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# An Analytical Study of Heat Transfer in Nucleate Boiling

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This paper deals with a theoretical prediction of heat transfer in a steady state nucleate boiling by applying the theory which was developed for the incidence of the first bubble in transient boiling. The predictions are compared with experimental data or empirical correlations of water and Neo SK oil # 350 (mixture of terphenyls). Some discussion is also presented for the preparing period of one cycle of bubbling.

## 1. Introduction

In recent years the importance of nucleate boiling heat transfer has increased, especially in relation to nuclear reactor engineering. Although there have been many investigations referring to the nucleate boiling, the mechanism of heat transfer in nucleate boiling is still unknown and indistinct; for examples, the nucleation, the bubble formation, the bubble growth, the frequency of bubbling, and the reason why the heat transfer characteristics are represented by the equation  $q_0 \propto \Delta T_s^n$  ( $n=3\sim 5$ ), and so on.

This paper deals with a theoretical analysis of heat transfer in a steady nucleate boiling, paying particular attention to the bubble formation. A theory<sup>1)</sup> which was derived for the prediction of the incidence of the first bubble in transient boiling will be extended to steady boiling, and it will be theoretically shown that the heat flux  $q_0$  is proportional to  $\Delta T_s^n$ . Some discussion will be also given to the frequency of bubbling.

## 2. An Analysis of Heat Transfer Characteristics in Nucleate Boiling

When a thin metal ribbon immersed in subcooled stagnant water is heated

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at exponential rates in time, the ribbon temperature will rise. However, boiling does not commence until the surface has exceeded the temperature level which is usually associated with the inception of steady boiling in a stagnant pool<sup>2</sup>). In such transient boiling, it is very interesting to investigate the incidence of the first bubble. However, the bubbles are formed periodically on the metal surface in steady state nucleate boiling. New bubbles are regenerated some time after the bubbles have detached from the surface. The phenomena of these two kinds of boiling suggest that there must be a circumstance in which nucleation can occur and the formation of new bubble can proceed, and that boiling does not commence until this circumstance has been established. Therefore, a theory related to the existence of such circumstances, which was derived for the prediction of the incidence of the first bubble in transient boiling, may be applicable to the successive formations of bubbles in steady boiling.

As is well known, the heat transfer characteristics of nucleate boiling can be correlated well by the relationship between the heat flux and the degree of overheating of the wall over the saturation temperature regardless of the degree of subcooling. This fact may show that the steady nucleate boiling is closely related to the temperature field near the surface.

Now, according to the kinematic theory of liquid<sup>3</sup>), the liquid should be overheated by the temperature difference  $T - T_s$ , in order to cause the boiling within the liquid. The embryonic bubble which can exist within the liquid has a critical radius  $r^*$  given by

$$r^* = \frac{2\sigma}{\Delta p} \quad (1)$$

In this occasion, the difference of the thermodynamical potential of the system which involves the bubble from that of absence of the bubble should have a maximum value at critical radius, and the bubble can grow further since the bubble has enough radius to pass through the potential barrier, which plays a role similar to that of the activation energy in the process of chemical reactions. Equation (1) can be changed to the following:

$$r^* = \frac{2\sigma}{p \left[ \exp \left\{ \frac{(T - T_s)\lambda}{kT_s T} \right\} - 1 \right]} \quad (2)$$

This equation shows that the critical radius  $r^*$  of embryonic bubble is a function of the overheating of liquid,  $T - T_s$ .

If it is assumed that the theory mentioned above can be applied for the

boiling which occurs on the surface of the heated wall and the critical radius  $r^*$  is a function of the overheating  $T_0 - T_s$  of the wall surface over the saturation temperature,  $r^*$  is given by the following equation:

$$r^* = \frac{2\sigma}{p \left[ \exp \left\{ \frac{(T_0 - T_s) \Delta}{k T_s T_0} \right\} - 1 \right]} \quad (3)$$

When we consider the circumstance required to cause nucleate boiling on the surface, it infers that the liquid near the surface must be overheated over the saturation temperature and therefore the heat energy must be stored in the liquid. Consequently, it is worthy to consider the heat energy  $E$  which is necessary to overheat the liquid layer within the critical radius  $r^*$  over the saturation temperature.  $E$  is given by

$$E = \int_0^{r^*} c\gamma(T - T_s)dy \quad (4)$$

Since  $r^*$  is defined by equation (3), the heat energy  $E$  is a function of the overheating  $T_0 - T_s$ , and  $E$  has a maximum value when the overheating  $T_0 - T_s$  takes a certain value. This characteristic was shown in the literature<sup>1)</sup> which discussed the incidence of the first bubble in transient boiling, and it inferred that the first bubble might occur when the heat energy  $E$  maximizes. Later, Lienhard<sup>4)</sup> interpreted thermodynamically that the first bubble occurs at the maximum value of  $E$ , assuming that the most likely overheating  $T_0 - T_s$  for the occurrence of the first bubble is at where the decrease of potential barrier ceases to be rapid and the increase of the thermodynamic availability begins to dominate. The prediction based on this theory agrees with Johnson et al's<sup>5)</sup> experimental results of exponential heat input transient boiling, regardless of whether it is saturated or subcooled.

Thus, the requirement of the circumstance mentioned above to cause nucleate boiling is to maximize the heat energy  $E$ , and this theory is also applicable to steady state nucleate boiling in which the heat flux  $q_0$  is constant in time.

As shown in Fig. 1, the critical radius  $r^*$  is so small that the distribution of temperature of liquid within  $r^*$  may be assumed

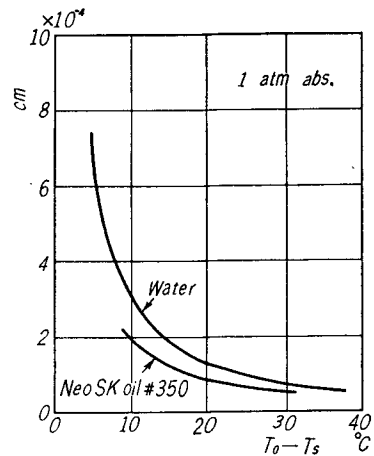


Fig. 1 Critical radius of embryonic bubble

linear and the temperature gradient is equal to  $-q_0/\lambda$ . Consequently, the heat energy  $E$  is given by the following equation:

$$E = c\gamma \left[ 2(T_0 - T_s) - \frac{q_0}{\lambda} r^* \right] \frac{r^{*2}}{2} \quad (5)$$

where the heat capacity  $c\gamma$  is defined at the saturation temperature and assumed to be constant.

Hence, one can evaluate  $E$  as a function of  $T_0 - T_s$  by using equations (3) and (5) under the condition of given heat flux  $q_0$ .

(1) The case of water at 1 atm abs.

The physical properties used for the calculations are defined at the saturation temperature and given as follows:

$$\begin{aligned} p &= 1.013 \times 10^6 \text{ dyn/cm}^2 \\ T_s &= 373^\circ \text{K} \\ A &= 6.7375 \times 10^{-13} \text{ erg} \\ \sigma &= 58.85 \text{ dyn/cm} \\ \lambda &= 0.586 \text{ kcal m}^{-1} \text{ h}^{-1} \text{ }^\circ\text{C}^{-1} \\ c &= 1.007 \text{ kcal kg}^{-1} \text{ }^\circ\text{C}^{-1} \\ \gamma &= 958.4 \text{ kg/m}^3 \end{aligned}$$

Fig. 2 illustrates the relationship between  $E/c\gamma$  and  $T_0 - T_s$ . Evidently, the heat energy  $E/c\gamma$  has a maximum value marked by an arrow. And it should be noted that the higher the heat flux  $q_0$ , the larger the overheating  $T_0 - T_s$  corresponding to the maximum of  $E$ . Consequently, one can obtain the relationship between the heat flux  $q_0$  and the most likely overheating  $\Delta T_s$  for given  $q_0$

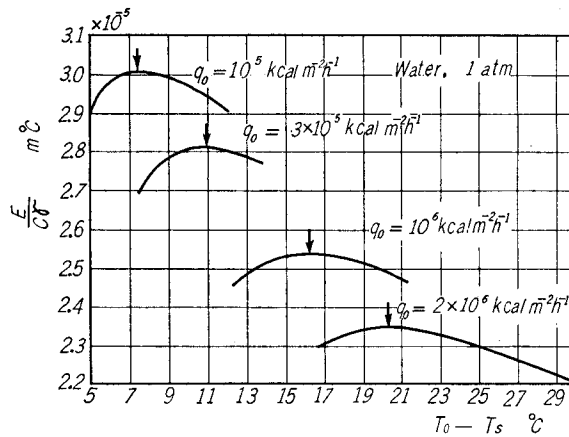


Fig. 2  $\frac{E}{c\gamma}$  vs.  $T_0 - T_s$  for water

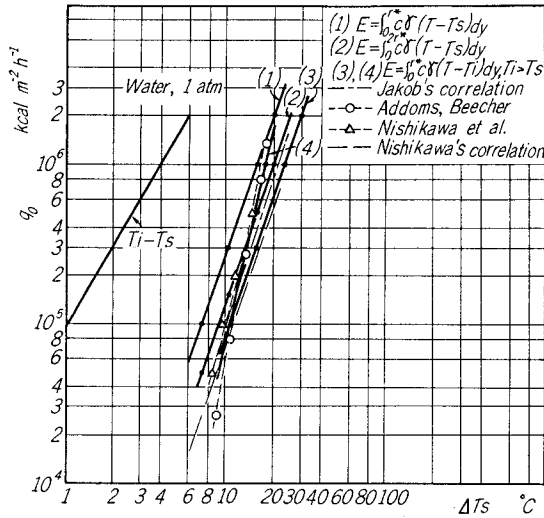


Fig. 3 Nucleate boiling heat transfer of water

as shown in Fig. 3, where  $\Delta T_s$  is defined as the overheating  $T_0 - T_s$  at where  $E$  takes the maximum.

In this figure, the full line marked (1) uses the integration

$$E = \int_0^{r^*} cr(T - T_s)dy$$

, while that marked (2)

$$E = \int_0^{2r^*} cr(T - T_s)dy \quad (6)$$

, in which  $E$  is defined as the heat energy which is necessary to overheat the liquid layer within  $2r^*$  instead of  $r^*$ .

Regardless of the definition of  $E$ , the following expression is obtainable:

$$q_0 \propto \Delta T_s^3 \quad (7)$$

This relationship is well known as the heat transfer characteristic in nucleate boiling. Comparing with the experimental results and the correlations, it is found that the full line marked (2) agrees quantitatively better with experiments than that marked (1).

Now, according to Isshiki and Tamaki's<sup>6)</sup> schlieren moving pictures, it has been found that the detached bubbles make a wake of which the temperature is high and that a thin liquid layer in which nucleation will occur remains in contact with the wall surface after the bubble detached from the surface, regardless of whether it is saturated or subcooled boiling. If it is true, we may assume that a liquid thin layer of high temperature always remains, being

undestroyed by the detached bubble in such a steady nucleate boiling. Hence,  $E$  is given by

$$E = \int_0^{r^*} c\gamma(T - T_i)dy \quad (8)$$

, instead of equation (4), in which  $T_i$  is a certain temperature averaged within  $r^*$  and higher than the saturation temperature  $T_s$ .

In Fig. 3, the full line marked (3) shows the results evaluated by the same method as the case of that marked (1) or (2). In this calculation,  $T_i - T_s$  was assumed to be a function of heat flux  $q_0$  as shown in Fig. 3. The full line marked (3) clearly shows the same relationship as equation (7), and it is close to Jakob's<sup>7)</sup> or Nishikawa's<sup>8)</sup> correlation. Of course, if it is assumed that  $T_i - T_s$  is not a function of  $q_0$  but constant, then the exponent  $n$  of  $\Delta T_s$  in equation (7) becomes larger than three; for example, when  $T_i - T_s = 1^\circ\text{C}$ ,

$$\begin{aligned} q_0 &= 10^5 \text{ kcal m}^{-2} \text{ h}^{-1} : \Delta T_s = 11^\circ\text{C} \\ q_0 &= 10^6 \text{ kcal m}^{-2} \text{ h}^{-1} : \Delta T_s = 18^\circ\text{C} \end{aligned}$$

hence

$$q_0 \propto \Delta T_s^{4.68}$$

This relationship which is shown by the full line marked (4) is close to Addoms's<sup>9)</sup>, Beecher's<sup>10)</sup> or Nishikawa's<sup>11)</sup> experimental data.

(2) The case of Neo SK oil #350 (mixture of terphenyls) at 1 atm abs.

The physical properties of Neo SK oil #350 used for the calculations are given as follows<sup>12)</sup>:

$$\begin{aligned} p &= 1.013 \times 10^6 \text{ dyn/cm}^2 \\ T_s &= 636^\circ\text{K} \\ A &= 9.439 \times 10^{-13} \text{ erg} \\ &\left( \begin{array}{l} \text{molecular weight} = 230.3 \\ \text{latent heat of evaporation} = 59 \text{ kcal/kg} \end{array} \right) \\ \sigma &= 18.1 \text{ dyn/cm} \\ \lambda &= 0.07164 \text{ kcal m}^{-1} \text{ h}^{-1} \text{ }^\circ\text{C}^{-1} \\ c &= 0.632 \text{ kcal kg}^{-1} \text{ }^\circ\text{C}^{-1} \\ \gamma &= 827 \text{ kg/m}^3 \end{aligned}$$

These values are defined at the saturation temperature which was investigated experimentally at our laboratory.

The Neo SK oil #350 is composed of 5% ortho-terphenyl, 50% meta-terphenyl, 40% para-terphenyl and 5% polymers. This organic liquid was selected since it is similar to the nuclear reactor coolant.

Fig. 4 illustrates the relationship between  $E/c\tau$  and  $T_0 - T_s$ . By using  $T_0 - T_s$  which corresponds to the maximum value of  $E$  marked by an arrow as the most likely overheating  $\Delta T_s$ ,  $q_0$  vs.  $\Delta T_s$  curves are obtainable as shown in Fig. 5. In this figure, the full line marked (1) shows the results evaluated by equation (4) or (5), and that marked (2) by equation (6). It is evidently shown that the relationship given by equation (7) still holds even for Neo SK oil #350, and that the coefficient of boiling heat transfer of the mixture of terphenyls is smaller than water as compared the full line marked (1) or (2) with that shown in Fig. 3.

The experiments of the nucleate boiling heat transfer for Neo SK oil #350 were performed<sup>12)</sup> by using a stainless steel ribbon (3 mm width  $\times$  100 mm length  $\times$  50  $\mu$ m thickness) immersed horizontally in the liquid and heated electrically. As is well known, the surface of the heated wall being in contact with the terphenyl becomes dirty during the experiments, and the heat transfer coefficient decreases from that of the fresh surface as shown in Fig. 5. However, the theoretical result, especially full line marked (2), agrees well with our experimental data performed with a fresh surface, regardless of the subcooling. Jordan and Leppert's data<sup>13)</sup> performed with horizontal tube of 0.135" o.d. looks closer to our data with dirty surface in spite of the fresh surface reported by them. Of course, since Neo SK oil #350 is a mixture of ortho-, meta-, para-terphenyl and polymers, it may not be

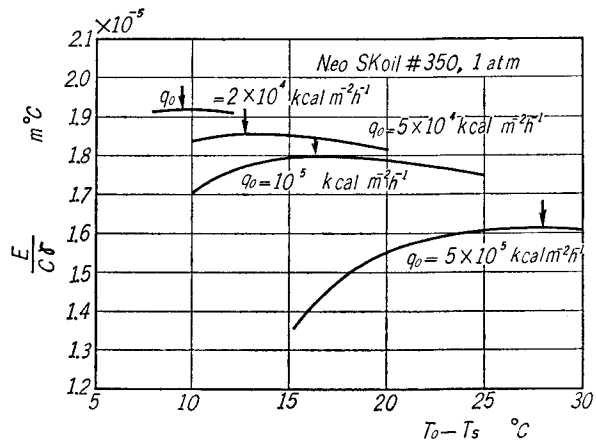


Fig. 4  $\frac{E}{c\tau}$  vs.  $T_0 - T_s$  for Neo SK oil #350

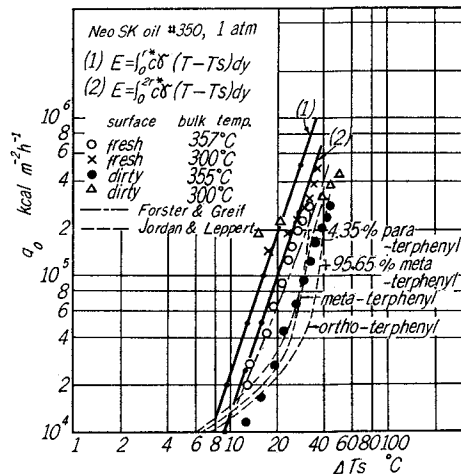


Fig. 5 Nucleate boiling heat transfer of Neo SK oil #350



qualified to be compared with pure compounds. In Fig. 5, the correlation by Forster's second equation<sup>14)</sup>, which has been reported to be applicable to many kinds of liquid, is illustrated by using the physical properties of Neo SK oil #350 mentioned above. Forster's correlation agrees well with full line marked (2).

From the examples mentioned above, generally speaking, it may be concluded that the theoretical prediction of boiling heat transfer characteristics developed in this paper coincides well with experimental data or empirical correlations, though the predicted overheating  $\Delta T_s$  gives a slightly low value at given heat flux  $q_0$ .

### 3. Some Discussions on Periodical Formations of Bubbles

As described in the above section, it should be noticed that there is a circumstance near the wall surface that is required to cause nucleation, and that boiling does not commence until such a circumstance has been established.

Thus, the bubbles are periodically regenerated on the surface, as the circumstance is periodically reestablished after the bubbles are detached from the surface. Therefore, one cycle of bubbling can be considered by a model as shown in Fig. 6.

Now, attention is paid especially to the preparing period. After the bubble detached from the surface, the bubbling elapses until the nucleation occurs. This is due to the destruction and disorder of the temperature field near the surface caused by the detached bubble. In the above section, the heat energy corresponding to the most likely overheating  $\Delta T_s$ , which will be denoted as

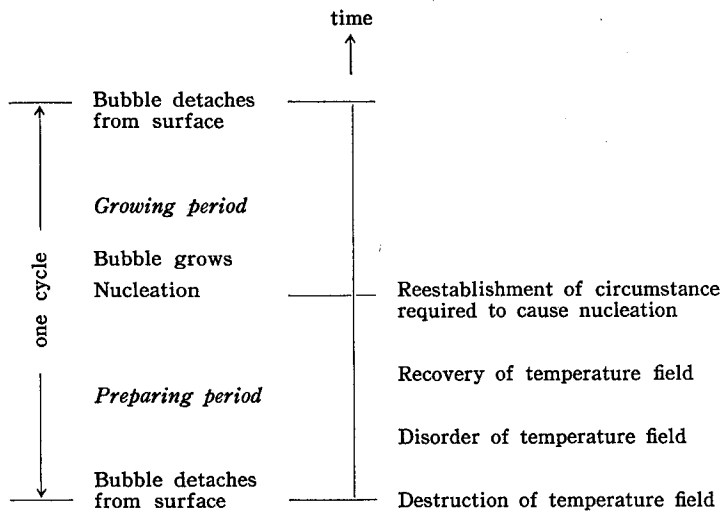


Fig. 6 One cycle of bubbling

$E_{\max}$ , was obtained as a function of heat flux  $q_0$ .  $E_{\max}$  is given by the heat energy which is required to overheat the liquid layer within  $r^*$  or  $2r^*$  corresponding to  $\Delta T_s$  over the saturation temperature  $T_s$  or a certain initial temperature  $T_i$  and is stored within the layer. Thus, we can evaluate  $E_{\max}/q_0$ , of which the dimension is time. This result is shown in Fig. 7. Evidently, the time  $E_{\max}/q_0$  decreases as  $q_0$  increases.  $E_{\max}/q_0$  is the time, which is necessary to overheat the liquid layer within  $r^*$  or  $2r^*$  to the most likely temperature over  $T_s$  or  $T_i$  under the conditions of constant heat flux  $q_0$  and of thermally insulated surface at  $y=r^*$  or  $2r^*$ . However, since the heat is transferred to the liquid through the surface at  $y=r^*$  or  $2r^*$ , it is more proper to consider the following transient heat conduction:

$$\left. \begin{aligned} \frac{\partial T}{\partial t} &= \frac{\lambda}{cr} \left( \frac{\partial^2 T}{\partial y^2} \right) \\ t = 0: T &= T_i \quad (\text{uniform}) \\ t > 0: -\lambda \left( \frac{\partial T}{\partial y} \right)_{y=0} &= q_0 \quad (\text{constant}) \end{aligned} \right\} \quad (9)$$

Namely, the heat is transferred from the wall surface at constant rate  $q_0$  in time, and the initial temperature of liquid very close to the surface is  $T_i$  at anywhere. Thus, the temperature  $T_0$  of the wall surface increases with time according to the following equation<sup>1)</sup>:

$$T_0 = T_i + \frac{q_0}{\sqrt{\lambda cr}} \frac{2}{\sqrt{\pi}} \sqrt{t} \quad (10)$$

In fact, the temperature of the wall surface varies periodically around the average temperature in steady nucleate boiling as pointed out by Hsu and Schmidt<sup>15)</sup>, Moore and Mesler<sup>16)</sup>. Although the disorder of the temperature field, of course, caused by the detached bubble is very complicated to be solved analytically, if it is assumed that the heat transfer to the liquid from the surface is simply given by the heat conduction equation (9), we can evaluate the time in which the surface temperature reaches the most likely temperature  $T_s + \Delta T_s$ .

Fig. 8 illustrates the time thus evaluated. In the case of the full line marked (1) or (2), we took the initial temperature  $T_i$  being equal to  $T_s$ ; the

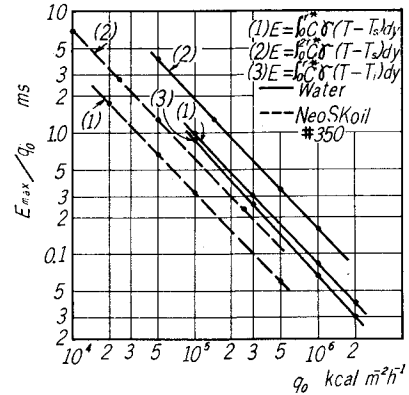


Fig. 7  $E_{\max}/q_0$  vs.  $q_0$

marked (3), the value which was shown in Fig. 3. As compared with Fig. 7, it is found that the time required to cause nucleation is longer than  $E_{max}/q_0$  but there is a similar tendency.

Isshiki and Tamaki<sup>6)</sup> found experimentally that the length of the preparing period is about 7 ms for the saturated boiling of water at atmospheric pressure under the low heat flux of around  $10^5$  kcal m<sup>-2</sup> h<sup>-1</sup>. Hsu and Graham<sup>17)</sup> measured the length of the preparing period and the growing period for 9°C subcooled boiling of water at 1 atm under the heat flux of  $2.25 \times 10^5$  kcal m<sup>-2</sup> h<sup>-1</sup> and pointed out that the preparing period was much longer than the growing period and the length of the preparing period varied quite randomly even if the heat flux was constant. Their 10 data out of 20 data show the length of preparing period ranges from 90 ms to 155 ms and the maximum is 387 ms and the minimum 0 ms. However, these facts are not surprising, because the heat transfer in nucleate boiling should be considered statistically.

The data of Hsu and Graham may suggest that the length of the preparing period for the subcooled boiling is longer than the saturated boiling. Because the liquid near the surface is cooled by the flow of bulk liquid rushing into the place where the bubble was and there must be a time lag until the liquid within  $r^*$  or  $2r^*$  exceeds the saturation temperature. But the heat rejected by the condensing bubble must be transferred to the surrounding liquid. Therefore, the liquid does not need to receive all the heat which comes from the wall surface to warm itself. Consequently, it may not be true that the time lag mentioned above is very long.

Unfortunately there has not yet been much data to compare with the theoretical prediction of the preparing period, especially in relation to the effect of heat flux or the subcooling.

#### 4. Conclusion

The heat transfer characteristics of steady nucleate boiling were theoretically predicted by applying the theory developed for the incidence of the first bubble in transient boiling. The theoretical predictions coincide well with experimental

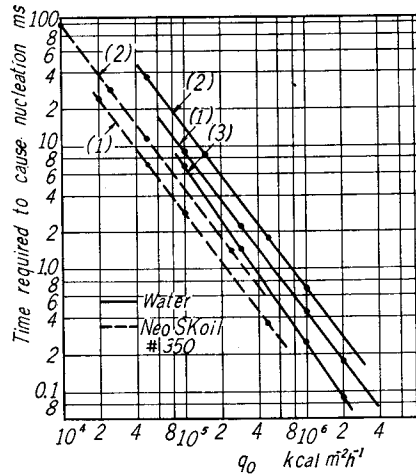


Fig. 8 Time required to cause nucleation

data or empirical correlations though the most likely overheating  $\Delta T_s$  at given heat flux  $q_0$  gives a slightly low value as compared with water or Neo SK oil #350 at 1 atm. Special attention were paid to the preparing period of one cycle of bubbling, and the length of the period was related to the heat flux. The preparing period decreases as heat flux increases. Some discussion on the formations of bubbles led to the necessity of further experiments for the preparing period in relation to the effects of heat flux and subcooling.

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#### Nomenclature

- $c$  : specific heat of liquid  
 $E$  : heat energy per unit surface area  
 $E_{\max}$  : maximum of  $E$   
 $k$  : Boltzmann constant,  $k=1.38044 \times 10^{-16}$  erg/ $^{\circ}$ K  
 $n$  : exponent of  $\Delta T_s$   
 $p$  : pressure  
 $q_0$  : heat flux  
 $r^*$  : critical radius of embryonic bubble  
 $t$  : time  
 $T$  : Temperature  
 $T_i$  : averaged temperature higher than  $T_s$ , initial temperature  
 $T_0$  : surface temperature of heated wall  
 $T_s$  : saturation temperature of liquid at  $p$   
 $y$  : distance from the surface, normal to the wall  
 $\gamma$  : specific weight of liquid  
 $\Delta p$  : pressure difference caused by surface tension  
 $\Delta T_s$  : temperature difference between the wall surface and saturation temperature; most likely overheating  
 $\lambda$  : thermal conductivity of liquid  
 $A$  : latent heat of evaporation referred to one molecule  
 $\sigma$  : surface tension

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