Scatter of Fatigue Life of Structural Steel and Its Influence on Safety of Structure

By

Ichiro KONISHI and Masanobu SHINOZUKA

Department of Civil Engineering

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1. Introduction

In the design of structure, it is most important to estimate the strength of material to be used and the load to be applied, and in order to estimate the safety of the structure it is necessary to investigate the characters of the distribution functions of strength and load.

From the point of this view, the fatigue experiment was carried out and the frequency distribution of the fatigue life of the structural steel was interpreted by the theory of the stochastic process.

2. Distribution of Fatigue Life of Steel

It has been well known that the frequency distribution of the fatigue life of steel or aluminum has wide scatter.

W. Weibull gave his empirical formula to the distribution of the fatigue life.[1) A. M. Freudenthal reported that it had approximately a logarithmic normal distribution.[2) Later, A. M. Freudenthal and E. J. Gumbel interpreted their experimental results by the theory of the distribution of the least value.[3) In our country, T. Yokobori regarded the phenomenon of the fatigue as a kind of the stochastic process and asserted from the standpoint of the metallographic physics that it was inevitable essentially for the distribution of the fatigue life to disperse widely.[4)

In all of these studies, it is remarkable that the distribution has the considerable positive skewness and wide scatter, but in our experiment the considerable positive skewness is not found and scatter is comparatively small.

Accordingly, so far as our experiment is concerned, the distribution of the fatigue life is interpreted more satisfactorily by the concept of stochastic process than by the logarithmic normal distribution or the distribution of the least value. And, as the small numbers of specimens were tested, it is difficult to infer the population.

In this paper, the following notations are used.

$\mu(N)$: the probability that the fracture occurs in unit cycle at N cycles.

$q(N)\,dN$: the probability that the fracture occurs between N and $N+dN$ cycles.

$p_l(N)=\int_N^\infty q(N)\,dN$: the probability that the fracture occurs after N cycles.

Then, we obtain the next equation.

$$q(N)\,dN=p_l(N)\,\mu(N)\,dN$$

$\mu(N)$ is derived from the above equation and represented by Eq. (1).

$$\mu(N)=-d(\ln p_l)/dN \tag{1}$$

From Eq. (1), it is found that $\mu(N)$ can be obtained from the slope of $N-\ln p_l$ diagram. p_l is calculated using the histogram obtained from the experiment or the relation $p_l=1-i/(1+n)$, where the latter p_l gives the probability of non-failure to the fatigue life N of i-th specimen among N specimens in order of the magnitude of N.

If $\mu(N)$ is constant, p_l is represented by Eq. (2).

$$p_l=\exp(-\mu N) \tag{2}$$

3. Experimental Result and Its Interpretation

For the purpose mentioned first, the specimen was tested under the same fabrication conditions as those of the member in the steel bridge. So, the specimen was not annealed, and the surface was left as it had been after the rolling. But to meet the statistical study, the exterior conditions of each specimen was manufactured to have same configuration as possible under the ordinary quality control.

The material of the specimen is structural steel SS 41. The specimen is shown by Fig. 1.

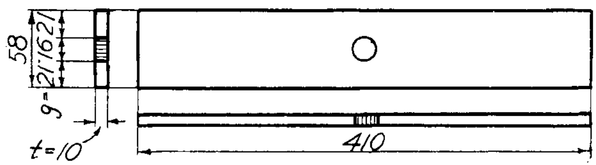

Fig. 1. Specimen for Fatigue Test.

The mechanical properties of the material are as follows: yield point $\sigma_s=27.7$ kg/mm^2, tensile strength $\sigma_B=44.5$ kg/mm^2, breaking strength on final area $\sigma_T=80.4$ kg/mm^2 and elongation $\varepsilon=28.5\%$ (G. L. = 150 mm).

The chemical composition of the material is shown by Table 1.

Losenhausen fatigue testing machine (type UHS) was used for pulsating tension test and the rate of the repetition of load was 800 cycles per minute.

Six kinds of experiment were carried out as follows.

The results of the experiments are shown in Table 3.

Table 1. Chemical Composition of Specimen

Element	C	Si	Mn	P	S	Cu
Content (%)	0.23	0.011	0.51	0.018	0.045	0.25

Table 2.

Experiment No.	1	2	3	4	5*	6**
Repeated stress (kg/mm²)	0～30	0～27	0～24	0～21	0～24 140,000 0～27 to fracture	0～27 50,000 0～24 to fracture
No. of specimens	20	20	25	20	20	20

* The repeated stress 0～27 kg/mm² was applied until the fracture after the repeated stress 0～24 kg/mm² was applied 140,000 cycles.

** The repeated stress 0～24 kg/mm² was applied until the fracture after the repeated stress 0～27 kg/mm² was applied 50,000 cycles.

Table 3. Results of Fatigue Test

Number i	Fatigue Life N					
	0～30 kg/mm²	0～27 kg/mm²	0～24 kg/mm²	0～21 kg/mm²	0～27 kg/mm² *	0～24 kg/mm² **
1	24040	44000	87270	201610	(82900)	5120
2	28160	68730	97620	206870	(103730)	35050
3	29810	70690	106680	210340	(105070)	46770
4	32280	70770	118600	211350	(107920)	54400
5	33080	78590	123500	254270	(127100)	56400
6	33540	80750	132950	294920	(127930)	59590
7	34630	84340	134610	313720	(128670)	61120
8	40960	86460	136940	365670	(133690)	62190
9	41090	89250	137340	386070	(138300)	62190
10	41380	94840	139380	415060	(139600)	63480
11	41670	105740	142390	424650	1500	63950
12	42040	105920	145450	435430	3000	69880
13	42900	106030	147400	441880	9190	75190
14	43330	107730	150000	466850	11280	82050
15	44450	108120	155310	541810	14090	85460
16	44690	109470	156110	559520	17900	93210
17	46460	110310	156790	622180	18100	105330
18	48600	111430	160300	667000	23920	107810
19	53550	126540	160430	669510	30070	112490
20	54100	128010	163500	862040	31280	124440
21			163970			
22			172630			
23			173180			
24			190040			
25			196320			

* The repeated stress 0～27 kg/mm² was applied until the fracture after the repeated stress 0～24 kg/mm² was applied 140,000 cycles.
The fatigue life of the specimen which fractured within the repetition of 140,000 cycles was represented in the parentheses.

** The repeated stress 0～24 kg/mm² was applied until the fracture after the repeated stress 0～27 kg/mm² was applied 50,000 cycles.

The relation between N and $\log p_l$ is illustrated in Fig. 2. The curve I, II, III, and IV correspond to experiment (1), (2), (3) and (4) respectively. From this diagram, it is found that μ is smaller at the begining of the curve and it becomes almost a constant value, μ_c, at the end.

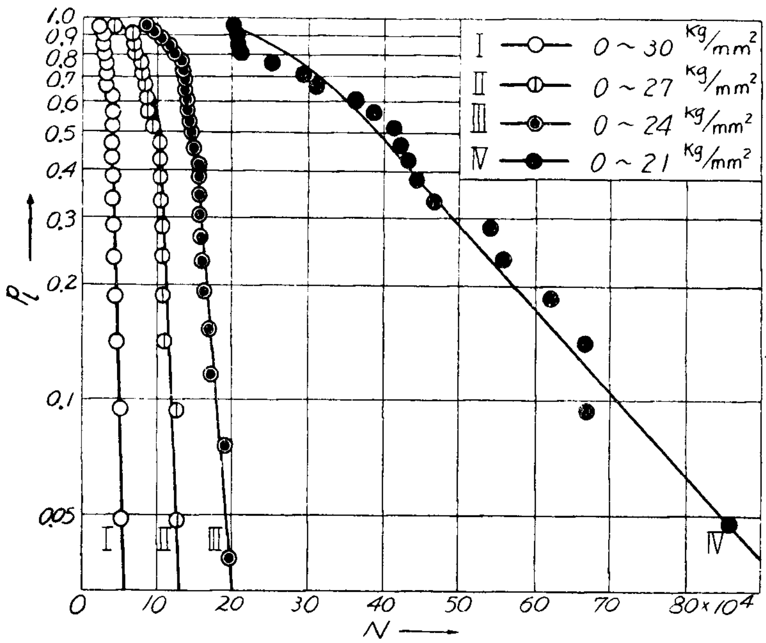

Fig. 2. Relation between N and p_l.
(fatigue test at constant stress levels)

The relation between μ_c and stress level, S, is approximately linear on the logarithmic scale as shown in Fig. 4 (a).

When the repeated stress $0 \sim 24\,\mathrm{kg/mm^2}$ is applied to a specimen, for example, the probability of non-failure of the specimen diminishes along the curve III, that is, we can estimate the probability of non-failure of the specimen to which the designated cycle of the repetition was applied.

Consequently, it can be said that we can estimate the probability of non-failure by such a curve when the constant repeated stress is applied to a specimen.

But, the problem is more complicated when the varying stress is applied to a specimen. The experiments (5) and (6) were carried out as the most simple cases in such a problem. The curves II and III are redrawn and the results of the experiments (5) and (6) are plotted in Fig. 3. In the case of experiment (5), for example, it is expected that the probability of non-failure of the specimen diminishes along the curve III to the value represented by the point A′ corresponding to the repetition of 140,000 cycles at the stress level 24 kg/mm², then it diminishes from the value represented by the point B′ (equal to the value represented by the point A′) to zero along the curve II, corresponding to the repetition at the stress level 27 kg/mm². Thus, the cycles of repetition at the stress level 0~27 kg/mm² are counted from the cycles corresponding to point A′, and the points in Fig. 3 are plotted under such a consideration. The same consideration can be made on the experiment (6). The figure shows a good agreement with the consideration mentioned above.

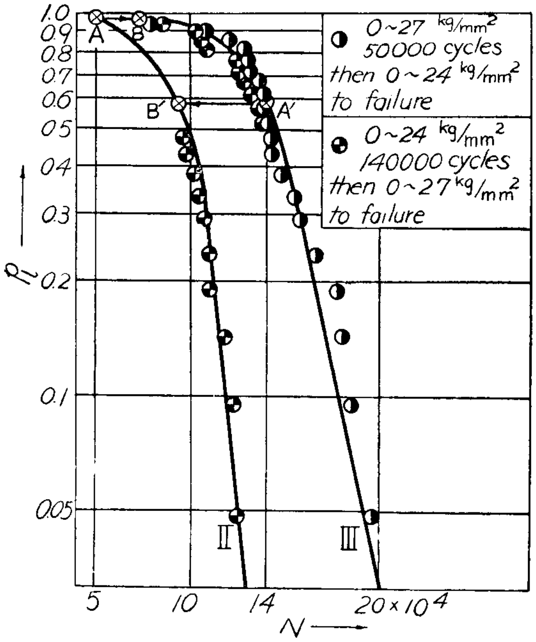

Fig. 3. Relation between N and p_l.
(fatigue test at two different stress levels)

Consequently, it is found that when two kinds of the repeated stress are applied to a specimen, the probability of non-failure is estimated using such two curves of constant stress levels.

The scatter of the fatigue life obtained from this experiment is considerably less than that reported by other investigators. This may be due to the following facts.

(1) The specimen used has a large stress concentration factor, 2.3.

(2) The surface of the specimen was finished roughly.

(3) The specimen was not annealed.

4. Probability of Non-failure due to Fatigue

In the design of the structural member, the fatigue life corresponding to the large value of p_l (nearly equal to unity) must be used. But, it is difficult to obtain $N-\ln p_l$ diagram in the region where p_l is sufficiently large because so many specimens and so much time are necessary for this purpose. Then, the following assumptions are used.

(1) $N-\ln p_l$ diagram starts from the point $N=0$, $p_l=1$ and is connected with the interpolated curve obtained experimentally at its begining.

(2) In this region, the curve is a straight line, and the value of μ is constant and can be calculated by using minimum fatigue life and its corresponding p_l and is expressed as μ_0.

Then, when the constant repeated stress is applied N cycles to a specimen, its probability of non-failure is expressed by Eq. (2) and when two kinds of the repeated stress $0\sim S_1\,kg/mm^2$ and $0\sim S_2\,kg/mm^2$ are applied N_1 and N_2 cycles respectively, its probability of non-failure, p_l, is given by Eq. (3). This is derived from the experiments (5) and (6).

$$p_l=\exp\{-(\mu_{01}N_1+\mu_{02}N_2)\} \tag{3}$$

where, μ_{01} and μ_{02} are the probabilities of the fracture occurrence in unit cycle at N cycles corresponding to the stress levels S_1 and S_2 respectively.

It is supposed that this consideration can be extended to the specimen subjected to the varying stress $0\sim S\,kg/mm^2$, where S is a variable. In this case, if the probability density function, $f(S)$, of S is given between $S_{min.}$ and $S_{max.}$ and the total cycles is given by N_T, the probability of non-failure is given by Eq. (4).

$$p_l=\exp\{-N_T\int_{S_{min.}}^{S_{max.}}\mu_0(S)\,f(S)\,dS\} \tag{4}$$

It must be mentioned that Eqs. (2), (3) and (4) are not valid when the value of p_l is smaller than the value which is used to calculate the value of μ_0 in the assumption (2).

The relation between $\log S$ and $\log \mu_0$ is approximately linear as shown by Fig. 4 (b), and this is conveniently used to calculate the value of p_l in Eqs. (2), (3) and (4).

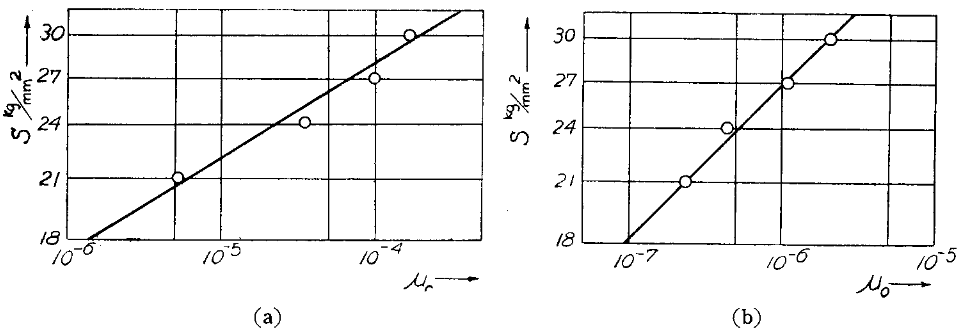

Fig. 4. Relation between μ and S.

5. Experimental Error

Several kinds of experimental errors are involved in the result represented by Table 3. They are (1) the error accompanied with the measurement of the cross sectional area of the specimen, (2) the error of accuracy of the loading, and (3) the error due to the fluctuation of the oil pressure of fatigue testing machine during testing.

We can estimate the scatter of the fatigue life due to these errors as follows.

(1) The error accompanied with the measurement of the cross sectional area of the specimen.

Under the assumption that the distribution of the error of measurement and estimation of length is a normal distribution, it is obtained from the data of measurement and the tolerance of dimension that the standard deviations, σ_g and σ_t, of the distribution of g and t, are equal to 0.1 mm and 0.05 mm respectively, where g is the width of one of the two minimum sections across the circular hole of the specimen and t is the thickness as shown in Fig. 1.

The distribution of the total sectional area, A, of these two minimum sections is approximately a normal distribution $N(\bar{A}, \sigma_A^2)$, where $\sigma_A^2=2(\bar{g}^2\sigma_t^2+\bar{t}^2\sigma_g^2)$ and $\bar{A}$ is the real value of A, and $\bar{g}$, $\bar{t}$ are the mean values of g, t respectively.

Maximum load $T=S_nA$ must be applied to the specimen in order that its minimum section is subjected to the maximum nominal mean stress S_n. But, due to the difference between $\bar{A}$ and A, the maximum mean stress S_1 which is produced actually at the minimum section of the specimen is not equal to S_n and represented by Eq. (5).

$$S_1=\frac{T}{\bar{A}}=\frac{S_nA}{\bar{A}} \tag{5}$$

Then it is found that the distribution of S_1 of the individual specimen is a normal distribution $N(S_n, \sigma_s^2)$, where σ_s is given by Eq. (6).

$$\sigma_s=\frac{S_n}{\bar{A}}\sigma_A=\frac{S_n}{\bar{A}}\sqrt{2(\bar{g}^2\sigma_t^2+\bar{t}^2\sigma_g^2)} \tag{6}$$

In general, each specimen has a different value of $\bar{A}$ and it can be assumed that the distribution of $\bar{A}$ is also a normal distribution. Consequently, from statistical theory, the distribution of S_1 becomes Cauchy's distribution. But, for convenience sake, it is assumed that S_1 is a random variable derived from a normal distribution $N(S_n, \sigma_1^2)$, where σ_1 is represented by Eq. (7).

$$\sigma_1=\frac{S_n}{A_{\text{min.}}}\sigma_A=\frac{S_n}{A_{\text{min.}}}\sqrt{2(\bar{g}^2\sigma_t^2+\bar{t}^2\sigma_g^2)} \tag{7}$$

This assumption gives the conservative result.

σ_A is equal to 1.96 mm², as the nominal dimensions 21 mm and 10 mm can be used as $\bar{g}$ and $\bar{t}$ respectively, and the minimum value, 410 mm, of A estimated from the measurement can be regarded as $A_{\text{min.}}$. Then, from Eq. (7), σ_1 is equal to 0.143, 0.129, 0.115 and 0.100 kg/mm² for $S_n=30$, 27, 24 and 21 kg/mm² respectively.

(2) The error of accuracy of the loading.

As the calibration of the dynamic load is not carried out, it is difficult to estimate the difference between the real oil pressure and its value indicated on the gauge. But, the difference gives no influence on the scatter of the fatigue life, as it gives the same error to the load of each specimen. So in this treatment, the error due to such a difference is not taken into account.

The gauge is graduated every 200 kg, then the maximum error of the load may be ± 100 kg, and the maximum error of the maximum mean stress may be about $\pm 100/400 = \pm 0.25$ kg/mm^2.

If it is assumed that the distribution of this error is normal, and the probability that the error of the maximum mean stress, S_2', is within the range of ± 0.25 kg/mm^2 is 0.99, the standard deviation, σ_2, of the distribution of S_2 is about 0.1 kg/mm^2. Therefore, the distribution of S_2 is a normal distribution $N(0, 0.097^2)$.

(3) The error due to the fluctuation of the oil pressure of fatigue testing machine during testing.

It is difficult to prevent the fluctuation of the oil pressures, i. e. the maximum load. The maximum load is controled to be in the range between lower limit load T_l and upper limit load T_u, that is, the automatic control of oil pressure prevents the load from decreasing below T_l and if the load exceeds T_u, the operation of testing machine is stopped.

The minimum load is controled in the same way.

Here, for simplicity, the fluctuation of the minimum load is neglected. This is expected to give conservative result, as it is found from the operation of testing machine that the phase and the frequency of the fluctuation of the maximum load are respectively equal to those of the minimum load.

Now, it is assumed that the effect of this fluctuating maximum load to the fatigue is equal to that given by the constant maximum load, T, under the same cycle of repetition. Namely, by introducing the constant maximum load T, we can estimate the degree of the fluctuation of oil pressure.

The distribution of the constant maximum load T can be estimated by Laplace's distribution. This can be understood by the method of control of testing machine and the actual observation of the fluctuation of the load. And then the distribution of the maximum mean stress S_3' corresponding to T, is also Laplace's distribution, $k \exp\{-k(S_3'-S_l)\}$, where S_l is the maximum mean stress corresponding to the lower limit load T_l. The value of $T-T_u$ is assumed to be less than 200 kg with the probability 0.99, that is, $S_3 = S_3' - S_l$ is less than 0.5 kg/mm^2 with the same probability. Using this value, we obtain $k = 9.22$ (kg/mm^2)$^{-1}$.

It is found, therefore, from the above considerations that the maximum mean stress S_a which is produced on the minimum section of the specimen is not equal to S_n but represented by next equation, as T_l is chosen equal to $S_n A$.

$$S_a = S_1 + S_2 + S_3 \tag{8}$$

From the statistical theory, it is understood that the distribution of $S = S_1 + S_2$ is a normal distribution $N(S_n, \sigma^2)$, where $\sigma^2 = \sigma_1^2 + \sigma_2^2$ and $\sigma = 0.173$, 0.161, 0.150 and 0.139 kg/mm² for $S_n = 30$, 27, 24 and 21 kg/mm² respectively. Then, the relation, $S_a = S + S_3$ is used instead of Eq. (8), where S is a random variable derived from normal distribution $N(S_n, \sigma^2)$ and S_3 is a variable from Laplace's distribution $k \exp(-kS_3)$.

The probability density, $f(S_a)\, dS_a$, of the distribution of S_a is represented by Eq. (9).

$$f(S_a)\, dS_a = \left\{ \int_{-\infty}^{S_a} k e^{-k(S_a - S)} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(S-S_n)^2}{2\sigma^2}} dS \right\} dS_a \tag{9}$$

Then, the distribution function $F(S_a)$ of S_a is given by Eq. (10).

$$\left.\begin{aligned} F(S_a) &= \int_{-\infty}^{S_a} f(X)\, dX = \Phi(S_a) - e^{\frac{k}{2}\{k\sigma^2 - 2(S_a - S_n)\}}\, \Psi(S_a) \\ \text{where,}& \\ \Phi(S_a) &= \int_{-\infty}^{S_a} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(X-S_n)^2}{2\sigma^2}} dX \\ \Psi(S_a) &= \int_{-\infty}^{S_a} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\{X-(S_n+k\sigma^2)\}}{2\sigma^2}} dX \end{aligned}\right\} \tag{10}$$

Now, we assume unique correspondence between the cycle of repetition N and the stress level S as shown in Fig. 5, where each of the plotted points represents the mean value of the fatigue life at each stress level, and the straight line is $S-N$ diagram for the probability of non-failure $p_l = 0.5$.

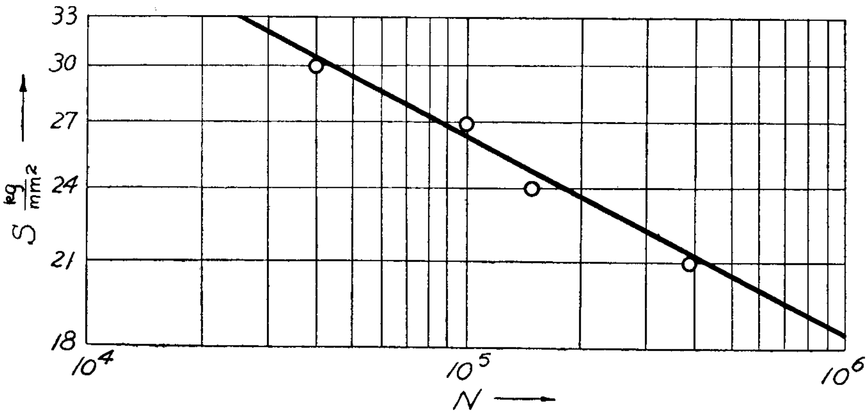

Fig. 5. $S-N$ diagram for $p_l = 0.5$.

From Fq. (10) and Fig. 5 the expected numbers of the specimens which fail in arbitrary intervals $(N, N+\Delta N)$ can be estimated, and these values are compared with the numbers obtained from the experiment as shown in Table 4. It is clear that the expected scatter of the fatigue life due to the experimental error is much less than that obtained from the experiment.

Consequently, it is found that the scatter of the fatigue life is not explained by the experimental error only. So the assumption of unique correspondence between N

Table 4.

<table>
<tr><td>a)</td><td colspan="18">$S_n=30$ kg/mm², $k=9.22$ (kg/mm²)⁻¹, $\sigma=0.173$ kg/mm²</td></tr>
<tr><td>S_a (kg/mm²)</td><td colspan="2">28.0</td><td colspan="2">28.5</td><td colspan="2">29.0</td><td colspan="2">29.5</td><td colspan="2">30.0</td><td colspan="2">30.5</td><td colspan="2">31.0</td><td colspan="2">31.5</td><td colspan="2">32.0</td></tr>
<tr><td>N (cycle)</td><td colspan="2">66 000</td><td colspan="2">59 000</td><td colspan="2">54 000</td><td colspan="2">48 000</td><td colspan="2">43 000</td><td colspan="2"></td><td colspan="2">34 000</td><td colspan="2"></td><td colspan="2">28 000</td></tr>
<tr><td>$F(S_a)$</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2">0.000 8</td><td colspan="2">0.309 0</td><td colspan="2">0.965 5</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Expected</td><td colspan="7">0</td><td colspan="2">6.2</td><td colspan="2">13.8</td><td colspan="7">0</td></tr>
<tr><td>Obtained</td><td colspan="3">0</td><td colspan="2">1</td><td colspan="2">2</td><td colspan="2">4</td><td colspan="4">7</td><td colspan="4">5</td><td colspan="1">1</td></tr>
<tr><td>b)</td><td colspan="18">$S_n=27$ kg/mm², $k=9.22$ (kg/mm²)⁻¹, $\sigma=0.161$ kg/mm²</td></tr>
<tr><td>S_a (kg/mm²)</td><td colspan="2">25.0</td><td colspan="2">25.5</td><td colspan="2">26.0</td><td colspan="2">26.5</td><td colspan="2">27.0</td><td colspan="2">27.5</td><td colspan="2">28.0</td><td colspan="2">28.5</td><td colspan="2">29.0</td></tr>
<tr><td>N (cycle)</td><td colspan="2">135 000</td><td colspan="2">120 000</td><td colspan="2">100 000</td><td colspan="2">90 000</td><td colspan="2">80 000</td><td colspan="2">74 000</td><td colspan="2">66 000</td><td colspan="2">59 000</td><td colspan="2">54 000</td></tr>
<tr><td>$F(S_a)$</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2">0.000 2</td><td colspan="2">0.294 5</td><td colspan="2">0.970 5</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Expected</td><td colspan="7">0</td><td colspan="2">5.9</td><td colspan="2">14.1</td><td colspan="7">0</td></tr>
<tr><td>Obtained</td><td colspan="1">0</td><td colspan="2">2</td><td colspan="2">8</td><td colspan="2">1</td><td colspan="2">4</td><td colspan="2">1</td><td colspan="2">3</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="1">1</td></tr>
<tr><td>c)</td><td colspan="18">$S_n=24$ kg/mm², $k=9.22$ (kg/mm²)⁻¹, $\sigma=0.150$ kg/mm²</td></tr>
<tr><td>S_a (kg/mm²)</td><td colspan="2">22.0</td><td colspan="2">22.5</td><td colspan="2">23.0</td><td colspan="2">23.5</td><td colspan="2">24.0</td><td colspan="2">24.5</td><td colspan="2">25.0</td><td colspan="2">25.5</td><td colspan="2">26.0</td></tr>
<tr><td>N (cycle)</td><td colspan="2">310 000</td><td colspan="2">270 000</td><td colspan="2">230 000</td><td colspan="2">200 000</td><td colspan="2">170 000</td><td colspan="2">150 000</td><td colspan="2">135 000</td><td colspan="2">120 000</td><td colspan="2">100 000</td></tr>
<tr><td>$F(S_a)$</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2">0.000 1</td><td colspan="2">0.288 0</td><td colspan="2">0.974 0</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Expected</td><td colspan="7">0</td><td colspan="2">7.2</td><td colspan="2">17.8</td><td colspan="7">0</td></tr>
<tr><td>Obtained</td><td colspan="7">0</td><td colspan="2">4</td><td colspan="2">8</td><td colspan="2">6</td><td colspan="2">3</td><td colspan="2">2</td><td colspan="1">2</td></tr>
<tr><td>d)</td><td colspan="18">$S_n=21$ kg/mm² $k=9.22$ (kg/mm²)⁻¹, $\sigma=0.139$ kg/mm²</td></tr>
<tr><td>S_a (kg/mm²)</td><td colspan="2">19.0</td><td colspan="2">19.5</td><td colspan="2">20.0</td><td colspan="2">20.5</td><td colspan="2">21.0</td><td colspan="2">21.5</td><td colspan="2">22.0</td><td colspan="2">22.5</td><td colspan="2">23.0</td></tr>
<tr><td>N (cycle)</td><td colspan="2">780 000</td><td colspan="2">690 000</td><td colspan="2">570 000</td><td colspan="2">470 000</td><td colspan="2">410 000</td><td colspan="2">360 000</td><td colspan="2">310 000</td><td colspan="2">270 000</td><td colspan="2">230 000</td></tr>
<tr><td>$F(S_a)$</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2">0.000 04</td><td colspan="2">0.273 5</td><td colspan="2">0.977 1</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Expected</td><td colspan="7">0</td><td colspan="2">5.5</td><td colspan="2">14.5</td><td colspan="7">0</td></tr>
<tr><td>Obtained</td><td colspan="1">1</td><td colspan="2">0</td><td colspan="2">3</td><td colspan="2">2</td><td colspan="2">5</td><td colspan="2">2</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="1">4</td></tr>
</table>

and S, must be rejected. In other words, the scatter of the fatigue life is inevitable essentially and $S-N$ diagram must be drawn by introducing the probability of non-failure p_l.

6. Conclusion

The summary and conclusion of this paper are as follows.

(1) The distribution of the fatigue life is interpreted satisfactorily by using the theory of the stochastic process.

(2) When the constant repeated stress is applied to a specimen, its probability of non-failure is estimated using the corresponding $N-\log p_l$ diagram, and when two kinds of the repeated stress are applied, the probability of non-failure is estimated by the corresponding two $N-\log p_l$ diagrams.

(3) The relation between stress level, S, and the probability of failure, μ_0, is linear on the logarithmic scale. This fact is used conveniently for the estimation of the probability of non-failure of the specimen subjected to the varying stress.

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