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CITATION:

SUGAWARA, Sugao ...[et al]. Dropwise Condensation. Memoirs of the Faculty of Engineering, Kyoto University 1956, 18(2): 84-111

ISSUE DATE:

1956-07-20

URL:

<http://hdl.handle.net/2433/280349>

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# Dropwise Condensation

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(Received January 31, 1956)

## Introduction

In 1930, Schmidt and his coworkers<sup>1)</sup> succeeded in inducing dropwise condensation on the mirror-polished chromium plate and discovered that the heat transfer coefficient of dropwise condensation was much higher than that of filmwise condensation. Nagle and his coworkers<sup>2)</sup> later carried out a research for the necessary conditions of a solid surface to induce dropwise condensation and reported that "dropwise condensation of steam is obtained only when the condensing surface is contaminated with a suitable promoter that prevents the condensate from wetting the surface", and obtained a patent for the use of promoters to induce dropwise condensation. Thus, the fact that the coefficient of heat transfer by dropwise condensation is much higher than that by filmwise condensation has attracted attention of the people concerned and the studies of dropwise condensation have been rapidly developed thereafter. However, the heat transfer by dropwise condensation, which is severely affected by various conditions, has not yet been sufficiently cleared both theoretically and experimentally as compared with the case of filmwise condensation. Therefore, we feel that a further study on the heat transfer should be made systematically as well as the study on effective methods to induce dropwise condensation on the practical condensing surface.

In Chapter 1, we shall study theoretically and experimentally the mechanism of dropwise condensation and discuss three essential factors to dropwise condensation; that is, the critical size and shape of a drop, the area covered by droplets, and the sweeping cycle. Adapting the knowledge obtained in Chapter 1, the theory of heat transfer by dropwise condensation shall be developed in Chapter 2, and a calculating method to estimate the heat transfer coefficient be proposed. Lastly, in Chapter 3, we shall discuss on various conditions related to the life of dropwise condensation and then propose an effective method to attain a longer life.

### Chapter 1. The Mechanism of the Dropwise Condensation.

When the water vapour condenses in drops on a vertical flat wall, the droplets

continue to grow both by condensation and by coalescence of other droplets until they grow to be of a critical size. The drop of a critical size, which starts near the top of the condensing surface, sweeps down the wall leaving a bare stripe which is exposed to vapour along the wall, and then the vapour condenses again on that bare stripe. In this way, the surface is periodically cleaned by the condensate. Fig. 1.1 shows clearly the adhering drops and the sweeping drops.

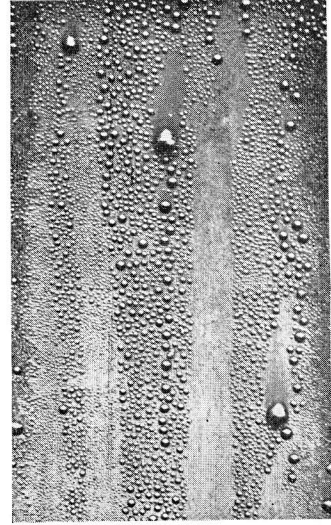


Fig. 1.1. Purely dropwise condensation.

Accordingly, the factors which are related to the mechanism of the dropwise condensation are: (1) the critical size and shape (contact angle) of a drop, (2) the area covered by droplets on the unit area of condensing surface, and (3) the sweeping cycle. This chapter deals with the theoretical and experimental study on the three kinds of factors mentioned above.

- (1) The critical size  $D_c$  and shape (contact angle  $\theta$ ) of a drop.

We consider of a drop which is adhered on a solid surface as shown in Fig. 1.2, where

- $\beta$ : the angle of inclination of the solid surface from the horizontal surface
- $\theta_a$ : the advancing contact angle
- $\theta_r$ : the receding contact angle
- $S_w$ : the surface tension of water.

Now, we consider of a case in which the element, whose breadth is  $dl$ , begins to move by gravitational force, in another word, when the drop begins to move, and then the equilibrium of the force becomes as follows:

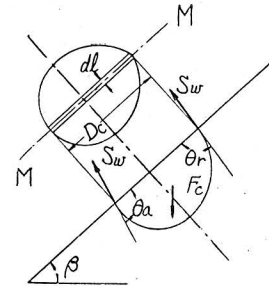


Fig. 1.2.

$$F_c dl \rho g \sin \beta = S_w (\cos \theta_r - \cos \theta_a) dl \tag{1.1}$$

where,  $\rho$ : the density of water

$g$ : acceleration of gravity

$F_c$ : the area of vertical section on  $M-M$  of drop, which is approximately given by the followings:

$$F_c \cong \frac{D_c^2}{8 \sin^2 \theta_m} \left[ \frac{\pi \theta_m}{90} - \sin 2\theta_m \right] \tag{1.2}$$

$\theta_m$ : the average contact angle,  $\theta_m = \frac{\theta_r + \theta_a}{2}$

Putting Eq. (1.2) into Eq. (1.1), the following is obtained :

$$D_c = \sqrt{\frac{S_w}{\gamma} \frac{1}{\sin \beta} \frac{16 \sin^3 \theta_m}{\frac{\pi \theta_m}{90} - \sin 2\theta_m} \sin \frac{\theta_a - \theta_r}{2}} \quad (1.3)$$

where,  $\gamma$  : the specific weight of water

$D_c$ : the critical diameter of a drop (we shall call it so).

In this way, the critical diameter of a drop is obtained. However, unless we know how the contact angle  $\theta_a$  and  $\theta_r$  vary with the angle of inclination and the character of solid surface, we can not adapt Eq. (1.3). Therefore, we experimented on the critical size and shape (contact angle) of a drop which begins to move on the metal surfaces promoted by three kinds of substances.

Measurements of the critical diameter and the two contact angles were taken by photographs of drops of city water in room temperature (about 20°C) which adhered downward by a spout and an injector. For example, Fig. 1.3 (photographs) was obtained.

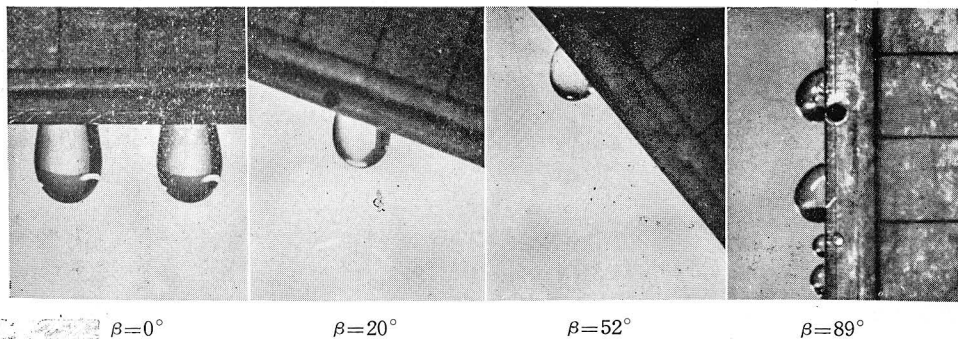


Fig. 1.3. Drops adhered on the surface promoted by silicon resin.

(a) The case where the promoter is silicon resin.

From the photographs in Fig. 1.3, Fig. 1.4a (the relations between the critical diameter  $D_c$  and the angle  $\beta$  of inclination) and Fig. 1.4b (the relations between the contact angle  $\theta_a$ ,  $\theta_r$  and  $\theta_m$  and the angle  $\beta$ ) are obtained. These figures clearly show that  $D_c$  (plotted by  $\circ$  points) is almost independent of the angle  $\beta$  and is about 3.6 mm; however, the volume or weight of drop is reduced as  $\beta$  is increased as shown in Fig. 1.3. It is also shown definitely that the advancing contact angle  $\theta_a$  (plotted by  $\circ$  points) slightly varies with  $\beta$ , but the receding contact angle  $\theta_r$  (plotted by  $\bullet$  points) is reduced greatly as  $\beta$  is increased.

Now, putting the contact angles,  $\theta_a$ ,  $\theta_r$  and  $\theta_m$ , which are measured by photographs, into Eq. (1.3) and calculating the critical diameter  $D_c$ , we obtain the points as shown by  $\times$  in Fig. 1.4a. Comparing  $\times$  points with  $\circ$  points, they show a satisfactory agreement, and it is clear that Eq. (1.3) is sufficiently accurate.

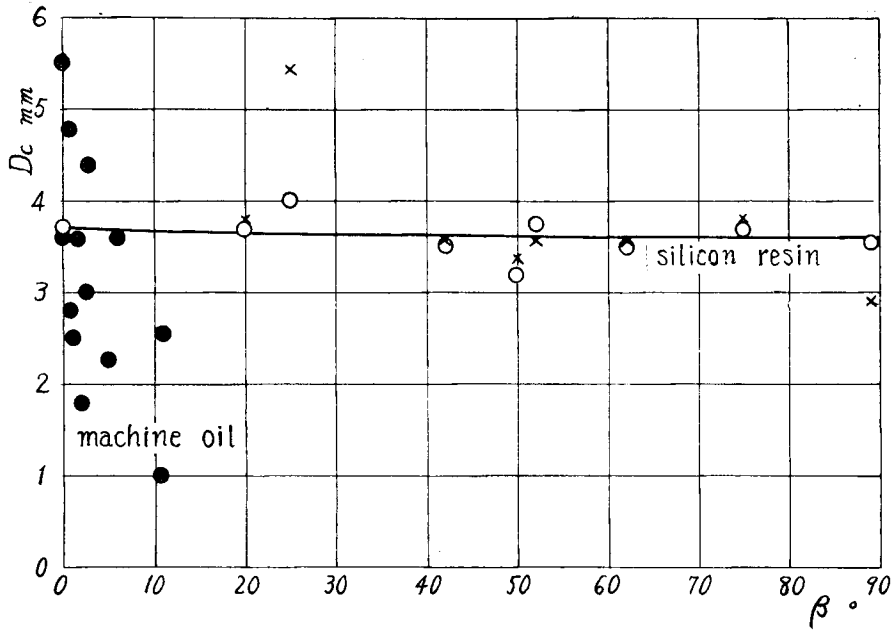


Fig. 1.4a The critical contact diameter. (Silicon resin and machine oil)

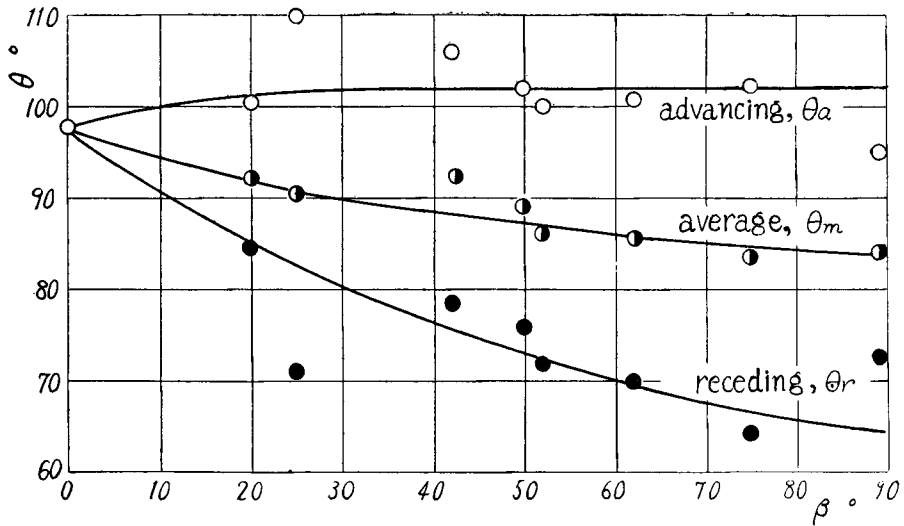


Fig. 1.4b The critical contact angle. (Silicon resin)

(b) The case where the promoter is a kind of machine oil.

In this case, as shown by ● points in Fig. 1.4a, the points are scattered, but it seems that the critical diameter is suddenly reduced as the angle of inclination is increased.

(c) The case where the promoter is oleic acid.

When the purely dropwise condensation occurred on the brass surface promoted by oleic acid, we stopped the condensation, dried the surface naturally, and then measured the critical size and shape of drop, which were adhering on the surface, by using the same method as mentioned in (a). The results obtained are shown in Fig. 1.5a and Fig. 1.5b. It is clear, as in the case of silicon resin, that  $D_c$  is almost independent of the angle  $\beta$ , and  $\theta_a$  is almost independent of  $\beta$ , but  $\theta_r$  is reduced as  $\beta$  is increased. The experimented points of  $D_c$  agreed sufficiently with the calculated points (plotted by  $\times$  points).

The temperature of the drop mentioned above was 20°C. Therefore, in order to estimate the critical size  $D_c$  in the case of other temperatures of drop, we assume that the temperature has no effect on the contact angle but affects only on the surface

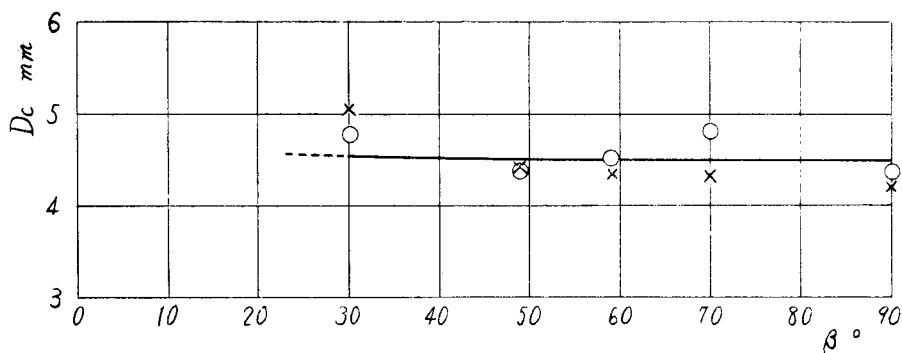


Fig. 1.5a The critical contact diameter. (Oleic acid)

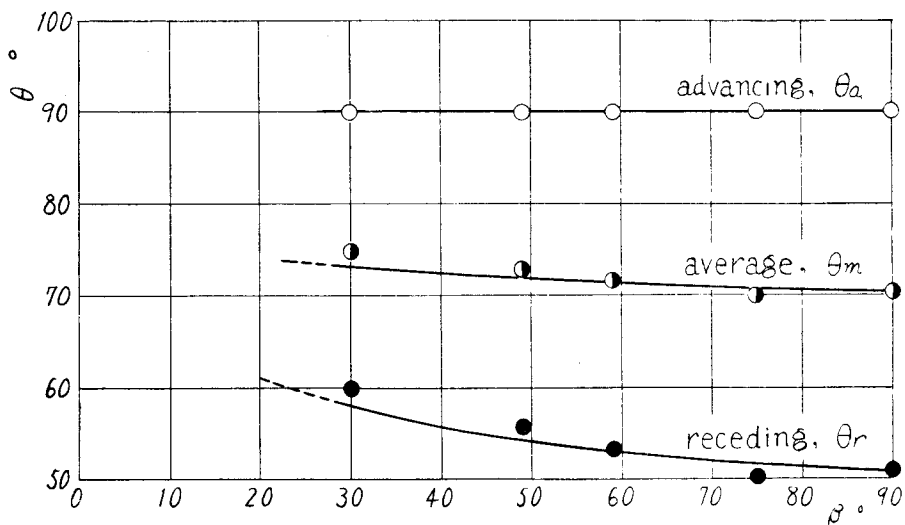


Fig. 1.5b The critical contact angle. (Oleic acid)

tension  $S_w$  and the specific weight of water  $\gamma$ . Then, the critical diameter  $D_c$  is proportional to  $\sqrt{\frac{S_w}{\gamma}}$  in Eq. (1.3). For example, when the temperature of drop is  $100^\circ\text{C}$ ,  $\sqrt{\frac{S_w}{\gamma}}$  is 0.247 cm corresponding to 0.273 cm in the case of  $20^\circ\text{C}$ . Accordingly, the critical diameter is reduced as the temperature of drop is increased. When the saturated steam, whose pressure is 760 mmHgabs., condenses on the vertical metal surface (the angle  $\beta$  of inclination is  $90^\circ$ ) which is promoted by oleic acid, the critical diameter  $D_c$  and the contact angles  $\theta_a$ ,  $\theta_r$  and  $\theta_m$  become as follows:

$$D_c = 3.9 \text{ mm}, \quad \theta_a = 90^\circ, \quad \theta_r = 50^\circ, \quad \theta_m = 70^\circ.$$

Comparing these results with Fatica et al's<sup>6)</sup> data:

$$D_c = 3.7 \text{ mm}, \quad \theta_a = 80^\circ, \quad \theta_r = 47 \sim 57^\circ, \quad \theta_m = 66^\circ$$

in the case where the steam ( $100^\circ\text{C}$ ) condenses on the copper surface promoted by oleic acid, it is clear that our results agree very well with Fatica et al's data.

- (2) The ratio  $A$  of the area covered by droplets to the unit area of condensing surface.

On the condensing surface, there are many droplets of various sizes shown in Fig. 1.6. But, if we divide the breadth of the condensing surface, we can make the droplets to be of almost same size in the divided strip. This divided strip corresponds exactly to the breadth of the sweeping way of a drop. Therefore, the part which contains small droplets belongs to the beginning stage of adhering period (the droplets continue to grow), while the part which contains larger drops belongs to the latter stage of adhering period

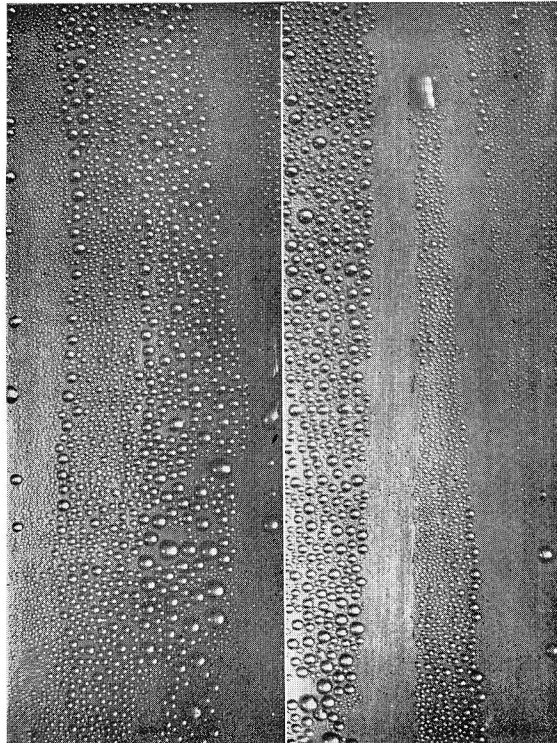


Fig. 1.6. The area covered by droplets.

and it will soon be swept by a drop. The photographs clearly show that the smaller drops distribute more densely and the larger drops, more sporadically.

When we count the numbers of drops on the unit area which contains drops of almost same size, the followings are obtained :

Mean diameter of drops	Unit area	No. of drops	The ratio A
0.23 mm	59.17 mm <sup>2</sup>	328	0.23
0.46 mm	59.17 mm <sup>2</sup>	135	0.38
0.85 mm	59.17 mm <sup>2</sup>	53	0.50

Fig. 1.7 shows the relation between the ratio  $A$  of the area covered by droplets to the unit area and the mean diameter  $D$ , and it also shows that  $A$  is reduced as  $D$  is decreased, while  $A$  is increased as  $D$  is increased but the increment of  $A$  is reduced and it approaches to a constant value. Ac-

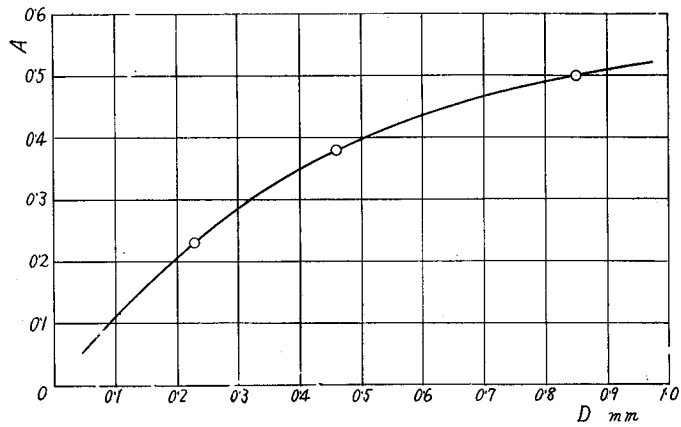


Fig. 1.7 The area covered by droplets on the unit surface area.

Accordingly, the ratio  $A$  is small at the beginning of adhering period and it is increased with the lapse of time and then it becomes a constant (about 0.5) at the end of the period.

Fatica et al<sup>6)</sup> reported that the ratio  $A$  is almost constant and is about 0.45, while Hampson et al<sup>7)</sup> reported that the ratio varies with time of the adhering period and is 0.54 at the end of the period.

Thus, about half the part of the unit area of the condensing surface is directly exposed to the steam; therefore, the temperature of the surface is very close to the temperature of the steam and the heat transfer coefficient of the dropwise condensation shows very high value as shown in Chapter 2.

(3) The sweeping cycle  $\tau$  (sec.).

The droplets continue to grow both by condensation and by coalescence of other droplets until they grow to be of a critical size, and then they are periodically swept down by the drop of the critical size which starts from

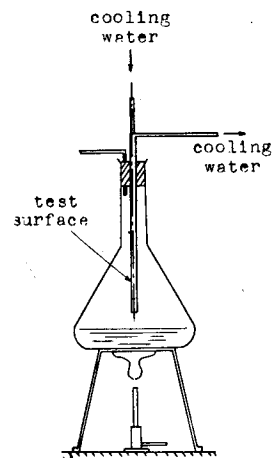


Fig. 1.8



near the top of the condensing surface. This periodical duration is called the sweeping cycle  $\tau$  (sec.).

By using the apparatus which is shown in Fig. 1.8, we counted the numbers of the sweeping in a given time at the positions 2.5 cm, 12.5 cm and 22.5 cm from the top of the condensing pipe which is promoted by oleic acid and cooled by the city water, and we measured the sweeping cycle  $\tau$ . The followings were obtained :

The flow rate of cooling water	Position 2.5 cm	Position 12.5 cm	Position 22.5 cm
3cc /sec	$\tau=7.7$ sec	4.4 sec	3.5 sec
42 cc/sec	$\tau=6.1$ sec	3.7 sec	3.0 sec

From these results, it is clear that (i) the sweeping cycle  $\tau$  is reduced as the flow rate of cooling water is increased, and (ii) the sweeping cycle  $\tau$  is shorter at the lower part of the condensing surface.

The fact of (i) is due to the reason that when the flow rate of cooling water is increased, the cooling rate of the condensing surface is increased and, therefore, the rate of condensation of the steam is increased.

The fact of (ii) may be due to the reason that, since the drop which starts to move from near the top of the condensing surface grows in size very quickly by coalescing with other droplets during its downward journey when the height of the condensing surface is so high, the breadth of the sweeping way of a drop widens at the lower part of the condensing surface and, therefore, these breadth overlap each other causing the lower part of the condensing surface being swept more often than the upper part. Naturally, when the condensing surface is so high, the lower part of the surface is swept by many drops which start to move from anywhere over the lower part; therefore, the lower the part the more often it is swept.

The critical diameter  $D_c$  is given by Eq. (1.3). When the solid surface is promoted by silicon resin or oleic acid, the advancing contact angle  $\theta_a$  is almost independent of the angle  $\beta$  of inclination of solid surface but the receding contact angle  $\theta_r$  is reduced as the angle  $\beta$  is increased.

The ratio  $A$  of the area covered by droplets to the unit area of the condensing surface is increased as the mean diameter of drop is increased, but the rate of increase becomes lesser and  $A$  approaches to a constant value (about 0.5). Accordingly,  $A$  is small at the beginning of the adhering period, and it increases with the lapse of time and then becomes constant at the end of the period.

The sweeping cycle  $\tau$  is reduced as the flow rate of cooling water is increased and the lower part of the condensing surface is swept more often than the upper part.

## Chapter 2. Theory of Heat Transfer by the Dropwise Condensation.

It has been reported by Gnam<sup>8)</sup> that the temperature of the condensing surface varies periodically and, therefore, the heat transfer rate by the dropwise condensation varies periodically. This is due to the fact that the condensing surface is cleaned periodically by the condensate as explained in Chapter 1. Actually, there are many droplets of various sizes and shapes on the condensing surface and some of them continue to grow while the others are being swept as already shown in Fig. 1.1. In this way, the parts which belong to the adhering period (in which the droplets continue to grow) are mixed with the parts which belong to the sweeping period (in which the droplets are being swept) on the condensing surface. Consequently, it is very difficult to analyze theoretically the heat transfer by the dropwise condensation. Fatica and Katz<sup>6)</sup> analyzed the heat transfer alone at the adhering period by assuming that the ratio  $A$  of the area covered by droplets to the unit area of condensing surface is constant during the adhering period. But the ratio  $A$  varies with time in the period as has been discussed in Chapter 1. Further, they did not analyze the heat transfer at the sweeping period.

In this chapter, we shall analyze theoretically the heat transfer by considering both the adhering and sweeping periods, and then propose a calculating method to estimate the coefficient of heat transfer by the dropwise condensation.

### 1. Heat transfer at the adhering period.

The average coefficient  $\alpha_{s1}$  of heat transfer at the adhering period is expressed by the following equation,

$$\alpha_{s1} = \frac{q_1}{\tau_1 \Delta T_{s1}} \quad (2.1)$$

where,

$q_1$ : the heat quantity transferred to the unit area of condensing surface at the interval  $\tau_1$  of the adhering period

$\tau_1$ : the interval of the adhering period

$\Delta T_{s1}$ : the average temperature drop from steam to condensing surface over  $\tau_1$ .

Accordingly, as the heat transfer resistance between the steam and the condensing surface, the followings are to be considered: (1) the resistance between the steam and the surface of a drop, (2) the resistance through a drop, (3) the resistance between the steam and the bare surface which is not covered by drops. Of these three, the resistance through a drop is very important as the resistance through the water film is important in the case of the filmwise condensation.

Now, we assume that the steam is stationary and it condenses on the solid surface

which is inclined by an angle  $\beta$ , then all the droplets are of the same size and shape, and the uniformly cover the condensing surface and continue to grow similarly, as shown in Fig. 2.1. Then, supposing that the heat transfer resistance between the steam and the surface of a drop is absent, the heat quantity  $dq_1$  transferred from the steam through the unit area of condensing wall to the cooling liquid at the interval  $dt$  becomes as follows:

$$dq_1 = K_d A \Delta T_t dt + K_f (1 - A) \Delta T_t dt \quad (2.2)$$

where,

$\Delta T_t$ : the total temperature drop from the steam to the cooling liquid

$K_d$ : the coefficient of heat transmission through the adhering drop

$K_f$ : the coefficient of heat transmission through the bare surface

$A$ : the ratio of the area covered by droplets to the unit surface area

and

$$\left. \begin{aligned} \frac{1}{K_d} &= \frac{1}{K_0} + \frac{1}{k_d} \\ \frac{1}{K_f} &= \frac{1}{K_0} + \frac{1}{k_f} \end{aligned} \right\} \quad (2.3)$$

$$\frac{1}{K_0} = \frac{1}{\alpha_l} + \frac{l_M}{\lambda_M} \quad (2.4)$$

where,

$1/k_d$ : the equivalent heat transmission resistance of the drop

$1/k_f$ : the heat transmission resistance between the steam and the bare condensing surface. When a non-condensable gas is contained in the steam, we assume that the bare surface is covered by the mixed gas of the steam and a non-condensable gas whose density is comparatively high, then we define the resistance through the mixed gas layer also with  $1/k_f$ .

$\alpha_l$ : the coefficient of heat transfer by flow of the cooling liquid

$l_M$ : the thickness of the condensing wall

$\lambda_M$ : thermal conductivity of the metal of the condensing wall

$K_0$ : the cooling rate (So we shall call it).

Putting Eq. (2.3) into Eq. (2.2), then

$$dq_1 = \frac{K_0 k_d A}{K_0 + k_d} \Delta T_t dt + \frac{K_0 k_f (1 - A)}{K_0 + k_f} \Delta T_t dt \quad (2.5)$$

The average temperature drop  $\Delta T_{s1}$  from the steam to the condensing surface at the interval  $\tau_1$  is expressed as follows:

$$\Delta T_{s1} = \frac{1}{\tau_1} \int_0^{\tau_1} [\Delta T_{sd} A + \Delta T_{sf} (1 - A)] dt \quad (2.6)$$

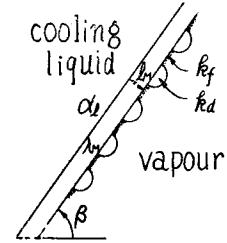


Fig. 2.1

where,

$\Delta T_{sd}$ : the temperature drop through a droplet

$\Delta T_{sf}$ : the temperature drop from the steam to the bare condensing surface.

Consequently,

$$\Delta T_{sd} = \frac{K_d}{k_d} \Delta T_t = \frac{K_0}{K_0 + k_d} \Delta T_t, \quad (2.7a)$$

$$\Delta T_{sf} = \frac{K_f}{k_f} \Delta T_t = \frac{K_0}{K_0 + k_f} \Delta T_t. \quad (2.7b)$$

Putting Eqs. (2.7a) and (2.7b) into Eq. (2.6), then we get

$$\Delta T_{s1} = \frac{K_0 \Delta T_t}{\tau_1} \left[ \int_0^{\tau_1} \frac{A}{K_0 + k_d} dt + \int_0^{\tau_1} \frac{1-A}{K_0 + k_f} dt \right]. \quad (2.8)$$

On the other hand, the increment  $dV$  of volume of the condensate on the area of the condensing surface corresponds to the heat quantity  $dq_1$ , that is:

$$dq_1 = L\gamma dV \quad (2.9)$$

where,

$L$ : the latent heat of the steam

$\gamma$ : the specific weight of the condensate

$V$ : the volume of condensate on the unit area of the condensing surface.

From Eqs. (2.5) and (2.9), we obtain

$$dt = \frac{L\gamma}{K_0 \Delta T_t} \left[ \frac{k_d A}{K_0 + k_d} + \frac{k_f (1-A)}{K_0 + k_f} \right]^{-1} dV, \quad (2.10)$$

then, putting this equation into Eq. (2.8), we obtain

$$\Delta T_{s1} = \frac{L\gamma}{\tau_1} \int_0^{V_c} \frac{Ak_f + K_0 + k_d(1-A)}{k_d(AK_0 + k_f) + k_f K_0(1-A)} dV. \quad (2.11)$$

On the other hand, the net heat quantity  $q_1$  transferred from the steam to the condensing surface at the interval  $\tau_1$  becomes as follows:

$$\begin{aligned} q_1 &= \int_0^{V_c} L\gamma dV \\ &= L\gamma V_c \end{aligned} \quad (2.12)$$

where,

$V_c$ : the net volume of condensate at the end of the adhering period on the unit area of condensing surface.

Accordingly, putting Eqs. (2.12) and (2.11) into Eq. (2.1), the average heat transfer coefficient  $\alpha_{s1}$  is given by the following:

$$\alpha_{s1} = \frac{V_c}{\int_0^{V_c} \frac{Ak_f + K_0 + k_d(1-A)}{k_d(AK_0 + k_f) + k_f K_0(1-A)} dV}. \quad (2.13)$$

Now, if we especially consider the case where the heat transfer resistance between the steam and the bare condensing surface is absent,  $\alpha_{s1}$  is given by the following by tending  $k_f$  to infinity,

$$\lim_{k_f \rightarrow \infty} \alpha_{s1} = \int_0^{V_c} \frac{V_c}{k_a + K_0(1-A)} dV. \quad (2.14)$$

Eq. (2.14) is the same as the one which has been treated by Fatica and Katz<sup>6)</sup>.

The volume  $V$  is given by the product of the numbers  $n$  of droplets (on the unit area of the condensing surface) and the volume  $v$  of a drop. Assuming that the drop is a segment of sphere whose average contact angle is  $\theta_m$ ,  $v$  is obtained by the following:

$$v = \frac{\pi}{24} D^3 \frac{2 - 3 \cos \theta_m + \cos^3 \theta_m}{\sin^3 \theta_m}. \quad (2.15)$$

Then,

$$V = \frac{n\pi}{24} D^3 \frac{2 - 3 \cos \theta_m + \cos^3 \theta_m}{\sin^3 \theta_m}, \quad (2.16)$$

where  $n$  varies with time at the interval of the adhering period, and  $D$  is the diameter of a drop on the contact surface.

Since we have assumed that all droplets are of the same size and shape on the condensing surface, all the droplets adhering on the unit condensing surface start to move at the same time when they assume the critical size and shape and then the adhering period ends. Accordingly,  $V_c$  (the net volume of condensate at the end of the adhering period) is given by the following:

$$V_c = \frac{n_c \pi}{24} D_c^3 \frac{2 - 3 \cos \theta_m + \cos^3 \theta_m}{\sin^3 \theta_m} \quad (2.17)$$

where,

$n_c$ : the numbers of droplets on the unit area of the condensing surface at the end of the adhering period.

While, the critical diameter  $D_c$  of a drop, which is adhering on the solid surface inclined by an angle  $\beta$  from the horizontal surface, as already mentioned in Chapter 1. (Eq. (1.3)), is given by the following equation:

$$D_c = \sqrt{\frac{S}{\gamma} \frac{1}{\sin \beta} \frac{16 \sin^3 \theta_m}{\pi \theta_m - \sin 2\theta_m} \sin \frac{\theta_a - \theta_r}{2}} \quad (2.18)$$

where,

$\theta_a$ : the advancing contact angle

$\theta_r$ : the receding contact angle

$\theta_m$ : the average contact angle,  $\theta_m = \frac{\theta_a + \theta_r}{2}$

$S$  : the surface tension of condensate

$\gamma$  : the specific weight of condensate.

Putting Eq. (2.18) into Eq. (2.17), then the net volume  $V_c$  of condensate at the interval of adhering period is obtainable.

The contact area  $a$  of a drop,

$$a = \frac{\pi}{4} D^2$$

and the ratio  $A$  of the area covered by droplets to the unit area of the condensing surface becomes as follows :

$$A = \frac{n\pi}{4} D^2. \quad (2.19)$$

Next, we shall analyze the equivalent heat transmission coefficient  $k_d$  through a drop. When we suppose the semi-sphere  $AB'C$ , as shown in Fig. 2.2, whose contact diameter is  $D$ , surface-temperature is  $T_1$  (anywhere) and base-temperature is  $T_0$  (anywhere) then the temperature distribution in the semi-sphere is given by the following from the theory of heat conduction :

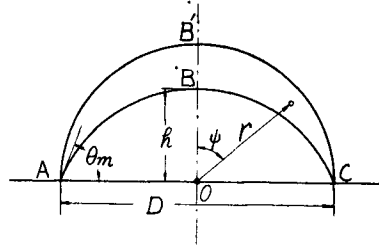


Fig. 2.2

$$\frac{T - T_0}{T_1 - T_0} = \theta = \sum_{m=0}^{\infty} \frac{(-1)^m (4m+3) [(2m)!]}{(2m+2) 2^{2m} (m!)^2} \frac{r^{2m+1}}{\left(\frac{D}{2}\right)^{2m+1}} P_{2m+1}(\mu^*) \quad (2.20)$$

where,

$T$  : the temperature at any position

$r$  : the distance from the origin to that position

$\mu^* = \cos \psi$

$\psi$  : the angle (spherical polar coordinates)

$P_{2m+1}(\mu^*)$  : the first Legendre function of the  $(2m+1)$  order

$m$  : the positive integer.

Accordingly, the heat quantity transferred from the surface to the base by heat conduction is given by the following :

$$Q_d = \int_0^{\pi} \lambda_d \left( \frac{\partial T}{\partial r} \right)_{r=\frac{D}{2}} \times 2\pi \left( \frac{D}{2} \right)^2 \sin \psi d\psi \quad (2.21)$$

where,

$\lambda_d$  : the thermal conductivity of condensate.

Now, putting Eq. (2.20) into Eq. (2.21),  $Q_d$  becomes as follows :

$$Q_d = \pi \lambda_d (T_1 - T_0) D \sum_{m=0}^{\infty} \frac{(4m+3) [(2m)!]^2 (2m+1)}{(2m+2) 2^{4m} (m!)^4}. \quad (2.22)$$

Next, we consider the spherical segment  $ABC$  whose contact diameter is  $D$  which is the same as that of the semi-sphere and contact angle is  $\theta_m$ , and we assume that the surface of the spherical segment agrees with an isothermal surface of the semi-sphere and that the heat quantity  $Q_d$  flows normally through the surface of a spherical segment, then the equivalent heat transmission coefficient of a drop (a spherical segment) may be expressed as follows :

$$k_d = \frac{Q_d}{\frac{\pi}{4} D^2 (T_h - T_0)} \tag{2.23}$$

where,

$T_h$ : the surface-temperature of a drop (the surface-temperature of spherical segment).

While, the temperature at  $r=h$  (the height of spherical segment) and  $\psi=0$  ( $\mu^*=1$ ) is obtainable by putting

$$h = \frac{D}{2} \tan \frac{\theta_m}{2} \tag{2.24}$$

and

$$\mu^* = 1$$

into Eq. (2.20). Then

$$\frac{T_h - T_0}{T_1 - T_0} = f(\theta_m).$$

Accordingly,

$$k_d = \frac{4\lambda_d}{Df(\theta_m)} \sum_{m=0}^{\infty} \frac{(4m+3)[(2m)!]^2 (2m+1)}{(2m+2)^2 4^m (m!)^4} \tag{2.25}$$

$$\equiv \frac{C\lambda_d}{Df(\theta_m)}$$

where,  $C \cong 15.058$ .

Fig. 2.3 shows the relation between  $\frac{k_d}{\lambda_d/D}$  and  $\tan \frac{\theta_m}{2}$ , and it clearly indicates that when  $\lambda_d$  and  $D$  are constant,  $k_d$  is increased as the contact angle is reduced. However, when the angle  $\beta$  of inclination approaches to  $90^\circ$  (vertical surface) the average contact angle  $\theta_m$  is reduced and then the critical diameter  $D_c$  is almost independent

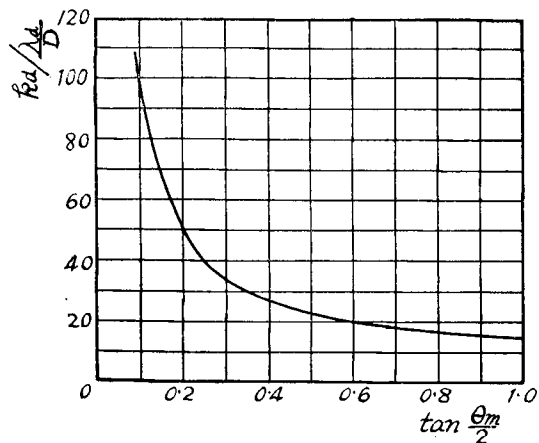


Fig. 2.3 Heat transmission through drop.

of  $\beta$ , as has been already mentioned in Chapter 1, and  $k_d$  of the drop adhering on the vertical surface is maximum. Of course,  $k_d$  is increased as  $D$  is reduced.

According to Eq. (2.13) or Eq. (2.14),  $\alpha_{s1}$  is a function of  $k_f$ ,  $K_0$ ,  $V$ ,  $k_d$  and  $A$ . But,  $k_f$  and  $K_0$  are constant at any time in the interval of adhering period, and  $V$ ,  $k_d$  and  $A$  vary with time. This is due to:

$$V = \text{function } (n, D, \theta_m) \quad (\text{from Eq. (2.16)})$$

$$k_d = \text{function } (D, \theta_m) \quad (\text{from Eq. (2.25)})$$

$$A = \text{function } (n, D) \quad (\text{from Eq. (2.19)}),$$

and the numbers  $n$  of droplets, the contact diameter  $D$  and the contact angle  $\theta_m$  vary with time. However, it may be assumed that  $\theta_m$  does not vary so much as compared with  $n$  and  $D$ , and then  $\theta_m$  is constant during the interval of the adhering period, and  $\theta_m$  is equal to the contact angle of a drop of the critical size.

As has been already mentioned in Chapter 1, the contact diameter  $D$  is small, the numbers  $n$  of droplets are large, and the ratio  $A$  of the area covered by droplets to the unit area is small at the beginning of the adhering period, but  $D$  increases,  $n$  reduces and  $A$  increases with lapse of time, and then  $D$  becomes  $D_c$ ,  $n$  becomes  $n_c$  and  $A$  becomes  $A_c$  (about 0.5) at the end of the period. Therefore, we treated the integration of Eq. (2.13) or Eq. (2.14) under the following assumptions:

$D$	$n$	$A$
(I) variable (increase)	variable (decrease)	constant
(II) variable (increase)	constant	variable (increase)
(III) constant	variable (increase)	variable (increase).

(1) The case of  $k_f = \infty$ .

Assumption (I),

$$\alpha_{s1} = \frac{V_c}{\frac{V_c A}{K_0(1-A)} - \frac{A}{K_0(1-A)}} \frac{V_c}{\frac{\lambda_d C A}{3K_0(1-A)f(\theta_m)} \left[ \ln \left\{ V + \frac{\lambda_d C A}{3K_0(1-A)f(\theta_m)} \right\} \right]_0^{V_c}} \quad (2.27)$$

Assumption (II),

$$\alpha_{s1} = \frac{V_c K_0}{\int_0^{D_c} \frac{\left( \frac{\pi}{4} n D^2 \right)^2}{\frac{\lambda_d C}{K_0 f(\theta_m) D} + 1 - \frac{\pi}{4} n D^2} dD} \quad (2.28)$$

Assumption (III),

$$\alpha_{s1} = \frac{n_c K_0}{\left[ \frac{4}{\pi D^2} \left( 1 + \frac{\lambda_d C}{K_0 D f(\theta_m)} \right) - n \right]_0^{n_c} - \frac{4}{\pi D^2} \left( 1 + \frac{\lambda_d C}{K_0 D f(\theta_m)} \right) \left[ \ln \left\{ \frac{4}{\pi D^2} \left( 1 + \frac{\lambda_d C}{K_0 D f(\theta_m)} \right) - n \right\} \right]_0^{n_c}} \quad (2.29)$$



Next, as an example, we calculate numerically the heat transfer coefficient  $\alpha_{s1}$  and compare the heat transfer coefficients reduced by these three assumptions :

The cooling rate  $K_0 = 2865 \text{ kcal/m}^2\text{h}^\circ\text{C}$ ,

The critical diameter  $D_c \approx 1.6 \text{ mm}$ ,

The advancing contact angle  $\theta_a \approx 95^\circ$ ,

The receding contact angle  $\theta_r \approx 85^\circ$ ,

The average contact angle  $\theta_m \approx 90^\circ$ .

This critical diameter satisfies Eq. (2.18) in the case in which the steam of  $100^\circ\text{C}$  condenses on a vertical condensing wall.

By using Eq. (2.27), (assumption (I))

$$\alpha_{s1} = 3.84 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D_c = 1.6 \text{ mm}, A = 0.35),$$

$$\alpha_{s1} = 2.90 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D_c = 1.6 \text{ mm}, A = 0.45).$$

By using Eq. (2.28), (assumption (II))

$$\alpha_{s1} = 4.23 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D_c = 1.6 \text{ mm}, A_c = 0.35),$$

$$\alpha_{s1} = 3.11 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D_c = 1.6 \text{ mm}, A_c = 0.45).$$

By using Eq. (2.29), (assumption (III))

$$\alpha_{s1} = 4.40 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D = 1.6 \text{ mm}, A_c = 0.35),$$

$$\alpha_{s1} = 3.33 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D = 1.6 \text{ mm}, A_c = 0.45).$$

From these calculations, the following facts are found :

(i) Independently of any of the assumptions,  $\alpha_{s1}$  is increased as  $A$  is reduced. This fact is due to reduction of the heat transfer resistance through the unit area of condensing surface, corresponding to the increase of the bare parts which are not occupied by droplets.

(ii) The heat transfer coefficient  $\alpha_{s1}$  calculated in the assumption (I) ( $A$  is constant) is smaller than that calculated in the other assumptions, (II) and (III). This fact shows that the heat transfer coefficient is much higher at the beginning of the adhering period since  $A$  is very small, and then it affects on the average value of heat transfer coefficient.

(iii) There is no difference between the assumption (II) and (III).

(2) The case of  $k_f \neq \infty$ .

Now, when we integrate Eq. (2.13) with the assumption (I),  $\alpha_{s1}$  becomes as follows :

$$\alpha_{s1} = \frac{V_c}{\left[ \ln \left\{ V + \frac{\lambda_d CA (AK_0 + k_f)}{3K_0 f(\theta_m) k_f (1-A)} \right\} \right]_0^{V_c} \left[ \frac{\lambda_d CA}{3K_0 f(\theta_m) k_f} - \frac{\lambda_d CA (AK_0 + k_f)}{3K_0 f(\theta_m) k_f (1-A)} \times \frac{Ak_f + K_0}{k_f K_0 (1-A)} \right] + \frac{V_c (Ak_f + K_0)}{k_f K_0 (1-A)}. \quad (2.30)$$

For example, when we calculate  $\alpha_{s1}$  under the condition mentioned above and

$$k_f = 5 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C}$$

then the followings are obtained :

$$\alpha_{s1} = 2.45 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D_c = 1.6 \text{ mm}, A = 0.35),$$

$$\alpha_{s1} = 2.11 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C} \quad (D_c = 1.6 \text{ mm}, A = 0.45).$$

From these results, it is clear that, when  $k_f$  is not infinity,  $\alpha_{s1}$  becomes smaller than that in the case of  $k_f = \infty$ . Consequently, when it is assumed that the condensing surface excepting that part covered with droplets is exposed directly to the steam and the heat transfer resistance from the steam to the bare condensing surface is negligible, then the heat transfer coefficient takes the maximum value. And we should pay special attention to these facts when a non-condensable gas is present in the steam.

From Eq. (2.11),

$$\begin{aligned} \Delta T_{s1}\tau_1 &= L\gamma \int_0^{V_c} \frac{Ak_f + K_0 + k_d(1-A)}{k_d(AK_0 + k_f) + k_fK_0(1-A)} dV \\ &= \frac{L\gamma V_c}{\alpha_{s1}}. \end{aligned} \quad (2.11')$$

Then, in the case  $k_f = \infty$ ,  $\Delta T_{s1}\tau_1$  takes the following values :

Assumption (I),

$$\Delta T_{s1}\tau_1 = 9.1 \text{ sec}^\circ\text{C} \quad (A = 0.35),$$

$$\Delta T_{s1}\tau_1 = 15.5 \text{ sec}^\circ\text{C} \quad (A = 0.45).$$

Assumption (II),

$$\Delta T_{s1}\tau_1 = 7.95 \text{ sec}^\circ\text{C} \quad (A_c = 0.35),$$

$$\Delta T_{s1}\tau_1 = 13.3 \text{ sec}^\circ\text{C} \quad (A_c = 0.45).$$

Assumption (III),

$$\Delta T_{s1}\tau_1 = 8.28 \text{ sec}^\circ\text{C} \quad (A_c = 0.35),$$

$$\Delta T_{s1}\tau_1 = 14.4 \text{ sec}^\circ\text{C} \quad (A_c = 0.45).$$

## 2. Heat transfer at the sweeping period.

When the drops grow to the critical size, they slide down sweeping other droplets on the condensing surface. In this section, it is assumed that  $k_f = \infty$ , the condensing wall is not so high (accordingly, the swept stripes do not overlap each other), the interval  $\tau_2$  of the sweeping period is short, and so the steam does not condense on the swept stripes, therefore the track cleaned by drops is a bare strip exposed to the steam.

Though the droplets at the lower part of the condensing surface grow to the critical size later than the droplets at the upper part, it may be assumed that when sliding drops arrive at a position on the surface the droplets which are adhering to

that position attain the critical size ; accordingly, the diameter of each droplets adhering to the surface remains constant and is the same as the diameter of a drop at the end of the adhering period.

A drop which has reached the maximum drop-size near the top of the condensing surface grows in size very quickly by coalescing other droplets during its downward journey, and gains appreciable velocity. Then, the increment  $dv$  of a drop may be expressed as follows :

$$dv = v_c D w n dt \tag{2.31}$$

where,

$v_c$  : the volume of a drop, having a contact diameter equal to the critical diameter, which is still adhering to the surface.

$D$  : the contact diameter of a sliding drop

$w$  : the velocity of a sliding drop

$n$  : the numbers of droplets adhering to the unit area.

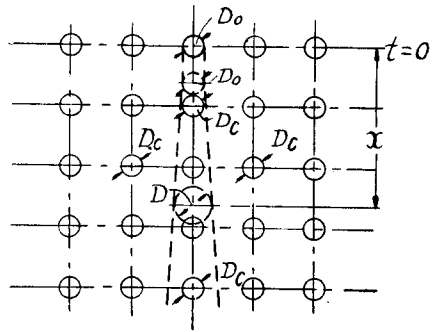


Fig. 2.4

as shown in Fig. 2.4.

On the other hand, from Eq. (2.15), by assuming  $\theta_m$  remains constant,

$$dv = \frac{\pi}{8} D^2 \frac{2 - 3 \cos \theta_m + \cos^3 \theta_m}{\sin^3 \theta_m} dD \tag{2.32}$$

and putting this equation into Eq. (2.31), the following is obtained :

$$\frac{D_c}{3} A w dt = \frac{\pi}{4} D dD \tag{2.33}$$

where,  $A$  is the ratio of the area covered by droplets to the unit area.

The equation of motion of a drop on the vertical condensing wall may be expressed as follows :

$$F_c dl \rho \frac{dw}{dt} = F_c dl \rho g - S(\cos \theta_r - \cos \theta_a) dl \tag{2.34}$$

where,

$$F_c \cong \frac{D^2}{8 \sin^2 \theta_m} \left[ \frac{\pi \theta_m}{90} - \sin 2\theta_m \right] \tag{2.35}$$

and  $\rho$  : the density of condensate.

By using Eqs. (2.33), (2.34) and (2.35),

$$\frac{d^2 D}{dt^2} + \frac{1}{D} \left( \frac{dD}{dt} \right)^2 = \frac{M}{D} - \frac{N}{D^3} \tag{2.36}$$

where

$$M = \frac{4D_c A g}{3\pi},$$

$$N = \frac{4D_c A}{3\pi} \times \frac{8S \sin^2 \theta_m (\cos \theta_r - \cos \theta_a)}{\rho \left\{ \frac{\pi \theta_m}{90} - \sin 2\theta_m \right\}}.$$

Now, we suppose a drop, whose contact diameter is  $D_0$  and is larger than the critical diameter  $D_c$ , is situated in the position shown in Fig. 2.4 at the time  $t=0$  and the position  $x=0$ . Until the drop coalesces with the first drop, the contact diameter  $D_0$  of the drop remains constant, then from Eqs. (2.34) and (2.35), the velocity  $w$  becomes

$$w = g \left( 1 - \frac{1}{z_1^2} \right) t \quad (2.37)$$

and distance  $x$ ,

$$x = \frac{1}{2} g \left( 1 - \frac{1}{z_1^2} \right) t^2 \quad (2.38)$$

where,

$$z_1 \equiv \frac{D_0}{D_c}.$$

While, the distance  $x$  is given by the following:

$$x = \sqrt{\frac{1}{n}} - \left( \frac{D_c}{2} + \frac{D_0}{2} \right). \quad (2.39)$$

Now, by using the followings mentioned in section 1,

$$\begin{aligned} D_c &\cong 1.6 \text{ mm}, \quad \theta_a \cong 95^\circ, \quad \theta_r \cong 85^\circ, \quad \theta_m \cong 90^\circ \\ A &= 0.45, \quad n_c = 0.2238 \times 10^6 \\ D_0 &= 1.67 \text{ mm}, \quad z_1 = 1.045 \end{aligned}$$

then, from Eqs. (2.38) and (2.39), the followings are obtained:

$$\left. \begin{aligned} &t = 0.07 \text{ sec} \\ \text{and from Eq. (2.37),} & \\ &w = 1.36 \text{ cm/s} \end{aligned} \right\} \quad (2.40)$$

Of course,

$$D = 1.67 \text{ mm}.$$

Next, by assuming that the sliding drop grows in size continuously thereafter, Eq. (2.36) is numerically integrated by means of Runge-Kutta's method by using Eq. (2.40) as the initial conditions. Then a knowledge is obtained that  $z$ ,

$$z \equiv \frac{D}{D_0},$$

quickly increases with time  $t$  as shown in Fig. 2.5 and, consequently, the swept track widens abruptly with the distance from the top of the condensing surface.

As for the distance  $x$ ,

$$x = \frac{3\pi D_0^3}{8D_c A} (z^2 - 1), \quad (2.41)$$

for the velocity of a drop,

$$w = \frac{3\pi D_0^3}{4D_c A} z \frac{dz}{dt}. \quad (2.42)$$

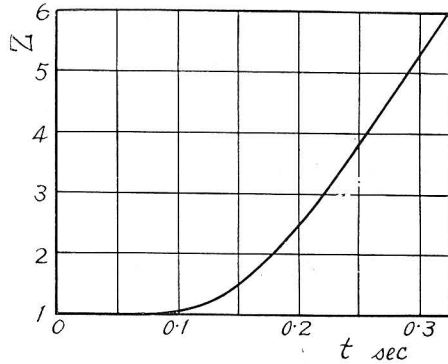


Fig. 2.5 Growth of a sweeping drop.

For example, when  $t=0.31$  sec,  $z=5.59$  ( $D=9.32$  mm)

then, from Eqs. (2.41) and (2.42),

$$x=14.3 \text{ cm}, \quad w=155 \text{ cm/s}$$

and consequently, the period of sweeping is very short and the downward velocity is very large even though the wall is not so high. Fig. 2.6 shows the sweeping drops on an actual condensing surface whose height is 30 cm.

The area  $B_0$  swept down by a drop (the area of bare stripe) may be expressed as follows:

$$dB_0 = Dw dt, \quad (2.43)$$

then,

$$B_0 = \frac{\pi D_0^3}{4D_c A} (z^3 - 1). \quad (2.44)$$

On the other hand, the area  $B_1$  which will be swept in future (the area still covered with droplets whose contact diameter is  $D_c$ ),

$$dB_1 = -Dw dt, \quad (2.45)$$

then,

$$B_1 = \frac{\pi D_c^3}{4D_c A} (z_m^3 - z^3). \quad (2.46)$$

The total area  $B_t$  swept by a drop will be

$$B_t = \frac{\pi D_0^3}{4D_c A} (z_m^3 - 1), \quad (2.47)$$

where,

$z_m$ :  $z$  at the lowest position of the condensing surface.

The quantity of heat transferred from the steam to the wall may be expressed as follows:

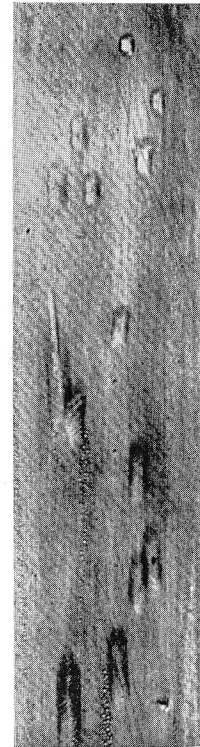


Fig. 2.6. The sweeping drops.

$$dq_2 = \left[ K_0 B_0 + \frac{K_0 k_d}{K_0 + k_d} a + \frac{K_0 \alpha_{s1} B_1}{K_0 + \alpha_{s1}} \right] \Delta T_t dt, \quad (2.48)$$

where,

$$a = \frac{\pi}{4} D^2.$$

Putting Eqs. (2.44) and (2.46) into Eq. (2.48),

$$q_2 = \frac{\pi}{4} K_0 \Delta T_t D_0^2 \int_0^{\tau_2} \left[ \frac{D_0}{D_c A} (z^3 - 1) + \frac{\frac{\lambda_d C}{K_0 D_0 f(\theta_m)} z^2}{z + \frac{\lambda_d C}{K_0 D_0 f(\theta_m)}} + \frac{\alpha_{s1}}{K_0 + \alpha_{s1}} \frac{D_0}{D_c A} (z_m^3 - z^3) \right] dt, \quad (2.49)$$

where,

$\tau_2$ : the interval of the sweeping period.

The average temperature drop  $\Delta T_{s2}$  from the steam to the condensing surface at the interval  $\tau_2$  is given by the following:

$$\begin{aligned} \Delta T_{s2} &= \frac{1}{\tau_2 B_t} \int_0^{\tau_2} \left[ \frac{K_0 \Delta T_t}{K_0 + k_d} a + \frac{K_0 \Delta T_t}{K_0 + \alpha_{s1}} B_1 \right] dt \\ &= \frac{\pi K_0 \Delta T_t D_0^2}{4 \tau_2 B_t} \int_0^{\tau_2} \left[ \frac{\frac{z^3}{K_0}}{\frac{\lambda_d C}{K_0 D_0 f(\theta_m)} + z} + \frac{1}{K_0 + \alpha_{s1}} \frac{D_0}{D_c A} (z_m^3 - z^3) \right] dt. \end{aligned} \quad (2.50)$$

Consequently, the heat transfer coefficient  $\alpha_{s2}$  at the sweeping period,

$$\alpha_{s2} = \frac{q_2}{\Delta T_{s2} \tau_2 B_t}, \quad (2.51)$$

is calculated by using Eqs. (2.49), (2.50) and (2.47).

For example, when

$$\tau_2 = 0.31 \text{ sec}, \quad z_m = 5.59$$

$$\alpha_{s1} = 3.11 \times 10^4 \frac{\text{kcal}}{\text{m}^2 \text{h}^\circ \text{C}} \quad (\text{the mean value obtained in section 1.})$$

then,

$$\alpha_{s2} = 3.18 \times 10^4 \text{ kcal/m}^2 \text{h}^\circ \text{C}$$

and

$$\Delta T_{s2} \tau_2 = 1.252 \text{ sec}^\circ \text{C}.$$

Thus, there is no appreciable difference between the heat transfer coefficient at the adhering period and that at the sweeping period. Consequently, although the parts belonging to the adhering period are mixed with the parts belonging to the sweeping period on the actual condensing surface, the average heat transfer coefficient for the total area of the condensing surface may be estimated with the heat transfer at the adhering period (given by Eq. (2.13)). And in the practical applications, it is much

more convenient to estimate the heat transfer coefficient  $\alpha_s$  with Eq. (2.13) since the calculation involved in it is quite simpler than Eq. (2.51).

However, we have led the theoretical equation (2.13) by assuming that all droplets exactly of the same size and shape at any point of the cycle; that the shape of droplets is a spherical segment and the droplets cover uniformly the condensing surface; and that the critical size of a drop at the end of the adhering period is exactly the same for all the droplets and nearly equal to the critical drop-size  $D_c$  given by Eq. (2.18). However, in the actual case, there are many droplets of various sizes on the condensing surface and many droplets, other than those formed near the top of the condensing surface, are swept very often before they attain the critical drop-size  $D_c$ . Consequently, so long as we follow the assumptions mentioned above, we should consider that the heat transmission coefficient  $k_d$  through a drop is, on the average, greater than that given by Eq. (2.25). Therefore, we propose the following equation to estimate the heat transfer coefficient instead of Eq. (2.13),

$$\alpha_s = \frac{V_c}{\int_0^{V_c} \frac{Ak_f + K_0 + \sigma k_d(1-A)}{\sigma k_d(AK_0 + k_f) + k_f K_0(1-A)} dV} \quad (2.52)$$

where,

$\sigma$ : the correcting factor.

For example, by using the results obtained in Chapter 1, (the case where the steam (100°C) condenses on the vertical brass-wall promoted by oleic acid)

$$D_c = 3.9 \text{ mm}, \quad \theta_a = 90^\circ, \quad \theta_r = 50^\circ, \quad \theta_m = -70^\circ$$

then the heat transfer coefficient  $\alpha_s$  becomes as follows:

In the case  $\sigma = 1$ ,

$$\alpha_s = 2.7 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C}.$$

In the case  $\sigma = 4$ ,

$$\alpha_s = 9.5 \times 10^4 \text{ kcal/m}^2\text{h}^\circ\text{C},$$

where

$$A = 0.4 \quad (\text{it is assumed that } A \text{ does not vary during the interval of the adhering period})$$

$$K_0 = 2500 \text{ kcal/m}^2\text{h}^\circ\text{C}.$$

In this way,  $\alpha_s$  is increased as  $\sigma$  is increased.

We consider that the value of the correcting factor  $\sigma$  is increased as the assumed condensing conditions on the metal surface, as mentioned above, deviate greatly from the actual condensing conditions. We shall report on this problem in a separate treatise.

The heat transfer coefficient  $\alpha_s$  by the dropwise condensation is obtainable by

Eq. (2.52). Then, assuming that  $A$  does not vary with time, the heat transfer coefficient can be estimated by the following equation :

$$\alpha_s = \frac{V_c}{\left[ \ln \left\{ V + \frac{\sigma \lambda_d C A (A K_0 + k_f)}{3 K_0 f(\theta_m) k_f (1-A)} \right\} \right]_{D_c} \left[ \frac{\sigma \lambda_d C A}{3 K_0 f(\theta_m) k_f} - \frac{\sigma \lambda_d C A (A K_0 + k_f)}{3 K_0 f(\theta_m) k_f (1-A)} \times \frac{A k_f + K_0}{k_f K_0 (1-A)} \right] + \frac{V_c (A k_f + K_0)}{k_f K_0 (1-A)} } \quad (2.53)$$

This theoretical equation contains various factors which affect heat transfer by the dropwise condensation, such as:  $D_c$ ,  $\theta_a$ ,  $\theta_r$ ,  $\theta_m$ ,  $A$ ,  $k_f$ ,  $K_0$ ,  $\tau$ ,  $\Delta T_s$  and etc.

### Chapter 3. The Life of the Dropwise Condensation.

There are two well-known ideal models of condensation of water vapour on a cooled metal surface; one is the filmwise condensation (the condensate is distributed as a continuous film on the metal surface) and the other is the dropwise condensation (the condensate is in the form of small drops which are segments of spheres). In the practical case, of course, both types of condensation usually occur simultaneously on the metal surface, in which case the droplets are distributed in irregular shapes and some parts are ideal dropwise while the others are filmwise. This type of condensation is called the "mixed type".

Fig. 1.1 shows the ideal dropwise condensation on the surface of the vertical brass-plate which is promoted by oleic acid, and Fig. 3.1 shows the mixed type of condensation mentioned above.

When the water vapour condenses on the test surface coated with oleic acid (generally known as a promoter), at first the condensate is distributed in very irregular shape of droplet (the size of which is larger than that of the ideal dropwise) because the surplus of oleic acid is present on the metal surface and the film of oleic acid is very thick, but when the surplus of oleic acid is gradually washed down by the condensate and the thickness of the film of oleic acid becomes very thin (according to Emmons<sup>3</sup>, the extent of thickness is several layers of oleic acid molecules), the ideal dropwise condensation begins, as shown in Fig. 1.1. Nevertheless, with the lapse of time, the test surface becomes more wettable to the water (condensate), the size of droplets becomes larger and their shapes irregular and, after the lapse of certain limited time,



Fig. 3.1. Mixed type condensation.



the ideal dropwise type breaks up and changes to the mixed type, as shown in Fig. 3. 1. This limited duration of time is called "life" of the purely dropwise condensation and, from the standpoint of the application, it is hoped for that the life is very long, for example, more than 1000 hours.

The life of the dropwise condensation is affected by various conditions, such as the following major factors :

- (a) The kind of metal and of promoter.
- (b) The degree of the surface finish of metal.
- (c) Initial cleanliness of metal surface before applying the promoter.
- (d) The method of application of the promoter.
- (e) Cleanliness of condenser-plant (steam-chest, steam pipes and steam boiler and feed water).
- (f) The condensing rate on the metal surface.
- (g) The pressure and temperature of the steam.
- (h) The temperature of the condensing surface.
- (i) The presence of a non-condensable gas.
- (j) The length and shape of the condensing surface.

Now, we shall discuss the effects of the factors above mentioned and find out an effective method to attain a longer life of the purely dropwise condensation.

- (a) This problem was researched by Nagle et al<sup>2)</sup> about twenty years ago and it was found that some contaminants were specifically effective on certain metals, e.g., benzylmercaptan on copper alloys, and others are quite generally effective, e.g., fatty acid (for example, oleic acid) on chromium, nickel, brass, etc. It is clear that a promoter should have a quality of an extremely low affinity for the vapour but a high affinity for the metal used; however, the promoter mentioned above has a weakness of poor affinity for the metal. Recently, Hampson<sup>4)</sup> reported that the mixed promoter (e.g., oleic acid and a light lubricating oil mixed) gives a longer duration than a single substance of the mixture, but even in this case, the duration of life is only forty to sixty hours. Therefore, we must study still further on the effective promoter.
- (b) In 1930, Schmidt et al<sup>1)</sup> succeeded in the dropwise condensation on a highly polished surface (the mirror finished chromium plate). But it might have been incorrect for them to have stated that only the mirror finishing of the surface without a promoter gives the dropwise condensation. Their achievement might have been due to formation of the Beilby layer or adsorption of some gas or greasy substance in the course of the polishing process. Nevertheless, according to these facts, it is clear that the mirror finishing is very effective to make the life long.
- (c) and (e) After we polished the test surface of a brass plate with No. 0000 emery

paper with the mixed powder of  $\text{CaCO}_3$  and  $\text{MgCO}_3$  and sufficiently washed the surface with the city water and benzene and naturally dried the surface, we polished the metal surface with a cloth containing a blur-preventing substance of glass. Upon conducting an experiment on this specimen, the dropwise condensation occurred contrary to our expectations. This might be due to the fact that the metal surface selectively adsorbed some greasy substance in the benzene or the cloth as mentioned in (b) or else it was due to uncleanliness of the condenser plant. The effects of the latter factor, namely, uncleanliness of the condenser plant, was examined by the following test; after we polished the surface with the No. 0000 emery paper and washed it, as mentioned above, we applied 18% solution of KOH on the test surface, and then the filmwise condensation occurred. However, when the cooling rate, namely condensing rate, was reduced, the filmwise type changed to the dropwise type. This may be due to the fact that, when the condensing rate is small, the test surface dries easily, therefore, the condensing surface is severely affected by some vapour of oil or greasy substance in the unclean steam-chest.

(f), (g) and (h) Fig. 3.2 shows an example of the change from the dropwise type

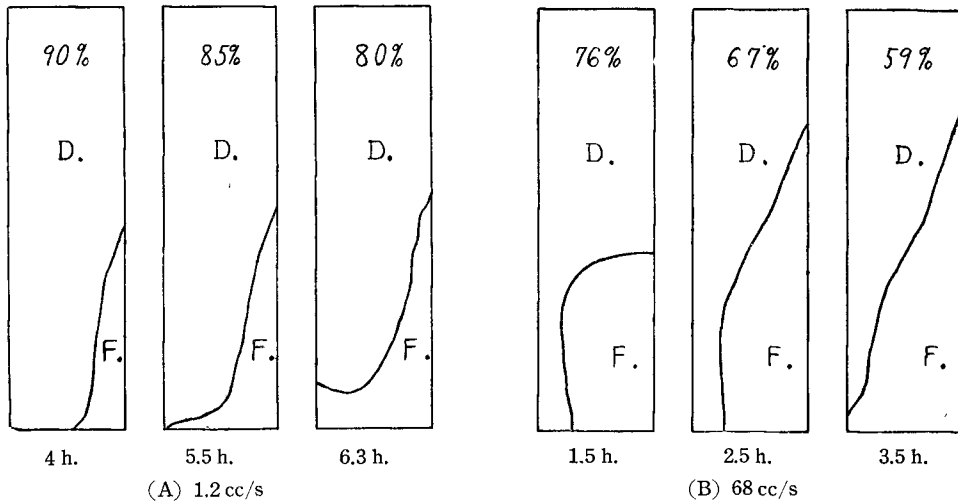


Fig. 3.2 Transition from Dropwise (D.) to Filmwise (F.).

to the filmwise type. (This dropwise condensation was obtained by the method without a promoter, as mentioned in (c) and (e)). In this figure, (A) deals with the case in which the cooling-water-flow-rate is 1.2 cc/sec and (B) 68 cc/sec. From these figures, it is clear that the life of the purely dropwise condensation is reduced as the cooling rate, namely the condensing rate, is increased.

As the pressure and temperature of the steam are increased, the condensing rate

(e.g. the heat load on the condensing surface) is slightly increased. Therefore, the life of the dropwise condensation is reduced.

The temperature of the condensing surface is affected by the pressure and temperature of the steam and the cooling rate (the temperature is reduced as the pressure and temperature of steam are reduced and also as the cooling rate is increased). Therefore, we must consider simultaneously the effects of factors (f), (g) and (h).

(i) Recently, Hampson<sup>4)</sup> reported that the presence of a non-condensable gas increases the life of the dropwise condensation, more than would be expected from the known reduction in the rate of condensation which results from the presence of the gas. But it is very dangerous, because the effects of a non-condensable gas on the heat transfer in the dropwise condensation is much severer than in the filmwise condensation.

(j) As shown in Fig. 3.2, the lower part of the condensing surface changes to the filmwise surface more rapidly than the upper part, because the lower part is washed down very often by the sliding drops which start near the upper part of the surface, as mentioned in Chapter 1. Consequently, the length of the condensing surface has a considerable bearing on the life of the dropwise condensation.

Lastly, we shall consider the factor (d), namely, the method of applying promoter, and discuss an effective method to attain a longer life.

The apparatus for testing the duration of the purely dropwise condensation is already shown in Fig. 1.8. When we let the steam condense on the cooled surface on which oleic acid was applied the purely dropwise condensation occurred after the surplus of oleic acid was washed down, and it had kept for more than 24 hours. (During this period, we did not change the water in the flask: therefore, the glass of the flask which was very clear at first became blurred). Next, we changed the water and washed the flask with 18% KOH solution and continued the experiment with the condensing surface remaining as it was, then one part of the cooling surface changed to the filmwise. But, when we poured some oleic acid into the water in the flask, the filmwise type changed to the purely dropwise type and this state had kept for more than 24 hours.

These facts show that the surplus of oleic acid washed down by condensate into the water in the flask or some oleic acid poured into the flask vapourized by the heating and the vapour was contained in the steam, and this vapour served to replenish oleic acid to the condensing surface (the glass surface of the flask which was very clear at first became blurred). As the result, the life of the dropwise condensation became longer and perhaps it might be permanent.

Further, we have found that the mixed type-condensing surface changes to the dropwise condensing surface when some oleic acid is poured into the steam pipe.

From these facts, we consider that an effective method of replenishing a promoter to the condensing surface under a continuous operation and a possibility of making the purely dropwise condensation permanent are discovered.

In order to prevent erosion of the condensing surface caused by  $\text{CO}_2$  or  $\text{O}_2$  in the steam, it has been attempted<sup>5)</sup> recently to make a film of octadecylamine continuously on the condensing surface by pouring amine into the steam with a chemical pump and to protect the surface from erosive condensate by adapting the non-wettable character of amine to the water, and its effectiveness for prevention of erosion on the practical condensing plant has been reported.

According to these operations, the possibility of making the purely dropwise condensation and of preventing the condensing surface can be expected. However, caution must be given to the fact that the greasy substances (for example, fatty acid) might flow into the feed water of the steam boiler with the condensate of steam condenser in the steam plant and the vapour of the greasy substances might pass through the turbine parts with the steam. Therefore, the condensate should be properly treated prior to feeding it into the steam boiler.

As the conditions for making the life of the purely dropwise condensation longer, the followings are important: (1) the mirror-finished polishing, (2) the initial cleanliness of the metal surface before applying a promoter and absence of moisture, (3) uncleanliness of the condenser plant, (4) low rate of condensing (low pressure and temperature of the steam and low flow rate of the cooling water), (5) presence of a non-condensable gas, and (6) low height of the condensing wall.

In practical applications, an effective method to induce the dropwise condensation continuously is the pouring of a promoter into the steam. The promoters on the condensing surface, then, are continuously replenished. Furthermore, this method is expected to prevent the condensing surface from erosion caused by carbonic acid gas or oxygen in the steam.

### Conclusion

The important factors related to the dropwise condensation are: (1) the critical size and shape of a drop, (2) the area covered by droplets, and (3) the sweeping cycle. The critical diameter and contact angle are given by Eq. (1.3). The ratio of the area covered by droplets to the unit surface area is small at the beginning of the adhering period, increases with the lapse of time and then becomes constant (about 0.5) at the end of the period. The interval of the sweeping is reduced as the flow rate of cooling water is increased and the lower part of the condensing surface is swept more often than the upper part.

The heat transfer coefficient by dropwise condensation is obtainable by Eq. (2.53),

which equation contains the various factors related to the heat transfer ; namely, the critical diameter and contact angle of drop, the area covered by droplets, the cooling rate, a non-condensable gas, the sweeping cycle, the temperature drop from the steam to the wall, and so on. We propose this theoretical equation as a calculating equation to estimate the heat transfer coefficient.

In practical applications, an effective method to induce the dropwise condensation continuously is the pouring of the promoter into the steam. Then, the promoters on the condensing surface are continuously replenished. In addition, it is expected that the erosion of the condensing surface caused by carbonic acid gas or oxygen in the steam is prevented by this method.

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