad x = 0 \tag{3}$$

$$t_1 = t_p = t_2 \quad \text{at} \quad x = \delta \tag{4}$$

$$\delta = 0 \quad \text{at} \quad \theta = 0 \tag{5}$$

$$t_1 = t_0 = t_2 \quad \text{at} \quad \theta = 0 \tag{6}$$

$$-\lambda_1 \frac{\partial t_1}{\partial x} + \lambda_2 \frac{\partial t_2}{\partial x} = Rr_p \quad \text{at} \quad x = \delta \tag{7}$$

where R is the drying rate. Expression of R is deduced as follows.

The following relation between δ and w was deduced geometrically about the schematical moisture distributions as shown in Figs. 2. (a), (b) and (c).

$$\frac{\delta}{L} = 1 - \left(\frac{w}{w_p}\right)^{1/n} \tag{8}$$

$n = 1$ at (a), $n = 2$ at (b), and $n = 3$ at (c).

By the definition of the drying rate R,

$$R = -\rho_1' L \frac{dw}{d\theta} = -\rho_1' n w_p \left(1 - \frac{\delta}{L}\right)^{n-1} \frac{d\delta}{d\theta} \tag{9}$$

$$\frac{\partial t_2}{\partial x} = 0 \quad \text{at} \quad x = L \tag{10}$$

To simplify the procedure, the temperature scale is chosen so that the temperature of the evaporating plane is zero. So the reduced temperature u_1 and u_2 are introduced.

$$u_1 = t_1 - t_p, \quad u_2 = t_2 - t_p \tag{11}$$

With the reduced temperature u, eqs. (1)~(8) are rewritten as follows.

$$\frac{\partial u_1}{\partial \theta} = \kappa_1 \frac{\partial^2 u_1}{\partial x^2} \quad 0 < x < \delta, \ \theta > 0 \tag{1'}$$

$$\frac{\partial u_2}{\partial \theta} = \kappa_2 \frac{\partial^2 u_2}{\partial x^2} \quad \delta < x < L, \ \theta > 0 \tag{2'}$$

$$\lambda_1 \frac{\partial u_1}{\partial x} = -H(\theta) = -h_p(u_a - u_{1\,x=0}) \quad \text{at} \quad x = 0 \tag{3'}$$

$$u_1 = 0 = u_2 \quad \text{at} \quad x = \delta \tag{4'}$$

$$\delta = 0 \quad \text{at} \quad \theta = 0 \tag{5'}$$

$$u_1 = u_0 (= t_0 - t_p) = u_2 \quad \text{at} \quad \theta = 0 \tag{6'}$$

$$-\lambda_1 \frac{\partial u_1}{\partial x} + \lambda_2 \frac{\partial u_2}{\partial x} = Rr_p \quad \text{at} \quad x = \delta \tag{7'}$$

$$\frac{\partial u_2}{\partial x} = 0 \quad \text{at} \quad x = L \tag{10'}$$

The values of h_p, which is the heat transfer coefficient at surface during the second falling rate period, can be taken as constant from our experimental results and λ_1 which is the effective thermal conductivity of the dried-up zone of the bed, is also constant. Furthermore λ_2 which is the effective thermal conductivity of the wetted zone of the bed, is a function of moisture content but varies slightly in the moisture range of this period, so it can be taken as nearly constant also.

3. Method of Analytical Solution

The heat-balance integral approximation is now introduced. The temperature distributions in the drying bed are assumed to be quadratic in x.

$$u_1(x, \theta) = \mathrm{A}(\theta)+\mathrm{B}(\theta)\,x+\mathrm{C}(\theta)\,x^2 \tag{12}$$

$$u_1(x, \theta) = A'(\theta)+B'(\theta)\,x+C'(\theta)\,x^2 \tag{13}$$

The following ordinary differential equations are lead by heat-balance integral method employing eqs. (1)′, (7)′, (9)′ (10)′, (12) and (13).

$$M_1 = \delta^2\frac{H(\theta)}{2\lambda_1}-\frac{1}{3\kappa_1}\frac{dM_1}{d\theta} \tag{14}$$

$$M_2 = \frac{(L-\delta)}{3\kappa_2}\frac{dM_2}{d\theta} \tag{15}$$

$$-\rho_1'nr_pw_p\left(1-\frac{\delta}{L}\right)^{n-1}\frac{d\delta}{d\theta}+\frac{\lambda_1}{\kappa_1}\frac{dM_1}{d\theta}+\frac{\lambda_2}{\kappa_2}\frac{dM_2}{d\theta} = H(\theta) \tag{16}$$

with the initial conditions;

$$\delta(\theta=0)=0, \quad M_1(\theta=0)=0, \quad M_2(\theta=0) = -u_0L \tag{17}$$

The quantities M_1 and M_2 appearing in these equations are defined as follows;

$$M_1(\theta) = \int_0^\delta u_1(x, \theta)\,dx \tag{18}$$

$$M_2(\theta) = \int_\delta^L u_2(x, \theta)\,dx \tag{19}$$

Despite of the great simplification introduced by the heat-balance integral method, a system of three resulting simultaneous ordinary differential equations remain nonlinear.

To solve the equations of this type, it is a standard approach to assume a solution in the form of a power series in powers of some small parameters. This series is substituted into the ordinary differential equations to obtain the equations for the coefficients in the series.

Before solving the equations, it is expedient to nondimensionalize the equations introducing the following variables;

$$\left.\begin{aligned} \sigma &= \frac{\delta}{L} \\ y &= \frac{\lambda_1\kappa_1 M_1}{\lambda_2\kappa_2 u_0 L} \\ v &= \frac{M_2}{u_0 L} \\ \tau &= \frac{\kappa_2}{L^2}\theta \\ a &= \frac{H(\theta)L}{2\lambda_2 u_0} \end{aligned}\right\} \tag{20}$$

and the following dimensionless parameters:

$$\left.\begin{aligned} \mu &= -\frac{\lambda_2 u_0}{\rho_1{'}\kappa_2 r_p n w_p} \\ \nu &= \frac{\kappa_1}{\kappa_2} \end{aligned}\right\} \tag{21}$$

In terms of the dimensionless quantities, eqs. (17)~(19) take the form;

$$(1-\sigma)^{n-1}\frac{d\sigma}{d\tau}+\mu\frac{dy}{d\tau}+\mu\frac{dv}{d\tau} = 2\mu a \tag{22}$$

$$\frac{\sigma^2}{3}\frac{dy}{d\tau} = (a\sigma^2-\nu y) \tag{23}$$

$$\frac{1}{3}(1-\sigma)^2\frac{dv}{d\tau} = -v \tag{24}$$

with initial conditions,

$$\sigma(\tau=0)=0,\quad y(\tau=0)=0,\quad v(\tau=0)=-1 \tag{25}$$

The parameter μ is chosen as the parameter with which the dimensionless variables are expanded and the resulting series of solutions are converged, since it lies between 0 and 1. This choice is in debt to Lighthill's method[4].

So the functions $\sigma(\tau)$, $y(\tau)$, $v(\tau)$ take the form of expansions.

$$\left.\begin{aligned} \sigma &= \sum_{i=0}^{\infty}\sigma_i(\nu,\tau)\mu^i \\ y &= \sum_{i=0}^{\infty}y_i(\nu,\tau)\mu^i \\ v &= \sum_{i=0}^{\infty}v_i(\nu,\tau)\mu^i \end{aligned}\right\} \tag{26}$$

Substituting these equations into eq. (22)~(24), it is found that the approximations of every order are given by the following first order linear equations,

$$\left.\begin{aligned}&(1-\sum\sigma_i\mu^i)^{n-1}\frac{d}{d\tau}\sum\sigma_i\mu^i+\mu\frac{d}{d\tau}\sum y_i\mu^i+\mu\frac{d}{d\tau}\sum v_i\mu^i=2\mu a\\&\frac{1}{3}\Big\{\sum\sigma_i\mu^i\Big\}^2\frac{d}{d\tau}\sum y_i\mu^i=a\Big\{\sum\sigma_i\mu^i\Big\}^2-\nu\sum y_i\mu^i\\&\frac{(1-\sum\sigma^2{}_i\mu^i)^2}{3}\frac{d}{d\tau}\sum v_i\mu^i=-\sum v_i\mu^i\end{aligned}\right\}\tag{27}$$

In these equations the unknown function $u_1(0, \theta)$ which must be found is included in the term of a. In this case "self consistent method" may be employed.

As the starting function of a, we prefer to take the solution under the conditions in which the specific heat of the bed, c_p, is negligible small as compared with the latent heat of evaporation, r_p.

After repetition by "self consistent method", the final solution of eq. (27) leads to the convergent form as follows.

$$M_1=\mu^2\frac{L^2h_pu_a}{2}\sigma_1^2\tag{28}$$

$$\sigma_1=-\Big\{\frac{L\lambda_1h_pu_a}{\lambda_2u_0}\tau+(e^{-3\tau}-1)\Big\}\tag{29}$$

The higher order solutions more than second order are negligible compared with the first order one.

Using eqs. (28) and (29), $A(\theta)$, $B(\theta)$, and $C(\theta)$ in eq. (12) can be obtained as follows:

$$A(\theta)=\frac{H(\theta)}{\lambda_1}\delta-C(\theta)\delta^2\tag{30}$$

$$B(\theta)=-\frac{H(\theta)}{\lambda_1}\tag{31}$$

$$C(\theta)=\frac{3}{4\lambda_1L_0}H(\theta)-\frac{3(\sigma_1\mu)^2Lh_pu_a}{4\delta^2\sigma\lambda_1}\tag{32}$$

where

$$H(\theta)=h_p\Big\{u_a-\frac{3}{4}(\sigma_1\mu)^2\frac{Lh_pu_a}{\sigma\lambda_1}\Big\}\frac{4\lambda_1}{4\lambda_1+h_pL\sigma}\tag{33}$$

Substituting eqs. (30)~(32) into eq. (12), the temperature distributions of the dried-up zone are calculated.

The distance of tne evaporating plane from the surface is obtained as follows:

$$\begin{aligned}\delta(\theta)&=L\sigma=L\Big\{\sigma_1\mu+\sigma_2\mu^2\Big\}\\&\fallingdotseq L\Big\{\sigma_1\mu+\frac{(n-1)}{2}(\sigma_1\mu)^2\Big\}\end{aligned}\tag{34}$$

For the wetted zone, the following results are obtained similarly.

$$A'(\theta) = -B'(\theta)\delta - C'(\theta)\delta^2 \tag{35}$$

$$B'(\theta) = -2C'(\theta)L \tag{36}$$

$$C'(\theta) = -\frac{3}{2}\frac{vu_0}{L^2(1-\sigma)} \tag{37}$$

where

$$v = \frac{4}{3}\frac{Lh_p u_a}{\lambda_2 u_0} \tag{38}$$

$$\sigma \fallingdotseq \mu\sigma_1 + \frac{(n-1)}{2}(\mu\sigma_1)^2 \tag{39}$$

Substituting eqs. (35)~(37) into eq. (11), the temperature distributions of the wetted zone are calculated.

4. Numerical Solution

The accuracy of the above analytical calculated results of eq. (12) with eqs. (30)~(32) and eq. (13) with eqs. (35) and (36) are examined by comparing with the numerical calculation.

We adopt the generalized numerical method suggested by W. D. Murray and F. Landis[5] for our problem.

It is assumed that the dried-up zone $(0<x<\delta)$ is divided into equally spaced increments of thickness $\Delta x_D=\delta/r$, increasing as the evaporating plane retreats. Similarly, the wetted zone $(\delta<x<L)$ is also divided into $(N-r)$ equally spaced intervals of thickness $\Delta x_{\overline{w}}=(L-\delta)/(N-r)$, decreasing with time. This is illustrated in **Fig. 4** for the special case of $N=2r=8$ network intervals.

When the basic differential equation (1)′ and (2)′ are written in a difference form of eqs. (40) and (41), all terms on the right-hand side should be expressed with a consistent approximation. A consistant three-point approximation will result in:

$$\frac{u_{1(n,m+1)}-u_{1(n,m)}}{\Delta\theta} = \frac{n}{\delta_m}\frac{u_{1(n+1,m)}-u_{1(n-1,m)}}{2}\frac{(\delta_{m+1}-\delta_m)}{\Delta\theta} + \kappa_1 r^2 \frac{u_{1(n-1,m)}-2u_{1(n,m)}+u_{1(n+1,m)}}{\delta_m^2} \tag{40}$$

$$\frac{u_{2(n,m+1)}-u_{2(n,m)}}{\Delta\theta} = \frac{N-n}{L-\delta_m}\frac{u_{2(n+1,m)}-u_{2(n-1,m)}}{2}\frac{(\delta_{m+1}-\delta_m)}{\Delta\theta} + \kappa_2(N-r)^2\frac{u_{2(n-1,m)}-2u_{2(n,m)}-u_{2(n+1,m)})}{(L-\delta_m)^2} \tag{41}$$

where suffixes n and m are corresponded to distance x and time θ respectively. Two boundary conditions eqs. (3)′ and (10)′ become:

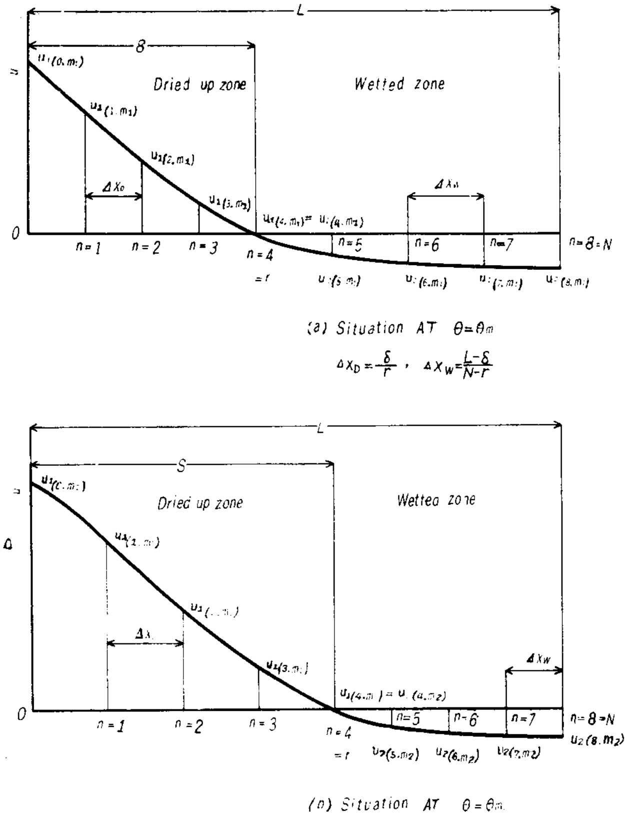

Fig. 4. Description of variables in numerical method.

$$\frac{u_{1(0,\,m+1)}-u_{1(0,\,m)}}{\Delta\theta}=\frac{\kappa_1}{\Delta x_D}\left\{\frac{u_{1(1,\,m)}-u_{1(0,\,m)}}{\Delta x_D}+\frac{h_p(u_a-u_{1(0,\,m)})}{\lambda_1}\right\} \tag{42}$$

$$\frac{u_{2(N,\,m+1)}-u_{2(N,\,m)}}{\Delta\theta}=\frac{2\kappa_2}{\Delta x_W{}^2}(u_{2(N-1,\,m)}-u_{2(N,\,m)})=0 \tag{43}$$

Eq. (7)' becomes:

$$\frac{\delta_{m+1}-\delta_m}{\Delta\theta}=-\frac{L}{\rho_1' nr_p w_p}\left(1-\frac{\delta_m}{L}\right)^{1-n}\left\{\lambda_1 r\frac{u_{1(r-2,\,m)}-4u_{1(r-1,\,m)}}{2\delta_m}\right.$$
$$\left.+\lambda_2(N-r)\frac{u_{2(r+2,\,m)}-4u_{2(r+1,\,m)}}{2(L-\delta_m)}\right\} \tag{44}$$

When $\delta=0$, some terms of eqs. (40) and (41) become either infinite or indeterminate. Therefore, the problem must be started with a small assumed intial value δ_0, and starting temperature must be assigned to the points in the dried-up zone. By the same argument, the solution must be stopped before $(L-\delta)=0$ and the final time period must be extrapolated.

In the numerical solution, the choice of the initial distance of evaporating plane δ_0 will have considerable effect on the solution time required; due to

Table. 1.

Run No.	3-90	Ac-7	S-22
material	$CaCO_3$*	Acricon**	Sand
L (m)	0.03	0.03	0.03
ρ_1' (kg/m³)	1596	526	1120
t_a (°C)	90.0	71.0	70.3
h_p (kcal/m². hr. °C)	18.5	5.92	12.8
t_p (°C)	63.6	49.5	50.0
r_p (kcal/kg)	560.5	568	568
w_p (kg/kg)	0.05	0.116	0.047
t_{mo} (°C)	55.1	44.5	46.8
c_{p_1} (kcal/kg. °C)	0.19	0.35	0.19
λ_1 (kcal/m.hr. °C)	0.231	0.0996	0.230
c_{p_2} (kcal/kg.°C)	$c_{p_1}+w$		

* fine power of calcium carbonate (ca. 1μ)
** beads of polythylmetaacrylate (42~60#)

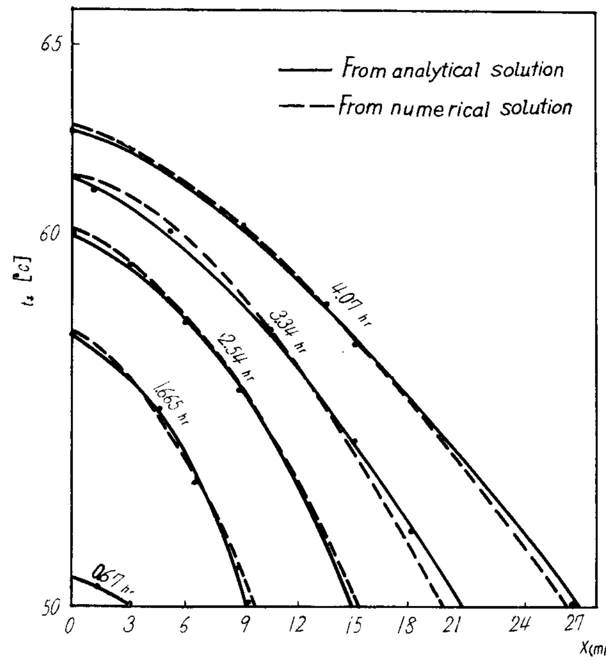

Fig. 5. Comparison of the approximated analytical solution with the numerical one (Run No. S-22).

stability requirements a small initial $\Delta\theta$, coresponding to the initial values of Δx_D, materially increases the solution time for early stages. The computations should be carried out with a time interval determined from $\frac{\Delta\theta}{\Delta x^2}=1/5$, which exceeds the stability criterion and provides improved accuracy. The computations were carried out with $N=2r=8$ and $\Delta\theta=1/4$ hr, and with a starting temperature distribution by the analytical solution at $\theta=1/4$ hr. For the evalution of the analytical solution and the numerical solution, the physical constants and the boundary conditions summerized in Table 1 were used. Both solutions about the Run Number S-22 are shown in **Fig. 5.**

The agreement of both calculated results shows the propriety of the analytical solutions.

5. Comparisons with the Experimental Results

The calculated results of the temperature of the bed versus time using the data in Table 1 are shown in **Fig. 6, Fig. 7** and **Fig. 8.** The experimental results of the temperature versus time are shown also in these figurs.

These are in good agreement. The calculated results of the distance $\delta(\theta)$ of the evaporating plane from surface versus time are shown in **Figs. 9** (1), (2) comparing with the experimental ones.

These are also in good agreement.

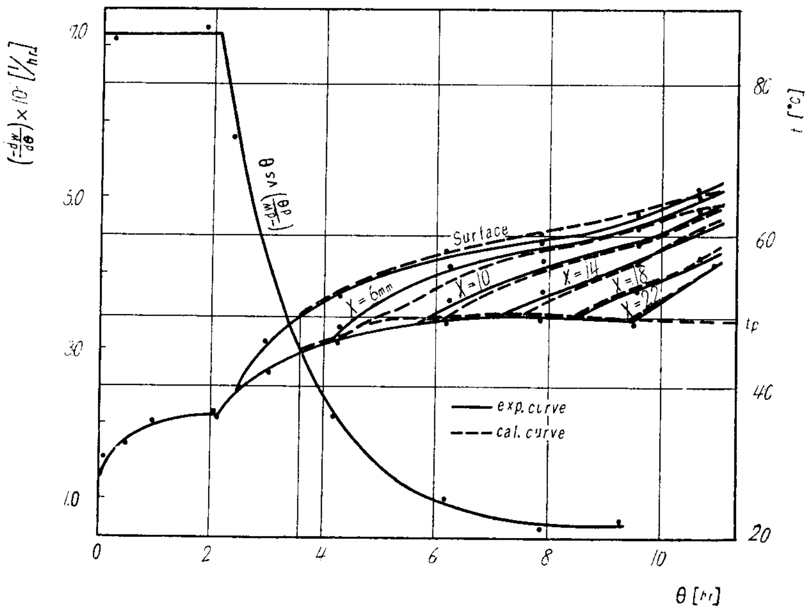

Fig. 6. Comparison of the analytical solution with the experimental temperature distribution (Run No. Ac-7).

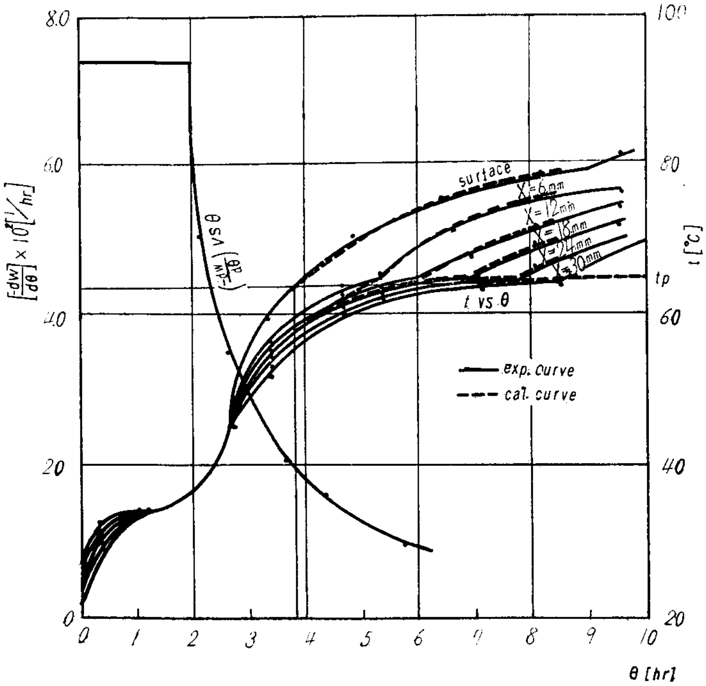

Fig. 7. Comparison of the analytical solution with the experimental temperature distribution (Run No. 3-90, fine powder of $CaCO_3$, t_a=90°C, H_a=0.0076, V_a=5.58 m/sec, L=0.03 m).

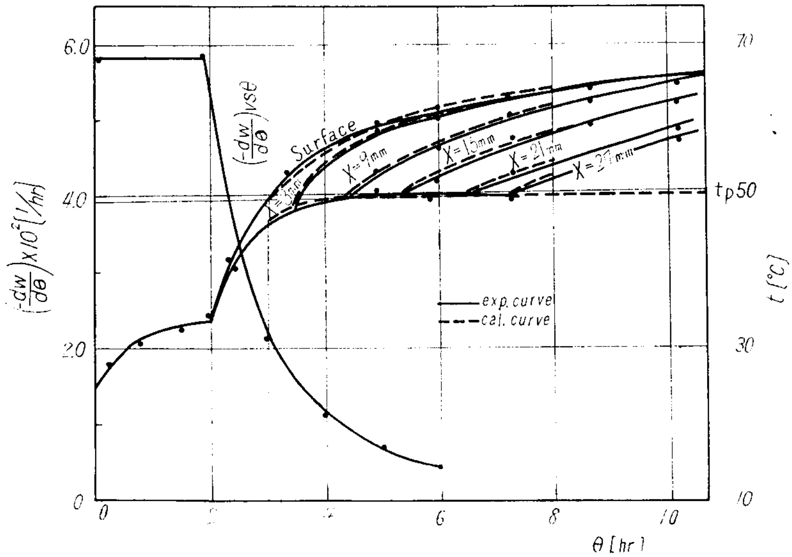

Fig. 8. Comparison of the analytical solution with the experimental temperature distribution (Run No. S-22, Sand (60~80#), t_a=70.3°C, H_a=0.0102, V_a=4.5 m/sec, L=0.03 m),

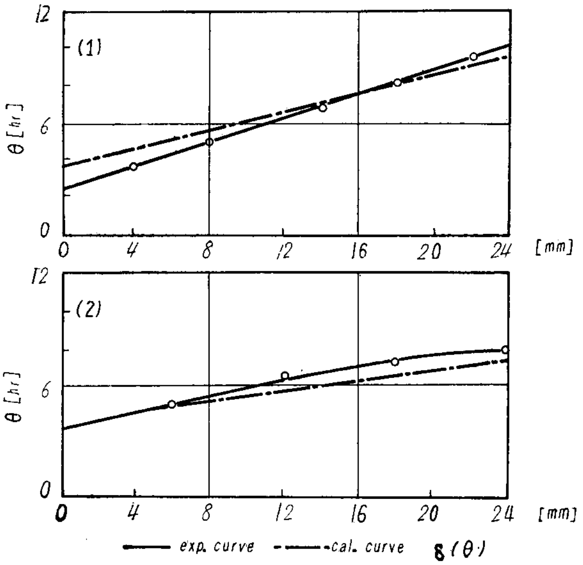

Fig. 9. Comparison of the calculated values of $\delta(\theta)$ vs. θ with the experimental ones (1) Run No. Ac-7, (2) Run No. 3-90.

Acknowledgment

The authors wish to express their appreciations to the student T. Fujitani for his help to this work.

Nomenculature

$H(\theta)$	:	heat flux at surface of the drying bed	[kcal/m²·hr]
h_p	:	overall heat transfer coefficient of air film	[kcal/m²·hr·°C]
L	:	depth of bed	[m]
r	:	latent heat of evaporation of water	[kcal/kg]
t	:	temperature	[°C]
t_p	:	asymptotic temperature	[°C]
u	:	reduced temperature ($=t-t_p$)	[°C]
w	:	average moisture content	[kg/kg]
w_p	:	average moisture content at the beginning of the second falling rate period	[kg/kg]
x	:	distance from surface	[m]

greek letter

δ	;	distance of evaporating plane from surface	[m]

κ	:	thermal diffusivity $\left(=\frac{\lambda}{c_p\rho'}\right)$	[m²/hr]
λ	:	effective thermal conductivity of bed	[kcal/m·hr·°C]
ρ_1'	:	bulk density of dried bed	[kg/m³]
θ	:	time	[hr]

suffix

0	:	initial value
1	:	value at dried-up zone
2	:	value at wetted zone
a	:	air
i	:	value at surface
m	:	material
p	:	value at w_p
δ	:	value at distance δ

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