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Estimation of the Temperature and Humidity of an Underground Air Current

By

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In planning underground air conditioning either by ventilation or by equipping with a cooler, it is essential to estimate the underground air temperature and humidity attained after the ventilation or the equipment is completed. The strict calculation of them, however, is very difficult and troublesome, because many factors, known and unknown, are involved. The authors have studied, for several years, the fundamental problems in connection with the temperature and humidity of an underground air current.

The present paper describes the method of estimation, proposed by them, in which the evaporation of water on the wall surface, one of the factors having a great influence on the air temperature, is taken into account. The theory of this estimation stands on the fact that the change in temperature of an air current becomes very small after a sufficient period of time, say a year, from the beginning of ventilation, and that the heat quantity transmitted to an air current from rock across a unit area in a unit time is approximately proportional to the heat conductivity of rock and the difference between the undisturbed earth temperature and the air temperature.

1. Introduction

Many investigators have treated the problem of estimating the underground air temperature and humidity, standing on their own assumptions or discussing in their own ways. The present authors, also, have published several papers on this problem, which will be outlined below.

Hiramatsu, first, discussed the factors related to the variation in the temperature of an underground air current¹⁾, and later Hiramatsu and Kokado analysed approximately the variation in the temperature with the distance and the lapse of time under the influence of the earth temperature assuming that the wall of the airway is dry²⁾. Subsequently, Hiramatsu and Kokado analysed the influence

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of seasonal as well as daily change in the temperature of intake air upon the underground air temperature³⁾. On the other hand, they showed theoretically the relation among the heat transfer coefficient, the mass transfer coefficient and the coefficient of friction drag⁴⁾.

Amano discussed the variations in temperature of an underground air current in a different way. He analysed the variations in rock temperature assuming that the temperature at any point on an air current did not vary with time⁵⁾, and calculated the heat quantity given to an air current by the rock⁶⁾. Making use of these results, he obtained the method for estimating the temperature of an air current after a sufficiently long period of time from the beginning of ventilation on the premise that the airway was dry⁷⁾. Subsequently, he showed how to estimate the temperature of an air current flowing through a wet airway, assuming that the temperature of the wall of an airway was lower than the dry bulb temperature by one third of the difference between the dry and wet bulb temperatures⁸⁾.

The present paper describes first the result of recent analysis on the temperature and humidity of an air current flowing through a wet airway, and second the method of approximate estimation of the temperature and humidity of an underground air current making use of all the results mentioned above.

2. Notation

- θ : rock temperature ($^{\circ}\text{C}$),
- θ_0 : initial rock temperature ($^{\circ}\text{C}$),
- θ_w : temperature of wall surface ($^{\circ}\text{C}$),
- Θ : air temperature ($^{\circ}\text{C}$),
- Θ_0 : mean temperature of intake air ($^{\circ}\text{C}$),
- λ : heat conductivity of rock ($\text{kcal}/\text{m}\cdot\text{h}\cdot^{\circ}\text{C}$),
- λ_a : heat conductivity of air ($\text{kcal}/\text{m}\cdot\text{h}\cdot^{\circ}\text{C}$),
- γ : specific gravity of rock (kg/m^3),
- γ_a : specific gravity of air (kg/m^3),
- c : specific heat of rock ($\text{kcal}/\text{kg}\cdot^{\circ}\text{C}$),
- c_p : specific heat of air under constant pressure ($\text{kcal}/\text{kg}\cdot^{\circ}\text{C}$),
- α : heat transfer coefficient ($\text{kcal}/\text{m}^2\cdot\text{h}\cdot^{\circ}\text{C}$),
- w : mean air velocity (m/h),
- G : rate of air flow (kg/h),
- G' : rate of air flow (m^3/h),
- f : specific humidity (kg/kg),

- z : distance of air flow (m),
 t : time elapsed (h),
 r : radial distance from the center axis of an airway (m),
 r_1 : radius or twice the hydraulic mean depth of an airway (m),
 ξ : coefficient of friction drag,
 β : mass transfer coefficient of water ($\text{kg}/\text{m}^2 \cdot \text{h}(\text{kg}/\text{m}^3)$),
 κ : $\lambda/c \cdot \gamma$ (m^2/h),
 h : α/λ ($1/\text{m}$).

3. Temperature Change of an Air Current Flowing through a Dry Horizontal Airway

The authors noticed, previously, that the rate of variation in the temperature of an air current becomes very small after a sufficient period of time, say about 10,000 h, from the beginning of ventilation. In order to prove this fact, the temperature of an air current was measured at three stations in the 22nd Level of the Besshi Copper Mine for two years from October 1957, when the ventilation was begun. Station 1 is near the East Shaft, and the distance between Station 1 and 2 is 300 m, while that between Station 2 and 3 is 383 m. Twice the hydraulic mean depth, r_1 , of this level is 1.28 m, and the rate of flow is $280 \text{ m}^3/\text{min}$. The temperature was measured once a week. The results are shown in Fig. 1. The

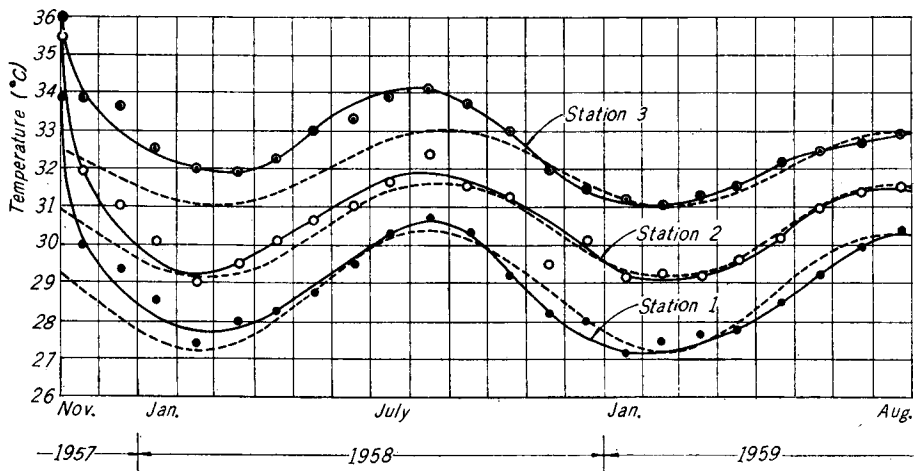


Fig. 1. Temperatures at Stations 1, 2 and 3 on the 22nd Level in the Besshi Mine.

variation in air temperature shown in Fig. 1 includes both the seasonal change and the change with time elapsed. In order to take out the latter change, the

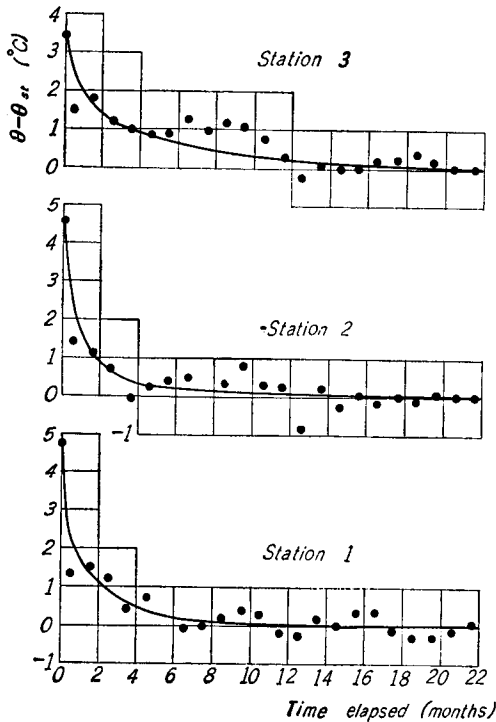


Fig. 2. The difference between the actual air temperature and the temperature assumed to show steady seasonal change θ_{st} .

temperature varying in a sine curve that coincides to the air temperature over the last half year (shown by the dotted curves) was deducted from the measured temperature. The difference is shown in Fig. 2. This figure shows that the rate of temperature change becomes rapidly lower with time, and that the temperature of an air current may be regarded as approximately constant after a year has elapsed from the beginning of ventilation.

Now, assuming that such a state has been obtained, let us analyze the heat quantity given to an air current across the dry wall surface of a unit length of level in a unit time, q kcal/m h. As the air temperature is assumed as constant, the rock temperature is given by the following equation²⁾.

$$\frac{\theta - \theta_0}{\theta_0 - \theta} = \frac{1 + h r_1 \log \frac{r}{r_1}}{1 + h r_1 \log \frac{r_2}{r_1}} + 2 r_1 h \sum_{n=1}^{\infty} \frac{u_0 \left(\frac{r}{r_1} x_n \right) u_0(x_n) \exp \left(-\kappa \frac{x_n^2}{r_1^2} t \right)}{\frac{r_2^2}{r_1^2} x_n^2 u_1^2 \left(\frac{r_2}{r_1} x_n \right) - \left(r_1^2 h^2 + x_n^2 \right) u_0^2(x_n)}, \quad (1)$$

where r_2 is the radial distance from the center of airway of the point at which the rock temperature is θ_0 , u_0 denotes the following expression:

$$u_0 \left(\frac{r}{r_1} x_n \right) = \frac{J_0 \left(\frac{r}{r_1} x_n \right)}{J_0 \left(\frac{r_2}{r_1} x_n \right)} - \frac{Y_0 \left(\frac{r}{r_1} x_n \right)}{Y_0 \left(\frac{r_2}{r_1} x_n \right)},$$

and x_n are positive roots of the equation

$$x_n u_0'(x_n) = r_1 h u_0(x_n).$$

The heat quantity given to an air current across the dry wall surface of a unit length of level in a unit time, q kcal/m·h, is given by

$$q = 2\pi r_1 \lambda \left(\frac{\partial \theta}{\partial r} \right)_{r=r_1}. \quad (2)$$

Considering Eq. (1) gives

$$q = 2\pi\lambda(\theta_0 - \Theta)\eta, \tag{3}$$

where

$$\eta = \frac{1}{1 + hr_1 \log \frac{r_2}{r_1}} + \sum_{n=1}^{\infty} \frac{2(r_1^2 h^2 + x_n^2) u_0^2(x_n) \exp\left(-\kappa \frac{x_n^2 t}{r_1^2}\right)}{\frac{r_2^2 x_n^2 u_1^2\left(\frac{r_2 x_n}{r_1}\right) - (r_1^2 h^2 + x_n^2) u_0^2(x_n)},$$

namely η is a value depending upon $\kappa t/r_1^2$ and hr_1 , as shown in Fig. 3. This figure shows that η decreases with time, the rate of which also decreases with time and that η can be taken as nearly constant for a sufficiently large value of t .

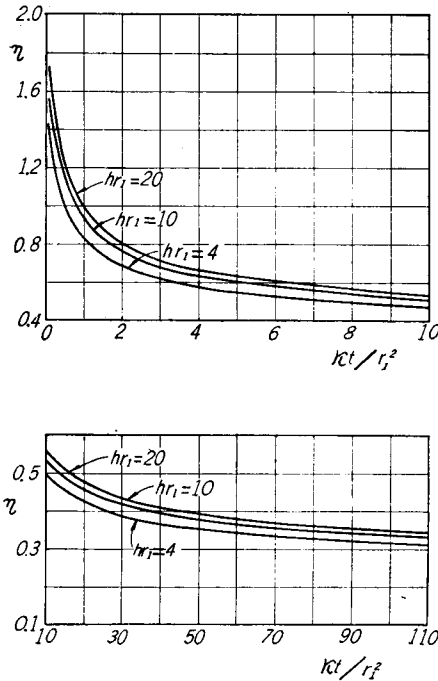


Fig. 3. Relation between $\kappa t/r_1^2$ and η .

When η is constant, from simple calculation, the temperature of an air current is given by

$$\Theta = \Theta_0 + (\theta_0 - \Theta_0) \left\{ 1 - \exp\left(-\frac{2\pi\eta\lambda}{Gc_p} z\right) \right\}. \tag{4}$$

4. Temperature Change of an Air Current Flowing through a West Horizontal Airway

4.1 Analysis

In general, the wall surface of an airway is partially wet. As the temperature of an air current in any cross section is uniform, and the wall temperature is

close to the air temperature, let us assume that in any cross section the wall temperature on wet parts is approximately equal to that on dry parts. Let U_w be the length of the total sum of wet portions out of the periphery $2\pi r_1$, and ψ be the ratio of U_w to $2\pi r_1$, i.e.

$$\psi = \frac{U_w}{2\pi r_1}. \quad (5)$$

Considering the transfer of heat across the wall surface, we obtain the following equation:

$$\frac{d\theta}{dz} = \frac{2\pi r_1 \alpha}{G c_p} (\theta_w - \theta). \quad (6)$$

On the other hand, taking the evaporation of water on the wet part of the wall surface into account, we have:

$$\frac{df}{dz} = \frac{U_w \beta}{G'} (f_w - f), \quad (7)$$

where f_w is the specific humidity on the wet wall surface, and it is equal to the saturated specific humidity for the temperature θ_w . Within a narrow range of temperature, the saturated specific humidity f_s is in a linear relation with the temperature, namely

$$f_s \doteq n_0 + n\theta, \quad (8-a)$$

or on the wet surface,

$$f_w \doteq n_0 + n\theta_w, \quad (8-b)$$

where n_0 and n are constants. There exists the following approximate relation between α and $\beta^{(4)}$:

$$\alpha = \beta c_p \gamma_a. \quad (9)$$

Considering the relation $\gamma_a G' = G$, we obtain, from Eqs. (7), (8) and (9),

$$\frac{df}{dz} = \frac{\alpha U_w}{G c_p} (n_0 + n\theta_w - f). \quad (10)$$

The heat given to an air current from rock across a small portion of wall surface, $2\pi r_1 dz$, in a short time dt is represented by $2\pi r_1 dz dt \lambda (d\theta/dr)_{r=r_1}$, which is consumed to raise the air temperature and to evaporate water. Therefore, we have

$$\frac{d\theta}{dz} + \frac{595}{c_p} \frac{df}{dz} = \frac{2\pi r_1}{G c_p} \lambda \left(\frac{d\theta}{dr} \right)_{r=r_1}, \quad (11)$$

where the latent heat of evaporation is taken as 595 kcal/kg. We have already pointed out that, after a sufficiently long period of time from the beginning of

ventilation, the heat given to an air current by the rock is proportional to both λ and $(\theta_0 - \Theta)$ and is affected little by the heat transfer coefficient⁹. Now, the presence of evaporation on the wall surface has a similar effect on the heat transmission to that of the great value of heat transfer coefficient. Consequently the heat quantity given to an air current by the rock depends on the product of λ and $(\theta_0 - \Theta)$, whether or not evaporation takes place. Therefore we have

$$2\pi r_1 \lambda \left(\frac{d\theta}{dr_1} \right)_{r=r} = 2\pi \eta \lambda (\theta_0 - \Theta). \tag{12}$$

From Eqs. (11) and (12),

$$\frac{df}{dz} = \frac{2\pi \eta \lambda (\theta_0 - \Theta)}{595 G} - \frac{c_p d\Theta}{G dz}. \tag{13}$$

Considering $U_w \neq 0$, and eliminating θ_w , the following equation is obtained from Eqs. (6), (10) and (13):

$$f = \left(\frac{G c_p n}{2\pi r_1 \alpha} + \frac{G c_p^2}{595 \alpha U_w} \right) \frac{d\Theta}{dz} + \left(n + \frac{2\pi \eta \lambda c_p}{595 \alpha U_w} \right) \Theta + n_0 - \frac{2\pi \eta \lambda c_p}{595 \alpha U_w} \theta_0. \tag{14}$$

Differentiating Eq. (14) with respect to z gives

$$\frac{df}{dz} = \left(\frac{G c_p n}{2\pi r_1 \alpha} + \frac{G c_p^2}{595 \alpha U_w} \right) \frac{d^2\Theta}{dz^2} + \left(n + \frac{2\pi \eta \lambda c_p}{595 \alpha U_w} \right) \frac{d\Theta}{dz}. \tag{15}$$

From Eqs. (13) and (15),

$$\frac{d^2\Theta}{dz^2} + LM \frac{d\Theta}{dz} + LN(\Theta - \theta_0) = 0, \tag{16-a}$$

where

$$\left. \begin{aligned} L &= \frac{2\pi r_1 \alpha \psi}{G c_p (595 n \psi + c_p)}, \\ M &= 595 n + c_p + \frac{\eta \lambda c_p}{r_1 \alpha \psi}, \\ N &= \frac{2\pi \eta \lambda}{G}. \end{aligned} \right\} \tag{16-b}$$

Solving Eq. (16), we obtain:

$$\begin{aligned} \Theta &= \theta_0 + C_1 \exp \left(-\frac{LM - \sqrt{L^2 M^2 - 4LM}}{2} z \right) \\ &+ C_2 \exp \left(-\frac{LM + \sqrt{L^2 M^2 - 4LM}}{2} z \right). \end{aligned} \tag{17}$$

The integration constants C_1 and C_2 are determined by first differentiating Eq. (17) with respect to z , and substituting it for $\frac{d\Theta}{dz}$ in Eq. (14), and second by putting $\Theta = \theta_0$, $f = f_0$ for $z = 0$ in this equation, as follows:

$$C_1 = -\frac{1}{\sqrt{L^2M^2-4LM}} \left\{ (\theta_0 - \Theta_0) \left(\frac{LM + \sqrt{L^2M^2-4LN}}{2} - \frac{1}{595n\psi + c_p} \right) + 595L(f_{s_0} - f_0) \right\},$$

$$C_2 = -\frac{1}{\sqrt{L^2M^2-4LN}} \left\{ (\theta - \Theta_0) \left(\frac{N}{595n\psi + c_p} - \frac{LM - \sqrt{L^2M^2-4LN}}{2} \right) - 595L(f_{s_0} - f_0) \right\},$$

where f_{s_0} is the saturated specific humidity for the temperature Θ_0 . Then the humidity f is given by:

$$f = n_0 + n\theta + \frac{\eta\lambda c_p}{595\alpha r_1\psi} (\theta - \theta_0) - \frac{G c_p (595n\psi + c_p)}{2\pi r_1 595\alpha\psi}$$

$$\times \left\{ C_1 \frac{LM - \sqrt{L^2M^2-4LN}}{2} \exp\left(-\frac{LM - \sqrt{L^2M^2-4LM}}{2} z\right) \right.$$

$$\left. + C_2 \frac{LM + \sqrt{L^2M^2-4LN}}{2} \exp\left(-\frac{LM + \sqrt{L^2M^2-4LN}}{2} z\right) \right\}. \quad (18)$$

In a special case, when the wall surface is completely wet, namely $U_w = 2\pi r_1$, or $\psi = 1$, the following equation is obtained from Eqs. (17) and (18):

$$\theta = \theta_0 + (\theta_0 - \Theta_0) (1 - e^{m_1 z}) + \frac{595(f_{s_0} - f_0)}{c_p + 595n} \frac{m_2}{m_1 - m_2} (e^{m_1 z} - e^{m_2 z}), \quad (19-a)$$

$$f = f_s - (f_{s_0} - f_0) e^{m_2 z}, \quad (19-b)$$

where

$$m_1 = -\frac{2\pi\eta\lambda}{G(c_p + 595n)}, \quad m_2 = -\frac{2\pi r_1\alpha}{G c_p}. \quad (19-c)$$

4.2 Example of Calculation

Now let us calculate, with the equations above obtained, the variation in temperature and humidity from Station 1 to 3 on the 22nd Level in the Besshi Mine which was previously cited, and compare the theoretical result with that measured. It was known by determination or estimation⁹⁾ that $\theta_0 = 41.3^\circ\text{C}$, $\lambda = 2.92 \text{ kcal/m}\cdot\text{h}\cdot^\circ\text{C}$, $r_1 = 1.28 \text{ m}$, $G = 19,820 \text{ kg/h}$, $z = 683 \text{ m}$, $\alpha = 8.3 \text{ kcal/m}^2\cdot\text{h}\cdot^\circ\text{C}$, $n = 1.68 \times 10^{-3} \text{ kg/kg}\cdot^\circ\text{C}$, $c_p = 0.24 \text{ kcal/kg}\cdot^\circ\text{C}$, $\eta = 0.385$. The yearly mean dry and wet bulb temperature at Station 1 after a year from the beginning of ventilation are 28.8°C and 28.0°C respectively. Thus $f_{s_0} = 24.4 \times 10^{-3} \text{ kg/kg}$, $f_0 = 22.9 \times 10^{-3} \text{ kg/kg}$. The ratio ψ was of course unknown. We attempted to calculate the temperature and humidity at Station 3 for various values of ψ using Eqs. (17), (18) and (19-a, b, c), the results being illustrated in Fig. 4. The measured temperature is, as shown in the figure, between the two calculated temperatures under the assumption of $\psi = 0$ and $\psi = 1$, and it is nearer to the latter rather than the former, while the measured humidity is very close to the calculated one under the assumption of $\psi = 1$. It is seen, from this figure, that the temperature decreases and the humidity increases rapidly as ψ increases from zero, and that the rate of decrease as well as increase drops rapidly with increase in ψ .

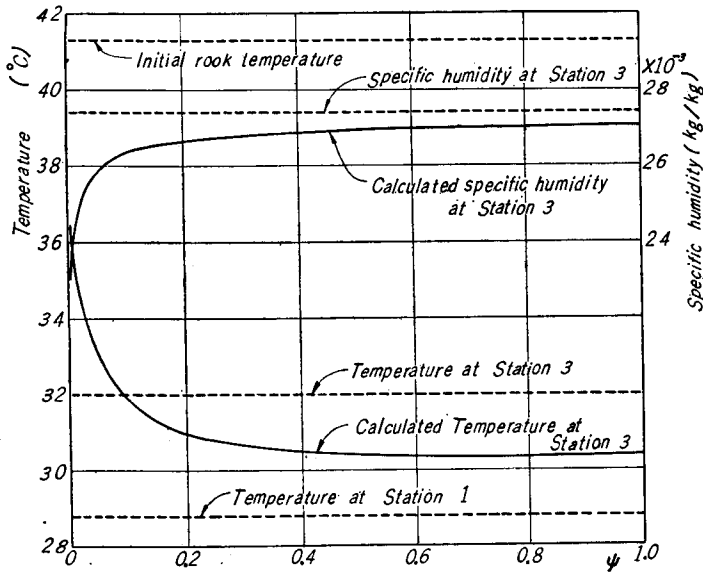


Fig. 4. Relation between ψ and the calculated temperature as well as specific humidity at Station 3.

5. Practical Estimation of the Temperature and Humidity of an Air Current

Eqs. (17) and (18) are too complicated to estimate the temperature and humidity of an air current. It is not easy to evaluate each factor accurately. Eqs. (19-a, b, c) for the case $\psi=1$, however, are somewhat simpler. Under these circumstances, the authors would like to present the following expedient means for the estimation of the temperature and humidity of an air current which are attained after a sufficient period of time, say a year.

Calculate first the temperature change $\Delta\theta$ by Eq. (4) assuming that the wall surface is dry i.e. $\psi=0$, the humidity remaining constant, and second calculate the change in temperature and humidity, $\Delta\theta_2$ and Δf_2 , by Eqs. (19-a, b, c) assuming that the wall surface is completely wet, i.e. $\psi=1$. Then estimate the probable change in temperature and humidity properly between $\Delta\theta_1$, $\Delta\theta_2$ and 0, Δf_2 respectively. It is supposed that even if the wall surface is slightly wet the actual temperature change may be nearer to $\Delta\theta_2$ rather than $\Delta\theta_1$.

6. Temperature Change of an Air Current Flowing through a Shaft

When a large amount of air is flowing through a shaft, the gradient of initial rock temperature influences little the temperature of an air current⁷⁾.

Let us assume, therefore, that the initial rock temperature is uniform along

its depth and is equal to the mean of the initial rock temperatures at the top and bottom of a shaft. Another matter to be noticed is that, Eqs. (6) and (11) are applicable to an air current flowing through a shaft if we add a term A/c_p , which is due to the variation in height, to the right side, A being the thermal equivalent of work, i.e. 1/427 kcal/m·kg.

Thus, when the wall surface is entirely dry, we have

$$\theta = \theta_0 + (\theta_0 - \theta_0) \left\{ 1 - \exp \left(-\frac{2\pi \eta \lambda}{G c_p} z \right) \right\} + \frac{AG}{2\pi \eta \lambda} \left\{ 1 - \exp \left(-\frac{2\pi \eta \lambda z}{G c_p} \right) \right\}, \quad (20)$$

and when the wall surface is completely wet, we have

$$\begin{aligned} \theta = \theta_0 + (\theta_0 - \theta_0) (1 - e^{m_1 z}) + \frac{595(f_{s0} - f_0)}{c_p + 595n} \frac{m_2}{m_1 - m_2} (e^{m_1 z} - e^{m_2 z}) \\ + \frac{A}{c_p} \left\{ \frac{1}{(m_1 - m_2)} (e^{m_1 z} - e^{m_2 z}) + \frac{G c_p}{2\pi \eta \lambda} \left(1 + \frac{m_2}{m_1 - m_2} e^{m_1 z} - \frac{m_1}{m_1 - m_2} e^{m_2 z} \right) \right\}, \end{aligned} \quad (21)$$

and

$$f = f_s - (f_{s0} - f_0) e^{m_2 z} + \frac{An}{c_p m_2} (1 - e^{m_2 z}). \quad (22)$$

When the shaft is inclining at an angle ϕ to the horizontal, substituting A in Eqs. (20), (21) and (22) by $A \sin \phi$ gives the equations applicable to this case.

7. Summary

- (1) The above mentioned are applicable for the estimation of temperature which has become nearly constant.
- (2) When a system of air current is in question, this is divided into several parts, for instance, air currents through vertical shafts, inclined shafts and levels, each one having uniform conditions, and the temperature change is estimated for each of them.
- (3) The temperature of the intake air, as well as the air flowing through underground airways shows the seasonal change. It must be noticed that in the calculation of temperature change described above the yearly mean temperature of intake air should be taken for θ_0 , and the calculated temperature θ gives also the yearly mean temperature at any point on an air current. The maximum or minimum temperature is obtained by adding or deducting the amplitude of the seasonal temperature change to or from the yearly mean temperature. This amplitude decreases approximately with z/r_1 in the rate

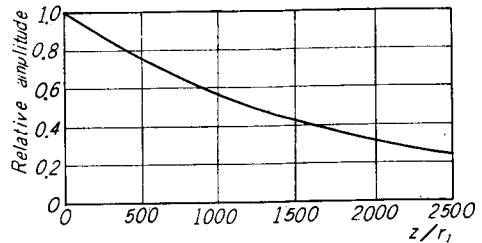


Fig. 5. Relation between z/r_1 and the relative amplitude of seasonal change of underground air temperature.

shown in Fig. 5³. In the main island of Japan, the amplitude of seasonal temperature change of intake air is about 11°C.

(4) When an air current passes a running machine, it is heated. If a machine, *L* H.P., is running continuously and a part of the power, ζL H.P., is converted into heat, the temperature rise $\Delta\theta$ due to this machine is given by

$$\Delta\theta = \frac{75 \times 3600 \times \zeta AL}{G c_p} \tag{23}$$

(5) The estimation of temperature change is practised between the two kinds of calculated temperature change under the assumptions of $\psi=0$ and $\psi=1$, making use of Eqs. (4), (19-a, b), (20), (21) and (22) as shown in Table 1.

Table 1. Equations to be used.

Air current flowing through	$\psi=0$		$\psi=1$	
	Temperature	Humidity	Temperature	Humidity
a level	Eq. (4)	Unaltered	(19-a)	(19-b)
a vertical shaft	Eq. (20)	Unaltered	(21)	(22)
a shaft inclining at an angle ϕ	Eq. (20), in which the notation <i>A</i> is substituted by (<i>A</i> sin ϕ)	Unaltered	Eq. (21), in which the notation <i>A</i> is substituted by (<i>A</i> sin ϕ)	Eq. (22), in which the notation <i>A</i> is substituted by (<i>A</i> sin ϕ)

For rougher estimation, however, η can be looked upon as a constant, about 0.38. Then those equations to be used are simplified as follows:

For Eq. (4),

$$\theta = \theta_0 + (\theta_1 - \theta_0) (1 - e^{-10 \lambda z / G}) \tag{4-A}$$

For Eq. (19-a),

$$\theta = \theta_0 + (\theta_1 - \theta_0) (1 - e^{-2 \lambda z / G}) - 500 (f_{s0} - f_0) e^{-2 \lambda z / G} \tag{19-a-A}$$

For Eq. (19-b),

$$f = f_s - (f_{s0} - f_0) e^{-0.077 \xi z / r_1} \tag{19-b-A}$$

For Eq. (20),

$$\theta = \theta_0 + (\theta_1 - \theta_0) (1 - e^{-10 \lambda z / G}) + \frac{G}{1000 \lambda} (1 - e^{-10 \lambda z / G}) \tag{20-A}$$

For Eq. (21),

$$\theta = \theta_0 + (\theta_1 - \theta_0) (1 - e^{-2 \lambda z / G}) - 500 (f_{s0} - f_0) e^{-2 \lambda z / G} + \left\{ \frac{r_1}{32 \xi} e^{-2 \lambda z / G} + \frac{G}{1000 \lambda} (1 - e^{-2 \lambda z / G}) \right\} \tag{21-A}$$

For Eq. (22),

$$f = f_s - (f_{s0} - f_0) e^{-0.077 \xi z / r_1} + \frac{2 r_1}{\xi} (1 - e^{-0.077 \xi z / r_1}) \times 10^{-4} \tag{22-A}$$

For the estimation of temperature and humidity in an inclined shaft, multiply the last term on the right side of each equation concerning a vertical shaft, namely Eqs. (20-A), (21-A) and (22-A), by $\sin \phi$.

(6) Other related formulas will be shown below.

$$\begin{aligned} \gamma &\doteq 1.2 \text{ kg/m}^3, \\ G &\doteq 3600 \gamma w F, \end{aligned}$$

where w is the mean air velocity in m/s, F the sectional area in m^2 .

$$r_1 \doteq 2 \frac{F}{U},$$

where U is the periphery of an airway in m.

$$\begin{aligned} \alpha &\doteq \lambda_a \xi w / 8\nu, \quad \text{or } \alpha \doteq 166 \xi w, \\ \xi &\doteq 0.015 \sim 0.02 \quad \text{for lined airways,} \\ \xi &\doteq 0.04 \quad \text{for airways without support,} \\ \xi &\doteq 0.1 \quad \text{for timbered airways.} \\ f &= \frac{0.622 p_w}{p - p_w} \end{aligned}$$

where p and p_w are the atmospheric pressure and the vapour tension respectively.

$$\begin{aligned} A &= \frac{1}{427} \text{ kcal/m} \cdot \text{kg}, \\ c_p &= 0.24 \text{ kcal/kg} \cdot ^\circ\text{C}, \\ \eta &\text{ is given by Fig. 3.} \end{aligned}$$

The value of n is as shown in Table 2.

Table 2. The value of n .

Pressure	25°~30°C	30°~35°C
720 mmHg	1.49×10^{-3} (1/°C)	1.98×10^{-3} (1/°C)
760 "	1.41×10^{-3} "	1.86×10^{-3} "
800 "	1.34×10^{-3} "	1.74×10^{-3} "

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