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# On the Cutting Mechanism for Soft Metals

By

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## Abstract

By orthogonal dry cutting of soft metals such as lead and Lipowitz alloy in low velocity with a specially designed cutting test equipment, the authors recognized the existence of the transitional region (to be called the "flow region" in this paper) between the rigid region of the workpiece and the plastic region of the steady chip and it should be taken into consideration in the theoretical analysis of cutting mechanism instead of the single shear plane. In this case the connecting line between the uncut surface of the workpiece and the back side of the chip is round.

The flow region associated with a simple continuous chip changes in shape and size according to cutting conditions, and sometimes periodically under the same cutting condition. One of the most important factors affecting the size of the flow region seems to be the cutting velocity: the higher the velocity, the narrower becomes the flow region because of the localization of flow.

Also in the case of a discontinuous chip, there exists a flow region during formation of one chip fragment. A fracture appears to occur on some surface near the ending boundary in the flow region where it is the easiest to slip, and is convex upwards.

Then, taking this range into consideration, the deformation figures of grid lattice are drawn by deriving the general expression of chip deformation under some assumptions.

Further, the authors have observed the square grid deformation under the machined surface; that is, the grid lines formerly perpendicular to the uncut surface are bent somewhat exponentially to the cutting direction, the degree of which is remarkable with dull cutting edges.

The chip-curl phenomenon in some cases is caused by the action of stress distribution at the tool-chip interface without temperature distribution in the flow region, and in this connection the tool-chip interface length depends remarkably upon the affinity characteristics between tool- and workpiece materials.

## 1. Introduction

Experimental researches have been made widely on machining with cutting tools from the end of the last century, and their scope has been expanded; as the result, the best cutting conditions, tool designs, machine tools, etc. have been determined which contributed for the enhancement of economic conditions in manufacture. On the other hand, theoretical researches have been made concerning fundamental cutting mechanism, tool wear, machinability, cutting fluid, etc. during these ten years or so. Among them the most fundamental orthogonal cutting mechanism of the simple continuous chip has been analysed pertaining to many factors affecting cutting phenomena, such as shear angle, friction coefficient, cutting force, cutting temperature, chip-curl, work-hardening, size effect, and so on, although the sufficient theory is not found yet.

In particular, many reports have been presented since Piispanen on the assumption that metal does not participate in strain-hardening and, in addition, on the consideration of the strain-hardening property of metal and the existence of weak points. The assumption, however, has been made in all of these theories that the formation of a simple continuous chip is a process of shear confined to a single plane extending from the cutting edge to the sharp chip- and work surfaces connection.

However, the authors reached the conclusion, by orthogonal dry cutting of soft metals, such as ductile lead and slightly brittle Lipowitz alloy in low velocity, that the transitional deformation of chip removal is not simply a process of shear on a single plane, but it is a process of slip and flow in a considerable wide range, and that the connecting line between the free surface of the workpiece and the back side of the chip is round instead of pointed. For the purpose of clarifying these points, soft metals with a side plane of view, on which square grids were described, were cut with tools of several shapes by specially designed cutting test apparatus, and the deformation of grid lines was observed during the cutting.

Further, taking this range into consideration, the figures of grid deformation are drawn and the equations of chip deformation are derived under some assumptions.

## 2. The Usual Theories and the New Concept on the Orthogonal Cutting Mechanism

### a. The Usual Theories

On the theoretical analysis of the orthogonal dry cutting mechanism in the case of a continuous chip, it is often assumed that the metal behaves like an ideal plastic body which can not be strain-hardened. The typical theories under this assumption were presented by Piispanen and Lee-Shaffer.

Piispanen<sup>1,2)\*</sup> analysed the cutting process through the card-model on the basis

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\* Numbers in parentheses refer to the Bibliography at the end of the paper.

of the principles of minimum energy and internal friction, assuming that slip occurs on the shear plane in which thin lamellae of metal slip one by one. After that similar considerations have been taken by Ernst-Merchant<sup>3)</sup> and Merchant<sup>4)</sup>.

Lee-Shaffer<sup>5)</sup> regarded the orthogonal cutting mechanism as the plane strain problem and, applying the principle of the perfect plastic flow to the chip between the shear plane and the tool face on the basis of Mises' theory, they analysed the cutting mechanism in the case of a continuous chip with or without built-up edge under the principle of the maximum shearing stress. The same problem has been developed in somewhat different manners by Hucks<sup>6)</sup>, Cook-Finnie-Shaw<sup>7)</sup> and Shaffer<sup>8)</sup>.

Under the above standpoint, the equations of the shear angle and cutting resistance have been derived and compared with the experimental results.

On the other hand, under the consideration of the work-hardening property of the metal, the orthogonal cutting process has been studied by Drucker<sup>9)</sup> and Lapsley-Grassi-Thomsen<sup>10)</sup>. Moreover, Shaw et al<sup>11)</sup> have developed the metal cutting theory, paying attention to the lattice fault of crystal, i.e. the anisotropy of metal and the existence of weak points.

The common viewpoint of the theories mentioned above, however, is the assumption made in them: the formation of a simple continuous chip is a process of shear or internal slip (no fracture) confined to a single plane, i.e. the "shear plane", extending from the cutting edge to the sharp chip-work surfaces conjunction as shown in Fig. 1. It should be noted that this single surface is regarded simply as a plane in almost all theories.

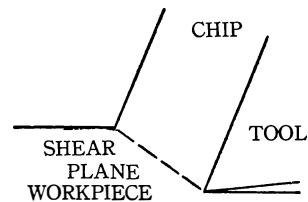


Fig. 1. The cutting process in the usual theories.

#### b. The New Concept

If the chip removal is a process of shear on the single plane which extends from the cutting edge to the sharp chip-workpiece surfaces conjunction, as is usually considered, it can be easily shown from the theoretical standpoint that strain and strain rate should be extremely large in comparison with those for testing material, especially when cutting with large negative rake angle tool, and in fact it is said that the experimental data for metal cutting and testing material do not coincide. Moreover the theoretical equations of the shear angle, which have been derived by many investigators under some assumptions in defiance of cutting speed, cutting heat, etc., do not strictly agree with the experimental results.

Aside from those assumptions, observing the process of chip removal with plastic flow of metal in detail, the

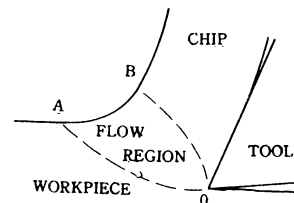


Fig. 2. The cutting process with the flow region.

authors recognized that, as is shown in Fig. 2, there exists always some transitional range AOB between the rigid region of the workpiece and the plastic region of the chip separating along the tool face in steady state, and that its free surface is round. This transitional range is called the "flow region" in this paper and is clearly observed when the soft metals of small strain-hardenable are cut in low speed, as is shown later in the experimental results.

The observation of a process of sharp wedge indentation into the plastic body shows that the projection at both sides of wedge does not occur with slip in a single plane, but with the pseudo-steady plastic flow. So does the cutting process, as the authors consider; that is, the shear deformation in large scale cannot occur suddenly on a single plane which is called the shear plane.

Thus the flow region mentioned above must be taken into consideration instead of the shear plane of former theories in the strict analysis of metal cutting process.

The above concept is different from the preflow region, the existence of which in the vicinity of free surface of the workpiece only is discussed by Finnie-Shaw<sup>12)</sup> as a cause of disagreement between shear angle estimated with theoretical equations and that observed with microscopic photographs, and also between data for metal cutting and those for testing material as to the stress-strain relation. The same concept is taken by Hoshi<sup>13)</sup>, in the case of a continuous chip with built-up edge; that is, the surface projection occurring before the shear process of chip removal is promoted with the indentation of built-up edge, and defined the preflow region co-existing with the deformation phenomenon of workpiece surface by the built-up edge.

In those cases they assume the shear process on the so-called conventional shear plane. The plastic flow, however, is not confined in the vicinity of the workpiece surface, but occurs at the surface OA further forward to that plane, and it ceases at the surface OB, as shown in Fig. 2. Hence chip removal should take place in this transitional range AOB. Under this concept of chip removal, the preflow region considered by Finnie-Shaw and Hoshi is contained in this flow region. Then the particular range of the so-called preflow region is not necessary to be considered. Furthermore, the shear angle and shear plane discussed in many cases become ambiguous and have no meaning but average value and position.

Nevertheless, in case the flow region is extremely narrow as a result of slip taking place sharply, for example with an increase of cutting velocity, this region tends to approach the so-called shear zone, and the above concept is consistent with usual theories.

### 3. Experimental Procedures

#### a. The Cutting Test Equipment

The cutting test equipment shown in Fig. 3 was specially designed for the purpose

of observing the orthogonal cutting process with the metallic microscope. The soft metals are cast into or installed to the workholder A, and its view-side is held on with a hard glass plate, through which the cutting process is observed. Then this is attached to the tool-holder B, and the orthogonal cutting tool C is fitted with some depth of cut. The apparatus thus equipped is installed on the metallic microscope. Then, cutting the soft metals in low velocity by managing the handle D with hand, the authors can observe the cutting process continuously through the ocular of the microscope and take microphotographs of cutting process sometimes.

Since a hard glass plate supports the cutting edge and the view-side of the workpiece on one plane, as shown in Fig. 4, the side-flow of chip and machined surface can not occur during the cutting; therefore, the authors consider that the nearly complete orthogonal cutting process can be observed clearly.

#### b. The Workpiece

The soft metals used as workpiece materials are ductile lead and slightly brittle Lipowitz alloy. Both are low melting metals and are easily cast into the workholder.

Lead [Pb] is of the twin structure and has the face-centred cubic lattice belonging to the isometric system, and is the very ductile blue-white metal of low melting point. Because of its low recrystallization temperature, it has no work-hardening property at the time of cutting process.

Lipowitz alloy is a kind of fusible alloy, the melting point of which is very low.

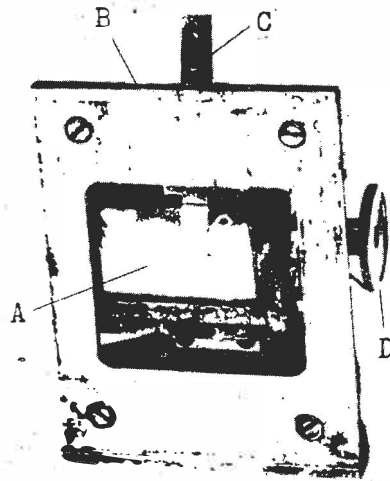


Fig. 3. The cutting test equipment.

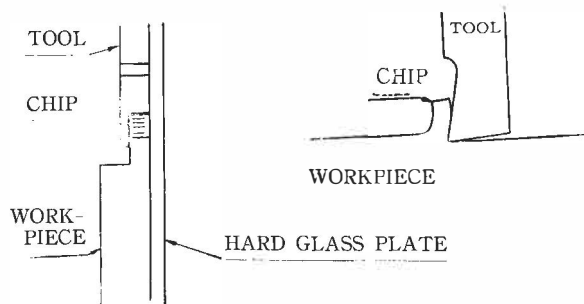


Fig. 4. The cutting state by the cutting test equipment.

It is soft, silver-white, and is composed of 50.0% of bismuth [Bi], 26.7% of lead [Pb], 13.3% of tin [Sn], and 10.0% of cadmium [Cd].

The physical and mechanical properties of these soft metals are shown in Table 1.

Table 1. The Properties of Lead and Lipowitz Alloy.

Material	Lead	Lipowitz Alloy
Melting Point ( $^{\circ}\text{C}$ )	327	65
Specific Gravity	11.4	7.9
Tensile Strength ( $\text{kg}/\text{mm}^2$ )	2	5
Hardness (Vickers)	6	140

On the view-side observed through a hard glass plate, square grids of 0.2 mm in a side-length are drawn to observe the cutting process. The authors observed the deformation of these square grids during the cutting. The carved seal of these square grids is made as shallow and accurate as possible.

#### c. The Tool

When soft metals are cut in low speed, no cutting heat is generated and no tool wear is recognized. Hence, in order to make various shapes easily, the carbon tool steel is chosen as the tool material. Fig. 5 shows the shapes of cutting edge used in the experiment (the names of the shapes are given in the figure in this paper).

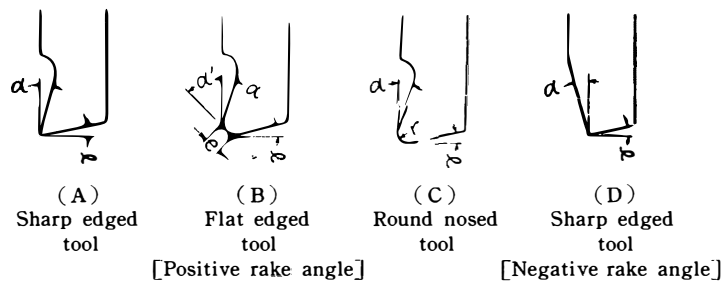


Fig. 5. Shapes of cutting edge.

#### d. The Consideration of Cutting Conditions

The weakest point in the above cutting test equipment is the fact that the cutting velocity is very low, that is, 0.01–0.03 m/min. Due to this fact, however, generation of the cutting heat is prevented which is one of the most troublesome factors affecting the cutting state. Therefore, since the effect of the cutting temperature can be neglected, it will be very simple to analyse the cutting mechanism theoretically from data obtained by the experiment.

It is clear from numerous observations of the cutting process that the cutting state scarcely varies even if the speed of rotation of the handle for cutting becomes either

a little faster or slower than the normal speed, and it appears also that the cutting state is not disturbed at all even if the handle for cutting is brought to a sudden stop for microphotography.

#### **4. Experimental Results**

The orthogonal dry cutting of soft metals in low speed with the cutting test equipment mentioned above led to the following results:

##### **a. The Cutting of Lead**

The initial cutting states of lead with tools of various shapes are shown in Fig. 6. In any of these cases when the tool touched and began to cut lead, the chip did not instantly start to flow out along the tool face, instead the free surface of the metal swelled gradually while the state of statical friction between the metal and tool face continued. Then at last, the chip began to flow out step by step along the tool face, and finally the steady state of cutting was reached. And the position at which the preflow started on the free surface was always fixed at somewhat constant position far before the so-called shear plane. The fact that, with the flat edged tool, the portion of metal above the horizontal grid line through cutting edge deforms larger than that below it is due to the effect of the free surface to the upper side.

After the steady cutting states had been reached, the continuous chip without built-up edge was produced as ordinarily, and the appearances of cutting processes were divided into the following two kinds:

- (1) A process of shear on an apparent single plane
- (2) A process of flow in a region

The cutting process progressing through shear on an apparent single plane is shown in Fig. 7. Observing this cutting state in detail, however, the so-called preflow was also recognized and the free surface of the metal already swelled a little far before the conventional shear plane. Therefore, this state can be regarded simply as a special case of very narrow flow region and, consequently, there exists the plastic flow before the conventional shear plane.

The cutting process progressing through flow in a region is shown in Fig. 8. It is evident from the grid deformation in the vicinity of cutting edge that there exists some transitional range, i.e. the flow region, between the rigid region of the workpiece and the plastic region of the steady chip separating along the tool face. The flow region associated with slip and flow varied in shape and size even on the same cutting condition. For instance, in the case of the cutting states with the sharp edged tool, the radii of curvature, with which the grid lines formerly parallel to the uncut surface become round in the flow region, grow usually larger as they move from the cutting edge to the free surface as A-1, but sometimes smaller as A-2. The latter case



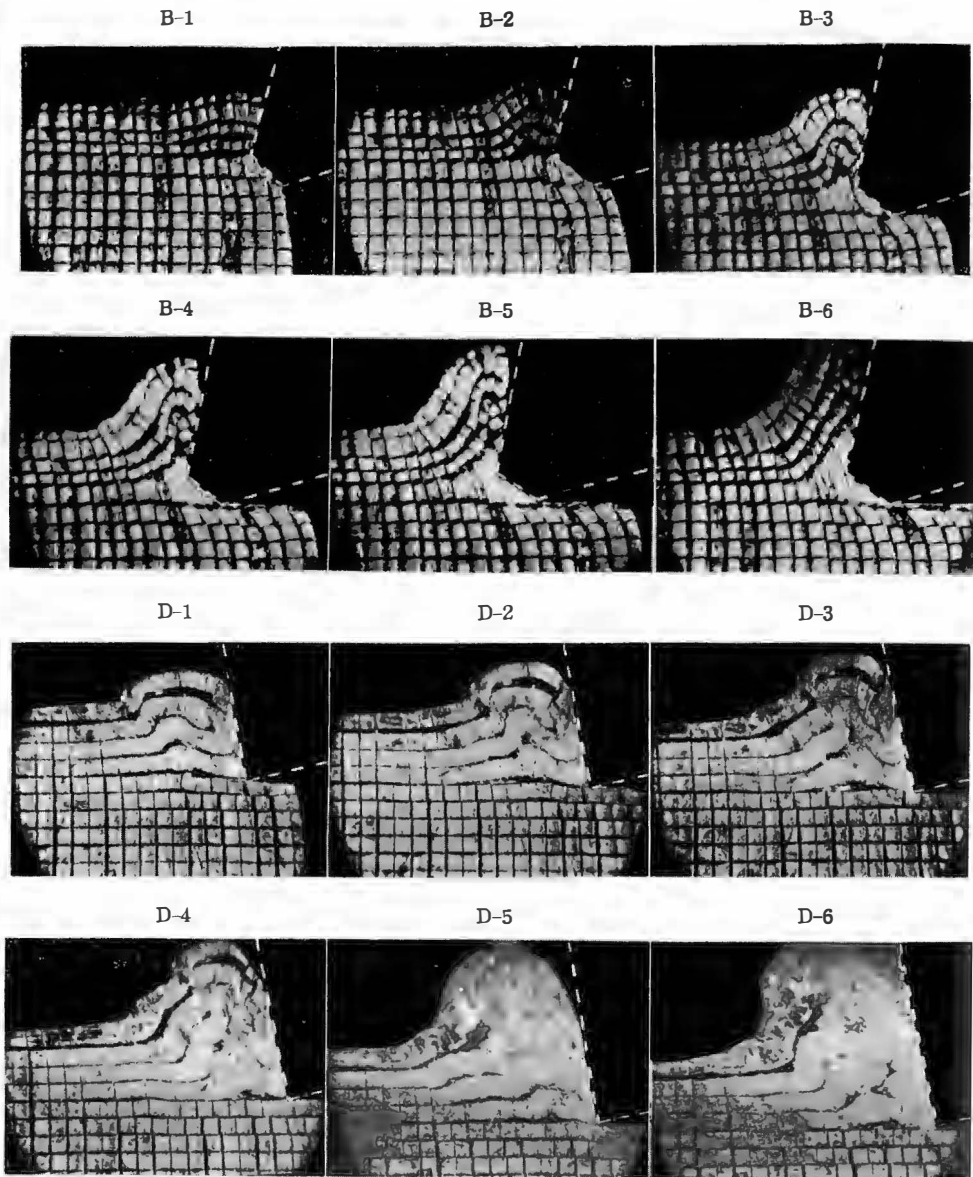


Fig. 6. Cutting of lead.  
I. The initial cutting state.

( $\times 12$ )

A : Positive rake sharp edged tool  
 B : Positive rake flat edged tool  
 C : Positive rake round nosed tool  
 D : Negative rake sharp edged tool  
 for Figs. 6 to 22.

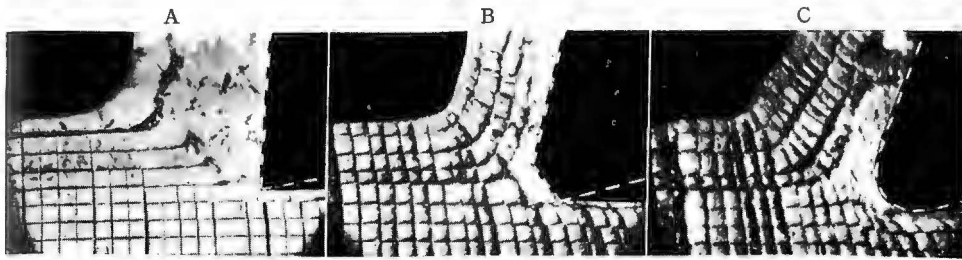


Fig. 7. Cutting of lead.

( $\times 12$ )

II. Cutting appearance of shear on an apparent single plane.

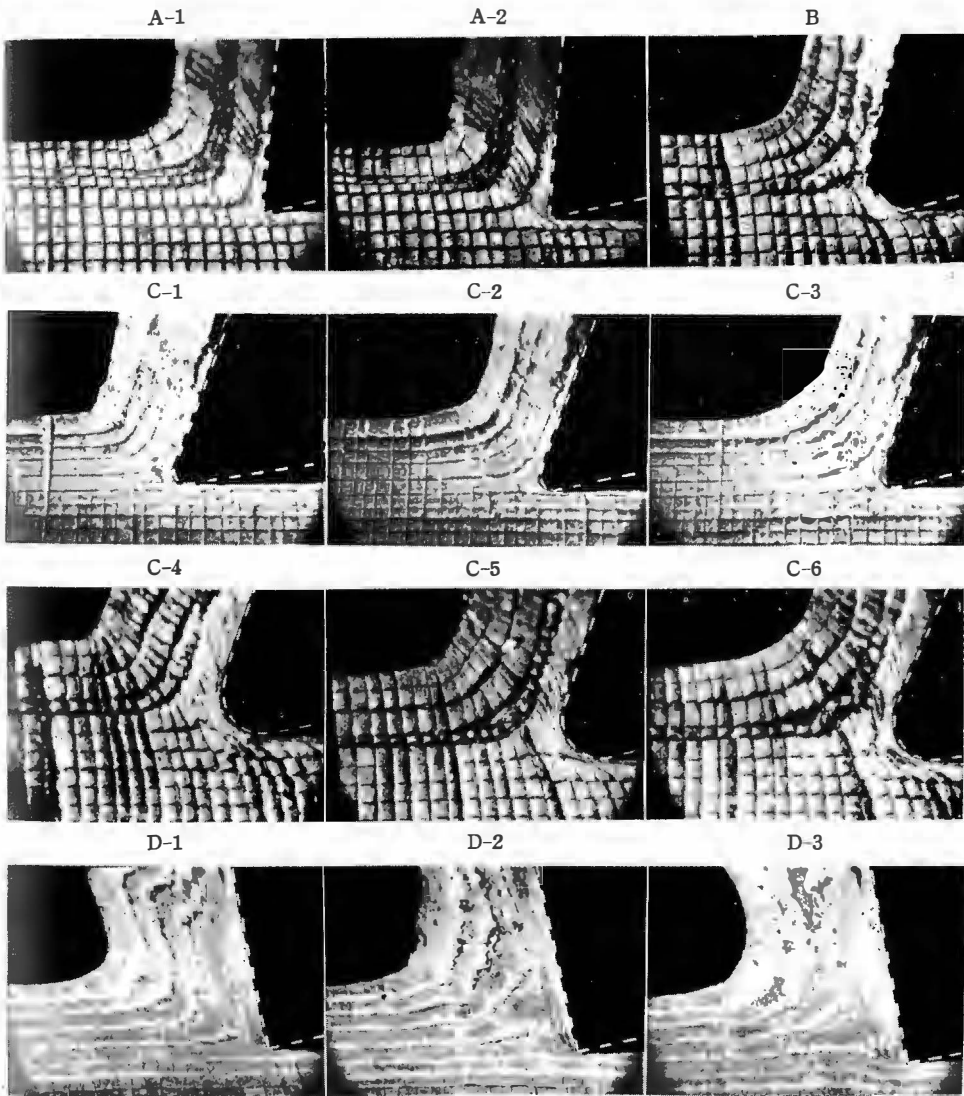


Fig. 8. Cutting of lead.

( $\times 12$ )

III. Cutting appearance of flow in a region.

was always associated with a pile of metal ahead of the cutting edge. With the flat edged tool, the radii of curvature of the grid lines in the flow region are large even in the vicinity of the cutting edge, differing from the cutting state with the sharp edged tool, and become much larger as they move from the cutting edge farther to the free surface. With the round nosed tool, also, there exists an ideal flow region, especially the state shown in C-3 is just like the stream lines of water. Furthermore, the same is true with the negative rake angle tool, and in this case the significantly large strain should be associated if cut by a process of shear confined to a single plane. From this point, it is reasonable to assume the existence of the flow region.

Sometimes the shape and size of the flow region change periodically even under the same cutting condition, as shown in Fig. 9. For instance, with the sharp edged tool, the flow region became gradually narrow as from A-1 to A-3. And at last the cutting state A-4 was reached, where the chip removal seemed to be of a process of shear in a somewhat single plane owing to disappearance of the flow region, but already at that instant the preflow was produced on the free surface of the metal far before the cutting edge, and at the next instant the cutting state reached the state A-1. And this cutting process continuously repeated itself after that.

The chip ordinarily is continuous as mentioned above because of the ductility of lead. But the chip was sometimes associated with a crack as in Fig. 10 when cut with a negative rake angle tool. The crack was always convex upwards and its degree was large at the side of the free surface and became small or disappeared at the side of the cutting edge. The instant the crack occurred, the authors in all cases recognized the existence of plastic deformation to some degree in the workpiece, especially near the uncut surface. The occurrence of such a crack in lead cutting was uncommon and thereafter the cutting state changed gradually again to the usual steady process of a simple continuous chip.

Now, from the general survey of the above cutting process, the chip contact length on the tool face is short and the chip curls after it passes this contact area.

The grid line formerly parallel to the uncut surface just under the cutting edge remained in a somewhat curved state downwards from the front of the edge to the lower part of the machined surface. Particularly with the round nosed tool, the grid line abruptly swelled in a mountain-shape just before the cutting edge, as shown in Fig. 8 C-4 to 6.

Furthermore, it is noticeable from the general survey of the cutting process with the flat edged and round nosed tools that the square grids just before and below the cutting edge are deformed much slenderly. Fig. 11 is an example of the cutting state with a round nosed tool. C-2 is a state where the tool was drawn back from the state C-1. It was recognized that the built-up edge did not exist at all. In the case

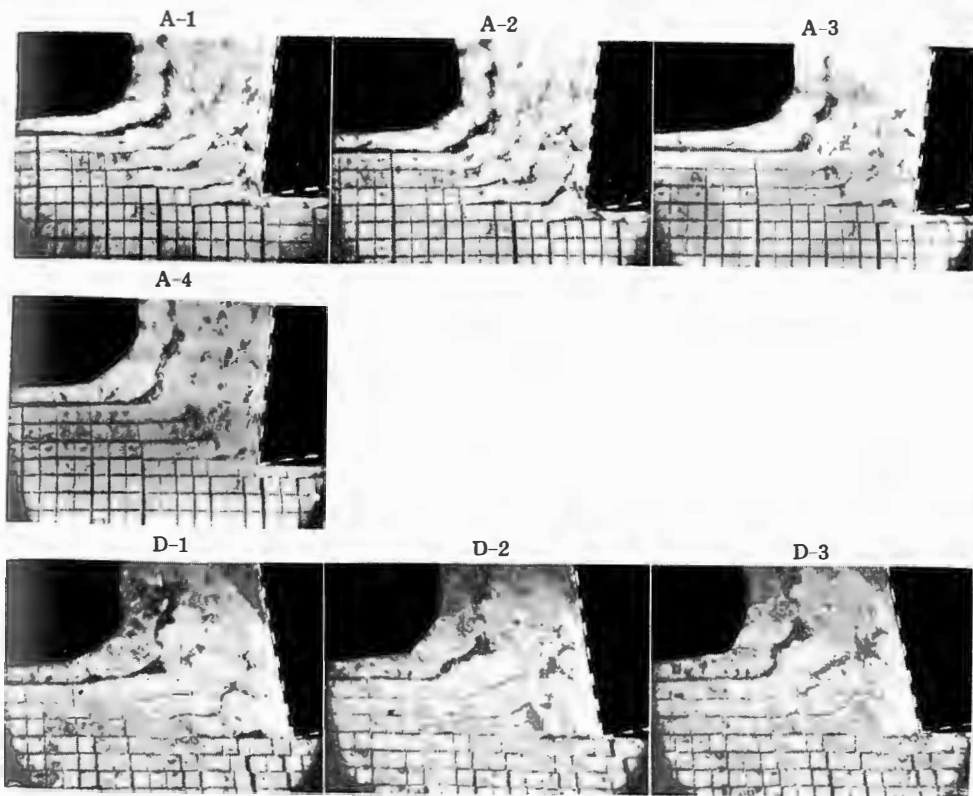


Fig. 9. Cutting of lead. (×12)  
IV. Periodic change of the flow region.

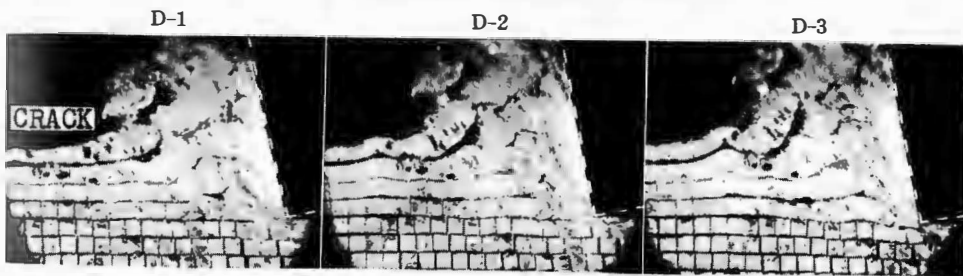


Fig. 10. Cutting of lead. (×12)  
V. Occurrence of crack.

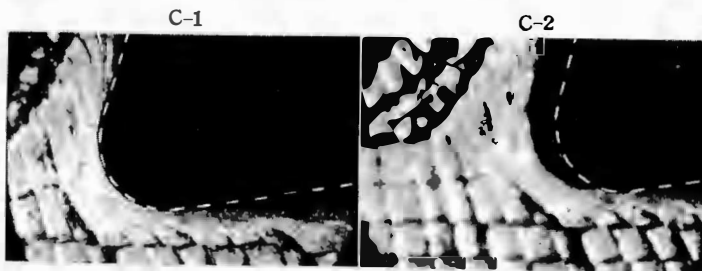


Fig. 11. Cutting of lead. (×24)  
VI. Slender deformation of square lattice in the vicinity of cutting edge.

of the large radius of the cutting edge, square grids in the vicinity of the nose deform greatly because of the large rolling action by the nose.

The machined surfaces are shown in Fig. 12. In any of these cases the grid lines formerly perpendicular to the uncut surface bent in large scale, and nearer to the machined surface, crystal grains are shifted largely in the cutting direction. The degree of bend of the grid line depends on the tool shape; that is, larger and deeper in the case of a negative rake than in the case of a positive rake, and with dull cutting edge than with sharp cutting edge.

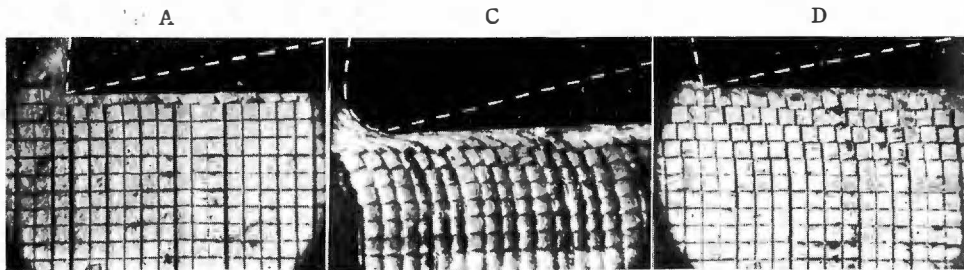


Fig. 12. Cutting of lead.  
VII. The machined surface.

( $\times 12$ )

Further, the grid lines formerly perpendicular to the uncut surface remained in chip bending convex upwards mostly, and especially with dull cutting edges the degree of bend of those was significant on the side of tool face.

Such large bend of grid lines in the machined surface and chip produced with dull cutting edge seems to occur as a result that square grids in the vicinity of cutting edge deform largely, hence crystal grains shift in large scale to the cutting direction on the machined surface and remain inseparable along the tool face.

#### b. The Cutting of Lipowitz Alloy

The initial cutting states of Lipowitz alloy with tools of various shapes are shown in Fig. 13. With the positive rake angle tool, differing from the cutting process of lead, the chip started to flow out along the tool face as soon as the tool began to cut and alloy was touched by the tool face, and soon the steady state was reached. So, the swelling-up of the uncut surface was not recognized. With the negative rake angle tool, on the other hand, the cutting process has a strong tendency to make a discontinuous chip from the beginning. After the workpiece completely touched the tool face as D-1, shear took place comparatively sharply as D-2 and a crack occurred as D-3. Already at this instant, the plastic flow had occurred in certain portion of the workpiece, and became gradually wide as D-4, then the chip cracked again as D-5, and the cutting state D-6 was reached. By repeating these processes, cracks are created gradually and the cutting is performed.

The steady cutting state being reached, a continuous type of chip without built-up edge was produced in many cases, but sometimes a discontinuous one was also produced.

The cutting processes associated with a simple continuous chip are the same as those of lead and are divided into two kinds: a process of shear on an apparent single plane and a process of flow in a region.

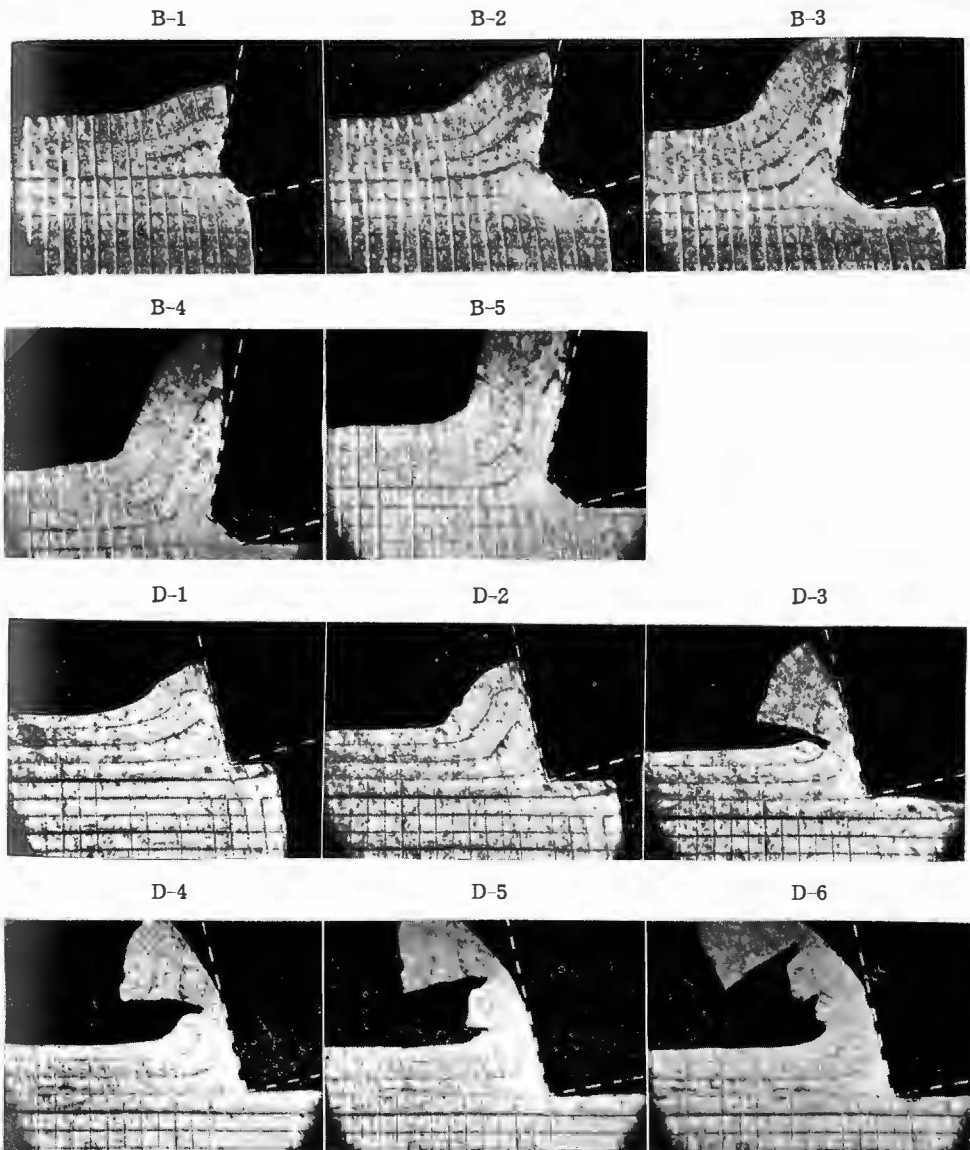


Fig. 13. Cutting of Lipowitz alloy.  
I. The initial cutting state.

( $\times 12$ )

The cutting process progressing through shear on an apparent single plane is shown in Fig. 14. Being observed in detail in this case also, it was revealed that the uncut surface had already swelled a little with the plastic flow far before the conventional shear plane. In other words, this state is a special case of very narrow flow region.

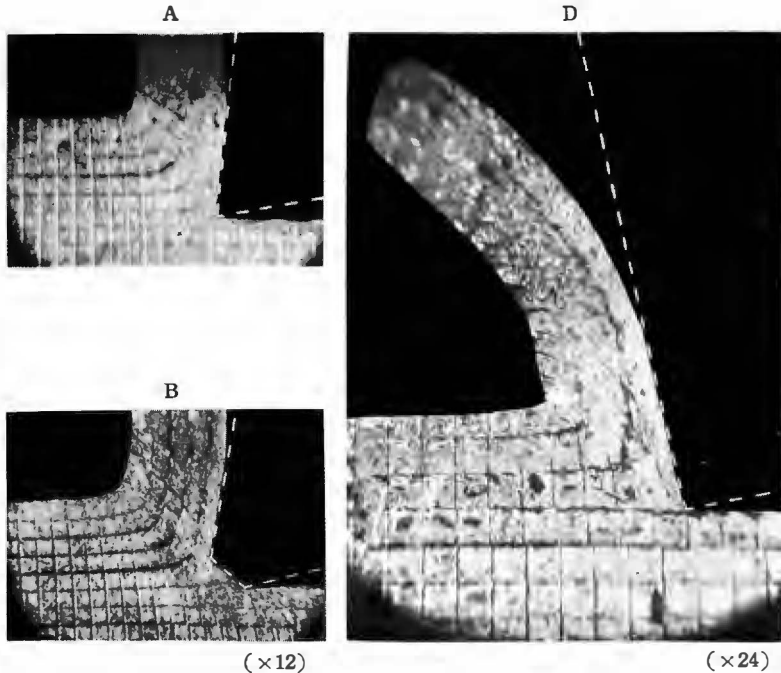


Fig. 14. Cutting of Lipowitz alloy.  
II. Cutting appearance of shear on an apparent single plane.

The cutting process progressing through flow in a region is recognized in many cases and shown in Fig. 15. The existence of the flow region in this case is evident, the same as in the case of lead cutting. This region varied in shape and size. The radii of curvature, with which the grid lines formerly parallel to the uncut surface become round in this region, grow usually larger as they move from the cutting edge to the free surface, but sometimes smaller or constant. The latter case was always associated with a pile of metal ahead of the cutting edge. Especially with the flat edged tool, the photographs show the appearance of them just like the stream lines of water, and the oblique part below the flat edge is exactly the same as stagnant water and the grids at this part deform much slenderly. In B-3 & 4, the change of colour is recognized in the portion from the workpiece to the chip and machined surface. This range is regarded as a region associated with the plastic flow, and the boundary-line of this range is consistent with the locus connecting points where the grid lines

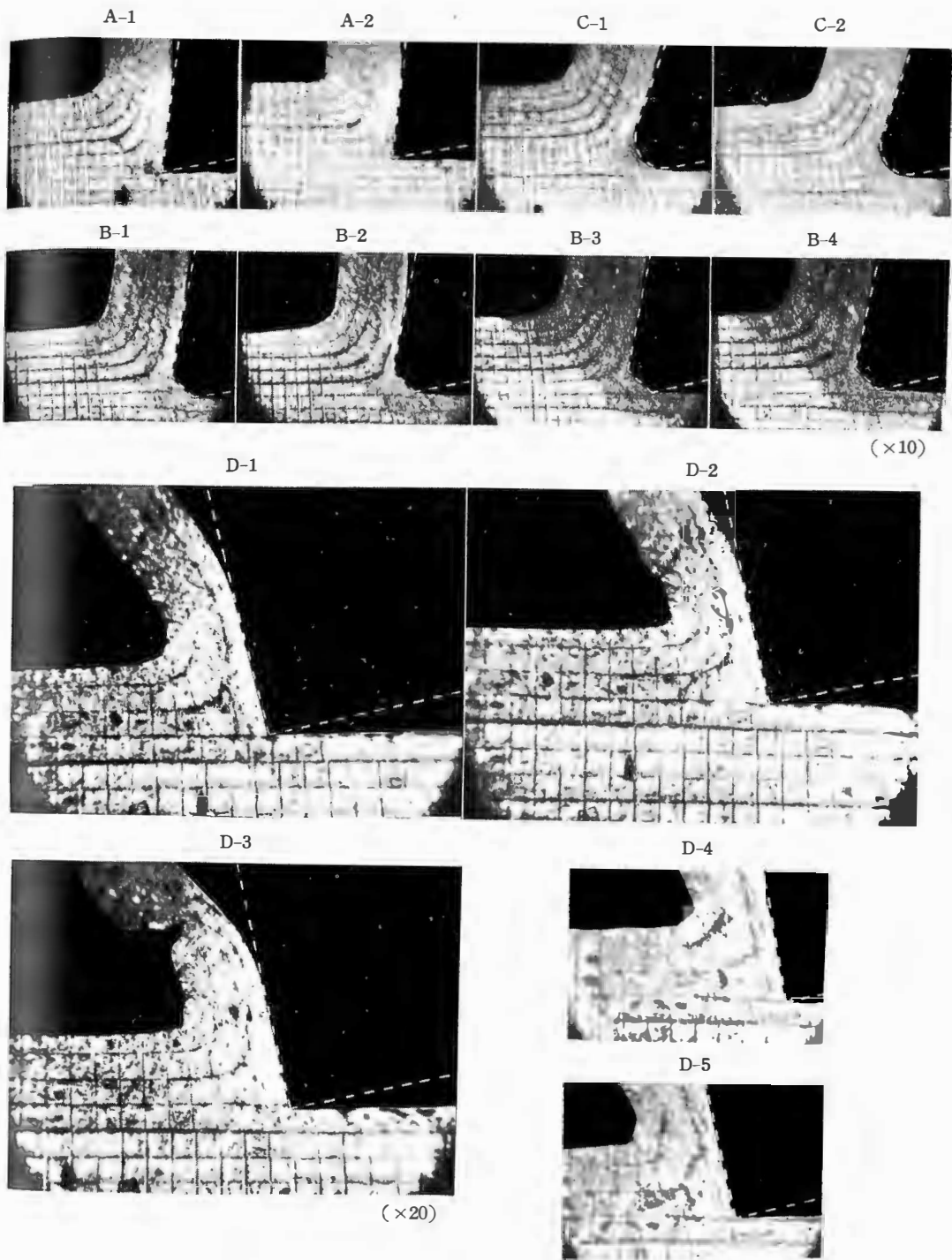


Fig. 15. Cutting of Lipowitz alloy.  
III. Cutting appearance of flow in a region.



parallel to the uncut surface began to bend. Moreover, the same ideal flow region also existed with the negative rake angle tool as D-1 to 3. The states D-4 & 5 are remarkable in comparison with the others; that is, the shape of flow region is a triangle also as shown in Fig. 16. This will be produced with large slip only at both sides of boundaries of the triangular flow region, within which the slip does not occur and the grid lines become straight. This triangular flow region is regarded as a special case in which the radii of curvature are infinitive. Being associated with such a region,

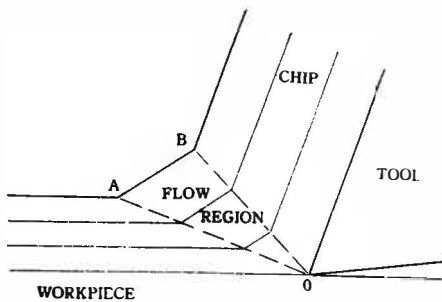


Fig. 16. The flow region of triangular shape.

the cutting proceeded from D-4 to D-5, and as the next stage, the cutting state changed to the process of shear on an apparent single plane as shown in Fig. 14 D, or to the process of flow in a region as shown in Fig. 15 D-1 to 3.

The size and shape of the flow region depend on cutting conditions, tool shapes, work materials, etc. Among them it seems to be the cutting velocity that has the most important bearing upon the size of the region. Fig. 15 A-1 is the cutting state in very low speed, while Fig. 15 A-2 is that in somewhat higher speed. It is clear from the comparison of them that the higher the cutting velocity the narrower the flow region becomes. This may be a result of the localization of flow in the flow region with an increase of cutting velocity as discussed later.

Now, from the general survey of the above cutting process, the chip contact length on the tool face is shorter than in the case of lead cutting and the chip begins to curl after separating from this contact area as shown in Fig. 17. The degree of chip-curl is larger than that in the case of lead. In most cases the grid lines formerly perpendicular to the uncut surface remained convex upwards and bent largely or disappeared at the side of the tool face, so the friction that occurs between the chip and the tool face seems to be large.

Lipowitz alloy is slightly brittle in comparison with lead and, consequently, the cutting was often associated with fractures or cracks. This tendency was stronger in the case of dull and negative raked tools and at high speed. Fig. 18 shows processes of discontinuous chip formation. With the flat edged and round nosed tools, once a crack suddenly occurred in a chip as B-2 & C-2 during the cutting which was associated with a continuous chip as B-1 & C-1, then a discontinuous chip was produced afterwards as shown in the following processes: B-3 to 4 & C-3. In any case, the instant a crack occurred, there existed already the plastic flow in the workpiece and the boundary-line of the flow was ahead of the crack. Furthermore, during

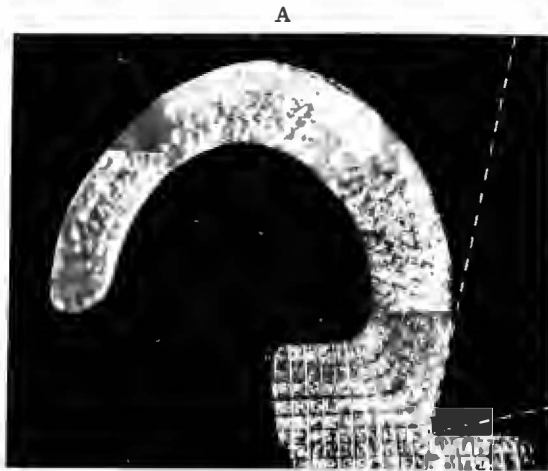


Fig. 17. Cutting of Lipowitz alloy. ( $\times 10$ )  
IV. The bend of chip.

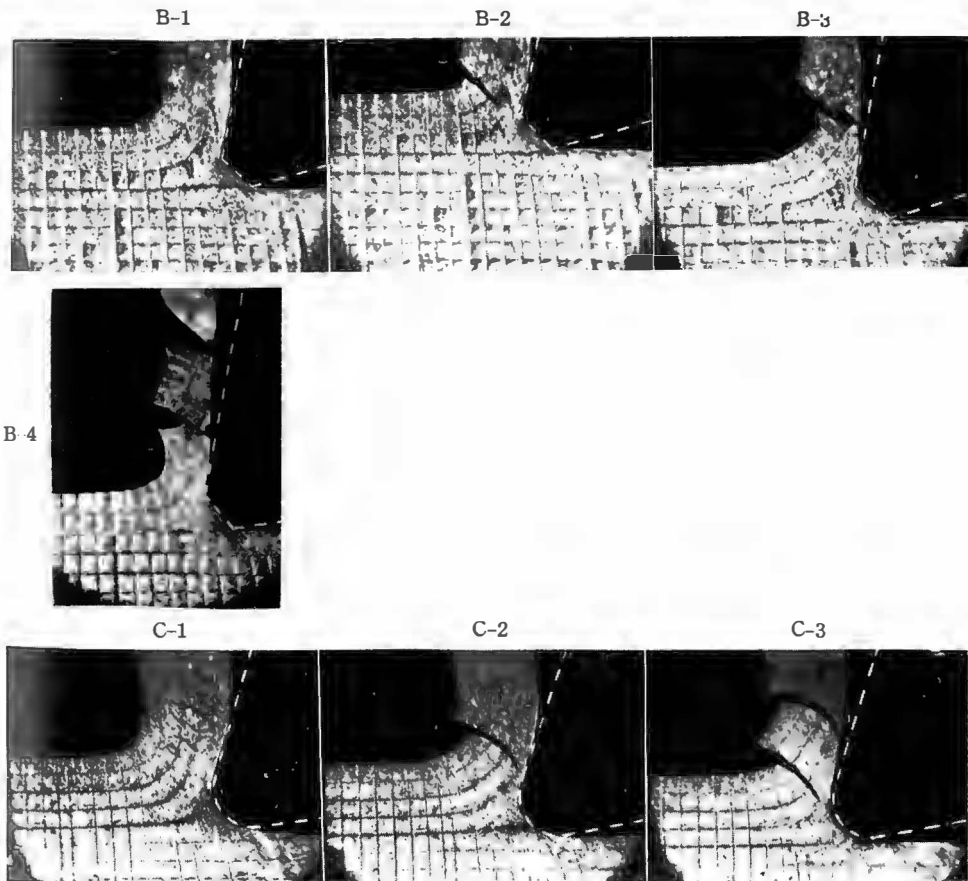


Fig. 18. Cutting of Lipowitz alloy. ( $\times 12$ )  
V. Discontinuous chip formation.

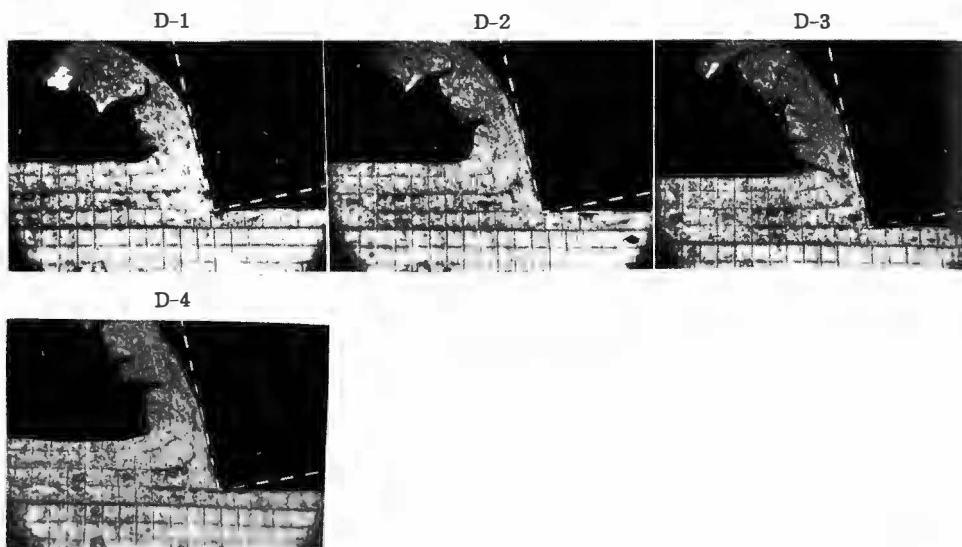


Fig. 19. Cutting of Lipowitz alloy. (×12)  
VI. Quasi-continuous chip formation.

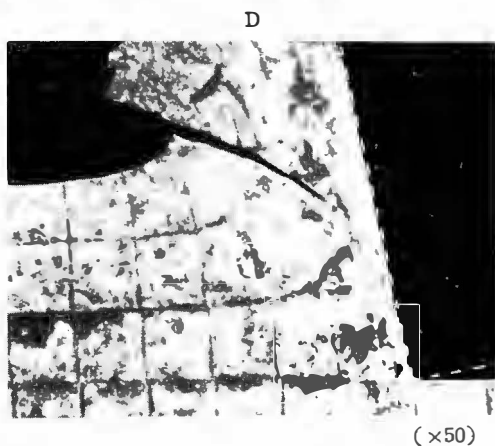


Fig. 20. Cutting of Lipowitz alloy. (×50)  
VII. Occurrence of crack.

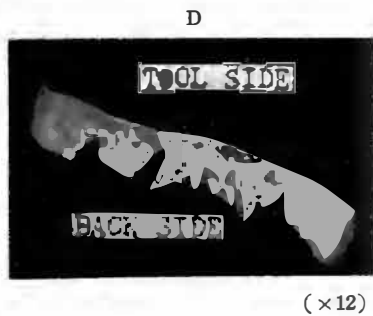


Fig. 21. Cutting of Lipowitz alloy. (×12)  
VIII. Quasi-continuous chip.

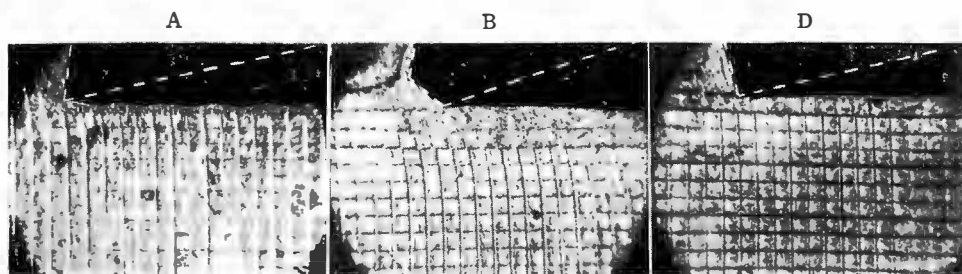


Fig. 22. Cutting of Lipowitz alloy. (×12)  
IX. The machined surface.

formation of one chip fragment, the flow region was observed as clearly as in the case of a continuous chip.

The cutting process with the negative rake angle tool has a strong tendency to accompany a crack in the chip, the process of which was observed in detail in Fig. 19. From the state D-1, where the flow occurred ahead of the crack, the cutting proceeded into the state D-2 which was associated with a narrow flow region, then in a little while the crack was produced as D-3. At this instant a part of the workpiece was already associated with the flow, and the state D-4 was reached, that is, the state D-1 was again reached. This course of formation of the chip fragment was repeated after that. The amount of crack is large at the back side of the chip, but the crack disappears midway to the tool face as shown in Fig. 20, so the chip fragments are connected at the side of tool face as shown in Fig. 21. This type of chip, therefore, cannot be strictly regarded as a discontinuous type, but should be called the "quasi-continuous type of chip".

Fractures or cracks are convex upwards in most cases as observed from the general survey of the above cutting states and, consequently, the slip can never occur on a plane. The amount of the slip is larger at the back side of the chip than at the separating side of it. Moreover, it is clear that fractures or cracks do not occur on the so-called shear plane, but that they occur on some surface in the flow region where it is the easiest to slip.

It is worthy of note that, differing from the general concept, the tendency to generate a discontinuous type of chip becomes strong with an increase of cutting velocity.

In the cutting states with dull tools, i.e. the flat edged and round nosed tools, the square grids just before and below the cutting edge deformed much slenderly just as in the case of lead cutting. This phenomenon seems to be somewhat related to the rolling action of the dull edge.

The machined surfaces are shown in Fig. 22. The grid lines formerly perpendicular to the uncut surface bent to the cutting direction just as in the case of lead. The degree of the bend depends on the sharpness of tool rather than the rake angle; that is, it is larger when cut with dull tools.

## **5. Discussions on the Cutting Mechanism for Soft Metals**

The following summary is made from the experimental results of the orthogonal dry cutting of soft metals in low velocity.

### **a. The Flow Region**

It appears from bending aspects of the grid lines ahead of the cutting edge that, as described before, the metal cutting is associated in most cases with slip and flow

in some region rather than with shear confined to a single plane. This region AOB shown in Fig. 2 exists between the rigid region of the workpiece and the plastic region of the steady chip, and the chip removal takes place through the plastic flow in the "flow region"—so called in this paper.

The shape and size of the flow region vary with cutting conditions, work materials, tool shapes, and so on. The typical shapes of the flow region are shown in Fig. 23. A shape (a) is the most ideal one, but the radius  $R_s$  of its free surface is in most cases different even on the same cutting condition as shown in Fig. 8 C-1 to 3 & C-4 to 6 and Fig. 15 D-1 to 3. The grid lines formerly parallel to the uncut surface are round in the flow region and their radii of curvature grow usually

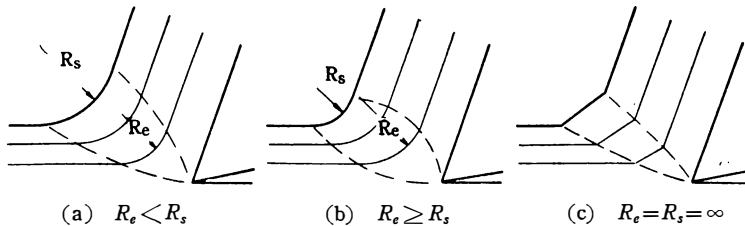


Fig. 23. Three shapes of the flow region.

larger from the cutting edge to the free surface as (a), but sometimes smaller or constant as (b). The latter case is always associated with a pile of metal. This part of pile is not a built-up edge, because there is no sticking on the tool face when cutting operation is stopped and the tool is drawn back as shown in Fig. 11. Such a pile, which is just like stagnant water in stream lines, is generally recognized during the cutting process with the tool of dull edges, but not with the tool of sharp edge.

In an extraordinary case, the radius of curvature at the free surface of the flow region is infinitive and the grid lines in the flow region are straight as shown in Fig. 23 (c), for example, Fig. 15 D-4 & 5. In this case, the flow region is triangle-shaped and the chip removal takes place smoothly as is evident from the analysis of the grid deformation discussed later.

Furthermore, observed in detail, the contour of the free surface of the flow region is not necessarily constant (circular), and changes periodically, for example (a)—(b)—(c)—(d)—(a)—(b)—... in Fig. 24, as often observed in the experiment (see Fig. 9). At a glance a state (a) looks like a process of slip in a single plane, but in reality that plane is only a conventional shear plane and the flow exists far ahead of that plane. As cutting proceeds, the point A, where the flow begins, scarcely changes its position, while the point B, at which the flow ceases, rises up gradually as a state (b), and then a state (c), associated with a typical shape of the flow region, is reached. Furthermore, B moves downwards and the flow region becomes very narrow as (d),

but never tends to be a single plane. Meanwhile the flow begins to occur afresh ahead of the cutting edge, and a state (a) is reached again. And this course of change is repeated from time to time. Such a periodical change of the shape of the flow region seems to have something to do with the tool chatter based upon self-excited vibration of

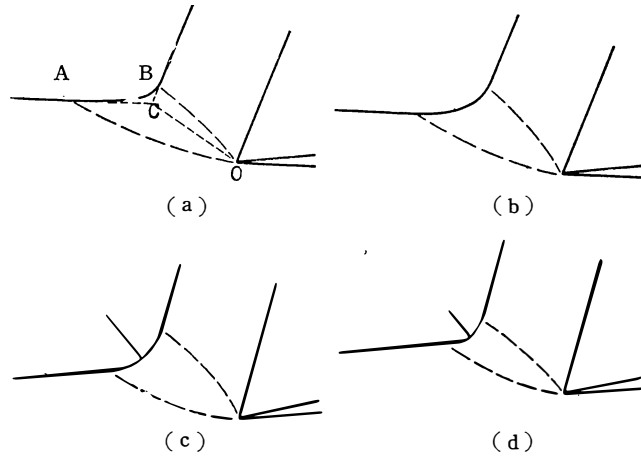


Fig. 24. Periodic change of the flow region.

tool, and to refer to the fact that, as pointed by Hill<sup>14)</sup>, in the case of a single shear plane, infinitely many steady states are possible instead of a unique steady state on the basis of a theorem on the maximum intensity of singularities in a material.

It is rarely observed that the steady cutting state associated with the ideal flow region (c) or (d) is always maintained without the existence of states (a) & (b).

Among many factors affecting the size of the flow region in the case of a continuous chip, the cutting velocity seems to have the most important influence. The higher the cutting velocity, the narrower the flow region becomes and the smaller the radius of curvature at the free surface becomes as mentioned already in the experimental results. From this fact, in the high speed range used in the ordinary machining, the flow will be localized in a very narrow region. This localization of flow with an increase of cutting velocity seems to be reasonable when considered from a study made on the speed effect by Zener-Hollomon<sup>15)</sup> that, with an increase of shear velocity, deformation occurs in a narrow region because of the localization of shear; and it is also rational when referred to the fact that Trigger-Chao<sup>16)</sup> stated that an increase in cutting speed would generally reduce plastic strain in the shear zone and result in the higher strain rate.

Furthermore, even in the case where the workpiece material is hardened by strain, the existence of the flow region seems to be theoretically valid in the view-point of mechanics; that is, the finite shear deformation in large scale can not occur suddenly<sup>17)</sup>.

From the facts mentioned above for theoretical analysis of metal cutting, it will be necessary to consider the existence of a flow region instead of a shear plane. Former theories are only a special case in which the flow region becomes very narrow and is supposed to tend to be a single plane because of the effects of cutting speed, etc.

### b. The Machined Surface

In the usual theoretical analysis of orthogonal cutting mechanism, it is generally considered that the work required to separate the chip from the metal (producing two new surfaces), as Merchant<sup>4)</sup> considered, can be negligible in comparison with the total work, and hence the part of work below the cutting edge does not participate in cutting; consequently, the grids deformation under machined surface, surface roughness of machined metal and work-hardened layer are not taken into consideration except in the experimental survey. Yet, Thomsen-Lapsley-Grassi<sup>18)</sup> pointed out that the work of deformation absorbed by the workpiece may be a major portion of the total cutting work when the chips are small, but that for large depth of cut the energy required to deform workpiece appears to become less significant and neglect of workpiece deformation may not introduce so much error in analysing forces in metal cutting.

From experimental results of the orthogonal cutting of soft metals, the machined surface is associated with roughness in the cutting direction and even the part of the workpiece below the cutting edge is affected by the cutting action, so the square grids deformation occurs in such a way that the grid lines formerly perpendicular to the uncut surface bend largely near the machined surface and the crystal grains shift in large scale as shown in Figs. 12 & 22.

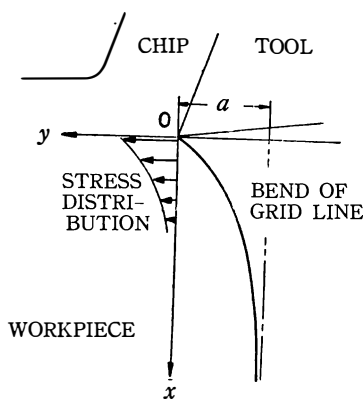


Fig. 25. The bend of grid line under machined surface.

The general survey of the photomicrographs of the machined surfaces reveals that the bending state of the grid line formerly perpendicular to the uncut surface under the machined surface is properly regarded as a somewhat exponential function as follows: supposing shifting coordinates with origin O as cutting edge,  $y$ -axis in cutting direction and  $x$ -axis perpendicular to  $y$ -axis downwards (see Fig. 25):

$$y = a(e^{-kx} - 1); \quad a, k > 0, \quad \text{for } x \geq 0,$$

where  $a$  and  $k$  are constants, depending on cutting conditions. The duller the cutting edge, the larger the  $a$  seems to be.

It is inferred also that the stress acting from the cutting edge to the machined surface distributes decreasing itself exponentially as it moves downwards under the cutting edge.

The energy required to produce new surfaces is not small enough to be negligible on the theoretical analysis of cutting mechanism.

It is recognized from the experimental results in the case of the dull edged tool that the grid lines formerly parallel to the uncut surface under the cutting edge

remained bending a little downwards from the front of the cutting edge to the part below it, and especially in the case of the round nosed tool a grid line just below the cutting edge swelled in a mountain-like shape in front of the edge. This phenomenon seems to have some connection with the side effect and the bending aspect of the grid lines formerly perpendicular to the uncut surface under the machined surface.

#### c. The Deformation and Bend of Chip

It is evident from the experimental results that the grid lines formerly perpendicular to the uncut surface become straight in a chip in some case but bend convex upwards in most cases and the bend is sudden especially at the tool side. This exhibits the extraordinary frictional effect on tool face, and whether a chip flows out along the tool face smoothly or not depends upon the shape of tool. In connection with this fact, the tool-chip interface length depends not only on the cutting conditions, tool shapes, etc., but especially on work- and tool materials. In the case of cutting of soft metals by carbon tool steel, the tool-chip interface length is smaller in the case of Lipowitz alloy than in the case of lead. It is evident from this that a factor determining the area of the tool-chip interface is the affinity characteristics between the tool- and work materials.

Now a chip produced through the flow region is in contact with the tool face for a while, then, after separating from this tool-chip interface, a chip begins to bend. Such a chip-curl is always recognized in the present experiment in spite of its very low cutting velocity. Hence the chip-curl phenomenon appears to be a result of only the action of stress distribution at the tool-chip interface, as presented by Henriksen<sup>19)</sup>, even if there exists no temperature distribution in the shear region as Hahn<sup>20)</sup> proposed.

#### d. The Formation of Discontinuous Type of Chip

As regards the theoretical analysis of discontinuous chip formation, there are some studies: a study based upon the card-model in the case of continuous chip by Piispanen<sup>2)</sup>, a derivation of plastic conditions as to a discontinuous chip fragment from the minimum energy theorem by Field-Merchant<sup>21)</sup>, and a study considering the slip-lines at the initial stage of discontinuous chip formation based upon Mises' theory by Lee<sup>22)</sup>. Further, Cook-Finnie-Shaw<sup>23)</sup> investigated the fundamental process of discontinuous chip formation experimentally and recognized that a chip fragment is produced through the periodic extrusion process into a parabolic shape associated with a rolling-down action of the chip along the tool face, instead of through the slip associated with a simple shear.

It is supposed in the above investigations that a crack or fracture occurs on the so-called shear plane where the shearing stress is the maximum as well as in the case of a continuous chip formation.



Now, during the cutting process of soft metals, in case a discontinuous type of chip is produced as often observed (see Fig. 18), the process of formation of one chip fragment was almost the same as that of the continuous chip formation offered by Field-Merchant, and not the periodic extrusion process discussed by Cook-Finnie-Shaw.

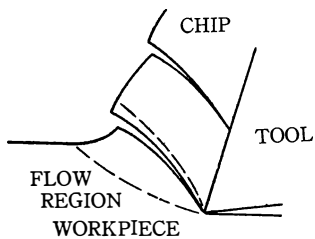


Fig. 26. Discontinuous chip formation.

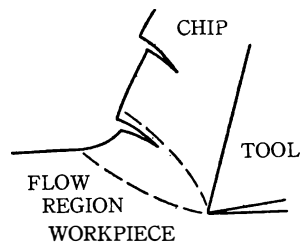


Fig. 27. Quasi-continuous chip formation.

Observing the process of formation of a chip fragment, the authors recognized the flow region as well as the continuous chip formation, and a fracture or crack occurs on some single surface in the flow region as shown in Fig. 26. As soon as a fracture or crack occurs, a part of workpiece before the fracture or crack is associated with plastic deformation of some degree. Then, flowing out along the tool face, this part widens the flow region, and after some cutting a fracture or crack occurs again on some surface in the flow region. Hence, the continual decrease of shear angle during the formation of one chip fragment, as presented in the usual theories, was not recognized at all.

Sometimes the authors recognized periodical cracks which, as shown in Fig. 27 (for example, Fig. 19), occurred on the back side of the chip but disappeared halfway without reaching the separating side of the chip. A chip with such periodical cracks on the back side runs in a line at the separating side as presented already in Fig. 21. This type of chip can never be called a complete discontinuous type because of incomplete fragments, nor a continuous type because of very large periodical cracks. Let this type of chip be called the "quasi-continuous chip" in this paper. Due to the difference of side where these cracks produce, this chip is quite different from the partially discontinuous chip discussed by Cook-Finnie-Shaw, which occurs with periodical cracks (shear type fracture) disappearing halfway at the separating side of the chip.

Observing the fact that the slip at the fracture between the workpiece and the chip fragment in the case of the discontinuous chip is large on the back side and that periodical cracks in the case of the quasi-continuous chip occur partially on the back side, the authors are led to the conclusion that the shear strain in cutting of soft metals becomes greater from the cutting edge toward the free surface.

Moreover, a fracture or crack does not occur on the so-called shear plane as is discussed in the usual theories, but it seems to occur on some surface in the flow region where it is the easiest to slip. This surface lies generally near the ending

boundary-line in the flow region and its shape in the majority of cases is convex upwards. A large slip on this surface takes place periodically by the break of energy balance and the chip fragments flow out in succession. Further detailed study is necessary to find out why a fracture or crack occurs convex upwards, in spite of the fact that it is easier to occur straightly from the cutting edge to the free surface.

Differing from the general concept, it is worthy of note that soft metals have a tendency to change the type of chip from continuous to discontinuous with an increase in cutting velocity.

6. Grid Deformation

Since it became clear from the experimental results and discussions mentioned above that there exists a flow region, though its size and shape vary, between the rigid region of the workpiece and the plastic region of the steady chip in the cutting process of soft metals, the authors try to analyse grid deformation geometrically and to derive the equation of chip deformation in the cutting process containing the flow region.

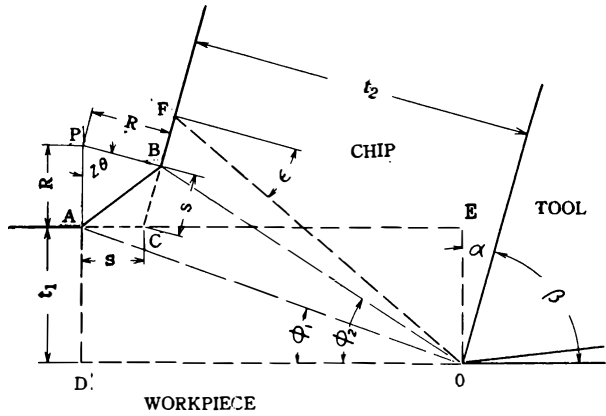


Fig. 28. Analysis of grid deformation. I. In the case of the flow region with a straight line free surface.

The following assumptions are made:

- (1) The steady chip slides uniformly along the tool face.
- (2) Grid deformation under the machined surface is neglected.
- (3) Concerning the flow region, both boundary-lines at the sides of workpiece and chip are straight lines, and the free surface is a straight line (see Fig. 28) or a circular arc (see Fig. 29).

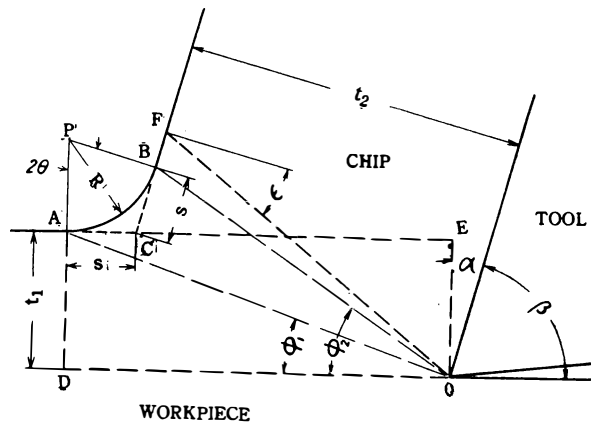


Fig. 29. Analysis of grid deformation. II. In the case of the flow region with a circular arc free surface.

The so-called (conventional) shear plane in former theories is represented by OC, and the flow region as AOB, as shown in Figs. 28 & 29. Fig. 28 shows a case in which the free surface of the flow region is straight and, consequently, the flow region is of triangular shape; and, for the sake of simplicity, a case of  $AC=BC$  ( $\equiv s$ ) which is considered to happen most easily will be discussed. Fig. 29 shows a case of the flow region associated with free surface of a circular arc. In both cases  $\phi_1$  and  $\phi_2$  are angles between cutting direction and boundary-lines OA and OB of flow region at work- and chip sides respectively.

Then, cutting with a depth of cut  $t_1$  by orthogonal cutting tool (rake angle  $\alpha$  and cutting angle  $\beta$ ), a part ADOE of the workpiece before the cutting deforms ADOFB during the cutting. Assumed that there is no change in density of the metal in the case of plastic deformation even though the thickness of the chip becomes  $t_2$ , the position of the point F can be determined and the angle  $\epsilon$  shown in Figs. 28 & 29 can be evaluated by the principle of the constancy of volume.

Putting a point at which a perpendicular line set on the tool face at a point B and the extended line of DA cross one another as P, it is denoted that

$$AP = BP = R, \quad \angle APB = 2\theta.$$

In case that the free surface of the flow region is a circular arc, R is its radius and it satisfies  $AC=BC$  ( $\equiv s$ ).

According to the principle of the constancy of volume,

$$\square ODAE = \triangle OAD + \triangle OAC + \triangle OBC + \triangle OBF + \triangle ABC \quad \text{or} \quad \sphericalangle ABC,$$

$$\text{i.e.} \quad \triangle OCE = \triangle OBC + \triangle OBF + \triangle ABC \quad \text{or} \quad \sphericalangle ABC. \quad (1)$$

Calculating the area of each term,

$$\left. \begin{aligned} \triangle OCE &= \frac{1}{2} t_1 (t_1 \cos \phi_1 - R \tan \theta) \\ \triangle OBC &= \frac{1}{2} R t_2 \tan \theta \\ \triangle OBF &= \frac{1}{2} t_2^2 \{ \tan \epsilon - \tan (\phi_2 - \alpha) \} \\ \triangle ABC &= R^2 \sin^2 \theta \tan \theta \\ \sphericalangle ABC &= 2\triangle PAC - \sphericalangle PAB = R^2 (\tan \theta - \theta) \end{aligned} \right\} \quad (2)$$

On the other hand, it is clear that

$$\left. \begin{aligned} R \tan \theta &= s \\ \theta &= \frac{\pi}{4} - \frac{\alpha}{2} = \frac{\beta}{2}. \end{aligned} \right\} \quad (3)$$

These expressions (2) & (3) substituted in Eq. (1), and defined as

$$\left. \begin{aligned} r &= \frac{t_1}{t_2} \quad (\text{cutting ratio}) \\ f &= \frac{s}{t_2} \quad (\text{ratio relative to the size of the flow region}), \end{aligned} \right\} \quad (4)$$

then, the following equations of chip deformation can be derived:

I) if the free surface of the flow region is a straight line (Fig. 28),

$$\tan \epsilon = \tan(\phi_2 - \alpha) + r^2 \cot \phi_1 - f \cdot (r+1) - f^2 \cos \alpha \quad (5)$$

II) if the free surface of the flow region is a circular arc (Fig. 29),

$$\tan \epsilon = \tan(\phi_2 - \alpha) + r^2 \cot \phi_1 - f \cdot (r+1) - 2f^2 \cot \frac{\beta}{2} \left(1 - \frac{\beta}{2} \cot \frac{\beta}{2}\right) \quad (6)$$

where  $\epsilon$ : angle between a perpendicular line to the tool face and the grid lines in the chip which were formerly perpendicular to the uncut surface.

$\phi_1$ : angle between cutting direction and starting boundary-line of the flow region.

$\phi_2$ : angle between cutting direction and ending boundary-line of the flow region.

$\alpha$ : rake angle.

$\beta$ : cutting angle.

$r$ : cutting ratio.

$f$ : ratio relative to the size of the flow region.

Then Fig. 30 shows some examples of figures of the grid deformation in the case of the cutting state which is associated with a simple continuous chip in which the triangular flow region with a straight free surface is taken into consideration. In this case, the slip (no fracture) occurs at the two positions of starting- and ending boundary-lines of the flow region, and such figure can be easily drawn geometrically whatever the value of  $s$  at the same cutting condition is chosen.

Extending this method of construction, figures of the grid deformation in the case of the flow region which is associated with a polygonal free surface can be drawn also as shown in Fig. 31.

On the other hand, in case the flow region is associated with a circular arc free surface, an example of figure of the grid deformation is shown in Fig. 32. In this case, the slip occurs in all range of the flow region.

Now, in order to inquire Eqs. (5) & (6) of the chip deformation derived above, if the flow region is replaced with a single plane, the cutting is a process of shear confined to a single plane, i.e. the shear plane, as in the usual theories, and therefore,

$$s = 0: f = 0,$$

$$\phi_1 = \phi_2 \quad (\equiv \phi, \text{ say}),$$

and

$$r = \frac{t_1}{t_2} = \frac{\sin \phi}{\cos(\phi - \alpha)}.$$

Substituting these values, Eqs. (5) & (6) both coincide to the equation of the chip deformation:

$$\tan \epsilon = \tan(\phi - \alpha) + \frac{\sin 2\phi}{2 \cos^2(\phi - \alpha)}, \quad (7)$$

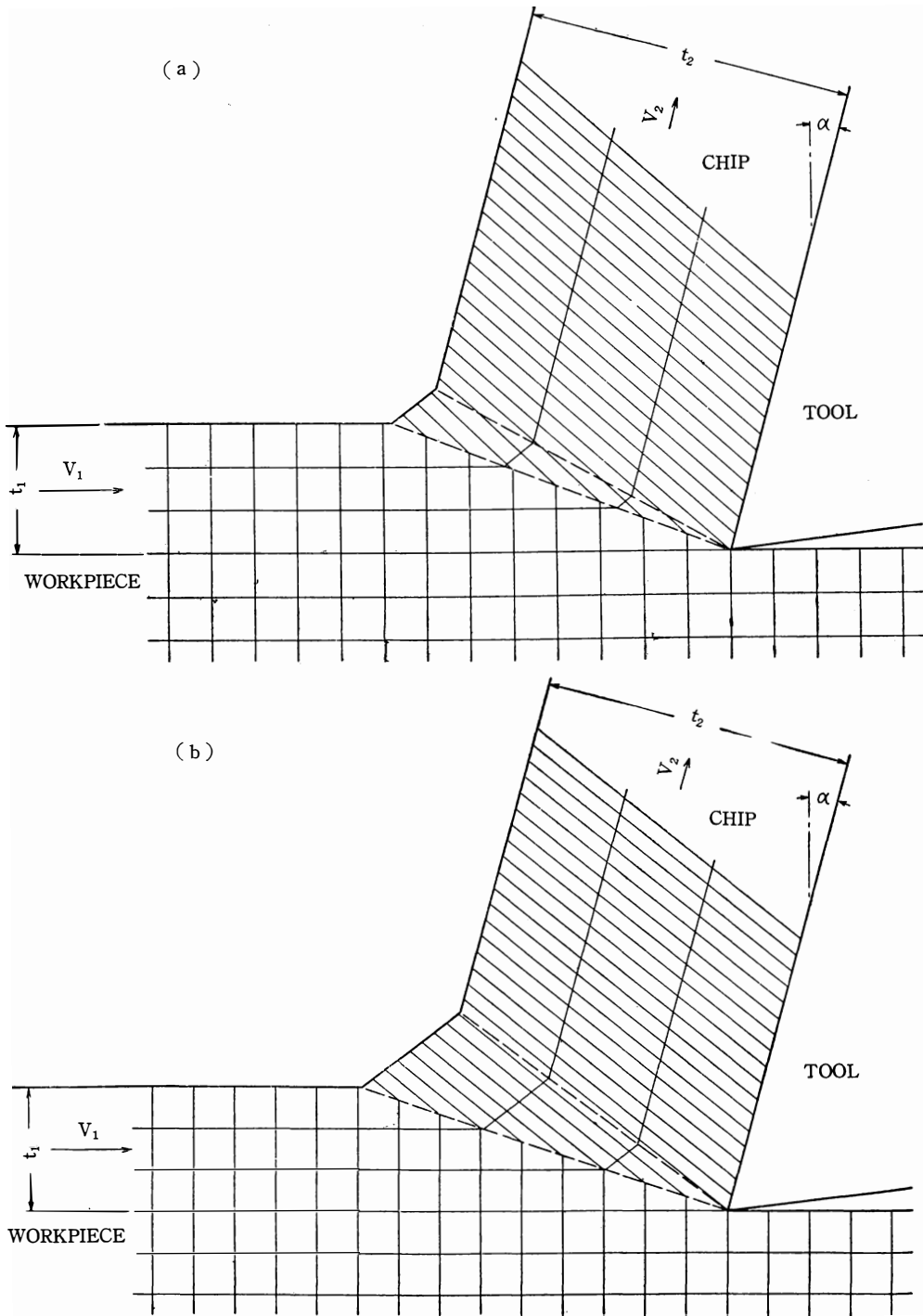


Fig. 30. Figures of grid deformation.

I. In the case of the flow region with a straight line free surface.

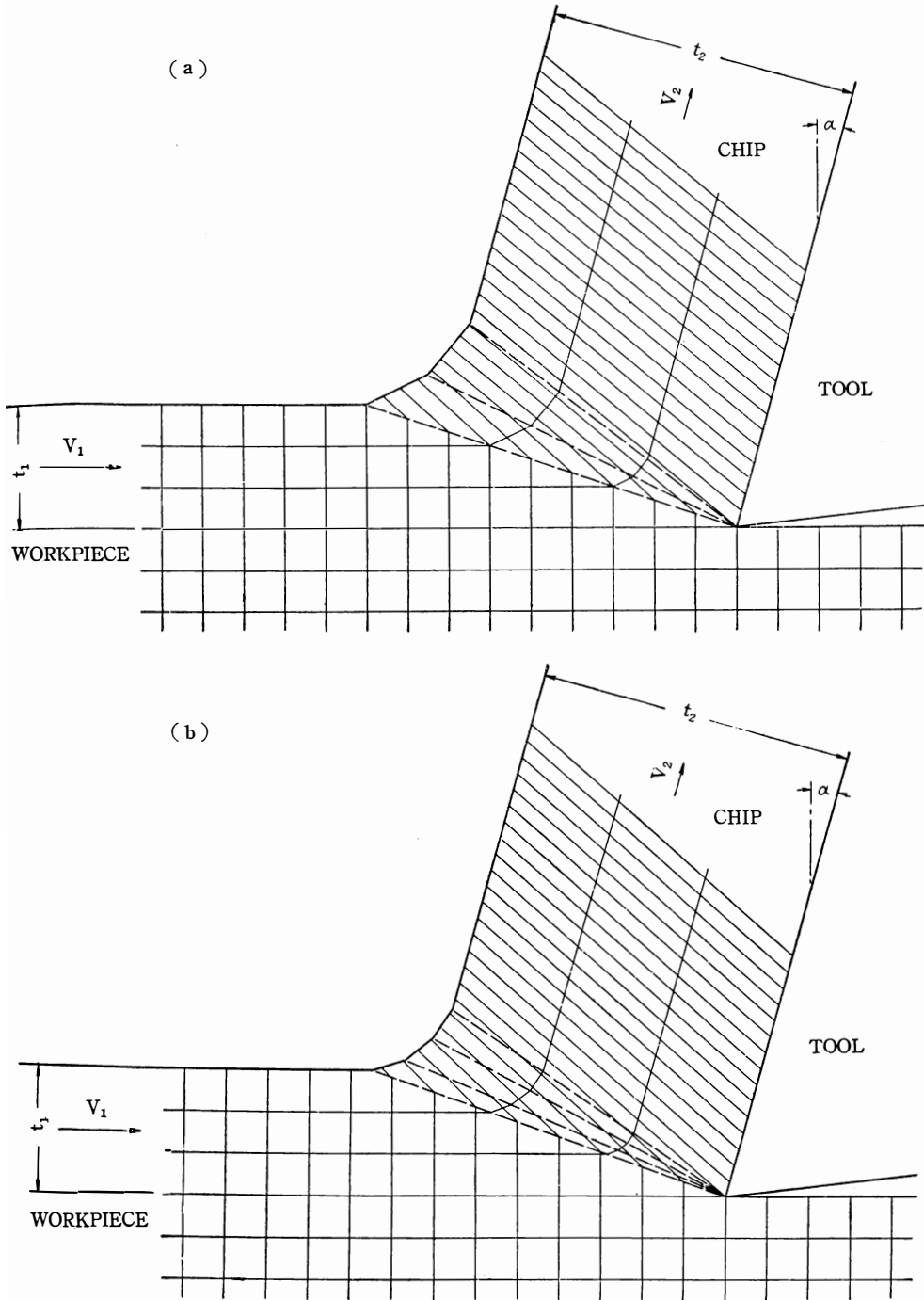


Fig. 31. Figures of grid deformation.

II. In the case of the flow region with a polygonal free surface.

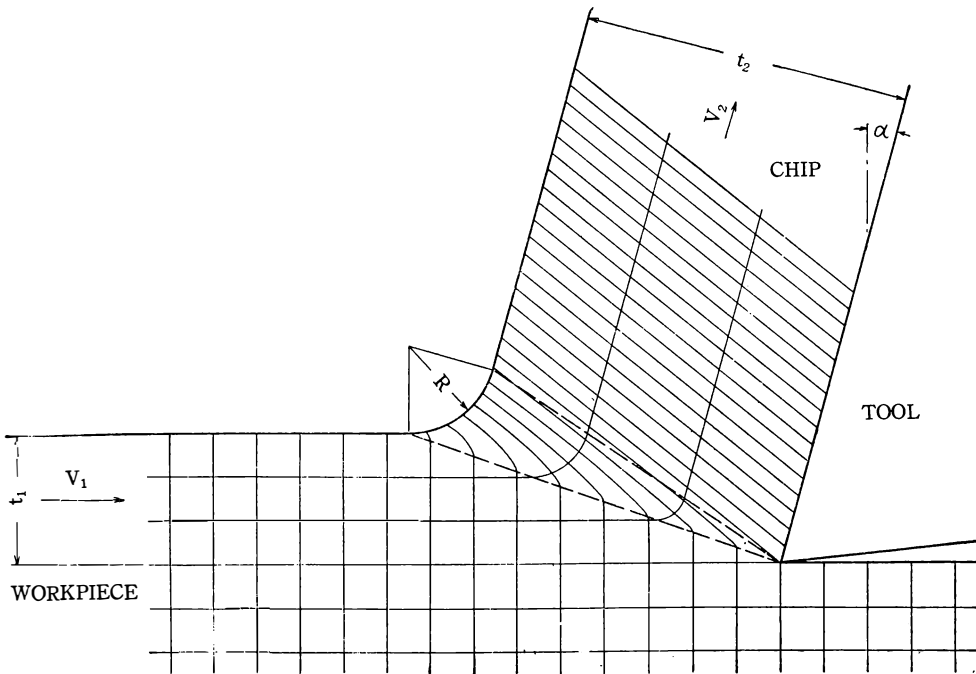


Fig. 32. A figure of grid deformation  
III. In the case of the flow region with a circular arc free surface.

which is reduced in the case of the usual orthogonal cutting with a simple continuous chip by Shaffer<sup>8)</sup>, who analysed the machining process as a study of the plane strain problem in which a chip is formed by the shearing action of a non-work-hardening material, proved that shear must occur along a straight line in order to satisfy continuity requirements, and drew the figure for the deformation of square grids.

The above justifies to consider that the expressions of the chip deformation given by Eqs. (5) & (6) are the general equations which are extended—a special case of which agrees with the expression of the chip deformation for the usually considered cutting process.

In reality, it is clear from the experimental results of dry cutting of soft metals in low velocity that the boundary-lines and a free surface contour of the flow region are curves rather than straight lines, so the assumptions made above do not strictly hold, and the flow region has various shapes, which makes it very difficult to derive the equation of the chip deformation and to draw the figure of square grid deformation.

## 7. Conclusions

Orthogonal dry cutting of soft metals such as lead and Lipowitz alloy in low velocity leads to the following conclusions:

(1) There exists the "flow region" between the rigid region of the workpiece and the plastic region of the steady chip when the cutting is associated with a simple continuous chip; that is to say, it is necessary to consider the flow process in a region rather than the shear process confined to a single plane in the cutting process. Under consideration of this region the connecting line between the uncut surface of the workpiece and the back side of the chip is round instead of sharply bent.

(2) The flow region associated with a simple continuous chip formation is various in shape and size according to cutting conditions, work materials, and so on. Sometimes it changes periodically during the cutting process even under the same cutting condition.

The cutting velocity is one of the most important factors affecting the size of the flow region. The higher the cutting velocity, the narrower the flow region becomes because of the localization of the flow.

(3) In the case of a discontinuous chip, the existence of the flow region is recognized during formation of one chip fragment as well as a continuous chip formation. The same is true with the quasi-continuous chip (so called in the present paper) which is associated with periodical cracks occurring on the back side of the chip and disappearing halfway without reaching the separating side of the chip.

Then, a fracture or crack appears to occur near the ending boundary-line in the flow region, and is convex upwards in shape. Its shear strain is larger at the free surface than at the tool side.

(4) The square grid deformation under the machined surface cannot be neglected; that is, the grid lines formerly perpendicular to the uncut surface bend somewhat exponentially, and the crystal grains near the machined surface shift in large scale, which forms the deformed layer. This tendency is greater in case the dull edged tool is employed.

Hence, the stress acting from the cutting edge to the machined surface will distribute decreasing itself exponentially downwards under the cutting edge.

(5) The tool-chip interface length depends upon the cutting conditions and the affinity characteristics between the tool- and work materials, and the friction effect on this interface is great.

(6) The chip-curl phenomenon appears to be a result of the action of only the stress distribution at the tool-chip interface, even if there exists no temperature distribution in the flow region.

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