An Approximate Analysis of the Transient Stability of One- or Two-Machine Systems

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By

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In this paper, first, we correct Hano's approximate analysis of power system stability in which he has neglected to consider the initial angular displacement and velocity of machines when he solved his approximate differential equation of angular motion of one or two machines in power systems. Next we propose better procedures, based on approximating the trigonometric function in the original nonlinear differential equation of angular motion of the machines by more appropriate triangles than Hano's, or by trapezoids. Then developing these approximate procedure, we derive a sort of stability criterion of one- or two-machine systems, the simple formulae for the critical switching time and so on, when the circuit breakers are reclosed or not reclosed after the fault has been cleared.

At last, comparing the calculated results of some transient stability problems by the approximate procedures with those by the conventional step-by-step method, we ascertain that the approximate analysis of system stability, especially the trapzoid-approximation is a good approximate analysis of system stability.

1. Introduction

In order to solve the equations of the angular motion of some machines in power systems and to predict the transient stability, power engineers have thought of many procedures, for example, numerical calculation by the step-by-step method, the computation by AC network analyzer, analog or digital computer. Moreover, in 1930, by an excellent idea, Hano introduced[1)2)3)] the piecewise linealized analysis of transient stability.

However, it is said that the calculation results from Hano's method, comparing with those by the step-by-step method, are too pesimistic, because power systems are predicted to be apt to incline to extreme unstability in the case of the former method.

So inspecting this method in detail, it has the basic defects that he has neglected to consider the initial angular displacement and velocity of the machines when he solved his approximate differential equations of their angular

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motion as mentioned later. Hence he could not help reaching the incorrect conclusion that the machines are stable only if the approximate equations have periodical solutions.

Therefore, first, we amend his wrong theory. Next we propose better procedures based on approximating the trigonometric function in the original nonlinear differential equation for the machines by the more suitable triangles than Hano's or the trapezoids. Moreover developing these procedures, we derive a kind of stability criterion of one- or two-machine systems, the expression for the angular dispacement in transient stability limit, the direct formula for the critical switching time and so on.

At last, comparing the calculated results by these approximations with those by the step-by-step procedure, the authors ascertain that their method, especially the trapezoid-approximation is a good approximate analysis for transient stability of power systems.

2. Differential Equations of Angular Motion for Two Machines

In Fig. 1, $\dot{A}$, $\dot{B}$, $\dot{C}$ and $\dot{D}$ are the overall four terminal network constants, which consist of the transient impedances of the two synchronous machines S_1 and S_2, the transformer impedances at the sending and the receiving end. and the line constants in the transient states, i.e. during line fault, after switching out the faulty line and after reclosing breakers. In the same Figure.

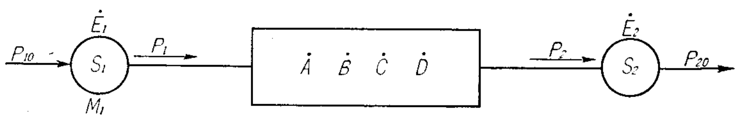


Fig. 1. Two-machine system diagram.

$P_{\nu 0}$: the mechanical input or output power for the synchronous machine S_ν before fault.

P_ν: the electrical input or output power of S_ν after fault.

M_ν: the inertia constant of S_ν.

$\dot{E}_\nu = E_\nu \underline{/\theta_\nu}$: the transient internal voltage of S_ν after fault, where E_ν and θ_ν are the magnitude and the phase angle of $\dot{E}_\nu$ respectively.

$\nu = 1$ and 2.

Next let us denote

$\dot{W} = \dot{W} \underline{/\theta_w}$, $\dot{W} = \dot{A}, \dot{B}, C, \dot{D}$; W and θ_w are the magnitude and the argument of $\dot{W}$ respectively, then, as well known, the transient electrical input or output power for S_1 and S_2 are given by, respectively

$$\left.\begin{aligned} P_1 &= \rho_{11}\cos(\theta_D-\theta_B)-\rho_{12}\cos(\theta+\theta_B), \\ P_2 &= \rho_{22}\cos(\theta_A-\theta_B)+\rho_{12}\cos(\theta-\theta_B) \end{aligned}\right\} \tag{1}$$

where

$$\rho_{11} = BE_1^2/D, \quad \rho_{12} = E_1E_2/B, \quad \rho_{22} = AE_2^2/B, \tag{2}$$

$\theta=\theta_1-\theta_2$: the electrical angular displacement between rotors of S_1 and S_2 or the voltages $\dot{E}_1$ and $\dot{E}_2$. (3)

As already known, the differential equation of angular motion for two machines in the power system shown in Fig. 1 are given as

$$\frac{d^2\theta}{dt^2} = \omega_0\left(\frac{P_{10}-P_1}{M_1}-\frac{P_{20}-P_1}{M_2}\right), \tag{4}$$

where

$$\omega_0 = 2\pi f_0$$

f_0: commercial frequency (5)

Eq. (4) is equivalent to the equation for one machine.
Next, substituting Eq. (1) into Eq. (4), yields

$$\frac{d^2\theta}{dt^2}+a_0\sin\theta+b_0\cos\theta = c_0, \tag{6}$$

where

$$\left.\begin{aligned} a_0 &= \omega_0\rho_{12}\sin\theta_B(1/M_1-1/M_2) \\ b_0 &= -\omega_0\rho_{12}\cos\theta_B \\ c_0 &= \omega_0[\{P_{10}-\rho_{11}\cos(\theta_D-\theta_B)\}/M_1-\{P_{20}+\rho_{22}\cos(\theta_A+\theta_B)\}/M_2] \end{aligned}\right\} \tag{7}$$

By the way, in reference 1), 2) and 3), denoting

$$\left.\begin{aligned} &\varphi' = \theta-\theta_0, \\ &\theta_0: \text{ initial angular displacement,} \end{aligned}\right\} \tag{8}$$

Hano introduced a fundamental equation of the same form as Eq. (6) from Eq. (4), and he linealized piecewise the fundamental equation by substituting the piecewise linear function corresponding to the ⓞ-triangle-approximation as illustrated in the next Article into $\sin\varphi'$ and $\cos\varphi'$ in the original equation, and then he investigated the transient stability of one- or two-machine systems through the solution φ' of the piecewise linealized approximate equations. However as pointed out previously, he has made the wrong conclusion that the rotating motion of the machines is stable only if φ' has a periodical solution, since he has failed to notice the effects of the initial values of φ' aud $\dot{\varphi}'=d\varphi'/dt$,

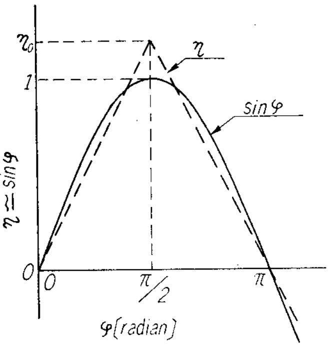


Fig. 2. Illustration of triangle-approximation.

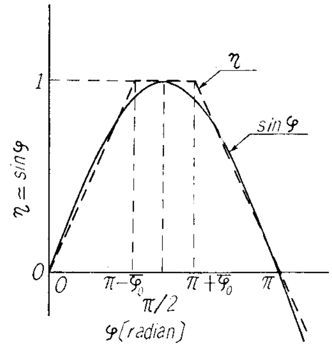


Fig. 3. Illustration of trapezoid-approximation.

On the other hand, we reduced the next convenient fundamental expression from Eq. (6), i.e.

$$\frac{d^2\varphi}{dt^2}+d_0\sin\varphi = c_0, \tag{9}$$

$$\left.\begin{aligned}\varphi &= \theta+\theta_a,\\ \theta_a &= \tan^{-1}(b_0/a_0),\\ d_0 &= \sqrt{a_0^2+b_0^2}.\end{aligned}\right\} \tag{10}$$

By means of approximating $\sin\varphi$ in the form of a triangle and a trapezoid as shown in Figs 2 and 3, we shall derive the piecewise linearized differential equations from Eq. (9). Next we shall discuss the transient stability of the systems with the solutions of the approximate equations. These will be illustrated in detail in the following Articles.

3. Approximate Differential Equations and their solutions

3.1. Case by Triangle-Approximation

From Fig. 2 and Eq. (9), we can derive the approximate differential equations and their solutions as shown in Table 1,
where

$\varphi_{\bar{0}}^{\,0}$ and $\dot{\varphi}_{\bar{0}}^{\,0}$: the initial values of φ and $\dot{\varphi}$ respectively, where $(-\pi/2)\leq\varphi\leq\pi/2$
$\varphi_{\bar{2}}^{\,0}$ and $\dot{\varphi}_{\bar{2}}^{\,0}$: the same values, where $\pi/2\leq\varphi\leq 3\pi/2$

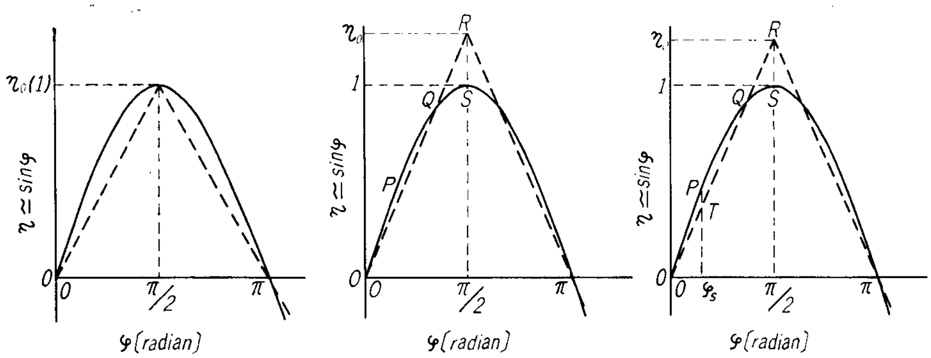
Moreover Table 2 denotes the values of η_0 in the three special cases as shown in Fig. 4(a), (b) and (c).

3.2. Case by Trapezoid-Approximation

From Fig. 3 and Eq. (9), we can introduce the approximate differential equations and their solutions as shown in Table 3,

Table 1. Approximate differential equations and their solutions—triangle-approximation.

		$-\frac{\pi}{2} \leqq \varphi \leqq \frac{\pi}{2}$	$\frac{\pi}{2} \leqq \varphi \leqq \frac{3\pi}{2}$
$\eta \cong \sin \varphi$		$\frac{2\eta_0}{\pi}\varphi$	$\frac{2\eta_0}{\pi}(\pi-\varphi)$
Approximate eifferential equations		$\frac{d^2\varphi}{dt}+\omega^2\varphi=c_0$ $\omega^2=2d_0\eta_0/\pi$	$\frac{d^2\varphi}{dt^2}-\omega^2\varphi=c_2$ $c_2=c_0-2\eta_0 d_0$
Solutions	φ	$\frac{c_0}{\omega^2}(1-\cos \omega t)+\varphi_0{}^{-0}\cos \omega t$ $+\frac{\dot{\varphi}_0{}^{-0}}{\omega}\sin \omega t$	$\frac{c_2}{\omega^2}(\cosh \omega t-1)+\varphi_2{}^{-0}\cosh \omega t$ $+\frac{\dot{\varphi}_2{}^{-0}}{\omega}\sinh \omega t$
	$\dot{\varphi}$	$\left(\frac{c_0}{\omega}-\omega\varphi_0{}^{-0}\right)\sin \omega t$ $+\dot{\varphi}_0{}^{-0}\cos \omega t$	$\left(\frac{c_2}{\omega}+\omega\varphi_2{}^{-0}\right)\sinh \omega t$ $+\dot{\varphi}_2{}^{-0}\cosh \omega t$



(a) ⓪-approximation. (b) ①$_0$-approximation ; area $0PQ$=area QRS. (c) ①$_1$-approximation ; area TPQ=area QSR.

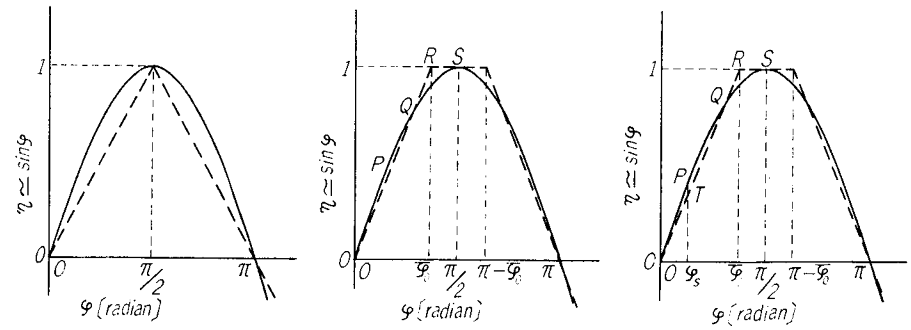
Fig. 4. Illustration of three special triangle-approximations.

Table 2. The values of η_0 for the three special triangle-approximations in Fig. 4.

Approximation	η_0
⓪	1
①$_0$	$4/\pi$
①$_1$	$(4\cos\varphi_s/\pi)/(1-2\varphi_s/\pi)^2$

Table 3. Approximate differential equations and their solutions—trapezoid-approximation.

		$-\bar{\varphi}_0 \leq \varphi \leq \bar{\varphi}_0$	$\bar{\varphi}_0 \leq \varphi \leq -\bar{\varphi}_0$	$\pi-\bar{\varphi}_0 \leq \varphi \leq \pi+\bar{\varphi}_0$	$\pi+\bar{\varphi}_0 \leq \varphi \leq 2\pi-\bar{\varphi}_0$
$\eta \simeq \sin\varphi$		$\varphi/\bar{\varphi}_0$	1	$(\pi-\bar{\varphi})/\bar{\varphi}_0$	-1
Approximate differential equations		$\frac{d^2\varphi}{dt^2}+\omega^2\varphi=c_0$ $\omega^2=d_0/\bar{\varphi}_0$	$\frac{d^2\varphi}{dt^2}=c_1$ $c_1=c_0-d_0$	$\frac{d^2\varphi}{dt^2}-\omega^2\varphi=c_2$ $c_2=c_0-(\pi/\bar{\varphi}_0)d_0$	$\frac{d^2\varphi}{dt^2}=c_3$ $c_3=c_0+d_0$
Solutions	φ	$(c_0/\omega^2)(1-\cos\omega t)$ $+\varphi_0{}^{-0}\cos\omega t$ $+(\dot{\varphi}_0{}^{-0}/\omega)\sin\omega t$	$\frac{c_1t^2}{2}+\varphi_1{}^{-0}+\dot{\varphi}_1{}^{-0}t$	$(c_2/\omega^2)(\cosh\omega t-1)$ $+\varphi_2{}^{-0}\cosh\omega t$ $+(\dot{\varphi}_2{}^{-0}/\omega)\sinh\omega t$	$\frac{c_3t^2}{2}+\varphi_3{}^{-0}+\dot{\varphi}_3{}^{-0}t$
	φ	$\left(\frac{c_0}{\omega}-\omega\varphi_0{}^{-0}\right)\sin\omega t$ $+\dot{\varphi}_0{}^{-0}\cos\omega t$	$c_1t+\dot{\varphi}_1{}^{-0}$	$\left(\frac{c_0}{\omega}-\omega\varphi_2{}^{-0}\right)\sinh\omega t$ $+\dot{\varphi}_2{}^{-0}\cosh\omega t$	$c_3t+\dot{\varphi}_3{}^{-0}$



(a) ⓪-approximation. (b) ②$_0$-approximation; area $0PQ$=area QSR. (c) ②$_1$-approximation; area TPQ=area QSR.

Fig. 5. Illustration of three special trapezoid-approximations.

Table 4. The values of $\bar{\varphi}_0$ for three special trapezoid-approximations in Fig. 5.

Approximations	$\bar{\varphi}_0$ (radian)
⓪	$\pi/2$
②$_0$	$\pi-2$
②$_1$	$\pi/2-\cos\varphi_s+\left\{(\pi/2-\cos\varphi_s)^2-\varphi_s{}^2\right\}^{1/2}$

where

$\varphi_{\bar{0}}^{-0}$ and $\dot{\varphi}_{\bar{0}}^{-0}$: the initial values of φ and $\dot{\varphi}$ respectively, where $(-\varphi_0) \leq \bar{\varphi} \leq \bar{\varphi}_0$

$\varphi_{\bar{1}}^{-0}$ and $\dot{\varphi}_{\bar{1}}^{-0}$: the same values, where $\bar{\varphi}_0 \leq \varphi \leq \pi - \bar{\varphi}_0$

$\varphi_{\bar{2}}^{-0}$ and $\dot{\varphi}_{\bar{2}}^{-0}$: the same values, where $\pi - \bar{\varphi}_0 \leq \varphi \leq \pi + \bar{\varphi}_0$

Table 4 denotes the values of $\bar{\varphi}_0$ in the three special cases shown in Fig. 5(a), (b) and (c), where Ⓞ-approximation in (a) perfectly coincides with Ⓒ-approximation in Fig. 4(a).

4. Stability Criterion, Critical Angular Displacement $\theta_{b\infty}$ and Critical Switching Time $\tau_{b\infty}$—Non-Reclosing Circuit Breakers

In the preceeding Articles 2 and 3, we could develop our theory without distinguishing the three transient states, i.e. the circuit modes during fault, after switching out faulty line and after reclosing cirduit breakers. However. hereafter, we have need of distinguishing these three states and so we supplement f, b and c as suffixes representing every constant and variable, except η_0, $\bar{\varphi}_0$ and φ_s, coming out in the approximate differential equations and their solutions during fault, switching out and after reclosing, in turn.

Next in this and the next Articles, we make the assumptions shown in Table 5, where

$\theta_{\nu m}$: the maximum value of the angular displacement, where $\nu = b$, c, and in the critical case the suffix m is displaced with ∞.

Table 5. Assumptions for initial angular displacement and the maximum ones in the case of switching out faulty line within critical switching time.

	triangle-approximation	trapezoid-approximation
Initial angular displacement ($\theta_{f0}{}^{-0} \equiv \theta_0$, $\varphi_{f0}{}^{-0} \equiv \varphi_{f0}$)	$-\pi/2 \leq \varphi_{f0} = \theta_0 + \theta_{fa} \leq \pi/2$	$-\bar{\varphi}_0 \leq \varphi_{f0} = \theta_0 + \theta_{fa} \leq \bar{\varphi}_0$
Maximum angular displacement—non reclosing breakers (θ_{bm}, φ_{bm})	$\pi/2 \leq \varphi_{bm} = \theta_{bm} + \theta_{ba} \leq 3\pi/2$	$\pi - \bar{\varphi}_0 \leq \varphi_{bm} = \theta_{bm} + \theta_{ba} \leq \pi + \bar{\varphi}_0$
Maximum angular displacement—reclosing breakers (θ_{cm}, φ_{cm})	$\pi/2 \leq \varphi_{cm} = \theta_{cm} + \theta_{ca} \leq 3\pi/2$	$\pi - \bar{\varphi}_0 \leq \varphi_{cm} = \theta_{cm} + \theta_{ca} \leq \pi + \bar{\varphi}_0$

4.1. Stability Criterion

If

$$\left.\begin{array}{l} \pi \leq \varphi_b \leq 3\pi/2 \text{ for triangle-approximation,} \\ \pi - \bar{\varphi}_0 \leq \varphi_b \leq \pi + \bar{\varphi}_0 \text{ for trapezoid-approximation,} \end{array}\right\} \quad (11)$$

the solutions of the approximate equations shown in Tables 1 and 3 give

$$\left.\begin{aligned}\varphi_b &= \frac{c_{b2}}{\omega_b^2}(\cosh\omega_b t_{b2}-1)+\varphi_{b2}^{-0}\operatorname{cose}\omega_b t_{b2}+\frac{\dot{\varphi}_{b2}^{-0}}{\omega_b}\sinh\omega_b t_{b2}\\ &= -\frac{c_{b2}}{\omega_b^2}+\frac{1}{2}\left(\frac{c_{b2}}{\omega_b^2}+\varphi_{b2}^{-0}+\frac{\dot{\varphi}_{b2}^{-0}}{\omega_b}\right)\varepsilon^{\omega_b t_{b2}}+\frac{1}{2}\left(\frac{c_{b2}}{\omega_b^2}+\varphi_{b2}^{-0}-\frac{\dot{\varphi}_{b2}^{-0}}{\omega_b}\right)\varepsilon^{-\omega_b t_{b2}},\\ \dot{\varphi}_b &= \frac{\omega_b}{2}\left(\frac{c_{b2}}{\omega_b^2}+\varphi_{b2}^{-0}+\frac{\dot{\varphi}_{b2}^{-0}}{\omega_b}\right)\varepsilon^{\omega_b t_{b2}}-\frac{\omega_b}{2}\left(\frac{c_{b2}}{\omega_b^2}+\varphi_{b2}^{-0}-\frac{\dot{\varphi}_{b2}^{-0}}{\omega_b}\right)\varepsilon^{-\omega_b t_{b2}},\end{aligned}\right\}\quad (12)$$

where

t_{b2}: the time taken from the moment when $\varphi_b-\varphi_b\geq\pi/2$ for the triangle-approximation or $\varphi_b\geq\pi-\bar{\varphi}_0$ for the trapezoid-approximation—reaches φ_{b2}^{-0}.

Therefore, the necessary and sufficient condition in order that φ_b has the maximum, i.e. θ has the maximum, hence the one- or two-machine systems are stable, is given by

$$\frac{c_{b2}}{\omega_b^2}+\varphi_{b2}^{-0}+\frac{\dot{\varphi}_{b2}^{-0}}{\omega_b}\leq 0. \quad (13)$$

This formula is the required stability criterion, which is able to be applied to both cases by the triangle- and trapezoid-approximations.

4.2. Critical Angular Displacement $\theta_{b\infty}$

The rotor motion of the machines reaches the stability limit, if the summation in the left side of Eq. (13) tends to zero. Then the critical angular displacement $\theta_{b\infty}$ is denoted by

$$\left.\begin{aligned}\theta_{b\infty} &= \varphi_{b\infty}-\theta_{ba} = -\frac{c_{b2}}{\omega_b^2}-\theta_{ba} = \pi-\frac{c_{b0}}{\omega_b^2}-\theta_{ba},\\ \text{where}\quad \varphi_{b\infty} &= \lim_{t\to\infty}\varphi_b.\end{aligned}\right\}\quad (14)$$

This formula, too, can be applied to both cases by the triangle- and trapezoid-approximation.

4.3. Critical Switching Time

(i) Case by Triangle-approximation

Using Eq. (13) and the solutions given in Table 3, we can derive the formulae to give the critical switching time $\tau_{b\infty}$ as presented in Table 6.

(ii) Case by Trapezoid-Approximation

With Eq. (13) and the solutions given in Table 3, we can derive the expressions to give the critical switching time $\tau_{b\infty}$ as shown in Table 7.

Table 6. Critical switching time $\tau_{b\infty}$—triangle-approximation.

	$\tau_{b\infty}$	m	n	h
$-\pi/2\leq\varphi_{f\tau b\infty}$, $\varphi_{b\tau b\infty}\leq\pi/2$	$\frac{1}{\omega_f}\cos^{-1}\left\{\frac{n}{m}\pm\sqrt{\left(\frac{n}{m}\right)^2-\frac{h}{m}}\right\}$	$\left(1-\frac{\omega_f^2}{\omega_b^2}\right)\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2$	$\left(\frac{c_{f0}}{\omega_f^2}-\frac{c_{b0}}{\omega_b^2}-\theta_{fa}+\theta_{ba}\right)\times\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)$	$\left(\frac{c_{f0}}{\omega_f^2}-\frac{c_{b0}}{\omega_f^2}-\theta_{fa}+\theta_{ba}\right)^2+\left(\frac{\omega_f}{\omega_b}\right)^2\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-2\left(\frac{c_{b0}}{\omega_b^2}-\frac{\pi}{2}\right)^2$
$\pi/2\leq\varphi_{f\tau b\infty}$, $\varphi_{b\tau b\infty}\leq3\pi/2$	$T_{f0}+\tau_{b2\infty}$; $T_{f_2}=\frac{1}{\omega_f}\cos^{-1}\left(\frac{c_{f0}/\omega_f^2-\pi/2}{c_{f0}/\omega_f^2-\varphi_{f0}}\right)$ $\tau_{2\infty}=\frac{1}{\omega_f}\log_e\left\{\frac{n}{m}\pm\sqrt{\left(\frac{n}{m}\right)^2-\frac{h}{m}}\right\}$	$\left(1+\frac{\omega_f}{\omega_b}\right)\left\{\frac{c_{f0}}{\omega_f^2}-\frac{\pi}{2}+\sqrt{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f^2}-\frac{\pi}{2}\right)^2}\right\}$	$\frac{c_{f0}}{\omega_f^2}-\frac{c_{b0}}{\omega_b^2}+\theta_{fa}-\theta_{ba}$	$\left(1-\frac{\omega_f}{\omega_b}\right)\left\{\frac{c_{f0}}{\omega_f^2}-\frac{\pi}{2}-\sqrt{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f^2}-\frac{\pi}{2}\right)^2}\right\}$

5. Stability Criterion, Critical Angular Displacement $\theta_{c\infty}$ and Critical Switching time $\tau_{c\infty}$—Reclosing Circuit Breakers

5.1. When No-Voltage Time $\tau_a=0$

In this case, our purpose is attained if only we substitute c for the suffix b of the symbol of every quantity coming out in Eqs. (13) and (14), Tables 6 and 7 shown in the preceding Article. For example, the stability criterion and the critical angular displacement $\theta_{c\infty}$ are reduced from Eqs. (13) and (14) respectively as follows:

$$\frac{c_{c2}}{\omega_c^2}+\varphi_{2c}^{-0}+\frac{\dot{\varphi}_{c2}^{-0}}{\omega_c}\leq 0, \tag{15}$$

and

$$\theta_{c\infty}=\pi-\frac{c_{c0}}{\omega_c^2}-\theta_{ca}. \tag{16}$$

5.2. When No-Voltage Time $\tau_a>0$

In this case, the stability criterion and the critical angular displacement $\theta_{c\infty}$

Table 7. Critical switching time $\tau_{b\infty}$—trapezoid-approximation.

	$\tau_{b\infty}$	m
$-\varphi_0 \leq \varphi_{f\tau b\infty}$, $\varphi_{b\tau b\infty} \leq \varphi_0$	$\frac{1}{\omega_f}\cos^{-1}\left\{\frac{n}{m} \pm \sqrt{\left(\frac{n}{m}\right)^2 - \frac{h}{m}}\right\}$	$\left(1-\frac{\omega_f^2}{\omega_b^2}\right)\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2$
$\varphi_0 \leq \varphi_{f\tau b\infty}$, $\varphi_{b\tau b\infty} \leq \pi - \varphi_0$	$T_{f0}+\tau_{b1\infty}$; $T_{f0}=\frac{1}{\omega_f}\cos^{-1}\left(\frac{\overline{c_{f0}/\omega_f^2-\bar{\varphi}_0}}{c_{f0}/\omega_f^2-\varphi_{f0}}\right)$ $\tau_{b1\infty}=\frac{n}{m} \pm \sqrt{\left(\frac{n}{m}\right)^2-\frac{h}{m}}$	$\omega_f^2\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)\left\{\frac{\omega_f^2}{\omega_b^2}\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)\right.$ $\left.-\left(\frac{c_{b0}}{\omega_b^2}-\bar{\varphi}_0\right)\right\}$
$\pi-\varphi_0 \leq \varphi_{f\tau b\infty}$, $\varphi_{b\tau b\infty} \leq \pi+\varphi_0$	$T_{f0}+T_{f1}+\tau_{b2\infty}$ $T_{f0}=\frac{1}{\omega_f}\cos^{-1}\left(\frac{\overline{c_{f0}/\omega_f^2-\bar{\varphi}_0}}{c_{f0}/\omega_f^2-\varphi_{f0}}\right)$ $T_{f1}=\left\{\sqrt{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0-\overline{2\pi-\varphi_0}\right)}\right.$ $\left.-\sqrt{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)^2}\right\}$ $\div\omega_f\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)$ $\tau_{b2\infty}=\frac{1}{\omega_f}\log\varepsilon\left\{\frac{n}{m}\pm\sqrt{\left(\frac{n}{m}\right)^2-\frac{h}{m}}\right\}$	$\left(1+\frac{\omega_f}{\omega_0}\right)\left\{\frac{c_{f0}}{\omega_b^2}-\bar{\varphi}_0+\sqrt{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)\left(\frac{c_{f0}}{\varphi_f^2}-\varphi_0-2\overline{\pi-\varphi_0}\right)}\right.$

n	h
$\left(\frac{c_{f0}}{\omega_f^2}-\frac{c_{b0}}{\omega_b^2}-\theta_{fa}+\theta_{ba}\right)$ $\times\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)$	$\left(\frac{c_{f0}}{\omega_f^2}-\frac{c_{b0}}{\omega_b^2}-\theta_{fa}+\theta_{ba}\right)^2$ $+\left(\frac{\omega_f}{\omega_b}\right)^2\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2$ $-2\left(\frac{c_{b0}}{\omega_b^2}-\bar{\varphi}_0\right)\left(\frac{c_{b0}}{\omega_b^2}-\overline{\pi-\bar{\varphi}_0}\right)$
$-\omega_f\left\{\frac{\omega_f^2}{\omega_b^2}\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)-\left(\frac{c_{b0}}{\omega_b^2}-\bar{\varphi}_0\right)\right\}$ $\times\sqrt[4]{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f}-\bar{\varphi}_0\right)^2}$	$\left(\frac{\omega_f}{\omega_b}\right)^2\left\{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)^2\right\}$ $+2\left(\frac{c_{a0}}{\omega_b^2}-\bar{\varphi}_0\right)(\pi-2\bar{\varphi}_0+\theta_{fa}-\theta_{ba})$ $-\left(\frac{c_{t0}}{\omega_b^2}-\bar{\varphi}_0\right)^2$
$\frac{c_{f0}}{\omega_b^2}-\frac{c_{b0}}{\omega_b^2}+\theta_{fa}-\theta_{ba}$	$\left(1-\frac{\omega_f}{\omega_o}\right)\left\{\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right.$ $-\sqrt{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\frac{c_{f0}}{\omega_f^2}-\varphi_0\right)\left(\frac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0-2\overline{\pi-\varphi_0}\right)}\left.\right\}$

are given by all the same expressions as those in Eqs. (15) and (16) respectively. On the other hand, the switching time $\tau_{c\infty}$ is introduced as shown Table 8, where we give only the case by the trapezoid-approximation for simplicity. However, as seen in this Table, compared with $\tau_{b\infty}$ in Tables 6 or 7 and $\tau_{c\infty}$

Table 8. Critical switching time $\tau_{c\infty}$—trapezoid-approximation.
$(\pi-\varphi_0 \leq \varphi_b \rvert_{t=\tau_{c\infty}+\tau_a},\ \varphi_c \rvert_{t=\tau_{c\infty}+\tau_a} \leq \pi+\varphi_0$

$-\bar{\varphi}_0 \leq \varphi_{f\tau_{c\infty}}$ $\varphi_{0\tau_{c\infty}} \leq \bar{\varphi}_0$	$\tau=\dfrac{1}{\omega_f}\left[\sin^{-1}\left\{\dfrac{\sqrt{\left(\frac{G}{H}\right)^2-2\left(\frac{C_{b0}}{\omega_f^2}-\bar{\varphi}_0\right)(\pi-2\bar{\varphi}_0)+\left(\frac{c_{f0}}{\omega_f^2}-\frac{c_{b0}}{\omega_b^2}-\theta_{fa}+\theta_{ba}\right)}}{\left(\frac{c_{f0}^2}{\omega_f^2}-\varphi_{f0}\right)}\right\}\right.$ $\left.-\delta_{b0}'\right],\ \delta_{b0}'=\tan^{-1}\left(\dfrac{\omega_b}{\omega_f}\tan\omega_b T_{b0}\right)$ $\tau=\dfrac{1}{\omega_f}\left[\cos^{-1}\left\{\dfrac{\frac{c_{b0}}{\omega_b^2}-\bar{\varphi}_0+\left(\frac{c_{f0}}{\omega_f^2}-\frac{c_{b0}}{\omega_b^2}-\theta_{fa}+\theta_{ba}\right)}{\left(\frac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)}\right\}\right.$ $\left.-\delta_{b0}''\right],\ \delta_{b0}''=\tan^{-1}\left(\dfrac{\omega_f}{\omega_b}\tan\omega_b T_{b0}\right)$ $T_{b0}=\tau_a-T_{b2}-\dfrac{1}{\omega_b\left\lvert\frac{c_{b0}}{\omega_b^2}-\bar{\varphi}_0\right\rvert}\left\{\sqrt{\left(\frac{G}{H}\right)^2-2\left(\frac{c_{b0}}{\omega_b^2}-\bar{\varphi}_0\right)}-\left\lvert\frac{G}{H}\right\rvert\right\}$ $G/H=\left\{\dfrac{c_{c0}}{\omega_c^2}-\dfrac{c_{b0}}{\omega_b^2}-\theta_{ba}+\theta_{ca}+\dfrac{1}{2}\left(\dfrac{c_{b0}}{\omega_b^2}-\bar{\varphi}_0\right)\left(\overline{1+\frac{\omega_b}{\omega_c}\varepsilon^{\omega_b T_{b2}}}\right.\right.$ $\left.\left.+\overline{1-\frac{\omega_b}{\omega_c}\varepsilon^{-\omega_b T_{b2}}}\right)\right\}\Big/\dfrac{1}{2}\left(\overline{1+\frac{\omega_b}{\omega_c}\varepsilon^{\omega_b T_{b2}}}-\overline{1-\frac{\omega_b}{\omega_c}\varepsilon^{-\omega_b T_{b2}}}\right)$	With these equations, $\tau=\tau_{c\infty}$ can be sought graphically.
$\bar{\varphi}_0 \leq \varphi_{f\tau_{c\infty}}$ $\varphi_{b\tau_{c\infty}} \leq \pi-\bar{\varphi}_0$	$\tau=-\dfrac{c_{f1}\tau_a-T_{b2}+\dot{\varphi}_{f1}^{-0}}{c_{f1}}+\sqrt{\left(\dfrac{c_f\tau_a-T_{b2}+\dot{\varphi}_{f1}^{-0}}{c_{f1}}\right)^2}$ $\overline{+\dfrac{2(\pi-2\bar{\varphi}_0+\theta_{fa}-\theta_{ba})-c_{b1}(\tau_a-T_{b2})^2-2\dot{\varphi}_{f1}^{-0}(\tau_a-T_{b2})}{c_{f1}}}$ $\tau=-\left(\overline{\tau_a-T_{b2}}+\dfrac{\dot{\varphi}_{f1}^{-0}}{c_{f1}}\right)+\sqrt{\left(\overline{\tau_a-T_{b2}}+\dfrac{\dot{\varphi}_{f1}^{-0}}{c_{f1}}\right)^2}$ $\overline{+\dfrac{2(\pi-2\bar{\varphi}_0+\theta_{fa}-\theta_{ba})-c_{b1}(\tau_a-T_{b2})^2-2\dot{\varphi}_{f1}^{-0}(\tau_a-T_{b2})}{c_{f1}}}$ $\dot{\varphi}_{f1}^{-0}=\omega_f\sqrt{\left(\dfrac{c_{f0}}{\omega_f^2}-\varphi_{f0}\right)^2-\left(\dfrac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)^2}$	With these equations, $\tau=\tau_{c\infty}$ can be sought graphically.
$-\bar{\varphi}_0 \leq \varphi_{f\tau_{c\infty}}$ $\varphi_{b\tau_{c\infty}} \leq \pi+\bar{\varphi}_0$	$\tau_{c\infty}=\dfrac{1}{\omega_f}\log_\varepsilon\left\{\dfrac{n}{m}\pm\sqrt{\left(\dfrac{n}{m}\right)^2-\dfrac{h}{m}}\right\},\quad \dot{\varphi}_{f2}^{-0}=\omega_f\sqrt{\dfrac{c_{f0}}{\omega_f^2}-\varphi_{f0}\Big)^2-\left(\dfrac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0\right)}$ $\overline{\times\left(\dfrac{C_{f0}}{\omega_f^2}-\varphi_0-2\overline{\pi-\bar{\varphi}_0}\right)}$ $m=\dfrac{1}{2}\left(\dfrac{c_{f0}}{\omega_f^2}-\varphi_0+\dfrac{\dot{\varphi}_{f2}^{-0}}{\omega_f}\right)\left\{\left(1+\dfrac{\omega_b}{\omega_c}\right)\left(1+\dfrac{\omega_f}{\omega_b}\right)\varepsilon^{\omega_b\tau_a}+\left(1-\dfrac{\omega_b}{\omega_c}\right)\left(1-\dfrac{\omega_f}{\omega_b}\right)\varepsilon^{-\omega_b\tau_a}\right\}$ $n=\dfrac{c_{c0}}{\omega_c^2}-\pi+\theta_{ca}-\theta_{ba}-\left(\dfrac{c_{b0}}{\omega_b^2}-\pi\right)\left[1+\dfrac{1}{2}\left\{\left(1+\dfrac{\omega_b}{\omega_c}\right)\varepsilon^{\omega_b\tau_a}+\left(1-\dfrac{\omega_b}{\omega_c}\right)\varepsilon^{-\omega_b\tau_a}\right\}\right.$ $\left.\times\left(\dfrac{c_{f0}}{\omega_b^2}-\pi+\theta_{fa}-\theta_{ba}\right)\right]$ $h=\dfrac{1}{2}\left(\dfrac{c_{f0}}{\omega_f^2}-\bar{\varphi}_0-\dfrac{\dot{\varphi}_{f2}^{-0}}{\omega_f}\right)\left\{\left(1+\dfrac{\omega_b}{\omega_c}\right)\left(1-\dfrac{\omega_f}{\omega_b}\right)\varepsilon^{\omega_b\tau_a}+\left(1-\dfrac{\omega_b}{\omega_c}\right)\left(1-\dfrac{\omega_f}{\omega_b}\right)\varepsilon^{-\omega_b\tau_a}\right\}$	

when $\tau_a=0$, it is impossible, in some cases, to express directly $\tau_{c\infty}$. In such cases, i.e. in the first or second columns in Table 8, we are obliged to determine the value of $\tau_{c\infty}$ on the graph, after calculating the values of τ by substituting the various numerical values into T_{b2} in Table 8, where

T_{b2}: the time taken for φ_b to vary from $\pi-\bar{\varphi}_0$ to the value of φ_b at the instant when circuit breakers are reclosed.

6. Numerical Examples

As one exercise, let us adopt the one-machine system sketched in Fig. 6, whose constants are indicated in Table 9.

6.1. Case of Non-Reclosing Circuit Breakers

In Fig. 7 are plotted the representative swing curves calculated by our approximate procedures previously mentioned and, for comparison, by the conventional step-by-step method, where it is assumed that two-wire ground fault

Table 9. System constants in Fig. 7.

Transmission line	Earth resistance : R_G	0.1 Ω/km
	Wire resistance : R	0.1 Ω/km
	Positive- and negative-phase-sequence reactance : L	1.3 mH/km
	Zero-phase-sequence reactance, I : L_0	4.5 mH/km/circuit
	Zero-phase-sequence reactance, II : L_{00}	7.5 mH/km/2 circuits
Sending end	Generator capacity : $[\mathrm{MVA}]_G$	250 MVA
	Terminal voltage of generator G before fault, reduced to the high-voltage side : E_G	154 kV
	Positive-phase-sequence steady-state reactance of G : x_1	75%
	Positiv-phase-sequence transient reactance of G : x_1'	35%
	Negative-phase-sequence reactance of G : x_2	55%
	Zero-phase-sequence reactance of G : x_0	15%
	Unit inertia constant of G : M_G	6
	Transformer caprcity : $[\mathrm{MVA}]_{TS}$	250 MVA
	Transformer reactance : x_{TS}	10%
Receiving end	Voltage of the infinite bus reduced to the high-voltage side : E_R	140 kV
	Power supplied to the infinite bus : P_R	160 MW (power factor=100%)
	Tranformer capacity : $[\mathrm{MVA}]_{TR}$	250MVA
	Transformer reactance : x_{TR}	10%

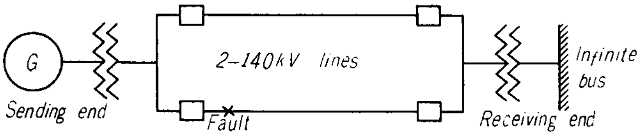


Fig. 6. One-machine system diagram with two-wire ground fault at the snding end of one of double circuits.

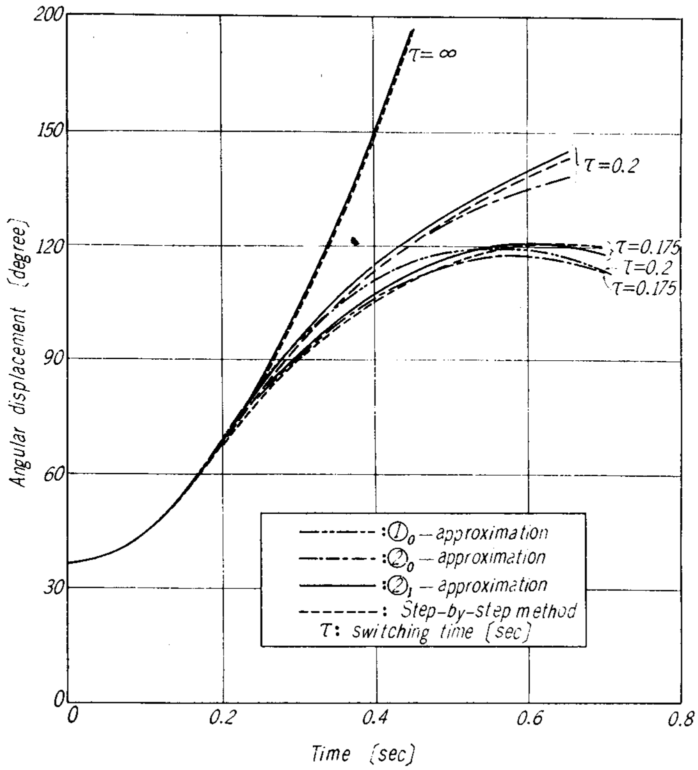


Fig. 7. Swing curves calculated by various procedures—non-reclosing circuit breakers.

Table 10. Calculated results of $\tau_{b\infty}$ and $\theta_{b\infty}$ by the various procedures.

	$\tau_{b\infty}$ [sec]	$\theta_{b\infty}$ [degree]
⓪-approximation	No exist	No exist
①$_0$-approximation	0.208	133.7
②$_0$-approximation	0.198	137.7
②$_1$-approximation (φ_0=1.18 radians)	0.187	136.1
Step-by-step method	0.175~0.2	138.1 (by equal-area caiterion)

occurs at the sending end of one of the double circuits of symmetrically arranged three-conductor transmission wires as shown in Fig. 6, and the faulty circuit is not reclosed after the fault has been cleared. Also Table 10 denotes the calculated results of the critical switching time $\tau_{b\infty}$ in seconds, and the critical angular dispacement $\theta_{b\infty}$ in degrees by the various procedures. As seen by these results, especially ②$_1$-approximation, where we assume the values of φ_s and so $\bar{\varphi}_0$ in equation shown in Table 4 as follows:

$$\varphi_s = \theta_{f0}+(\theta_{fa}+\theta_{ba})/2=0.538 \text{ [radian]},$$
$$\therefore\ \bar{\varphi}_0 = 1.18 \text{ [radian]}, \tag{17}$$

give better results than the other approximations, if compared with the results from the step-by-step procedure.

6.2. Case of Reclosing Circuit Breakers

Table 11 shows the calculated results of the critical switching time $\tau_{c0\infty}$ and $\tau_{c15\infty}$ in seconds, which present $\tau_{c\infty}$ when $\tau_a=0$ and 15 in cycles respectively, and the critical angular dispacement $\theta_{c\infty}$ in degrees by the step-by-step method (containing equal-area criterion) and ②$_1$-approximation, where it is assumed

$$\bar{\varphi}_0 = \bar{\varphi}_0 \quad \text{in Eq. (17)},$$

and the faulty condition is the same as that in Section 6.1. Here, too, we see that ②$_1$-approximation gives the good results.

Table 11. Calculated resuls of $\tau_{c0\infty}$ and $\tau_{c15\infty}$ and $\theta_{c\infty}$ by the ②$_1$-approxlmation and the step-by-step method.

	$\tau_{c0\infty}$ [sec]	$\tau_{c15\infty}$ [sec]	$\theta_{c\infty}$ [degree]
②$_1$-approximation (φ_0=1.18 radians)	0.278	0.211	149.4
Step-by-step method	0.275~0.3	0.2~0.225	153.2 (by equal-area criterion)

7. Conclusion

In the preceding Articles, we have illustrated our approximate analysis of transient stability of one- or two-machine systems. As ascertained by the numerical examples, through our approximate analysis, especially ②$_1$-approximation, we are able to estimate power system stability with suitable accuracy.

References

1) I. Hano: J. I. E. E. J., **50**, 535 (1930)
2) I. Hano: ibid. **52**, 16 (1932)
3) I. Hano: ibid. **52**, 942 (1932)