Studies on the Mixing of High Viscous Liquids

NAGATA, Shinji; YANAGIMOTO, Masamichi; YOKOYAMA, Tōhei

NAGATA, Shinji ...[et al]. Studies on the Mixing of High Viscous Liquids. Memoirs of the Faculty of Engineering, Kyoto University 1957, 18(4): 444-460

Studies on the Mixing of High Viscous Liquids

By

Shinji NAGATA, Masamichi YANAGIMOTO and Tōhei YOKOYAMA

Department of Chemical Engineering

(Received July 19, 1956)

1. Introduction

Several interesting results were reported on the mixing of homogeneous liquid by H. Kramers et al.[1] and by J. G. van de Vusse[2]. These studies are concerned mainly with the mixing of low viscous liquids. As the mixing of low viscous liquids would be accomplished easily, there are few difficulties in this kind of mixing except in the case where larger volumes of tanks are concerned.

Whereas in the case of high viscous liquids, neither rapid nor homogeneous mixing can be attained easily. Especially by the use of inadequate impellers, considerable lack of uniformity takes place. Such an inequality can hardly be removed either by the increase in impeller speed or by the elongation of mixing time.

In this report, the authors deal mainly with liquid of high viscosity and intend to clarify the effective types of mixing impellers and their proper sizes to the mixing vessel. Correlations are also proposed between several factors (e.g. impeller speed, vessel diameter, viscosity etc.) and mixing speed.

2. Experimental procedures

Millet-jelly is used as mixing liquid adjusting its viscosity by adding some water. Iodine and potassium iodide solution (260 g I_2/l) is prepared and is added to the sample in such proportion as indicated in Table 1. Thus the viscous iodine solution

Table 1. Vessels Used.

Vessel dia. (cm)	Content of vessels (c.c.)	I_2 Solution added (c.c.)	$Na_2S_2O_3$ Solution added (c.c.)	Materials of construction
10 (†)(*)	800	0.25	3	Poly vinyl chloride
15 (†)	2,500	0.75	9	Glass
20 (†)	6,280	2.0	24	Glass
30 (†)(*)	21,200	6.5	78	Poly vinyl chloride

(†) Vessels with flat bottom, (*) Vessels with dished bottom.

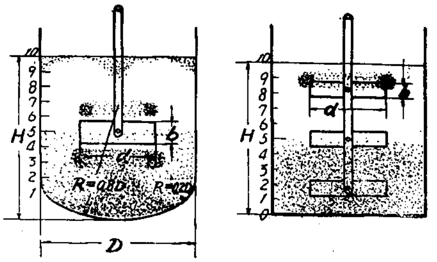
is obtained. On the other hand, sodium thiosulphate is dissolved in the millet-jelly as viscous as the mixing liquid. Its concentration is equal to 60 g $Na_2S_2O_3 \cdot 5H_2O/l$.

The sample solution containing iodine is agitated to a steady state and then the sodium thiosulphate solution corresponding to 1.4 times equivalent is poured along the impeller shaft within 3 to 5 seconds using an injector. The time required for complete mixing or the time when the reduction of iodine is over, is measured and denoted as θ. The ratios of reducing liquid to be injected are shown in Table 1.

With an inadequate impeller, a dead corner or a ring form like a doughnut comes out and is not mixed with the other parts of the container for a long time (Refer to Fig. 1). Therefore the time when the larger part of liquid is faded and decolorized, is taken as the time required for mixing.

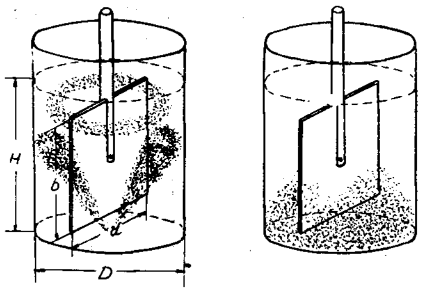
3. Experimental conditions

In this study, two sorts of vessels are used, one is a cylindrical vessel with a flat bottom as shown in Table 1, and Fig. 1 and 2, while the other is that with a dished bottom. The liquid depth, H, was fixed to be equal to the vessel diameter, i.e., $H=D$.



(a) Cylindrical vessel with dished bottom.

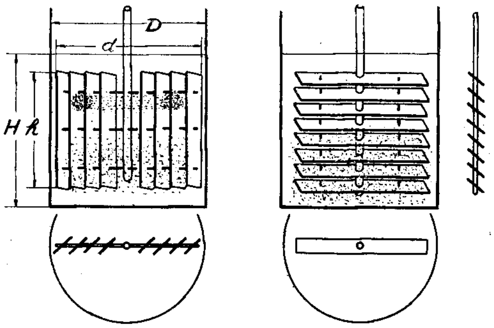
(b) Cylindrical vessel with flat bottom.



(c) Paddle mixer.

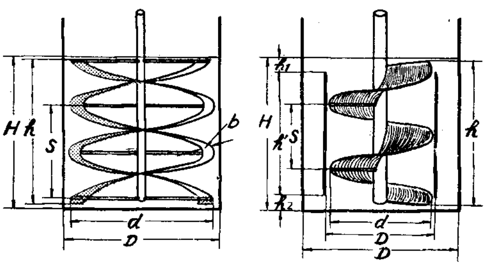
(d) Paddler mixe.

Fig. 1. State of mixing produced by several types of paddle mixers.



(a) Gate type mixer.

(b) Louver boards type mixer.



(c) Ribbon mixer (R).

(d) Screw mixer with draft tube (Q).

Fig. 2. Types of mixing impellers used.

Impellers used are as follows;

(1) Paddle mixer (denoted as *Pa*, refer to Fig. 1 (c), (d))
(2) Ribbon mixer (denoted as *R*, refer to Fig. 2 (c))
(3) Screw mixer with draft tube (denoted as *Q*, refer to Fig. 2 (d))

The dimensions of these mixers are shown in Table 2.

4. Results obtained

4.1. Preliminary test

As mentioned in the previous paper(3), if a large impeller be used in the mixing of low viscous liquids, the cylindrically rotating zone becomes somewhat large. With an impeller of large size, especially of a large width, it was observed that only the corner sections of the impeller contributed to agitation, and the mixing between upper and lower halves of the container got worse. Therefore, it was recommended to use impellers of comparatively small width in the mixing of low viscous liquid.

The higher the viscosity of liquid, the smaller the radius of the cylindrically rotating zone becomes. Therefore, keeping steps with liquid viscosity, the effective impeller width (2 lc) becomes larger as shown by Fig. 8 in the previous paper(3). Thus an impeller with larger sizes both in diameter and width makes itself an efficient mixer for viscous liquid.

If a paddle mixer of small size as shown by Fig. 1 (a) or a propeller as shown by Fig. 3 (d) are used for the mixing of high viscous liquids, only the narrow ranges of liquid near the impeller are mixed leaving the other parts unmixed. Even in the case of the large impeller, lack of uniformity occurs from time to time as shown by Fig. 1 (c) and (d). Uniform mixing is apt to be attained by setting the impeller near the bottom of the vessel.

Although the authors have also tested various types of impellers as shown by Fig. 3, many of them have not proved to be effective.

In the case of the large rectangular paddle, it may be supposed that co-rotation of liquid with paddle would occur, but actually it is not the case and a nearly satisfactory result is obtained. It can scarcely be expected to have particularly promising results by using a gate-type mixer or louver-boards-type mixer. (refer to Fig. 2 (a), (b)).

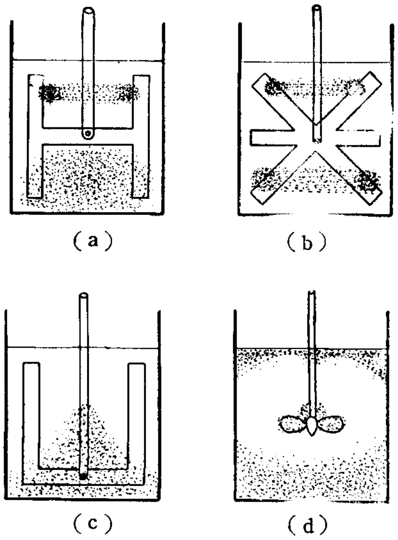


Fig. 3. Various types of mixing impellers tested.

In designing the gate-or louver boards-type mixers, care must be taken to set up the mixer with many blades. For instance, a mixer having minor blades like that shown by Fig. 1 (b), would be an example of poor design.

Especially, the anchor-type mixer as shown by Fig. 3 (c) is not a good mixer for high viscous liquid as it leaves a wide range of unmixed part.

Therefore the authors intend to advance their studies confining only on the efficient mixers as shown in Table 2.

Table 2. Impellers Used (Referto Fig. 2)

Type of impeller / Vessel dia. (cm)	Paddle mixer (Pa)	Ribbon mixer (R)		Screw mixer with drafft tube (Q)		
		(R-1)	(R-2)	(Q-1)	(Q-2)	(Q-3)
10	$d=0.8D$ $b=0.8D$	$d=0.94D$ $b=0.11D$ $s=0.745d$ $h=0.9D$	$d=0.94D$ $b=0.11D$ $s=1.12d$ $h=0.9D$	$D'=0.7D$ $d=0.9D'$ $s=0.715d$ $h=0.9D$ $h'=0.8D$ $h_1=h_2=0.1D$	$D'=0.7D$ $d=0.93D'$ $s=1.385d$ $h=0.9D$ $h'=0.8D$ $h_1=h_2=0.1D$	$D'=0.6D$ $d=0.9D'$ $s=0.67d$ $h=0.9D$ $h'=0.7D$ $h_1=h_2=0.15D$
20	$d=0.8D$ $b=0.8D$	(R-3) $d=0.95D$ $b=0.1D$ $s=1.0d$ $h=0.95D$		—	—	—
30	$d=0.8D$ $b=0.8D$	(R-3') $d=0.95D$ $b=0.1D$ $s=1.0d$ $h=0.95D$		—	—	—

4. 2. $N-\theta$ diagram

As stated above, by cross-plotting the relations between time required for complete mixing, θ, and impeller speed, N, in r.p.m., Fig. 4 and as the like are obtained. Replotting these relations on a logarithmic paper, Fig. 5, 6, 7 and 9 are obtained.

In cases of the ribbon mixer, R, and screw mixer with draft tube, Q, the slopes of the lines are nearly equal to (-1). In Fig. 6, there is an indication that the slope become steeper than (-1) (refer to the dotted line 2), but when the injection rate of sodium thiosulphate solution is proportional to the impeller speed, the slope of the curve agrees fairly well with (-1) as shown by the curve (3) in Fig. 6. Other results are not properly measured, because the time required for complete mixing has been measured after the completion of addition of reducer, without regards

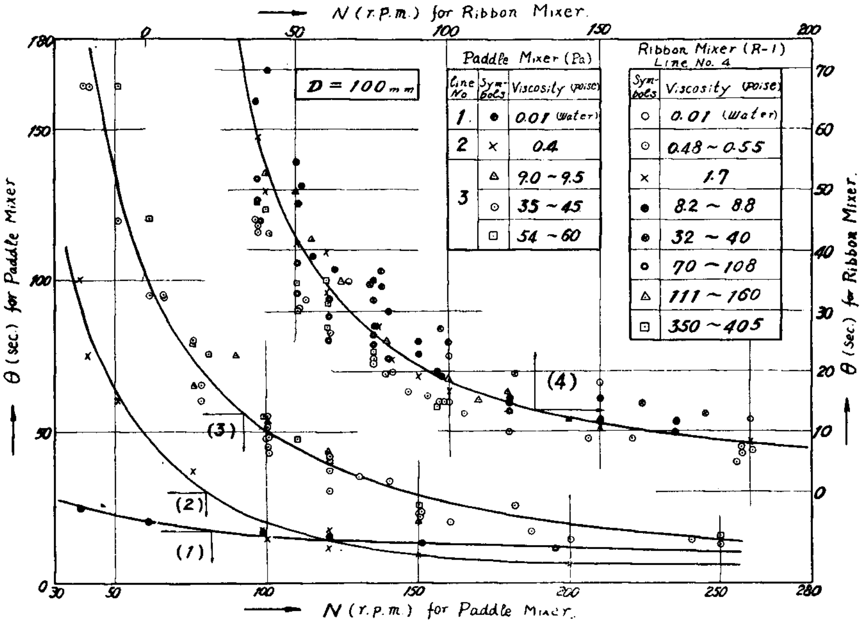


Fig. 4. Time required for complete mixing (θ), versus impeller speed (N).

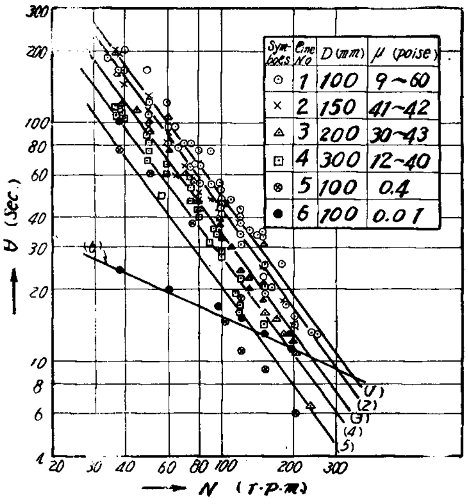


Fig. 5. N-θ diagram of paddle mixer (P_a) (Effect of vessel diameter).

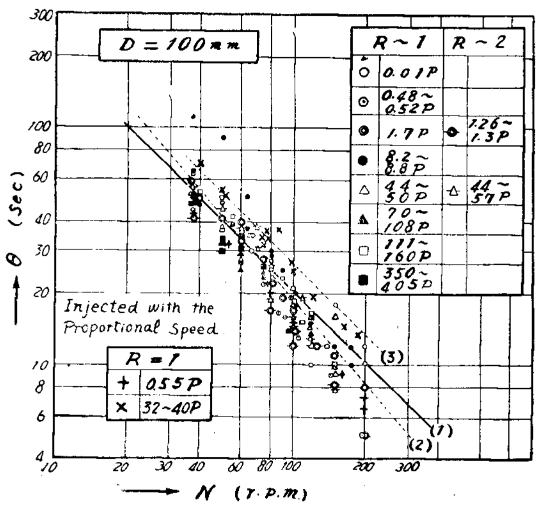


Fig. 6. N-θ diagram of ribbon mixers (R) (Effect of liquid viscosity).

to the impeller speed. Therefore, provided all of the measurements were carried out under the above formula, θ in the higher ranges in N would be rather larger, and the slope would agree well with (-1). Thus the following relation is obtained.

$$N\theta \fallingdotseq k_1 = \text{constant}. \qquad (1)^*$$

But, in the case of the paddle mixer, the slope appears somewhat steeper as shown by Fig. 5 and the relation should be expressed as follows,

$$\theta = k_1' N^{-1.4}. \qquad (1')$$

Thus, there are some differences between the results of the paddle mixer and those of the ribbon- or screw-mixers. The reason for those differences may be based on the difference in the mode of mixing taken place.

4.3. Effect of liquid viscosity

The effect of liquid viscosity can hardly be detected in the range of 1 to 400 poise examined as shown by Fig. 6, therefore the effect of viscosity can be neglected thus,

$$N\theta = k_2 \mu^0 \quad \text{(i.e., independent of viscosity).} \qquad (2)$$

But in the range of turbulent flow region, it deviates largely. For example, in the case of the paddle mixer, the curves (5) and (6) in Fig. 5 are obtained. As for water, the slope of $N-\theta$ curve becomes so gentle that the impeller speed would cause little effect on θ. In the case of the ribbon mixer, lack of uniformity appears between blades or on the bottom of the vessel. In the liquid of low viscosity such as water, a cylindrical zone of the liquid around the impeller shaft remains unmixed with the surrounding. Thus the co-rotation with the impeller takes place and the circulation flow of upward and downward direction diminishes, and the circumferential flow predominates.

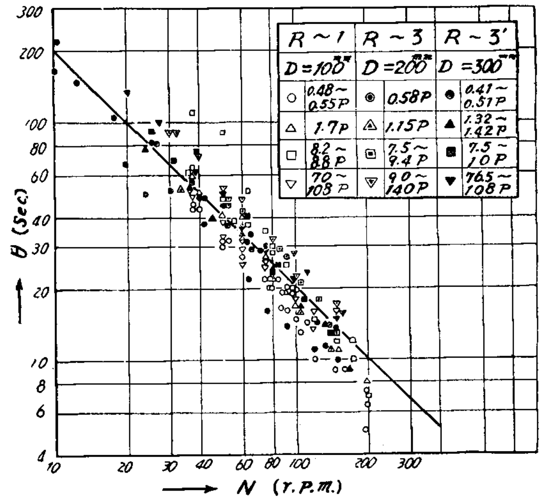


Fig. 7. N-θ diagram of ribbon mixers (R) (Effect of vessel diameter).

As it is difficult to discuss the mixing ranging from turbulent to laminar regions, the authors intend to confine their studies in the laminar flow region hereafter. (namely in the region where the slope of $(P.g_c/\rho n^3 d^5)$ versus $(d^2 n\rho/\mu)$ is taken to be equal to (-1), as shown later by Fig. 12).

4.4. Effect of vessel diameter, *D*, —Method of scale up of the mixing vessel

As shown by Fig. 7, in the ribbon mixer (probably the same

---

* Kramers et al.[1] obtained the same results on the mixing velocity of KCl solution into water.

in the screw mixer with draft tube), variations of the vessel diameter do not affect the mixing velocity. Therefore the following simple relation is obtained.

$$N\theta = k_3 D^0 \quad \text{(independent of vessel diameter).} \qquad (3)$$

In the case of the paddle mixer, $N-\theta$ lines come down in parallel with the increase in vessel diameter as shown by Fig. 5, and the following relation is obtained.

$$\theta = k_3' D^{-0.56} N^{-1.4}. \qquad (3')$$

This may be due to the essential difference of the mechanism of mixing between paddle-and ribbon-mixers.

4.5. Flow patterns of ribbon and screw mixers

The path of the liquid flow produced by ribbon mixers is quite different from that produced by paddle mixers. As can be seen from their construction, the former shows a circulation as shown by Fig. 8, while the latter, shows mainly a tangential rotation.

When the ribbon mixer is rotated to force liquid upwards, the liquid goes up around the vessel wall side along the spiral path and comes down the central zone of the vessel almost vertically along the shaft. Of course, when the mixer rotates reversely, the direction of circulation is reversed. As a rule for the purpose of mixing, the former rotation may be recommended.

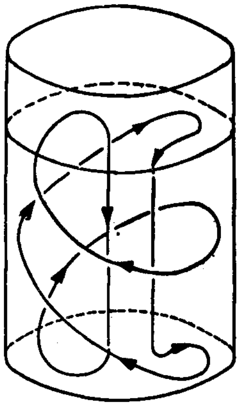


Fig. 8. Flow pattern produced by the ribbon mixer (R).

The path of the liquid flow produced by the screw mixer with draft tube can be followed as that of the ribbon mixer.

In case of right rotation as recommended above, the liquid goes up vertically outside the draft tube, and comes down inside of it. In this case, right rotation does faster mixing than the reverse one.

4.6. Effect of pitch, blade width and impeller diameter

4.6.1. Ribbon mixer

Three kinds of impellers are compared; $S=0.745$; ($R-1$), $S=1.12$; ($R-2$) as shown in Table 2 and $S=1.5$; ($R-2'$) which is not given in Table 2. (S means pitch of the spiral).

Time required for complete mixing of ($R-1$) and ($R-2$) would nearly be equal each other as shown by the plots in Fig. 6. As to the impeller ($R-2'$), it has the tendency to show lack of uniformity in mixing, so that impellers having small pitches would be taken as suitable, but the smaller the pitch of spiral, the larger would be the power consumption as shown later (see Fig. 10). Therefore it may be recommended to use the ribbon mixer having pitch of $S=1$ as a most efficient one, and so the mixers ($R-3$) for $D=20$ cm and ($R-3'$) for $D=30$ cm are constructed in this basis. The impeller diameter should be taken as long as possible on the standpoint of construction and the width of the ribbon would be optimum in the size of $b=0.1D$.

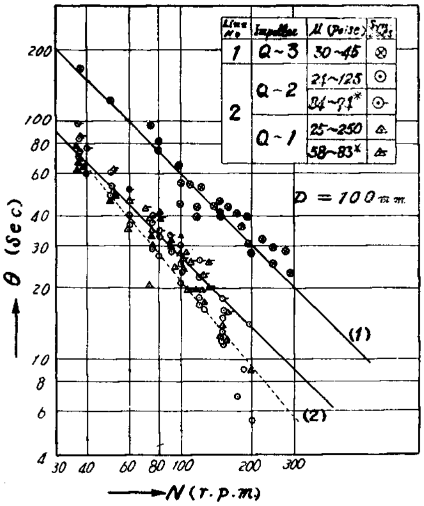


Fig. 9. N-θ diagram of screw impellers with draft tube (Q).
* Results of reverse rotation.

4.6.2. Screw mixer with draft tube

Two screw mixers having pitches of $S=0.715$; ($Q-1$) and $S=1.385$; ($Q-2$) were tested. There are few differences in the state of mixing and the time required for complete mixing between these two mixers for viscous liquid (more than 10 poise) as shown by Fig. 9.

But in mixing of liquid of slightly lower viscosity, there is apt to be a lack of uniformity between impeller blades, especially for a mixer of larger pitch ($Q-2$).

On the other hand, power consumption becomes larger with decreasing pitch in the same way as the ribbon mixer (see Fig. 10). Therefore it may be decided that the optimum pitch is equal to 1 for practical use.

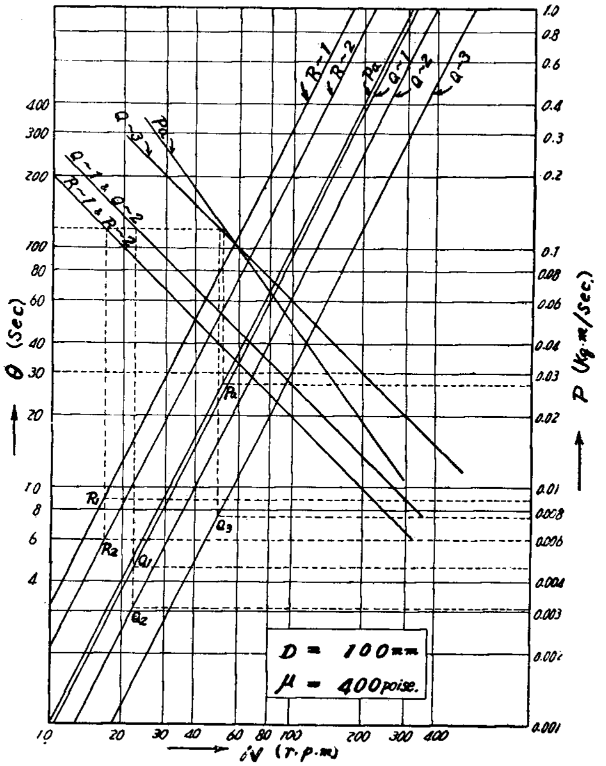


Fig. 10. Comparison of power consumed by various types of impellers for complete mixing.

4.6.3. Size of draft tube

Diameter, D', of the draft tube for screw mixer, is taken in the ratio as $D'=0.7D$. This is based on the idea that the sectional area of the tube is equal to that of the annular space outside of it. Experimental result shows that $Q-1\,(D'=0.7D)$ takes shorter time for mixing than $Q-3\,(D'=0.5D)$ as shown by Fig. 9 and 10.

4.7. Comparison of the types of mixing impellers based on their power consumptions

To compare the characteristics of various types of mixers, only the $N-\theta$ diagram mentioned above is not sufficient. It is recommended to compare on the basis of their power consumption. Power consumptions of various types of mixers; P_a, $(R-1)$, $(R-2)$, $(Q-1)$, $(Q-2)$ and $(Q-3)$ in the vessel of $D=10\,\text{cm}$ are plotted against their rotational speed as shown by Fig. 10. This diagrom is denoted as N-P lines. In addition to these N-P lines, N-θ lines are also drawn in Fig. 10 for the sake of comparison.

For example, abscissa reading, N, at the points of intersection of the horizontal line at $\theta=120$ sec. and the N-θ lines show the rotational speed of various impellers where complete mixing can be attained within 120 sec. Following the dotted lines vertically, the points of intersection with the N-P lines can be read off as R_1, R_2, Q_1, Q_2, Q_3 and P_a whose ordidate values (on the right hand side scale) show their power consumptions.

As shown by the diagram, the order of the power consumption is determined as follows;

$$(Q-2)<(Q-1)<(R-2)<(Q-3)<(R-1)<P_a.$$

Multiplying the mixing time θ (sec.) by the power consumption P (Kg-m/sec.), the mixing energy $P\cdot\theta$ (Kg-m) can be obtained. Fig. 11 shows the result.

The power consumstion at laminer flow range are shown by the following relation.

$$\frac{P\cdot g_c}{\rho n^3 d^5}=C_1\left(\frac{d^2 n\rho}{\mu}\right)^{-1} \qquad (4)$$

The observed data of the impellers used are plotted on Fig. 12. It is obvious that that Power number, $(Pg_c/\rho n^3 d^5)$, and Reynolds number, $(d^2 n\rho/\mu)$, can be correlated by Eq. (4) in the range of Raynolds number

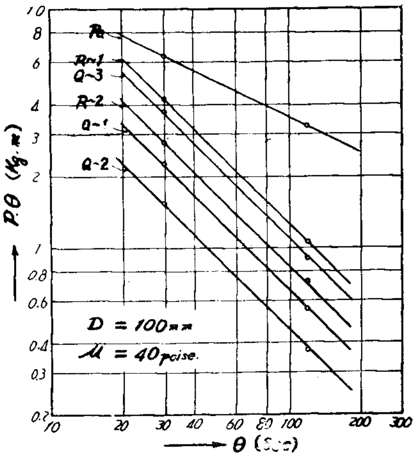


Fig. 11. Mixing energy required for various types of impellers.

less than 100. Therefore the authors confined their studies in the range of $Re < 100$, i.e., perfect laminar flow range. Eq. (4) can be rearranged thus,

$$P = C_2 \mu n^2 d^3 \qquad (5)$$

5. Some considerations on the experimental results

As stated in [4.1] (refer to Fig. 1 (c), (d)), the liquid flow produced by the paddle mixer, P_a, is very complex and is inferior in reproducibility, so that it is difficult to discuss on the mechanism of mixing.

Whereas in tne case of the ribbon mixer, R, and screw mixer with draft tube, Q, the liquid flow is steady and considerably clear and the results of experiments have good reproducibility. Therefore the authors confine their discussion on these two types of mixers hereafter.

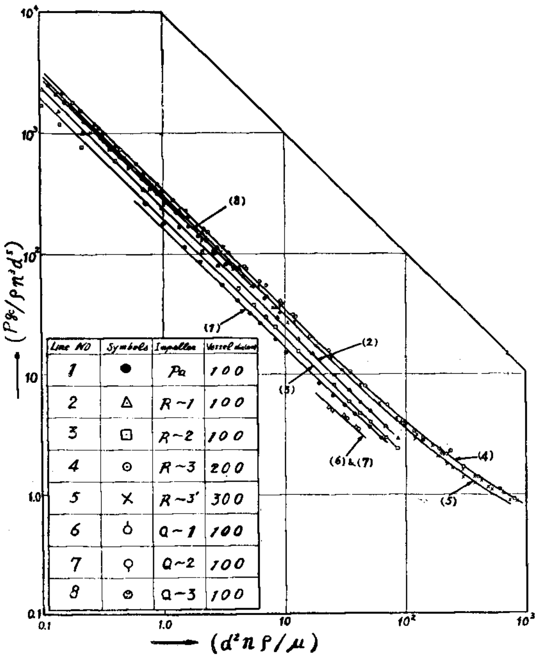
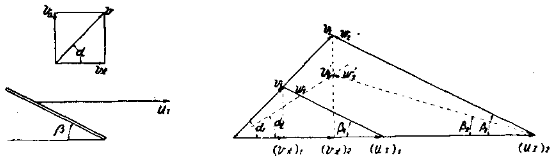


Fig. 12. Power characteristics of several types of impellers.

5.1. Similarity in mixing conditions

As mentioned in Eq. (3), it was clarified experimentally that the time required for complete mixing was nearly equal at the same impeller speed regardless of the vessel diameters. The reason may be explained as follows.



(a) (b)

Fig. 13. Velocity triangles of mixing impeller operated in viscous liquid.

Fig. 13 (a) shows a part of impeller blade. u_I is the linear velocity of an impeller at a certain impeller speed n and v_t, v_a and v represent the tangential, axial, and combined velocities, respectively. Let the relative velocity between impeller and liquid in the horizontal direction be noted as u_r, that is,

$$u_r = u_I - v_t \qquad (6)$$

As u_I is proportional to dn,

$$u_I = k_0 dn \qquad (7)$$

Assuming the following relations,

$$v_t = k_1' u_I = k_1 dn \qquad (8)$$

$$v_a = k_2' u_I = k_2 dn \qquad (9)$$

where k_1, k_2, k_1' and k_2' may not always be constant.

Let T be the resistance gained on an impeller in laminar flow range, then the following relation holds.

$$T = c\mu d u_r \qquad (10)$$

where c is a proportional constant.

Then the power consumption of the impeller, P, is shown, as,

$$P = T \cdot u_I = c\mu d u_r u_I . \qquad (11)$$

From Eq. (6), (7) and (8),

$$u_r = (k_0 - k_1) dn . \qquad (12)$$

Substituting this u_r in Eq. (11), the following relation is derived.

$$P = c\mu n^2 d^3 (k_0 - k_1) k_0 . \qquad (13)$$

From the experiments in laminar flow range, Eq. (5) was obtained.

$$P = C_2 \mu n^2 d^3 . \qquad (5)$$

Therefore, from these two relations, $(k_0 - k_1)$ is proved to be constant. As mentioned above k_0 is constant, so k_1 must also be constant.

On the other hand, it is rather hard to prove directly whether the k_2 in Eq. (9) would be a constant or not. Suppose that v_t varies proportionally with u_I, and v_a does not, then the direction of the absolute velocity of liquid, v, would vary with the impeller speed. Dotted lines in Fig. 13 shows this case. In this figure, β, the direction of the relative speed (w) of liquid to impeller, varies from β_1 to β_2, that is, separation of flow from the surface of impeller blade takes place so that eddies may ocour. In such a case, the behavior of liquid flow may approach to tubulent step by step and the power data may not agree with Eq. (5). In the present studies, the range is confined completely in laminar flow where Eq. (5) holds good. Therefore, it is reasonable to consider that no separation of flow would take place and the coefficient k_2 in Eq. (9) must be constant.

Accordingly, the absolute velocity of liquid, v, becomes proportional to impeller diameter and rotational speed n.

Thus,

$$v = kdn = K'Dn . \qquad (14)$$

In the geometrically similar mixers, v is proportional to D in an equal impeller speed, while the length L of the path of circulation flow must be proportional to D, because the vector of liquid velocity is similar as mentioned above.

If the assumption that the mixing may be performed by the circulation of liquid is correct, then the time required for complete mixing, θ, would be proportional to the time τ that is required for one circulation of liquid.

The time of one circulation, τ, can be calculated at an equal impeller speed as follows,

$$\tau = L/v = k'D/k''D = \text{constant}\,. \tag{15}$$

This result means that τ is independent of vessel diameter. Therefore, if the mixing be completed by m times circulations, then the mixing time, θ, is equal to $m\tau$, i.e.,

$$\theta = m\tau\,. \tag{16}^*$$

As τ is constant, so θ is also constant.

From the above explanations, it is clear that for the series of geometrically similar mixers of the type of ribbon and screw operated in high viscous liquid, the condition of equal mixing velocity can simply be reduced to equal impeller shaft speed.

5. 2. Relation between impeller speed and mixing time

When impeller diameter is constant, liquid velocity becomes proportional to the rotational speed, n, of the impeller as is obvious from Eq. (14). On the other hand, the circulation period, τ, is equal to L/v where L is taken as constant in a same vessel and v is proportional to n. Therefore,

$$\tau = kn^{-1}. \tag{15'}$$

From Eq. (16) and (15'), the next relation is derived.

$$\left.\begin{aligned} &\theta = mkn^{-1} \\ \text{or}\qquad &n\theta = mk = \text{constant} \end{aligned}\right\} \tag{17}$$

Thus, the experimental result shown by Eq. (1) can be explained reasonably.

5. 3. Number of rotation of impeller, *Z*, and number of circulation of liquid, *m*, required for complete mixing

As shown by Eq. (17), the result of $n\theta = \text{constant}$ is obtained. n is the rotational speed of an impeller in revolution per second and θ is the time in second required for complete mixing.

Let $n\theta$ be noted as Z, then Z is the number of rotation of an impeller in the time interval required for complete mixing.

Eq. (17) means that, during Z revolutions of the mixing impeller, the liquid

---

* van de Vusse proposed this idea and represented m as a dimensionless mixing time.

circulates m times and mixing is finished independent of the impeller speed. Calculating from the results of Fig. 6 about ribbon mixer, the following results are obtained.

$$\left.\begin{aligned} N &= 30\ \text{r.p.m.}, & n &= 0.5\ (1/\text{sec.}), & \theta &= 66\ \text{sec.}, & n\theta &= 33 \\ N &= 180\ \text{r.p.m.}, & n &= 3\ (1/\text{sec.}), & \theta &= 11\ \text{sec.}, & n\theta &= 33 \end{aligned}\right\}. \qquad (18')$$

In the case when the dotted line data are adopted, $n\theta$ is equal to $38\sim29$ $(=0.5\times75\sim3\times9.5)$ and may be approximated to be equal to 34.

Also, in regards to the screw mixer ($Q-1$, $Q-2$; $D'=0.7D$), refering to Fig. 9, the following results are obtained.

$$\left.\begin{aligned} N &= 30\ \text{r.p.m.}, & n &= 0.5\ (1/\text{sec.}), & \theta &= 90\ \text{sec.}, & n\theta &= 45 \\ N &= 180\ \text{r.p.m.}, & n &= 3\ (1/\text{sec.}), & \theta &= 15\ \text{sec.}, & n\theta &= 45 \end{aligned}\right\}. \qquad (18'')$$

In the case of screw mixer $Q-3$ ($D'=0.5D$), nearly a constant value is obtained as follows.

$$n\theta = 0.5\times200\sim3\times34 = 100\sim102\,. \qquad (18''')$$

As to the measurement of the circulation velocity of liquid, the following method is adopted. A speck of cotton, dyed and tightend with vinyl acetate resin, is introduced in the mixing liquid and the number of circulations during a comparatively long time is measured. The circulation periods, τ, observed by this method are shown in Table 3. Therefore $n\tau$ is equal to the number of revolutions of the impeller for one circulation of liquid and $Z/n\tau$ is equal to m, the number of circulations of liquid required for complete mixing.

Table 3. Number of Revolutions of Mixing Impellers ($Z=n\theta$) and Number of Circulations of Liquid (m) Reguired for Cemplete Mixing.

Type of mixer	Vessel dia. (mm)	N rpm (n rps) / Viscosity (p)	15 (0.25)		30 (0.5)		50 (0.833)		100 (1.66)		150 (2.5)		$(n\tau)$ avg.	Z	$m=Z/n\tau$
			τ (sec.)	$n\tau$	τ (sec.)	$n\tau$	τ (sec.)	$n\tau$	τ (sec.)	$n\tau$	τ (sec.)	$n\tau$			
R-1	100	40	—	—	—	—	12.9	10.75	6.7	11.2	—	—	11.0	33	3.0
R-2	100	40	—	—	—	—	13.7	11.4	7.2	12.0	—	—	11.7	33	2.8
R-3′	300	22	51.5	12.9	23.4	11.7	16.35	13.6	—	—	—	—	12.7	33	2.6
Q-1	100	40	—	—	—	—	17.6	15.3	8.8	14.7	—	—	15.0	45	3.0
Q-2	100	40	—	—	—	—	19.6	17.0	9.1	15.2	6.5	16.25	16.1	45	2.9
Q-3	100	40	—	—	—	—	—	—	22.5	37.6	14.3	35.8	36.7	100	2.73

As shown on the last column in Table 3, approximately three time of circulation of liquid would be sufficient for complete mixing.

5.4. Power consumption per unit volume of liquid

Power consumption in the range of perfect laminar flow is shown by Eq. (5). For geometrically similar vessels, the following equation holds.

$$P = C_3 \mu n^2 D^3 . \qquad (5')$$

For the mixing of same liquid, viscosity, μ, may be included into constant and Eq. (5′) becomes,

$$P = C_4 n^2 D^3 .$$

The power per unit volume, P_v, is as follows,

$$P_v = C_5 n^2 . \qquad (19)$$

Thus, the power per unit volume is equal when the liquid is mixed with equal impeller speed and is independent of the vessel diameter, D.

On the other hand, it was ascertained that the mixing time would be equal at the same rotational speed in the case of similar vessels as shown by Eq. (3). Therefore, the condition of similarity for mixing of high viscous liquids is also based on equal power per unit volume. But the relation between vessel diameter and impeller speed is greatly different from that of turbulent flow range.[4)]

5.5. Mixing energy and mixing intensity

The mixing efficiency of various types of impeller has been compared. Let the mixing energy E that is equal to $P \cdot \theta$ be compared in both ribbon- and screw-mixers. From Eq. (5) and (18), the following relation is derived.

$$E = P \cdot \theta = C_2 \mu n^2 d^3 (Z/n) = C' \mu n d^3 . \qquad (20)$$

Then the mixing energy per unit volume E_v is,

$$E_v = E/(\pi/4) D^3 = C'' \mu n . \qquad (21)$$

The value of E_v corresponds to mixing intensity and is proportional to viscosity of liquid and impeller speed. Therefore, for speedy mixing, larger impeller speed is desired and so much larger mixing energy is necesary as shown by Eq. (21). And also, for the same mixing time the impeller speeds are equal irrespective of liquid viscosity, but the mixing energy is larger in proportion with liquid viscosity.

5.6. Loss of head in mixing vessel

From Eq. (16),

$$m = \theta/\tau = \theta Q/V \qquad (16')$$

where Q is the volumetric rate of circulation in m^3/sec., and V is the volume of vessel in m^3.

On the other hand, the power consumption P is shown by the following equation.

$$P = \gamma Q H_m \qquad (22)$$

where γ is the specific weight of liquid in Kg/m^3 and H_m is the loss of head in m for the circulation of liquid in the vessel. Values of Q and H_m are all taken the mean values for the whole vessel. From Eq. (16′), (20), (21) and (22), the following relation is obtained.

$$m=\frac{P\cdot\theta}{\gamma H_m V}=\frac{E}{\gamma H_m V}, \qquad (23)$$

$$=\frac{C''\mu n}{\gamma H_m}. \qquad (23')$$

As shown in Table 3, m is nearly equal to a constant value of 2.6~3.0. Considering this result to be significant, the following relation is obtained.

$$H_m=(C''/m\gamma)\mu n=K\mu n. \qquad (24)$$

Thus, the higher the viscosity or the impeller speed, the larger the loss of head.

On the same vessel, the volume V is constant, so that $P\cdot\theta$ is proportional to H_m from Eq. (23). As the values of $P\cdot\theta$ are in the order, $(Q-2)<(Q-1)<(R-2)<(Q-3)<(R-1)$ as shown by Fig. 11, so the losses of head become larger in these order. The completion of mixing can be attained by the three times circulation in each impeller, but the losses of head for those circulations are different from each other and the impellers of proper sizes and construction are recommended.

6. Example of calculation of the mixing energy for ribbon mixer

In Fig. 12, the lines for $(R-3)$ and $(R-3')$ are to overlap each other, but the slight differences in the construction cause the inconsistency. Taking the mean value, Eq. (5) can be represented as follows.

$$P\cdot g_c=300\mu n^2 d^3. \qquad (5)$$

Replacing the representative length from d to D and substituting g_c by 9.8, the following equation is obtained for ribbon mixer.

$$P=26\mu n^2 D^3. \qquad (22)$$

Therefore the mixing energy $P\cdot\theta$ can be obtained, combining Eq. (22) with Eq. (18′).

$$P\theta=860\mu n D^3 \quad [\text{Kg. m}] \qquad (23)$$

or

$$P\theta=11.5\mu n D^3 \quad [\text{P.S. sec.}]. \qquad (23')$$

In case of $n=0.5$ r.p.s., $\mu=100$ poise and $D=1$ m

$$P\theta=11.5\times10\times0.5\times1^3=57.5 \quad [\text{P.S.-sec.}].$$

In order to complete mixing in 10 and 60 seconds, the power requirement is reached to 5.75 and 0.96 P.S. respectively.

7. Conclusion

(1) Time required for complete mixing was observed for high viscous liquids with various types of mixing impellers. The most homogeneous and speedy mixing can be obtained with the ribbon mixer and the screw mixer with draft tube which are usually employed.

(2) Using these two types of mixers, time required for complete mixing is inversely proportional to impeller speed, and independent of liquid viscosity and vessel diameter. Generally complete mixing is almost accomplished in three times of circulation.

In order to operate in the same mixing velocity in various sizes of vessels, the geometrically similar impellers are to be used and rotated at equal impeller speed.

(3) The number of revolutions of a mixing impeller required for complete mixing and that required for one circulation of liquid remain constant regardless of the impeller speed.

(4) Mixing energy becomes proportional to liquid viscosity and increases proportionally with mixing speed.

(5) Simplified explanations were given to these experimental results.

Notations Used

b	:	Width of an impeller (refer to Fig. 1 and 2).	[m]
C_1, C_2 etc.	:	Proportionality constants.	[–]
D	:	Vessel diameter.	[m]
D'	:	Diameter of a draft tube (refer to Fig. 2 (d)).	[m]
$E=P\cdot\theta$	:	Mixing energy.	[Kg. m]
E_v	:	Mixing energy per unit volume of liquid.	[Kg. m/m^3]
g_c	:	Gravitational conversion factor.	[kg. m/Kg. sec.^{2}]
H	:	Liquid depth.	[m]
H_m	:	Loss of head due to friction.	[m]
h	:	Impeller height (refer to Fig. 2).	[m]
h'	:	Height of a draft tube.	[m]
h_1, h_2	:	Clearance of draft tube from liquid surface and from the bottom of a vessel resp.	[m]
k_1, k_1', k_2 etc	:	Proportional constant.	[–]
L	:	Length of the path of liquid flow in a mixing vessel.	[m]
m	:	Number of circulation of liquid requird for complete mixing.	[–]
N	:	Impeller speed in r.p.m.	[1/min.]
n	:	Impeller speed in r.p.s.	[1/sec.]

P : Power consumption of impellers. [Kg.m/sec.]

Q : Volumetric rate of flow. [m^3/sec.]

S : Pitch of an impeller (refer to Fig. 2 (c), (d)). [–]

T : Resistance of liquid to an impeller. [Kg]

u_I : Linear velocity of an impeller. [m/sec.]

u_r : Relative velocity between an impeller and liquid. [m/sec.]

V : Volume of mixing vessel. [m^3]

v, v_t, v_a : Liquid velocity, its tangential and axial component resp. [m/sec.]

w : Liquid velocity relative to an impeller. [m/sec.]

Z : Number of revolution of impeller for complete mixing. [–]

γ : Specific weight of liquid. [Kg/m^3]

ρ : Density of liquids. [kg/m^3]

μ : Viscosity of liquid. [kg/m. sec], [poise]

θ : Time required for complete mixing. [sec.]

τ : Period of circulation of liquid in mixing vessels. [sec.]

Literature Cited

1) H. Kramers et al.: Chem. Eng. Sci., **2**, 35 (1953).
2) J. G. van de Vusse: Chem. Eng. Sci., **4**, 178, 209 (1955).
3) S. Nagata, N. Yoshioka and T. Yokoyama: Mem. of the Faculty of Eng., Kyoto Univ. **17**, 175 (1955).
4) S. Nagata et al.: Chem. Eng. (Japan), **15**, 59 (1951).