The Skew Network Difference Equation for the Orthotropic Parallelogram Plate and Its Application to the Experimental Study on the Model Skew Composite Grillage Girder Bridge

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Abstract

The skew network difference equation for the differential equation of the deflc-tion surface of the orthotropic parallelogram plate

$$B_x\frac{\partial^4 w}{\partial x^4}+2H\frac{\partial^4 w}{\partial x^2\partial y^2}+B_y\frac{\partial^4 w}{\partial y^4}=p$$

were proposed for the special case $H/(B_x \cdot B_y)^{1/2}=1$ and for the special boundary condition that the plate is supported simply at the opposite two skew sides and supported by flexible edge girders at the other two sides. These difference equations were applied to the theoretical analysis of the experimental study on the model skew composite grillage girder bridge, and it was found that this numerical analysis was very effective. To calculate the influence coefficients of the deflection and bending moment of the girders, the electronic digital automatic computer UNIVAC-120 was used.

1. Introduction

The number of researches on the skew girder bridge seems to be less than that on the skew isotropic plate. That is, the experimental study on the skew I-beam bridge with five main girders made by N. M. Newmark, C. P. Siess and W. H. Peckham can be pointed out as the first study in this field[1]. In this study, the values calculated by the right girder bridge theory in which the load distributing action of the slab is taken into consideration were used as the theoretical values which correspond to the measured values, and the theoretical values calculated taking the skew angle into consideration are not used.

Next, there are two theoretical researches on the skew girder bridge. The first is that by N. M. Newmark, C. P. Siess and T. Y. Chen[2] and the second that by the first of the authors and H. Yonezawa[3]. The former research is based on the theory of the isotropic parallelogram plate supported by flexible girders and gives the influence coefficients of the bending moment and deflection which are calculated by the difference equation method, using the skew network. The latter research is based on the theory of the orthotropic parallelogram plate and the calculation is made by the difference equation method using the rectangular network, and also the applicability of this calculation method to the skew girder bridge is made clear by the experimental study of the cast iron skew model girder bridge, but this research does not give the influence coefficients. So far as the authors know, there are the above three researches based on the plate theory on the skew girder bridge.

The authors have developed the skew network difference equation for the orthotropic parallelogram plate which is simply supported at the opposite two sides and is free or supported by flexible girders at the other two sides, and as the continuation of the previous research on the model right composite grillage girder bridge[4], an experimental study of the skew composite grillage girder bridge was planned in order to check the applicability of the authors' method to the analysis of the skew girder bridge.

2. Outline of the Model Composite Grillage Girder Bridge

The general plan and section of the model skew composite grillage girder bridge are shown in Fig. 1 and the details of the model bridge are as follows:

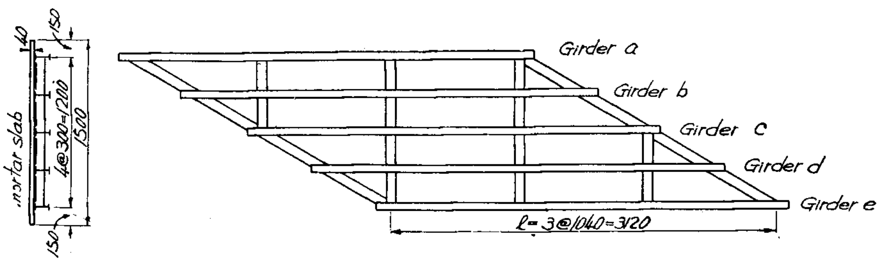


Fig. 1. General plan and section of model skew bridge

1) skew angle: 60 degree, 2) number of main girder: 5, 3) span: 312 cm, 4) spacing of main girder: 30 cm, 5) spacing of cross girder: 104 cm, 6) mortar slab is 4 cm thick and is connected by shear connectors recommended by G. Wastlund, 7) section: main girder, 2-flange plates 60×8, 1-web plate 120×6; cross girder, 2-flange plates 50×8, 1-web plate 80×6.

The moments of inertia of the main and cross girders are all the same and are 10374 cm^4 and 4820 cm^4 respectively, when converted to the moment of inertia of mortar section. The effective width of the flange of the main and cross girders are all assumed as 30cm (spacing of main girder) andt he ratio of the modulus of elasticity of steel to that of mortar is assumed as 10 (E_s=2,100,000 kg/cm^2 and E_c=210,000 kg/cm^2).

The model is the same as the previously tested model girder bridges[4] except the span and number of the cross girders. Because the cross girder was expected to be arranged to pass through the end of the edge girders, the span inevitably became 312 cm and the arrangement of the cross girders became as shown in Fig. 1.

3. Loading and Measurement

The load was applied to the fifteen points shown in Fig. 2, and these points correspond to the quarter- and mid-span points of the girders. The load consists of two or three steel ingots which are about 1.01 or 1.02 tons. In order to apply this load, the 300×520 mm steel plate and many sheets of newspaper were used.

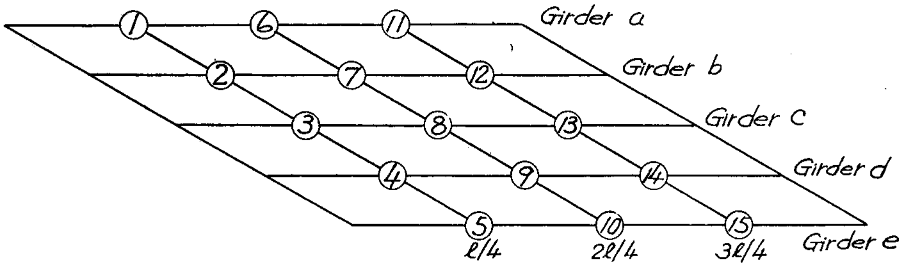


Fig. 2. Loading point

The strain of the girder was picked up by electric wire resistance strain gages and measured by the strain indicators made by Baldwin and Japanese makers. The deflection was measured by dial gages.

I. Skew Network Difference Equation for the Orthotropic Parallelogram Plate and Its Application to the Model Skew Composite Grillage Girder Bridge.

The effectiveness of the method of analysis in which the right girder bridge is assumed as the orthotropic rectangular plate has been made clear by many experimental researches and also the method of analysis of the skew girder bridge in which the orthotropic parallelogram plate was solved by difference equation of rectangular network was acertained by the first of the authors as described above. From this point of view, the authors have further developed the skew network difference equation for the orthotropic parallelogram plate. The detail of the induction is omitted and only the result is described as follows:

a) Notation (see Fig. 3)

l = span of bridge or of orthotropic parallelogram plate, center to center of supports

B_x = flexural rigidity of the orthotropic plate in x direction

B_y = flexural rigidity of the orthotropic plate in y direction

ν_x, ν_y = Poisson's ratio for the materials in the plate in x and y directions, taken as zero in the numerical data given here

$E_b I_b$ = product of modulus of elasticity of edge girder material and the moment of inertia of the girder cross section

P = concentrated load

p = load per unit of area uniformly distributed over the plate

q = load per unit of length uniformly distributed along a girder

w = deflection of plate, positive downward; with subscript indicating the deflection at a particular point denoted by the subscript

x, y = rectangular coordinates

u, v = skew coordinates

λ_x, λ_y = distance between points or lines of the network as defined in Fig. 3

φ = skew angle

$\alpha = (B_y/B_x)^{1/2}$, an abbreviation

$K = \lambda_y/\lambda_x$, an abbreviation

$A = K^2(1+\alpha \tan^2\varphi)$, an abbreviation

$B = \alpha K \tan\varphi$, an abbreviation

$C = K^2(1-\nu_x)$, an abbreviation

$D = B(A+C)$, an abbreviation

$J = K^4(E_b I_b/\lambda_y B_x)$, a dimensionless number proportional to relative stiffness

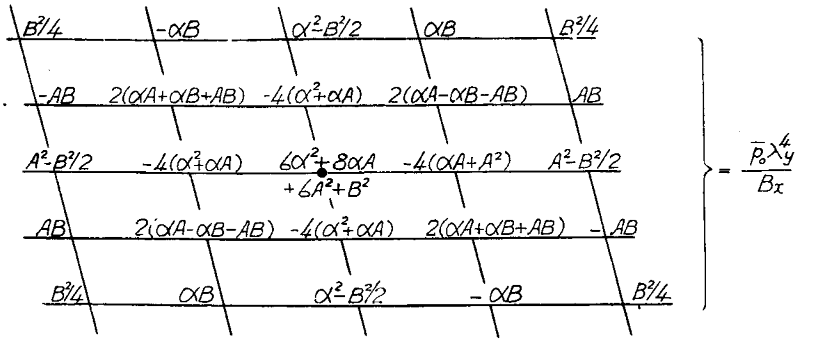
b) Skew network difference equation

The skew network difference equation for the fundamental differential equation of the orthotropic parallelogram plate

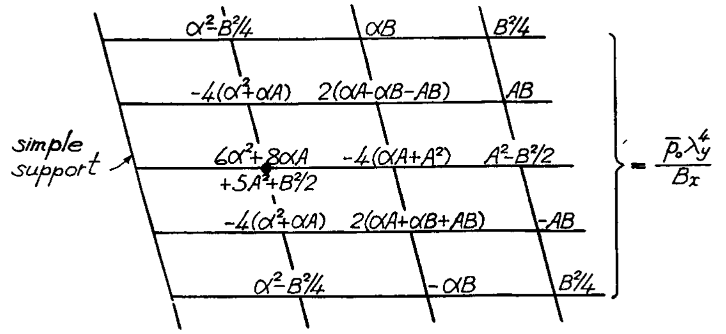
$$B_x\frac{\partial^4 w}{\partial x^4}+2H\frac{\partial^4 w}{\partial x^2 \partial y^2}+B_y\frac{\partial^4 w}{\partial y^4}=p$$

are as follows for the special assumption of $H/(B_x \cdot B_y)^{1/2}=1$ and the boundary condition that the plate is simply supported at the opposite skew sides and is supported by flexible edge girders at the other two sides.

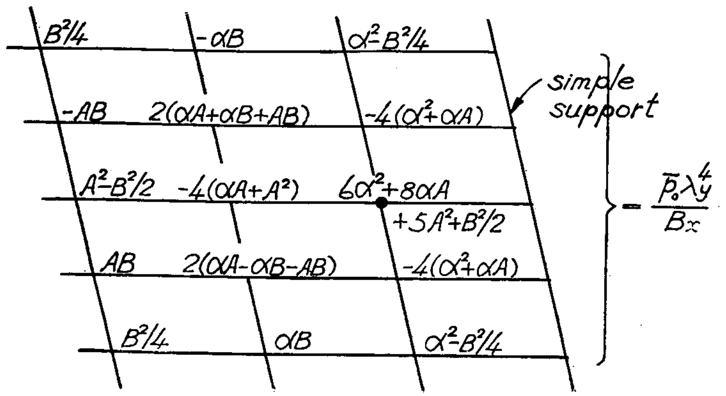
1) general interior point: shown in eq. (1)
2) interior point near left simple support: shown in eq. (2)
3) interior point near right simple support: shown in eq. (3)
4) interior point near edge girder: shown in eq. (4)
5) interior point near sharp corner: shown in eq. (5)


B²/4 -αB α²-B²/2 αB B²/4
-AB 2(αA+αB+AB) -4(α²+αA) 2(αA-αB-AB) AB
A²-B²/2 -4(α²+αA) 6α²+8αA +6A²+B² -4(αA+A²) A²-B²/2
AB 2(αA-αB-AB) -4(α²+αA) 2(αA+αB+AB) -AB
B²/4 αB α²-B²/2 -αB B²/4
= P̄₀λy⁴ / Bx

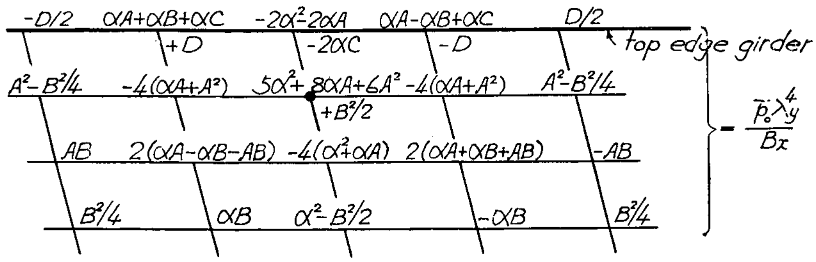
(1)


α²-B²/4 αB B²/4
-4(α²+αA) 2(αA-αB-AB) AB
simple support
6α²+8αA +5A²+B²/2 -4(αA+A²) A²-B²/2
-4(α²+αA) 2(αA+αB+AB) -AB
α²-B²/4 -αB B²/4
= P̄₀λy⁴ / Bx

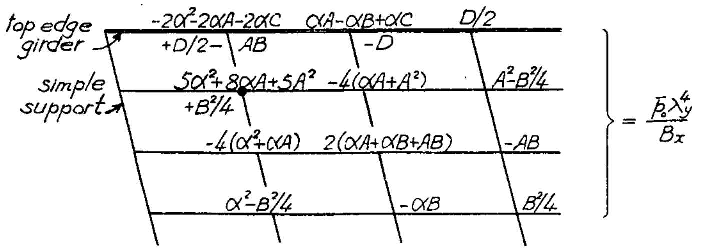
(2)


B²/4 -αB α²-B²/4
-AB 2(αA+αB+AB) -4(α²+αA)
simple support
A²-B²/2 -4(αA+A²) 6α²+8αA +5A²+B²/2
AB 2(αA-αB-AB) -4(α²+αA)
B²/4 αB α²-B²/4
= P̄₀λy⁴ / Bx

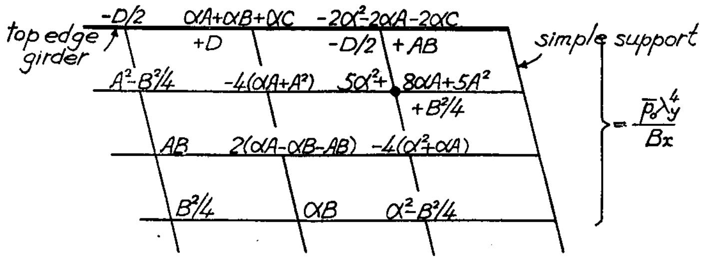
(3)


-D/2 αA+αB+αC +D -2α²-2αA -2αC αA-αB+αC -D D/2
top edge girder
A²-B²/4 -4(αA+A²) 5α²+8αA+6A² +B²/2 -4(αA+A²) A²-B²/4
AB 2(αA-αB-AB) -4(α²+αA) 2(αA+αB+AB) -AB
B²/4 αB α²-B²/2 -αB B²/4
= P̄₀λy⁴ / Bx

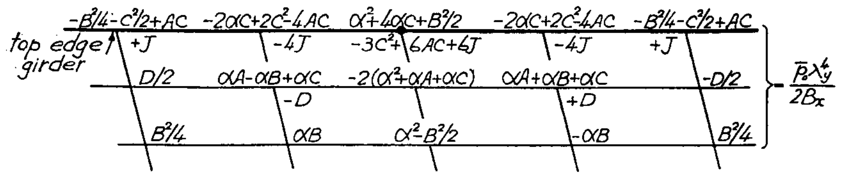
(4)



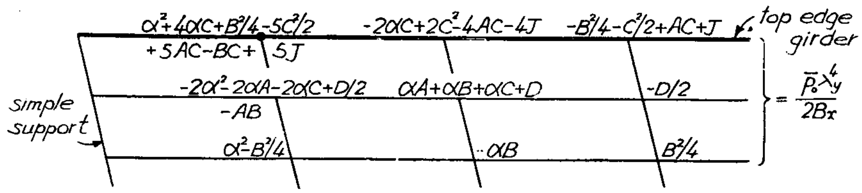
(5)



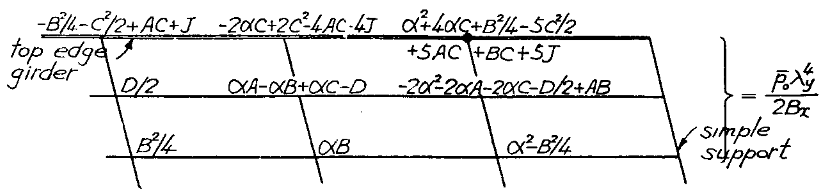
(6)



(7)



(8)



(9)

6) interior point near blunt corner: shown in eq. (6)

7) general edge point: shown in eq. (7)

8) edge point near sharp corner: shown in eq. (8)

9) edge point near blunt corner: shown in eq. (9)

In these equations, the quantity $\bar{p}_0$ is the equivalent combined effects in terms of load per unit of area of all the loads that act at the point considered (O). Thus, if at point O, there act a uniformly distributed load of p_o per unit of area, a line load of q per unit of length in x direction, and a concentrated load of P_o, $\bar{p}_o$ is given by

$$\bar{p}_o = p_o + \frac{q_o}{\lambda_y} + \frac{P_o}{\lambda_x \lambda_y} = p_o + \frac{q_o}{\lambda_y} + \frac{KP_o}{\lambda_y^2}.$$

If point O lies on an exterior edge of the plate, $\bar{p}_o$ is given by

$$\bar{p}_o = p_o + \frac{q_o}{\lambda_y/2} + \frac{P_o}{\lambda_x \lambda_y/2} = p_o + \frac{2q_o}{\lambda_y} + \frac{2KP_o}{\lambda_y^2}.$$

If we assume $B_x = B_y$, that is, $\alpha = 1$, the above nine equations become equal to those given by N. M. Newmark, C. P. Siess and T. Y. Chen.

c) Theoretical calculation for the model composite grillage girder bridge

We assume the model girder bridge as the orthotropic parallelogram plate which has $B_x = 10374\,E_c/30 = 345.8\,E_c$, $B_y = 4820\,E_c/104 = 46.346154\,E_c$, $H = (B_x \cdot B_y)^{1/2}$, $\varphi = 60°$ and also is simply supported at the opposite skew sides and supported by flexible edge girders ($E_b I_b = 10374\,E_c$) at the other two sides. It was acertained by the authors that the assumption $H/\sqrt{B_x B_y} = 1$ was effective for such a model composite grillage girder bridge[4], and therefore the same assumption was used.

Let us divide the orthotropic parallelogram plate and denote each point as shown in Fig. 3.

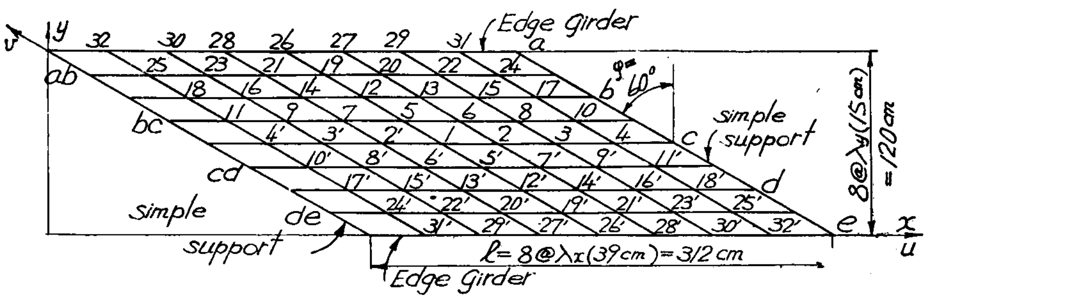


Fig. 3.

The values of the above notations necessary to obtain the difference equations are as follows:

$\varphi = 60°(\tan\varphi = 1.732\,051)$,	$\alpha = (46.346\,154/345.8)^{1/2} = 0.366\,096$,
$K = 15/39 = 0.384\,615$,	$A = 0.310\,398$,
$B = 0.243\,883$,	$C = 0.147\,929$,
$D = 0.111\,778$,	$J = (0.384\,615)^4(30/15) = 0.043\,766$.

The unknown terms are the deflections of $9 \times 7 = 63$ points. In this calculation, if we may consider the symmetrical and skew-symmetrical loading states, it is possible to reduce the number of unknown terms which are 32 for the symmetrical loading and 31 for the skew-symmetrical loading. The eq. (1)~(9) being applied to each case, we obtain the 32×32 elements and 31×31 elements of the stiffness matrix for these two cases respectively.

We calculated the inverse matrix (flexibility matrix) of the above stiffness matrix by electronic computer UNIVAC-120 which belongs to the Harima Shipbuilding and Engineering Work Co. Ltd., AIOI, Japan.

To describe the elements of these matrix requires so much space that it will be omitted.

From the inverse matrix thus obtained, we can obtain the influence coefficients for the deflections of the above 32 points, and therefore can calculate that for the bending moment of each point from which the influence coefficients for the bending moment of the girder can be obtained by multiplying $2\lambda_y = 30$ cm. Table 1 shows the influence coefficients of the deflection in $2l/8$, $4l/8$, $6l/8$ sections of girders a, b and in $2l/8$, $3l/8$, $4l/8$ sections of girder c. Also Table 2 gives the influence coefficients of the bending moments in the above sections of each girder.

It must be remembered that these tables can only be applied to the case of the following conditions:

a) $B_y/B_x = 0.134\,026$, b) width/span $= 120/312 = 0.384\,615$, c) $H/(B_x \cdot B_y)^{1/2} = 1$

d) $E_b I_b = 30 \cdot B_x$, e) $\varphi = 60°$.

II. Result of Measurement and Its Comparison with the Theoretical Values

The stress of the lower flange was measured at $l/4$, $2l/4$ and $3l/4$ sections of each girder and the deflection was measured at the mid-span section of each girder. The result of the measurement is shown in Table 3 and 4 with the theoretical values. These theoretical values were calculated by the influence coefficients given in Table 1 and 2.

III. Consideration of the Result

It is generally recognized from Table 3 and 4 that the experimental values agree considerably well with the theoretical values calculated by the authors' method. Thus, the theory of the orthotropic parallelogram plate can be applied to the analysis of the skew grillage girder bridge with considerable accuracy.

Also, it is generally known by the experimental stress analysis of the existing highway skew girder bridges that the measured values of the stress and deflection of the skew girder can not be interpreted by any analytical method for the right

Table 3. Measured Values of Stress of the Skew Composite Grillage Girder Bridge and Its Comparison with the Theoretical Values (unit: kg/cm^2/t)

State of loading		$l/4$ section					$l/2$ section					$3l/4$ section	
		a	b	c	d	e	a	b	c	d	e	a	c
1	Measured Values	204	115	70	31	21	112	57	36	—	—	21	−31
	Theoretical Values	323	181	93	43	15	131	60	18	4	1	22	−21
2	Measured Values	69	138	62	35	29	134	57	30	—	—	59	−21
	Theoretical Values	107	214	109	54	23	135	66	23	8	4	47	−18
3	Measured Values	21	45	92	52	41	61	95	45	—	—	55	—
	Theoretical Values	29	62	138	84	47	80	81	16	2	1	57	−9
6	Measured Values	109	156	120	62	34	299	146	57	17	—	89	−25
	Theoretical Values	129	188	138	74	30	368	156	50	12	−1	99	−33
7	Measured Values	48	66	91	62	46	124	139	72	41	14	123	−16
	Theoretical Values	40	66	102	84	51	154	188	90	36	20	124	−17
8	Measured Values	14	—	24	66	62	60	69	142	69	60	64	24
	Theoretical Values	9	6	25	75	75	58	93	158	93	58	75	25
11	Measured Values	26	68	75	57	39	225	143	60	40	39	232	−21
	Theoretical Values	42	79	82	57	29	279	124	62	23	10	280	−15
12	Measured Values	14	11	29	38	41	53	47	68	62	41	74	41
	Theoretical Values	9	11	26	43	46	43	52	72	54	35	111	39

Table 4. The Measured Values of the Deflection of Each Girder and its Comparison with the Theoretical Values (unit: 0.01mm/t)

State of loading		$l/2$ section				
		a	b	c	d	e
1	Measured Values	64.8	35.1	16.6	6.8	2.0
	Theoretical Values	75.0	36.7	15.8	6.2	2.1
2	Measured Values	57.3	35.1	19.1	10.6	4.0
	Theoertical Values	55.7	36.0	18.6	8.8	4.3
3	Measured Values	31.4	30.6	26.0	16.1	10.2
	Theoretical Values	32.4	30.6	23.9	15.6	10.6
6	Measured Values	102.9	57.9	30.2	13.7	6.3
	Theoretical Values	112.6	61.1	28.5	12.3	5.1
7	Measured Values	63.9	47.7	32.7	21.7	13.0
	Theoretical Values	61.1	52.7	34.3	19.7	12.3
8	Measured Values	26.4	30.6	38.2	30.6	26.4
	Theoretical Values	28.5	34.3	41.3	34.3	28.5
11	Measured Values	55.5	44.1	28.2	13.2	6.8
	Theoretical Values	57.0	46.4	25.5	12.7	6.6
12	Measured Values	28.4	31.4	25.4	21.9	17.0
	Theoretical Values	27.0	28.6	26.9	20.6	16.1

girder bridge, inspite of the measured values for the right girder bridge being explainable by the application of the theory of the orthotropic rectangular plate, and that the skew angle becomes sharper, the difference between the experimental and theoretical values is larger and the measured values can not be explained. This fact teaches us that it is necessary to introduce the skew angle into the analysis of the skew girder bridge and the authors' method is an effective procedure for the analysis of the skew girder bridge.

4. Conclusion

The authors have developed the skew network difference equation for the orthotropic parallelogram plate and the influence coefficients of the deflection and bending moment of the plate were obtained by an electronic automatic computer for the special value of the plate and boundary condition and also, the experimental values for the model skew composite grillage girder bridge were compared with the theoretical values calculated by the above method. As a result, it was made clear that the authors' method can explain well the experimental values.

We shall plan to calculate the influence coefficients of the deflection and bending moment of the orthotropic parallelogram plate, and also to contribute to the structural analysis of the skew girder bridge. This is the first paper of the authors' research in this field.

Acknowlegment

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References

1) N. M. Newmark, C. P. Siess and W. H. Peckham : University of Illinois Bulletin, No. 375 (1948).
2) N. M. Newmark, C. P. Siess and T. Y. Chen : University of Illinois Bulletin, No. 439 (1957).
3) N. Naruoka und H. Yonezawa : Bauingenieur, 32 (1958), S. 391.
4) M. Naruoka und H. Yonezawa : Preliminary Publication of 5th International Congress for Bridge and Structural Engineering, Lisbon 1956, p. 391.

Table 1. Influence Coefficients for Deflection in Girders
(unit: $10^{-5}Pl^2/B_x$ per $0.5P$)

	Transverse Location of Load	Longitudinal Position of Load						
		1/8	2/8	3/8	4/8	5/8	6/8	7/8
$\delta_{a,l/4}$	a	1530	2687	3010	2799	2250	1515	722
	a b	1309	2093	2248	2000	1514	928	376
	b	1027	1550	1607	1370	980	558	208
	b c	763	1105	1108	909	620	335	121
	c	544	764	744	589	387	202	71
	c d	377	515	488	375	238	121	42
	d	255	339	312	233	144	72	25
	d e	165	212	190	139	85	42	15
	e	88	116	106	79	51	28	12
$\delta_{a,l/2}$	a	1472	2799	3792	4198	3656	2126	1256
	a b	1385	2497	3153	3175	2554	1624	676
	b	1225	2076	2443	2280	1713	1009	389
	b c	1009	1624	1796	1581	1122	628	234
	c	790	1209	1271	1063	726	394	145
	c d	590	884	874	704	467	249	92
	d	426	602	586	460	300	159	60
	d e	293	401	382	295	193	106	43
	e	174	245	239	192	177	79	36
$\delta_{a,3l/4}$	a	780	1514	2140	2426	2688	2330	1250
	a b	756	1423	1927	2179	2076	1542	693
	b	704	1274	1641	1732	1503	997	410
	b c	626	1085	1323	1306	1048	645	255
	c	531	881	1020	949	718	422	165
	c d	431	687	760	675	490	282	111
	d	337	518	552	474	337	194	79
	d e	254	379	395	334	239	142	62
	e	174	265	281	245	183	116	55
$\delta_{b,l/4}$	a	818	1550	2005	2076	1798	1274	630
	a b	815	1493	1798	1720	1371	880	378
	b	806	1395	1521	1344	998	596	240
	b c	719	1134	1169	984	697	399	156
	c	577	854	850	694	477	266	103
	c d	434	616	599	479	323	179	70
	d	313	432	414	327	220	123	49
	d e	218	296	282	224	153	88	38
	e	137	194	192	159	115	72	34
$\delta_{b,l/2}$	a	700	1370	1936	2280	2227	1732	915
	a b	701	1358	1880	2137	1944	1373	643
	b	705	1344	1811	1967	1636	1067	469
	b c	695	1279	1632	1634	1279	795	339
	c	652	1141	1366	1281	964	584	246
	c d	576	957	1086	974	717	431	183
	d	483	769	840	736	539	327	144
	d e	389	604	648	566	420	264	122
	e	302	474	515	460	354	233	113
$\delta_{b,3l/4}$	a	281	558	812	1009	1095	997	613
	a b	285	570	830	1026	1130	986	566
	b	297	596	868	1067	1133	986	514
	b c	314	627	898	1070	1074	842	407
	c	329	642	888	1005	936	674	312
	c d	333	630	834	892	780	533	244
	d	326	594	754	769	646	432	201
	d e	310	551	702	668	549	369	177
	e	300	518	619	602	495	339	169
$\delta_{c,l/4}$	a	389	764	1064	1209	1146	881	468
	a b	401	793	1082	1176	1050	751	356
	b	432	854	1117	1141	955	642	298
	b c	485	934	1130	1066	836	535	240
	c	543	976	1053	892	690	428	188
	c d	523	843	870	743	545	335	148
	d	445	674	687	584	429	266	165
	d e	365	528	541	466	350	223	105
	e	279	422	445	394	305	202	99
$\delta_{c,l3/8}$	a	375	744	1068	1271	1267	1020	564
	a b	386	778	1120	1305	1247	948	489
	b	417	850	1217	1366	1236	888	438
	b c	470	955	1332	1402	1185	805	381
	c	535	1053	1394	1337	1060	690	318
	c d	568	1041	1258	1154	890	571	262
	d	546	936	1069	964	740	477	223
	d e	493	812	905	816	633	416	200
	e	444	718	796	726	574	387	192
$\delta_{c,l/2}$	a	295	589	862	1063	1116	949	552
	a b	305	623	924	1144	1182	970	538
	b	332	694	1039	1281	1282	1005	533
	b c	380	801	1192	1439	1361	1000	503
	c	445	893	1337	1541	1337	893	445

Table 2. Influence Coefficients for Bending Moment in Girders
(unit: $10^{-4}Pl$ per $0.5P$)

	Transverse Location of Load	Longitudinal Position of Load						
		1/8	2/8	3/8	4/8	5/8	6/8	7/8
$M_{a,l/4}$	a	230	512	325	205	124	67	28
	a b	210	279	192	118	68	33	10
	b	144	171	105	64	34	14	3
	b c	87	91	56	32	15	4	1
	c	47	46	28	14	5	0	−1
	c d	24	22	11	4	−1	−2	−2
	d	11	9	0	−1	−3	−3	−2
	d e	4	1	−2	−4	−5	−4	−3
	e	−1	−3	−6	−7	−6	−4	−2
$M_{a,l/2}$	a	95	207	361	583	351	217	81
	a b	107	252	353	384	224	115	41
	b	117	215	276	244	139	68	23
	b c	102	175	193	156	84	44	14
	c	83	127	127	91	50	23	7
	c d	60	135	80	54	29	13	−4
	d	37	55	44	31	17	7	1
	d e	26	33	27	23	7	3	1
	e	13	16	12	20	7	1	1
$M_{a,3l/4}$	a	14	35	74	156	254	444	183
	a b	20	54	108	187	276	281	104
	b	29	74	135	197	228	176	63
	b c	38	89	140	176	169	58	42
	c	44	91	129	142	121	74	28
	c d	44	112	109	109	85	50	20
	d	41	72	86	81	60	36	15
	d e	36	59	68	87	44	28	12
	e	28	46	52	47	36	23	11
$M_{b,l/4}$	a	138	286	343	298	214	126	52
	a b	137	289	329	174	121	57	16
	b	135	340	183	104	52	17	1
	b c	129	186	98	45	14	−2	−6
	c	89	98	41	10	−6	−38	−8
	c d	50	42	9	8	−15	−13	−8
	d	23	11	−9	−17	−18	−15	−9
	d e	6	−6	−18	−29	−20	−16	−8
	e	−7	−17	−25	−25	−22	−16	−7
$M_{b,l/2}$	a	44	95	157	248	266	197	94
	a b	46	97	398	548	239	138	52
	b	46	105	185	299	170	82	26
	b c	54	123	202	223	113	50	17
	c	63	129	170	148	72	31	10
	c d	63	112	126	92	46	20	6
	d	55	86	84	57	28	12	4
	d e	41	65	50	29	17	9	3
	e	26	37	31	21	9	6	2
$M_{b,3l/4}$	a	−24	−45	−56	−52	−21	50	62
	a b	−23	−39	−41	−24	60	125	90
	b	−19	−28	−18	18	87	200	81
	b c	−13	−8	21	75	149	292	67
	c	−1	20	63	119	142	134	54
	c d	14	70	100	135	143	103	44
	d	27	68	110	133	124	86	38
	d e	37	93	147	128	107	73	34
	e	61	90	113	118	97	68	34
$M_{c,l/8}$	a	73	147	204	220	190	130	61
	a b	74	154	205	198	150	84	34
	b	78	173	202	162	98	42	9
	b c	84	199	180	106	43	7	−5
	c	92	219	109	40	1	−14	−13
	c d	97	137	45	−1	−22	−24	−15
	d	68	62	2	−26	−34	−29	−27
	d e	27	10	−55	−42	−41	−32	−17
	e	−1	−24	−46	−52	−45	−33	−18
$M_{c\ 3l/8}$	a	41	83	129	165	167	129	66
	a b	41	86	144	178	161	108	52
	b	43	94	171	191	145	79	28
	b c	45	106	210	184	106	46	12
	c	50	149	245	148	57	37	2
	c d	68	147	175	78	26	4	−2
	d	70	119	104	39	7	−4	−3
	d e	51	78	54	14	−5	−9	−6
	e	35	40	19	−3	−12	−10	−6
$M_{c,l/2}$	a	14	29	50	80	104	99	59
	a b	14	32	58	103	130	111	58
	b	15	31	74	143	159	114	50
	b c	17	47	100	198	172	95	35
	c	23	75	135	251	135	75	23