Fundamental Studies on Impact Crushing

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By

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When calculated by means of Rosin, Rammler and Sperling's Equation $R = e^{-(x/\bar{x})}$, the residue of the crushed product R is equal to 1, in the case $x/\bar{x}=0$, and becomes 0 in the case $x/x=\infty$. Whereas, the actual particle size of the crushed product must be always smaller than the initial particle size x_0 of the raw material, or R must be equal to 0 in the case $x/\bar{x}=x_0/\bar{x}<\infty$. Therefore, the above equation does not show good agreement with the measured values of extremely coarse grains. The authors have presented the following equation as an expression for the size distribution of the crushed product and to represent the relations among the net work input to crushing, A_r, the surface increase $\varDelta S$, and the size distribution of the crushed product:

$$R = e^{-(x'/\bar{x}')^n}$$

where

$$1/x' = 1/x - 1/x_0$$

and

$$\varDelta S = k_1 A_r = k_2 W_0/x'$$

$$R = e^{-(x'/\bar{x}')^n} = e^{-k(A_r/W_0)^n \cdot x'^n}$$

They presents also

$$A_r = \sum_1^z \cdot A_{r_z} = A_{r_1} \cdot \sum_1^z z \cdot e^{-\frac{z-1}{z_m}}$$

as an expression of the relation between the net work input to crushing, A_r, and the number of blows, z.

They have confirmed that these equations give good agreements with the corresponding experimental results.

1. Introduction

The so-called laws of crushing used generally, namely Rittinger's, Kick's Bond's and many other laws, have been discussed for many years. They were derived from different points of view and led to different forms. Certain experimental data are found to be in good agreement with Rittinger's Law, while others seem to support Kick's Law, and so on. The product obtained by a crushing operation may be expressed generally in terms of the new surface area produced or of the size distribution. Most investigators always carried out careful measure-

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ments for finding the surface area of the crushed product and utilized different methods for measuring the work consumed to get definite amounts of surface increase. However, there still remains some question as to whether the value obtained by these investigators gives the net work or the actual work done. Measurements of the surface area of crushed products and of net work input are always accompanied by countless difficulties. At the present time, there is no method for absolute surface measurement and not even an approximate method for measurement of the net work input to crushing.

Most laws of crushing hitherto used give relations between the work required for crushing and the magnitude of the increased surface area of the crushed product. But it is to be regretted that the work required for crushing is not yet distinctly defined nor exactly expressed in any form. Strictly speaking, we can hardly apply such laws to any practical case, in so for as the defined work is ambiguous.

In the law developed by the present authors, the surface increment is considered to be directly proportional to the net work input to crushing, and their law may be considered to coincide with Rittinger's, if the work described in his law be defined as the net work input. The authors assumed the proportional work as the net work input for crushing by drop weight impact expressed in the equations in this paper, and derived the equations showing a relation between the net work input to crushing, the surface increment, and the size distribution of the crushed product.

2. Relation between Net Work Input to Crushing, Surface Increase and Size Distribution of Crushed Product[7]

(a) *Size* distribution of the crushed product

Rosin, Rammler, Sperling and Bennett[8] proposed an equation for size distribution

$$R = e^{-(x/\bar{x})^n} \tag{1}$$

where R is the residue of crushed product, x the particle size and $\bar{x}$ that in the case $R=1/e$. If this equation is plotted in a log−log *vs* log paper, it shows a straight line where constant n is the slope of this line. By this equation, the residue of crushed product $R=1$ in the case $x/\bar{x}=0$, and $R=0$ in the case $x/\bar{x}=\infty$. As the particle size of the crushed product x is, of course, always smaller than the initial particle size x_0, and $R=0$ in the case $x/\bar{x}=x_0/\bar{x}<\infty$, it is clear that Eq. (1) is not suited for extremely coarse grains on account of the properties of the equation.

Therefore, the authors have adopted the following modified equation as an expression for the size distribution of the crushed product

$$R = e^{-(x'/\bar{x}')^n} \tag{2}$$

where
$$1/x' = 1/x - 1/x_0 \tag{3}$$

From this equation, we obtain $R=0$ in the case $x=x_0$, and Eq. (2) becomes applicable in the case of extremely coarse grains. For the fine grains, $x \ll x_0$, $x' \approx x$ and Eq. (2) becomes nearly equal to Eq. (1).

(b) Relation between the net work input to crushing and the surface increase

The surface of the particle is generally expressed by the relation

$$S = \beta \cdot G/\rho \cdot 1/x$$

where x is the representative length of a particle size, G the weight, ρ the density, and β the shape factor. In this paper, the authors assume the value of β to be constant. Let dR be a part of the residue at a particle size x, and dS be the corresponding surface area, then we get

$$dS = -\beta/\rho \cdot W_0 dR \cdot 1/x$$

in which W_0 is the total weight of the powdery sample. Before crushing, a group of such particles possessing a uniform size x_0 has the surface area $dS_0 = -\beta/\rho \cdot W_0 dR \cdot 1/x_0$. Then, after crushing, the surface increase due to crushing may be shown as

$$\begin{aligned} d\Delta S &= d(S-S_0) = -\beta/\rho \cdot W_0 dR \cdot (1/x - 1/x_0) \\ \Delta S &= S - S_0 = -\beta/\rho \cdot W_0 \int (1/x')\, dR \end{aligned} \tag{4}$$

where $1/x' = 1/x - 1/x_0$ by Eq. (3).

Let the energy absorbed in the unit-volume of material possessing a particle size x by crushing be a_x, and now we assume the relation $a_x \propto 1/x$. Then, the value of a_x may be considered to represent the energy absorbed by crushing from an infinite size to a size x.

The energy absorbed by crushing from an initial size x_0 to a size x may be shown by

$$a_{x-x_0} \propto (1/x - 1/x_0) = 1/x'$$

If we represent the net work input to crushing by A_r, it is given by the following equation

$$\begin{aligned} A_r &= -\sum (a_{x-x_0} \cdot W_0 dR) = -\int a_{x-x_0} \cdot W_0 dR \\ &\propto -W_0 \int (1/x')\, dR \end{aligned} \tag{5}$$

From Eq. (4) and Eq. (5), we obtain

$$\Delta S = k_1 \cdot A_r \tag{6}$$

The surface increment is directly proportional to the net work input to crushing, and this relation coincides with Rittinger's law, if the work required for crushing in his law be defined as the net work input to crushing.

(c) Relation between the net work input to crushing, the surface increase and the size distribution of the crushed product

Eq. (4) may be written in the form

$$\Delta S = \beta/\rho\cdot W_0\cdot n\cdot(1/\bar{x}')^n\left[\frac{x'^{n-1}}{n-1}-\frac{(1/\bar{x}')^n\cdot x'^{2n-1}}{(2n-1)\cdot 1!}+\frac{(1/\bar{x}')^{2n}\cdot x'^{3n-1}}{(3n-1)\cdot 2!}-\cdots\right]_{x'\min}^{x'\max}$$

Let $x'_{\min}=0$ and $x'_{\max}=\alpha\cdot\bar{x}'$, so that

$$\Delta S = \beta/\rho\cdot W_0\cdot n\cdot(1/\bar{x}')\cdot\left\{\frac{\alpha^{n-1}}{n-1}-\frac{\alpha^{2n-1}}{(2n-1)\cdot 1!}+\frac{\alpha^{3n-1}}{(3n-1)\cdot 2!}-\cdots\right\}$$

and when β, ρ, n and α=const., we obtain

$$\Delta S = k_2\cdot W_0/\bar{x}' \tag{7}$$

Hence, combining Eq. (6) and Eq. (7), we get

$$\Delta S = k_1 A_r = k_2 W_0/\bar{x}' \tag{8}$$

By means of Eq. (8), Eq. (2) may be written in a different form

$$R = e^{-(x'/\bar{x}')^n} = e^{-k(A_r/W_0)^n\cdot x'^n} \tag{9}$$

In the range where Eq. (1) may be used as an approximate form of Eq. (2), Eq. (9) may be written in an approximate form as follows:

$$R = k^{-(x/\bar{x})^n} = e^{-k(A_r/W_0)^n\cdot x^n} \tag{10}$$

The authore present Eqs. (8) and (9) which is some respects are similar to Rittinger's and Rammler's equations.

3. Relation between Net Work Input, Drop Height, Drop Weight and Number of Blows

(a) Relation between the net work input, the drop height and the drop weight in case of one blow[5,7)]

The crusher used in our experiments is shown in **Fig. 1.** When a ball possessing a weight W_2 falls from height h on a plunger W_1 which rests on a cubic sample W_0, the momentum of the ball is expressed by $W_2/g\cdot\sqrt{2gh}$. Let the velocity of the plunger and ball moving vertically at the instant of collision be V. Since the momentum before impact is set as equal to that after impact, the velocity V is expressed by

$$V = 2W_2\sqrt{2gh}/\{2(W_1+W_2)+W_0\} \tag{11}$$

The authors assumed that the work dissipated in the crusher, and the work

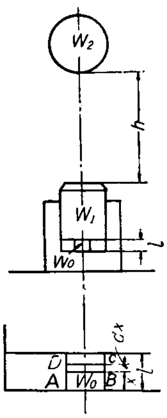

Fig. 1.

due to rebounding of the ball, are negligibly small. Let the work delivered to the sample by impact be A and the corresponding maximum deflection δ, then we have

$$A = V^2(W_1+W_2)/(2g)+V^2 W_0/(6g)+\delta(W_1+W_2) \quad (12)$$

We denote as σ the maximum stress produced in the sample in the ideal case where it is assumed that the total applied work is stored as elastic energy in it. The stress σ is not actually produced.

Then the strain energy of the sample is $\sigma^2 \cdot f \cdot l/(2E)$, and this is equal to the external work A stored in the sample. Let $\delta=\sigma l/E$, we obtain

$$\sigma = (W_1+W_2)/f+(1/f)[(W_1+W_2)^2+V^2 Ef\{3(W_1+W_2) + W_0\}/(3lg)]^{0.5}$$

Let $W_0 \ll W_1$ and $W_0 \ll W_2$

$$\sigma = \sigma_0[1+\sqrt{1+2EfW_2^2 h/\{l(W_1+W_2)^3\}}] \quad (13)$$

wnere $\sigma_0=(W_1+W_1)/f$ is the stress produced by a static load, f the cross sectional area of the sample, l the cubic size of the sample and E modulus of elasticity.

The ratio of the net work input to crushing to the external work A_r/A varies according to the various kinds of crushing conditions. The authors assumed that the ratio A_r/A was directly proportional to the ideal stress σ above mentioned, and derived the following equations.

From the assumption

$$A_r/A = \sigma/c\sigma_0 \quad (14)$$

we have

$$A_r = A\sigma/c\sigma_0 = [1+\sqrt{1+2EfW_2^2 h/\{l(W_1+W_2)^3\}}]\cdot A/c \quad (15)$$

where c is a constant. When $\sigma/\sigma_0 \gg 2$, Eq. (15) may be written in an approximate form

$$A_r = A\sigma/c\sigma_0 \approx h^{1.5} W_2^3 (W_1+W_2)^{-2.5}(2Ef/l)^{0.5} c^{-1} \quad (16)$$

in which $\sigma/\sigma_0 \geqq 2$, $1 \geqq \sigma/\sigma_0 \geqq 0$, $1 \geqq A_r/A \geqq 0$, $c=\sigma/\sigma_0 \cdot A/A_r$. The value of c is assumed to be far greater than 2 in ordinary cases, but the exact value of c cannot be easily determined at this stage. By Eqs. (15) and (16), they calculated the value of $A_r \cdot c$, and obtained the relation between the net work input to crushing, the surface increase and the size distribution of the crushed product. The authors have found that the value of c is constant in an ordinary range of impact tests.

Strictly speaking, as Eq. (14) is an approximate equation, it is not adaptable to extreme cases in which σ has an extremely small or large value. When the greatest stress produced in a specimen is kept at a value smaller than the limitting stress, crushing will not occur, and A_r/A will remain at zero. As A_r cannot exceed A, even in the other extreme case where large stress occurs, the ratio A_r/A remains in the range $(0\sim1)$. The authors have adopted the following equation as an expression for the general form of Eq. (14):

$$A_r/A = 1-e^{-\frac{\sigma-\bar{\sigma}}{c\sigma_0}} \tag{17}$$

where $\bar{\sigma}$ is the lowest stress which causes crushing or rupture of materials. In the range $\sigma<\bar{\sigma}$, crushing does not occur and external work changes into elastic energy, and net work input to crushing does not, consequently, appear. Therefore, in the case of the range $\sigma\geqq\bar{\sigma}$, the ratio of net work to applied energy lies between 0 and 1. For the range $0\ll\sigma-\bar{\sigma}\ll c\sigma_0$, Eq. (17) may be written in an approximate form, namely as Eq. (14).

(b) Relation between the net work input and the number of blows[6)]

The authors presented the following equation as the relation between the net work input to crushing and the number of blows of the drop weight:

$$A_{r_z} = A_{r_1}\cdot z\cdot e^{-\frac{z-1}{z_m}} \tag{18}$$

$$A_r = \sum_1^z A_{r_z} = A_{r_1}\cdot\sum_1^z z\,e^{-\frac{z-1}{z_m}} \tag{19}$$

where A_{r_z} is the net work input to crushing due to one blow named number z. Let A_{r_1} be that due to the first blow, then $\sum_1^z A_{r_z}$ is the sum total of net work due to z blows and z_m is the number of blows when A_{r_z} becomes maximum. Eqs. (18) and (19) are shown in **Fig. 2.** As the initial net work A_{r_1}, we may take the value of A_r giben by Eq. (15).

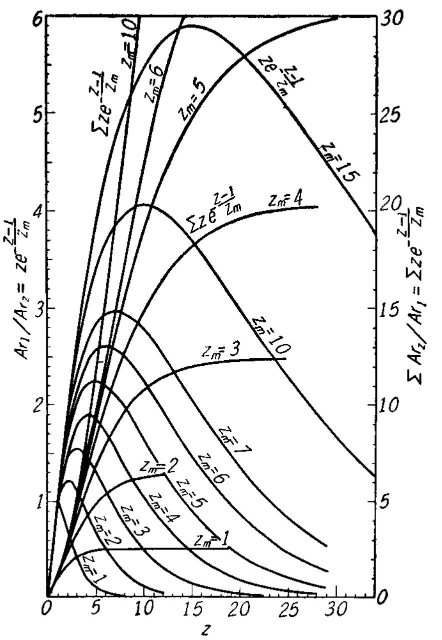

Fig. 2. Variation of Ar_z/Ar_1 or $\sum_1^z Ar_z/Ar_1$ relative to z (Curve calculated by means of Eqs. (18) and (19)).

Hence combining Eq. (8) and Eq. (19), the relation between the surface increase of the crushed product and the number of blows is obtained as follows:

$$(\varDelta S)_z = S_z - S_0 = k_1 A_{r_1} \sum_1^z z e^{-\frac{z-1}{z_m}}$$

$$= \{(\varDelta S)_{z_m} / \sum_1^{z_m} z e^{-\frac{z-1}{z_m}}\} \cdot \sum_1^z z e^{-\frac{z-1}{z_m}} \tag{20}$$

where S_z is the surface area of the crushed product after z blows have been applied, S_0 is the initial surface area of the sample, $(\varDelta S)_z$ is the surface increase in the case of z, and $(\varDelta S)_{z_m}$ is that in the case of z_m,

$$(\varDelta S)_{z_m} = S_{z_m} - S_0 = k_1 A_{r_1} \sum_1^{z_m} z e^{-\frac{z-1}{z_m}} .$$

Further, combining Eq. (9) and Eq. (19), the relation between the size distribution of the crushed product and the number of blows is obtained as follows:

$$R_z = e^{-k(A_{r_1} \cdot \sum_1^z z e^{-\frac{z-1}{z_m}} / W_0)^n \cdot x'^n} \tag{21}$$

Let W_0 and x' be constant, then we have

$$R_z = e^{-k'(A_{r_1} \cdot \sum_1^z z e^{-\frac{z-1}{z_m}})^n} \tag{22}$$

Now, if we denote the pass-through D_z, we have

$$D_z = 1 - R_z \approx k'(A_{r_1} \cdot \sum_1^z z e^{-\frac{z-1}{z_m}})^n$$

$$D_z^{1/n} \approx k'^{1/n} \cdot A_{r_1} \cdot \sum_1^z z e^{-\frac{z-1}{z_m}}$$

$$= \{D_{z_m}{}^{1/n} / \sum_1^{z_m} z e^{-\frac{z-1}{z_m}}\} \cdot \sum_1^z z e^{-\frac{z-1}{z_m}} \tag{23}$$

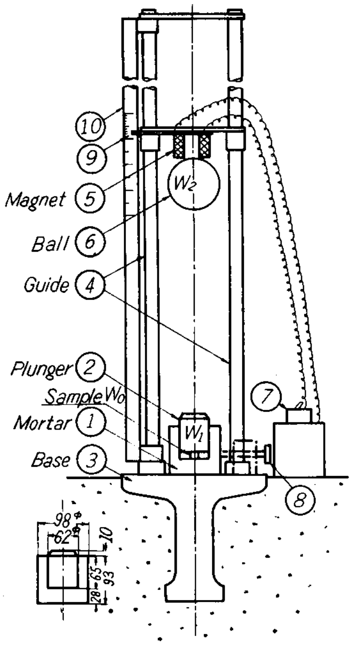

Fig. 3. Drop weight crusher apparatus.

4. Experimental Results

(a) Description of the apparatus[5)]

The drop weight crusher apparatus used by the authors is shown in **Fig. 3.** Before testing, the ball is held in position at a given height by the magnet and, when released, it strikes the cylindrical plunger which rests on the sample in a cylindrical cavity in the mortar. As the blow is repeated several times, the height of the sample gradually decreases, and the position of the ball should be regulated to have an exact height of fall. No permanent deformrtion was detected in any part of the crushing apparatus.

A slight degree of bounce of the ball is observed after impact, but it may be assumed to be negligibly small in this experiment. The base, mortar, plunger and ball are made of hardened tool steel. The base is 9.69 Kg in weight, the mortar 4.01 Kg, the plungers 3.27, 2.11, 1.68, 0.94 and 0.47 Kg, the balls 4.41, 3.11, 1.80, 1.39, 1.04 and 0.440 Kg. The five plungers above mentioned are 62 mm in diameter and have different heights. The hammers used in the drop hammer crusher are 5.73, 3.90, 2.87, 2.60, 1.44, 1.30 and 0.67 Kg in weight. In the case of multiple blows, the plunger is withdrawn after each impact, the sample is agitated and the size distribution of the crushed product is measured by means using total particles of the sample. It is then placed again in the mortar, and is the sample again on next blow.

(b) Effect of drop height, drop weight and plunger weight in the case of one blow[7)]

Size distribution curves of crushed Hakuun-stoneware 15 mm cubic and #10/20 mesh/inch powdery samples in the drop ball impact tests are shown in **Figs. 4 and 5** expressed as a function of the drop height and the drop weight.

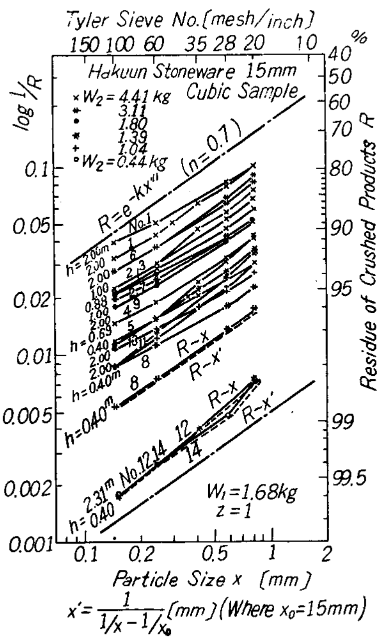

Fig. 4. Size distribution curve of crushed product (R vs x, or R vs x') (Impact crushing condition: Drop ball weight W_2 [Kg], plunger weight W_1 [Kg], drop height h [m], number of blow $z=1$, crushing chamber or mortar cavity 62 $\phi\times$65 mm).

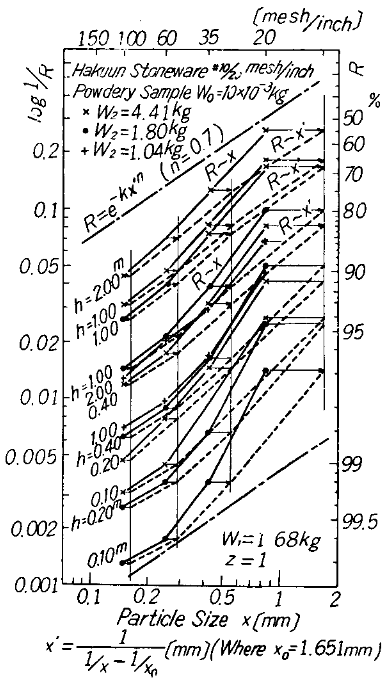

Fig. 5. R vs x, R vs x'.

In **Figs. 6 and 7**, the residue at a particle size corresponding to #100 mesh/inch Tyler sieve is plotted *vs* the net work input to crushing calculated by means of Eq. (16). It will be seen that Eqs. (2), (9) and (16) give good agreements with the results of the experiments. In the range of our experiments, the value

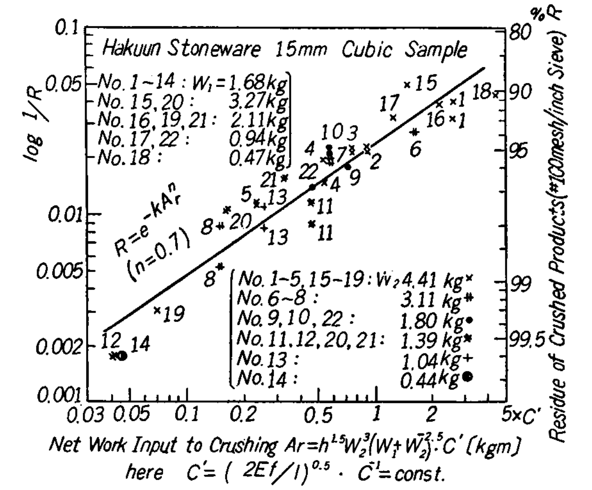

Fig. 6. Variation of residue relative to net work input to crushing, R vs A_r (Impact crushing condition: Fig. 4, No. 1, 6, 9, 11, 13, 15, 16, 20, 21: h=2.00 m, No. 2, 7, 17, 22: h=1.00, No. 5, 8, 12: h=0.400, No. 3: h=0.880, No. 4: h=0.690, No. 10: h=1.71, No. 14: h=2.31, No. 19: h=0.200 m).

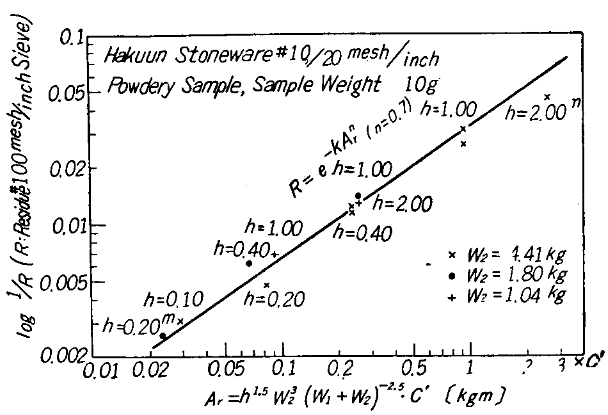

Fig. 7. R vs A_r (cf. Fig. 5).

of c in Eq. (16) is taken to be constant, regardless of the varieties of crushing conditions. The index n in these equations may be constant for the given material of the samples regardless of their shapes, cubic or powdery, and n is 0.7 for Hakuun-stoneware in our cases.

Figs. 8 and 9 show the results of impact tests on unglazed porcelain specimens, **Figs. 10 and 11** of tests on brick specimens, **Figs. 12 and 13** on porcelain insulator specimens, and **Figs. 14 cnd 15** on bakelite specimens.

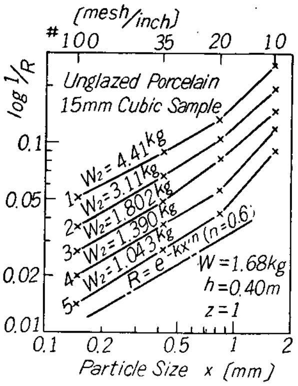

Fig. 8. R vs x.

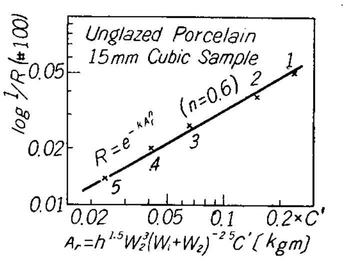

Fig. 9. R vs A_r (cf. Fig. 8).

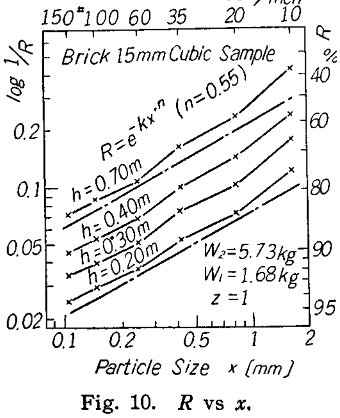

Fig. 10. R vs x.

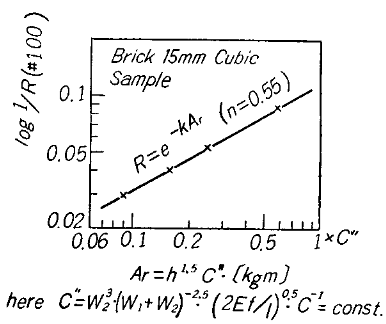

Fig. 11. R vs A_r (cf. Fig. 10).

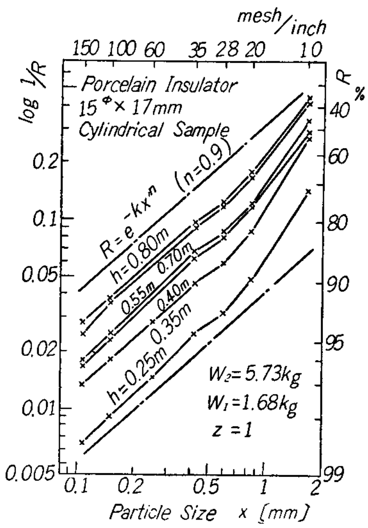

Fig. 12. R vs x.

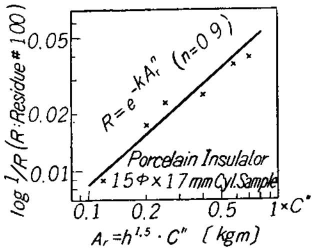

Fig. 13. R vs A_r (cf. Fig. 12)

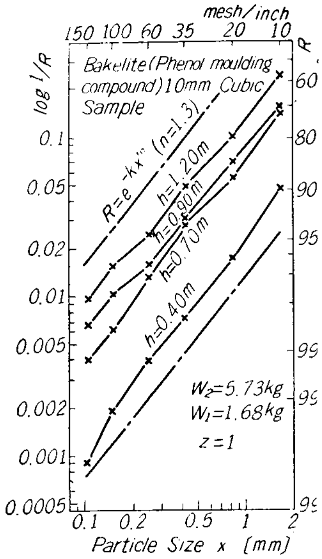

Fig. 14. R vs x.

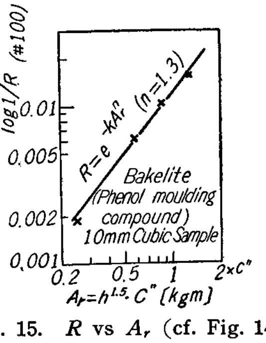

Fig. 15. R vs A_r (cf. Fig. 14

(c) **Effect of the number of blows**

The relations between the size distribution of the crushed product and the number of blows is shown in **Fig. 16.** In **Figs. 17 and 18,** the experimental values of $D_z^{1/n}$ are plotted *vs* number of blows z, where the value of n is determined from the slope of the curve in **Fig. 16.** In these figures, the calculated curves may be drawn by means of Eq. (23). The relation between R_z and $\sum A_{r_z}$ may be drawn as shown in **Fig. 19.**

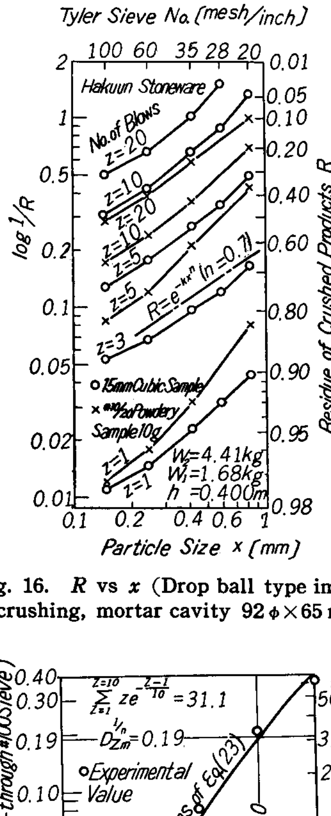

Fig. 16. R vs x (Drop ball type impact crushing, mortar cavity 92 ϕ × 65 mm).

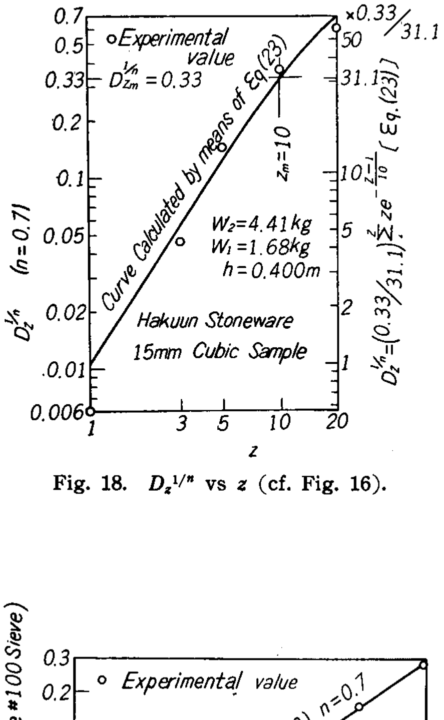

Fig. 18. $D_z^{1/n}$ vs z (cf. Fig. 16).

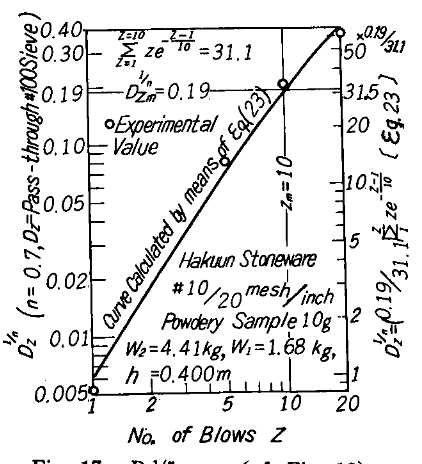

Fig. 17. $D_z^{1/n}$ vs z (cf. Fig. 16).

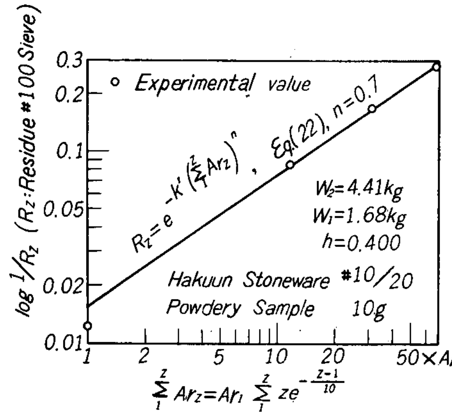

Fig. 19. R_z vs $\sum_1^z Ar_z$ (cf. Figs. 16 and 17).

Figs. 20~24 show the experimental results for bakelite specimens, **Figs. 25~27** show those for brick specimens and **Figs. 28~30** for porcelain insulator specimens. We see that Eqs. (9), (19), (21) and (23) give good agreements with

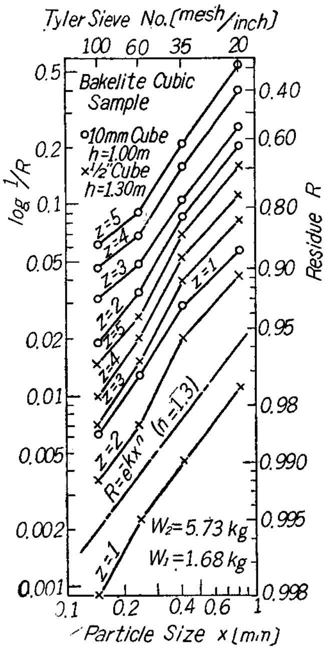

Fig. 20. R vs x.

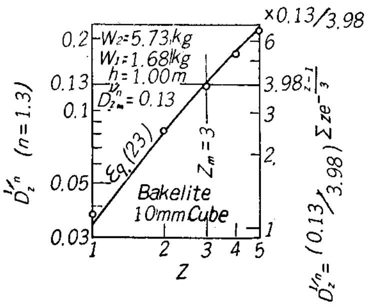

Fig. 21. $D_z^{1/n}$ vs z (cf. Fig. 20).

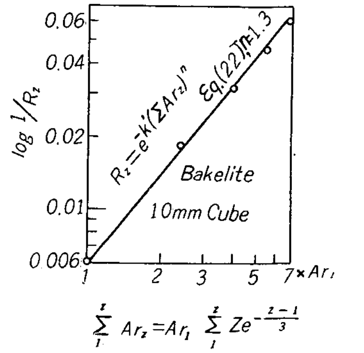

Fig. 22. R_z vs $\sum Ar_z$ (cf. Figs. 20 and 21).

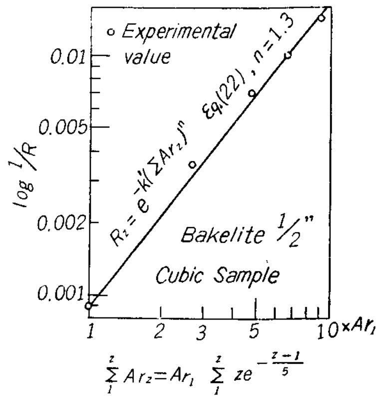

Fig. 23. $D_z^{1/n}$ vs z (cf. Fig. 20).

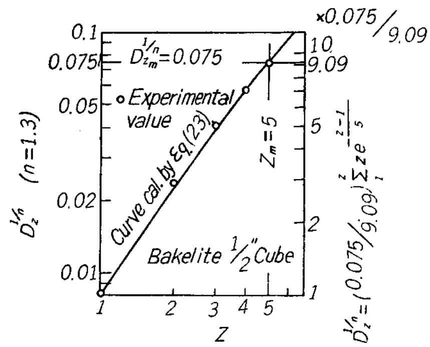

Fig. 24. R_z vs $\sum Ar_z$ (cf. Figs. 20 and 23).

these experimental results. The value of n in these equations may be considered constant regardless of the difference in the number of blows where the same materials are used in the impact compression crushing.

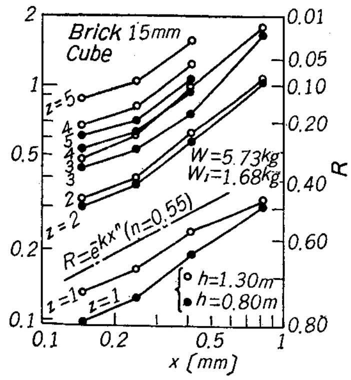

Fig. 25. R vs x.

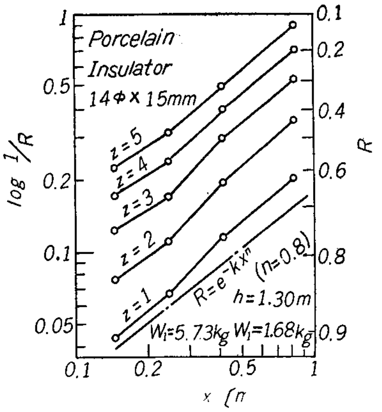

Fig. 28. R vs x.

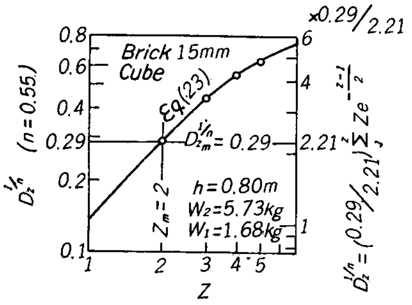

Fig. 26. $D_z^{1/n}$ vs z (cf. Fig. 25).

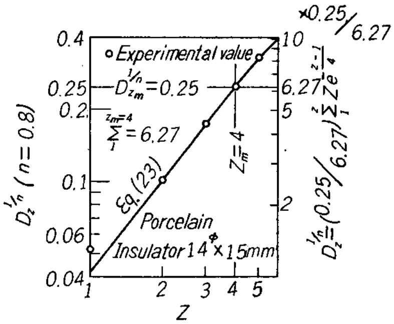

Fig. 29. $D_z^{1/n}$ vs z (cf. Fig. 28).

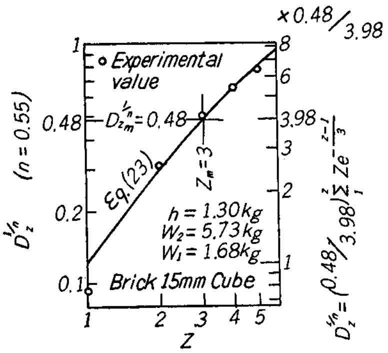

Fig. 27. $D_z^{1/n}$ vs z (cf. Fig. 25).

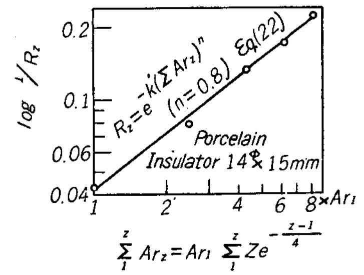

Fig. 30. R_z vs $\sum Ar_z$ (cf. Figs. 28 and 29).

(d) Comparison of the results experimental obtained by other researchers

(i) Experiments of Gross and Zimmerley,' and Koster's study

It was recognized that the new surface area of a crushed material produced by the crushing test of Gross[2] and Zimmerley was proportional to the work done, in accordance with the results of Rittinger's Law. But, it was found, at the same time, that the proportionality constant of Rittinger's Law might be determined by the distance through which the force acted in breaking, and the nature of the fracture surface. For practical purposes, it was desirable[2] to know this constant for various substances and to know how it might vary with the speed and method of breaking. The work done as noted down by Gross and Zimmerley was not, of course, identified with the net work imput to crushing. According to Koster's study[4], the proportionality constant of Rittinger's law might be dependent on the rate of load application and the necessary work for fracture might be decreased by rapid loading. According to Gross[2], instantaneous loading due to the explosive shattering might be considered effective. These results are similar to the authors' concepts in Eq. (14).

(ii) Tanaka's experiments

The experimental values of energy required for crushing described in Tanaka's paper[9] are directly proportional to the number of blows, or to the enternal work as shown in **Figs. 31 and 32.** Owing to the variations in the circumstances of the materials to be crushed by repeated blows of the hammer, the work input to crushing may not be considered to be proportional to the number of blows, as a general rule. The energy denoted in his paper does not, consequently, correspond to the net work input to crushing, and his formula[9] does not express the relation between the surface increase and the net work input. But, his experimental results[9] concerning the surface increase and the number of blows give good agreements with the authrs' equations as shown in **Figs. 33 and 34.**

(iii) Hönig's experiments

According to Hönig's experimental results[3], the surface increase due to crushing by a small drop weight with a high drop height was considerably larger than that due to crushing by a large drop weight with a low drop height. Also, the surface increase due to the work 15 Kg cm/cm^3 by one blow was larger than that due to the total work 20 Kg cm/cm^3 by four blows[3]. Such results were also confirmed by the authors' experiments.

(iv) Other researchers' experiments

Andreasen's experimental results[1] sometimes do not doincide with Rittinger's law, but they are in fair agreement with those of Reytt. According to Reytt, the fineness of the crushed product will sometimes increase more rapidly than

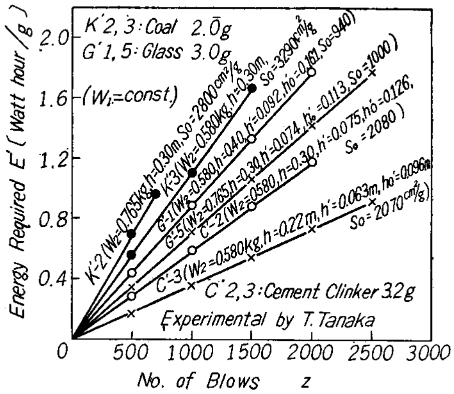

Fig. 31. Energy reguired vs number of blows (Experimental by T. Tanaka).

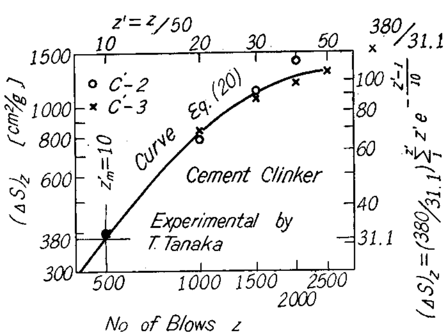

Fig. 33. $(\Delta S)_z$ vs z (cf. Figs. 31 and 32).

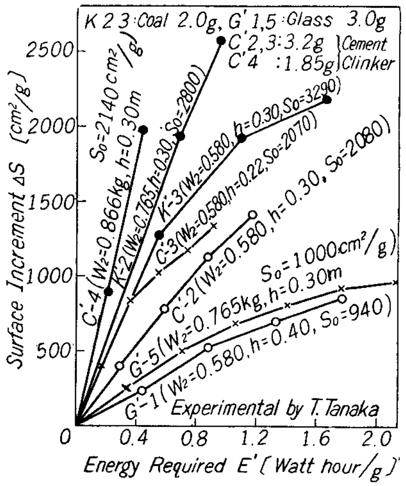

Fig. 32. ΔS vs E' (cf. Fig. 31).

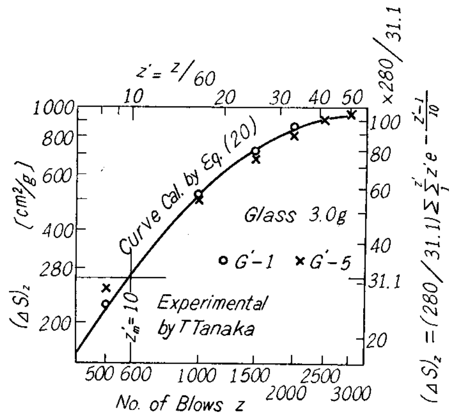

Fig. 34. $(\Delta S)_z$ vs z (cf. Figs. 31 and 32).

the energy expended[1]. By the authors' equation, we seen such feature only in the first stage of the crushing process, and the curve of the surface area is concept and Kick's law are tenable only in these cases, at the beginning of crushing. The surface increment per one blow increases with the number of blows and the curve becomes nearly straight for medium crushing and tends to become convex upward with successive crusning. Since the work input to crushing is taken to be proportional to the number of blows in Rittinger's and Kick's Law and even in Tanaka's formula, such formulas do not correlate with experimental relations obtained in a wider range.

(e) Effect of the particle size, the weight of the sample and the number of layers of particles in the crushing chamber[7)]

(i) Effect of the cubic size

Under the condition of constant values of h, W_2, W_1, E and c, and assuming the initial particle to be a cube, and putting $z=1$, $f/l=l$ and $W_0 \propto l^3$ in Eqs. (8), (16) and (21), we obtain

$$\Delta S \propto A_r \propto l^{0.5}, \quad \Delta S/W_0 \propto A_r/W_0 \propto l^{-2.5}$$
$$R = e^{-k(A_r/W_0)^n \cdot x'^n} = e^{-k' \cdot l^{-2.5n} \cdot x'^n} \tag{27}$$

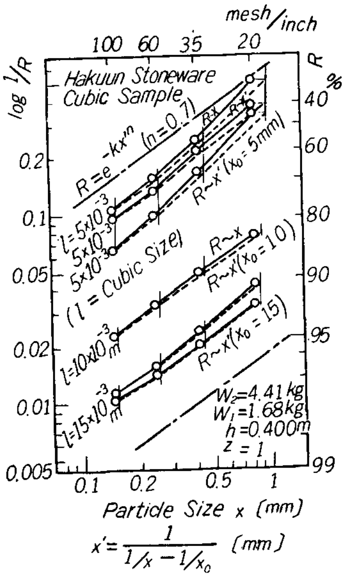

Fig. 35. R vs x, R vs x' (Cubic sample).

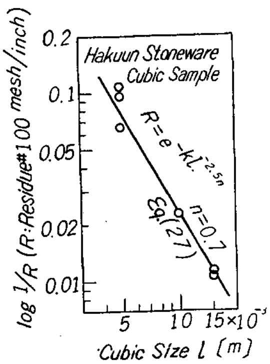

Fig. 36. Variation of residue relative to cubic size (cf. Fig. 35).

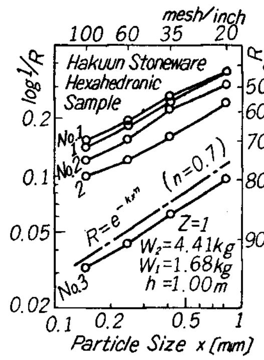

Fig. 37. R vs x (Hexahedronic sample, No. 1: $f=10\times10\times10^{-6}$ [m²], $l=5\times10^{-3}$[m], No. 2: $f=15\times15\times10^{-6}$[m²], $l=5\times10^{-3}$[m], No. 3: $f=15\times15\times10^{-6}$ [m²], $l=10\times10^{-3}$[m]).

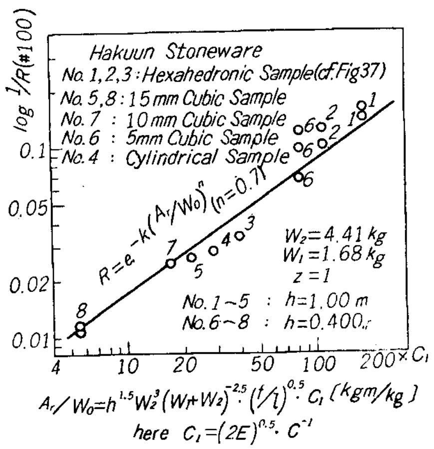

Fig. 38. R vs A_r/W_0 (Cubic, hexahedronic and cylindrical sample, No. 1, 2, 3: cf. Fig. 37, No. 4: $f=\frac{\pi}{4}(12.7)^2\times10^{-6}$ [m²], $l=5\times10^{-3}$ [m]).

Figs. 35 and 36 show the experimental results on Hakuun-stoneware cubic samples, **Figs. 37 and 38** on Hakuun-stoneware hexhedronic samples, and **Figs. 39 and 40** on bakelite hexahedronic samples. **Figs. 41 and 42** show Hönig's experi-

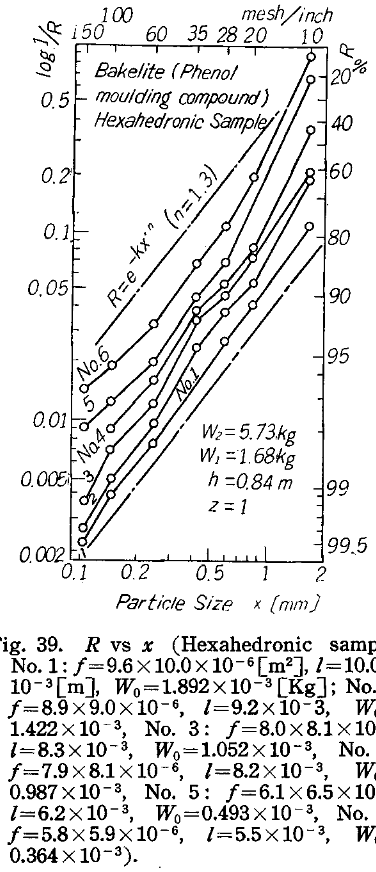

Fig. 39. R vs x (Hexahedronic sample, No. 1: $f=9.6\times10.0\times10^{-6}$[m²], $l=10.0\times10^{-3}$[m], $W_0=1.892\times10^{-3}$[Kg]; No. 2: $f=8.9\times9.0\times10^{-6}$, $l=9.2\times10^{-3}$, $W_0=1.422\times10^{-3}$, No. 3: $f=8.0\times8.1\times10^{-6}$, $l=8.3\times10^{-3}$, $W_0=1.052\times10^{-3}$, No. 4: $f=7.9\times8.1\times10^{-6}$, $l=8.2\times10^{-3}$, $W_0=0.987\times10^{-3}$, No. 5: $f=6.1\times6.5\times10^{-6}$, $l=6.2\times10^{-3}$, $W_0=0.493\times10^{-3}$, No. 6: $f=5.8\times5.9\times10^{-6}$, $l=5.5\times10^{-3}$, $W_0=0.364\times10^{-3}$).

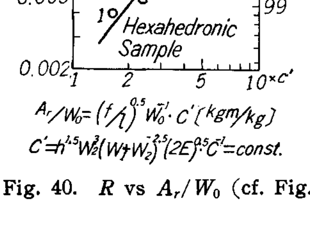

Fig. 40. R vs A_r/W_0 (cf. Fig. 39).

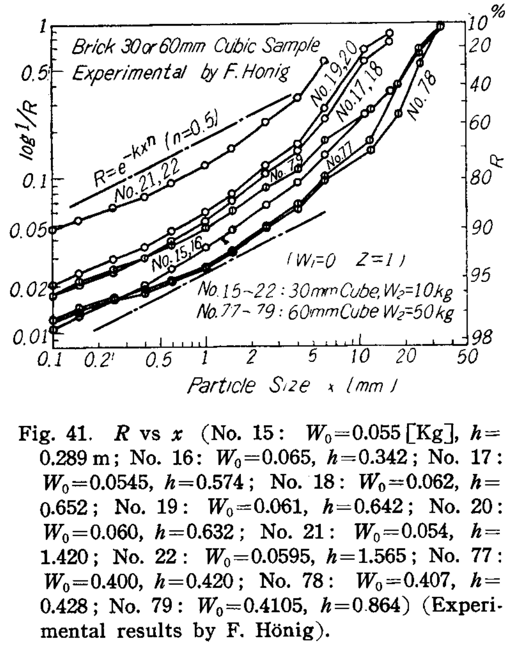

Fig. 41. R vs x (No. 15: $W_0=0.055$[Kg], $h=0.289$ m; No. 16: $W_0=0.065$, $h=0.342$; No. 17: $W_0=0.0545$, $h=0.574$; No. 18: $W_0=0.062$, $h=0.652$; No. 19: $W_0=0.061$, $h=0.642$; No. 20: $W_0=0.060$, $h=0.632$; No. 21: $W_0=0.054$, $h=1.420$; No. 22: $W_0=0.0595$, $h=1.565$; No. 77: $W_0=0.400$, $h=0.420$; No. 78: $W_0=0.407$, $h=0.428$; No. 79: $W_0=0.4105$, $h=0.864$) (Experimental results by F. Hönig).

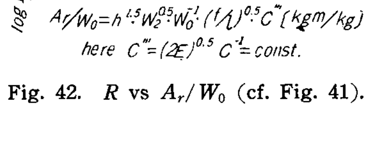

Fig. 42. R vs A_r/W_0 (cf. Fig. 41).

mental results[3] on brick cubic samples. The authors have found close similarity between these experimental results and the authors' equations, and the value of c in Fig. (16) may be considered constant in the range of experiments.

(ii) Effect of the weight of the sample, or the number of layers of particles in the crushing chamber

If we consider that the sample set under crushing consists of cubic particles possessing a uniform size, Eqs. (16) and (21) may be applicable to this case. Since the cross sectional area of the crushing chamber is constant, the number of layers of particles in it is taken to be proportional to the total weight of sample. Putting h, W_2, W_1, E, c and $f=$const., $z=1$ and $l \propto W_0$ in Eqs. (16) and (21), we get

$$\Delta S \propto A_r \propto W_0^{-0.5}, \quad \Delta S/W_0 \propto A_r/W_0 \propto W_0^{-1.5}$$

$$R = e^{-k(A_r/W_0)^n \cdot x'^n} = e^{-k' \cdot W_0^{-1.5n} \cdot x'^n} \tag{28}$$

When the weight of the sample is extremely small and there remains only one layer of particles, the number of particles may be taken to be increased in proportion to the weight of the sample. In this case, $f \propto W_0$ and we obtain

$$\Delta S \propto A_r \propto W_0^{0.5}, \quad \Delta S/W_0 \propto W_0^{-0.5}$$

$$R = e^{-k(A_r/W_0)^n \cdot x'^n} = e^{-k' \cdot W_0^{-0.5n} \cdot x'^n} \tag{29}$$

The results of experiments on Hakuun-stoneware #10/20 mesh/inch powdery samples possessing 0.1~100 g are shown in **Figs. 43 and 44.** Eqs. (28) and (29) give good agreements with these experimental results.

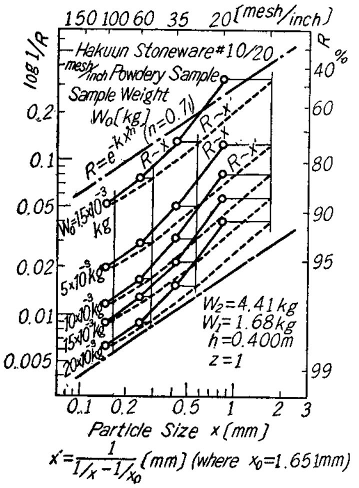

Fig. 43. R vs x, R vs x' (Powdery sample).

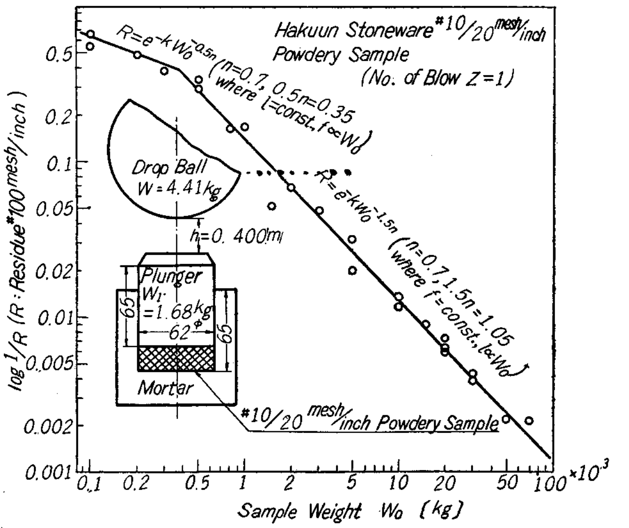

Fig. 44. Variation of residue relative to powdery sample weight (cf. Fig. 43).

5. Conclusions

The authors have presented Eq. (2) as an expression for the size distribution of the crushed product, and Fqs. (8) and (9) to repesent the relations among the net work input to crushing, the surface increase and the size distribution. Also they have assumed Eq. (16) as decribing the relation between the net work input to crushing and the impact crushing conditions, and Eq. (19) the relation between the net work input and the number of blows. Other equations in this paper are derived from the equations above mentioned. The authors have confirmed that these equations give good agreement with the corresponding experimental results.

Nomenclature

A	:	Work delivered to the sample by impact	Kgm
A_r	:	Net work input to crushing	Kgm
A_{r_z}	:	Net work input to crushing due to one blow named number z	Kgm
A_{r_1}	:	That due to the first blow	Kgm
$\sum_1^z A_{r_z}$	:	The sum total of net work due to z blows	Kgm
c	:	Constant	1
D	:	Pass-through of the crushed product	1
E	:	Young's modulus of the sample	Kg/m^2
f	:	Cross sectional area of sample	m^2
h	:	Drop height	m
l	:	Height of sample	m
n	:	Inclination of the size distribution curve	1
R	:	Residue of the crushed product	1
S_0	:	Initial surface area of sample	m^2
S_z	:	Surface area of crushed product after z blows	m^2
ΔS	:	Surface increase	m^2
V	:	Velocity of the plunger and ball moving vertically at the instant collison	m/s
W_0	:	Weight of sample	Kg
W_1	:	Weight of plunger	Kg
W_2	:	Weight of drop ball or drop hammer	Kg
x	:	Particle size of the crushed product	mm
x_0	:	Initial particle size of sample	mm
$\bar{x}$	:	Particle size in the case $R=1/e$	mm
x'	:	Modified particle size by Eq. (3)	mm
$\bar{x}'$	:	That in the case $R=1/e$	mm

z	:	Number of blows	1
z_m	:	Number of blows when A_{r_z} becomes maximum	1
σ	:	Maximum stress produced in the sample in the ideal case where it is assumed that the total applied work is stored as elastic energy in it	Kg/m^2
σ_0	:	Stress produced by a static load	Kg/m^2
σ	:	Lowest stress which causes crushing or rupture of materials	Kg/m^2
$\alpha, \beta, k, k', k_1, k_2$	:	Coefficients	

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