Statistical Representation of S-N Curve on the Fatigue Test Results

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Introduction

It is generally recognized that fatigue test results are widely scattered. Consequently, in treating fatigue test data, it is desirable to represent them statistically and, of late, the studies in this line are frequently reported[1)~5)]. In all of these studies, fatigue tests are carried on many test pieces on each stress level, and the distribution of the number of cycles to fracture N is statistically studied; and the probabilities of fracture P are obtained by application of the most probable distribution function and the S–N–P curves are drawn. However, to draw S–N–P curves by these methods, many a fatigue tests must be performed on many specimens on numerous stress levels and it is felt that conducting such experiments in most cases is practically difficult because of the exceedingly long duration of time and huge expenditure required. On the contrary, the method the authors propose here is a method which does not necessitate performance of many tests on the same stress level, and yet gives the probabilities of fracture P. In other words, the S–N–P curves are obtained from the whole fatigue test results obtained on different stress levels, even with a single test performed for each stress level.

1. Determination of S–N Curve

For the purpose of obtaining the most probable S–N curve, it is quite useful to represent fatigue test results with an equation. Although there are many equations which represent the S–N relations[1)2)5)6)], the following equation is applied in this study:

$$\sigma - \sigma_w = AN^m \quad (m < 0) \tag{1}$$

where N is a number of cycles to fracture under a repeated stress σ (kg/mm^2), σ_w is a constant which represents the endurance limit (kg/mm^2), and A and m are arbitrary constants.

Taking the common logarithm of both sides of Eq. (1), then we have

$$\log_{10}(\sigma-\sigma_w)=\log_{10}A+m\cdot\log_{10}N \tag{2}$$

By transforming variables by the following substitution,

$$\left.\begin{aligned}\log_{10}(\sigma-\sigma_w)&=y\\ \log_{10}A&=a\\ \log_{10}N&=x\end{aligned}\right\}, \tag{3}$$

the following relation is obtained:

$$y=a+mx \tag{4}$$

Here the correlation between x and y becomes linear.

Before determining the constants a and m, the value of a constant σ_w must be determined. For the different sets of values of σ and N to be obtained from fatigue test results, it is best to determine the value of σ_w so as to best satisfy the linear relation of Eq. (4). In other words, the correlation coefficient r is computed between x and y, and the value of σ_w is determined to make the absolute value of r to be maximum. For an arbitrary value of σ_w, x and y computed from n experimental results are denoted σ_w as x_i and y_i $(i=1, 2, 3, \cdots, n)$ and the correlation coefficient r is given as follows:

$$r=\frac{\sum_{i=1}^{n}x_iy_i-\frac{1}{n}\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{\sqrt{\sum_{i=1}^{n}x_i^2-\frac{1}{n}(\sum_{i=1}^{n}x_i)^2}\sqrt{\sum_{i=1}^{n}y_i^2-\frac{1}{n}(\sum_{i=1}^{n}y_i)^2}} \tag{5}$$

The constants a and m are computed by the least square method for the value of σ_w thus determined and are given by the following equations:

$$\left.\begin{aligned}a&=\frac{\sum_{i=1}^{n}x_i^2\sum_{i=1}^{n}y_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}x_iy_i}{n\sum_{i=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2}\\ m&=\frac{n\sum_{i=1}^{n}x_iy_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2}\end{aligned}\right\} \tag{6}$$

Thus, the *S-N* curve can be drawn.

2. Scatter of Fatigue Test Results

The conventional method of computing the probability of failure P is: the results obtained by numerous tests with many specimens on each stress level are arranged in an increasing order of the magnitudes of fatigue lives and then the probability P

is calculated[1)3)5)]. In this study, however, the values of P obtained from the whole fatigue test results for the various stress levels are computed. The most probable S–N curve for the test results can be determined by the method described in the preceding paragraph. The S–N equation thus determined is assumed as Eq. (1). The S–N curve of Eq. (1) does not generally pass through the plotted points of experimental values in the S–N diagram. But by changing the value of any one of the three parameters σ_w, A, and m in Eq. (1), we can make the S–N curve pass through a test point. Therefore, we can reduce the scatter of test results to the scatter of any one of the three parameters.

In the case in which the value of A and m are constant and only the value of σ_w varies with an increase of P, a vertical distance between the original S–N curve and the new S–N curve (which has been made to pass through each test point by changing the value of σ_w only,) is constant and independent of the value of N as shown in Fig. 1, if the measurement of the ordinate σ in the S–N diagram is made with a linear scale. Ransom[4)] states that the values of the endurance limits are widely scattered about a mean of their values. Therefore, for such test results showing such a scatter of test points, the method in which the value of σ_w alone is changed can be applied.

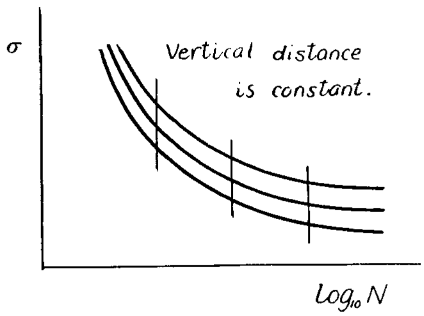

Fig. 1. S–N curves in which the endurance limit σ_w is only varied.

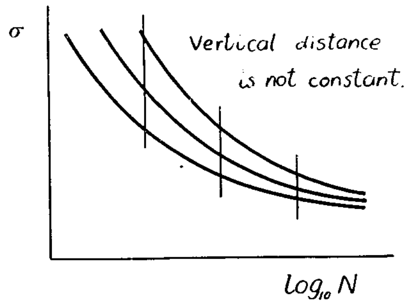

Fig. 2. S–N curves in which the parameter A is only varied.

Now, when the values of σ_w and m are constant and the value of A only varies with an increase of P, a vertical distance between the original S–N curve and the new S–N curve (which has been made to pass through each test point by changing the value of A,) decreases with an increase of the value N as shown in Fig. 2, if the measurement of the ordinate σ is made with a linear scale as above. Also in the case where the values of σ_w and A are constant and the value of m only varies with an increase of P, the similar tendencies as shown in Fig. 2 are obtained. In many cases, the endurance limit scarcely shows any scatter of its value, but their fatigue lives on each stress level are widely scattered. For the cases of this type, the method in which the value of A only (or the value of m only) is changed can be applied.

However, in general, it seems better to vary adequately the values of all the three parameters σ_w, A, and m in the S–N equation with an increase of P. As mentioned above, it is known that both the methods of changing the value of A only and m only indicate the similar tendencies. In this study, the method of calculating the value P in the case where the value of m is kept constant and other two values σ_w and A vary with P while maintaining a definite relation between them. (The method in which the values σ_w and m are changed keeping the value A as a constant is similar to this.)

3. Relation between the Parameter A and σ_w

The S–N equation determined by the least square method is denoted as follows:

$$\sigma-\sigma_{w0}=A_0N^m \qquad (7)$$

Taking into consideration the condition of scatter of test points, a scatter band is drawn in the S–N diagram. Now, making the value of the parameter m constant and assuming the equation of the lower boundary of the scatter band as

$$\sigma-\sigma_w'=A'N^m, \qquad (8)$$

and taking the two points (σ_1, N_1) and (σ_2, N_2) on its boundary adequately, then the following relations are obtained from Eq. (8).

$$\left.\begin{aligned}\sigma_1-\sigma'_w=A'N_1^m\\ \sigma_2-\sigma'_w=A'N_2^m\end{aligned}\right\} \qquad (9)$$

From these relations, the values of the constants σ_w' and A' are calculated as follows:

$$\left.\begin{aligned}\sigma_w'&=\frac{\sigma_1-\left(\dfrac{N_1}{N_2}\right)^m\sigma_2}{1-\left(\dfrac{N_1}{N_2}\right)^m}\\ A'&=\frac{\sigma_1-\sigma_2}{N_1^m-N_2^m}\end{aligned}\right\} \qquad (10)$$

Accordingly, when the two points (σ_1, N_1) and (σ_2, N_2) are determined, the constants σ_w' and A' of the lower boundary of the S–N scatter band can be determined.

Next, when the equation of the upper boundary of the scatter band is taken as

$$\sigma-\sigma_w''=A''N^m, \qquad (11)$$

the two constants σ_w'' and A'' are calculated in a similar manner described above by taking the two points (σ_3, N_3) and (σ_4, N_4) adequately on the upper boundary. Hence, we have

$$\left.\begin{aligned} \sigma_w{}'' &= \frac{\sigma_3-\left(\dfrac{N_3}{N_4}\right)^m \sigma_4}{1-\left(\dfrac{N_3}{N_4}\right)^m} \\ A' &= \frac{\sigma_3-\sigma_4}{N_3{}^m-N_4{}^m} \end{aligned}\right\} \quad (12)$$

Then, the three sets of values (σ_{w0}, A_0), $(\sigma_w{}', A')$, and $(\sigma_w{}'', A'')$ are plotted into the diagram, Fig. 3, which represents the value of A in ordinate and σ_w in abcissa. It is assumed that the relation between A and σ_w can be represented by the two straight lines shown in Fig. 3. Then, the relations between A and σ_w become as follows:

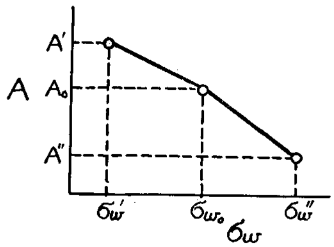

Fig. 3. Relation between A and σ_w.

$$\left.\begin{aligned} A &= A_0+\frac{A'-A_0}{\sigma_w{}'-\sigma_{w0}}(\sigma_w-\sigma_{w0}) \quad (\text{for } \sigma_w \leqq \sigma_{w0}) \\ A &= A_0+\frac{A''-A_0}{\sigma_w{}''-\sigma_{w0}}(\sigma_w-\sigma_{w0}) \quad (\text{for } \sigma_w \geqq \sigma_{w0}) \end{aligned}\right\} \quad (13)$$

In the next stage, the values of σ_w and A corresponding to each experimental value are determined from Eq. (1) applying the relation of Eq. (13). Then, the scatter of experimental values can be reduced to the scatter of value of the parameter σ_w.

In the method discussed above, an appropriate estimation of the shape of a scatter band must be allowed. Also, in the method in which the values of both parameters σ_w and A are changed, some relation between them must be assumed in order to represent the scatter of test results as the scatter of values of σ_w. This relation can be obtained by estimating the shape of the scatter band. The estimation of the scatter band depends upon the judgment with the eye and has no theoretical basis. However, it may be permissible similarly as Eq. (1) is applied at the S-N equation without any theoretical basis.

4. Determination of Probability of Failure

The method to compute the probability of failure P from the scatter of the value z which represents one of the three parameters σ_w, A, and m shall be explained. The n values of z, corresponding to each experimental value, are obtained by substituting experimental values into Eq. (1) and these n values are arranged in an increasing order of their magnitudes and numbered from 1 to n. The expected value of the probability of failure P_ν corresponding to the value of z numbered as ν i.e. z_ν, (that is, the probability that the value of z is less than or equal to z_ν) can be computed by the following equation[1].

$$P_\nu=\frac{\nu}{n+1} \quad (\nu=1, 2, 3, \cdots, n) \tag{14}$$

The relation between z and P is indicated by plotting the n points (z_ν, P_ν) computed above into a diagram which represents the value of P in ordinate and z in abcissa as shown in Fig. 4, and the most fitted equation of the curve for the computed values is sought. The type of the function P which is suitable for the cumulative distribution function of the random variable z is considered to be something like the following:

$$P=1-e^{-\varphi(z)} \tag{15}$$

where $\varphi(z)$ expresses an increasing function of z. Many equations have been introduced as the function of $\varphi(z)$ in Eq. (15)[1)3)5)], but in this study the following equation is adopted as the cumulative distribution functions of z:

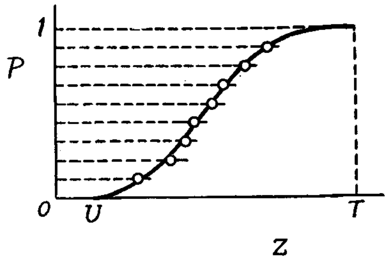

Fig. 4. Relation between P and z: plotted points are the calculated values and the solid line represents the cumulative distribution curve of z.

$$P=1-e^{-k\left(\frac{z-U}{T-z}\right)^b} \tag{16}$$

which satisfies the following conditions:

$$\left.\begin{array}{lll} P=0^* & \text{at} & z=U \\ P=1 & \text{at} & z=T \end{array}\right\} \tag{17}$$

where T and U are the constants showing the upper and lower limit of z, and k and b are arbitrary constants. Taking twice the common logarithm of both sides of Eq. (16), we have

$$\log_{10}\log_{10}\frac{1}{1-P}=\log_{10}k+\log_{10}\log_{10}e+b\log_{10}\left(\frac{z-U}{T-z}\right). \tag{18}$$

Putting

$$\left.\begin{array}{l} \log_{10}\log_{10}\dfrac{1}{1-P}=Y \\ \log_{10}k+\log_{10}\log_{10}e=K \\ \log_{10}\left(\dfrac{z-U}{T-z}\right)=X \end{array}\right\}, \tag{19}$$

the following linear correlation between X and Y is obtained.

$$Y=K+bX \tag{20}$$

These constants T and U are determined so as to satisfy the relation of Eq. (20)

* The S-N curve for $P=0$ is important depending upon applications for design, etc.—especially in estimation of a safety factor.

most fittedly: that is, the correlation coefficient r between X and Y computed from the n sets of values (z_ν, P_ν) are calculated for the various values of T and U, and the most probable values of T and U, which make the value of r maximum, are determined. The constants k and b are determined by the least square method from the values of X and Y which are calculated from the n values (z_ν, P_ν), using the value of T and U determined above.

Thus, the cumulative distribution function P of a random variable z being determined and the value of z corresponding to a given arbitrary value of P being computed, we can now draw the S–N curves with parameter P (S–N–P curves).

5. Numerical Examples

(1) Method in which the Parameter σ_w is varied

The numerical example is indicated in which the values of parameters A and m are kept constant, another parameter σ_w varies with P, and the scatter of test results is reduced to the scatter of the parameter σ_w. This computation process is applied to the rotating bending fatigue test results of the rail steel (0.7% carbon steel). These test results are shown in Table 1 and the S–N diagram in Fig. 5. The eight

Table 1. Fatigue Test Results of Rail Steel.

No.	Stress σ (kg/mm^2)	Number of Cycles to Failure N
21	44.7	44.5×10^3
77	42.9	71.0
4	41.4	155.0
27	38.7	252.0
37	36.7	367.0
3	36.2	909.0
30	35.7	399.0
78	34.8	1580.0

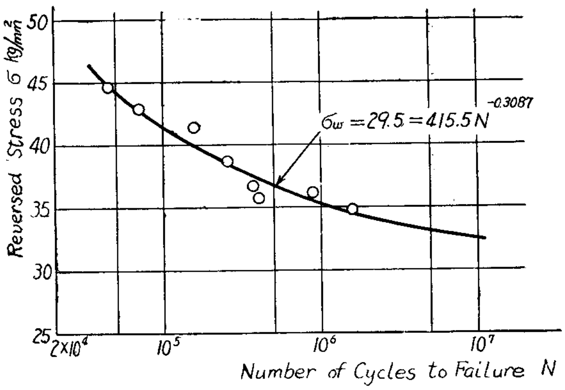

Fig. 5. S–N curve of Rail Steel computed by the Least Square Method.

plotted points in the diagram indicate the respective test results. As the results of computations by the least square method, the most probable S–N equation is obtained as follows:

S–N equation: $\sigma-29.5=415.5\,N^{-0.3087}$

Endurance limit: $\sigma_w=29.5$ kg/mm^2

and the correlation coefficient becomes: $r=-0.95537$

The solid line shown in Fig. 5 is the S–N curve thus computed. Fig. 6 shows the relation between the endurance limit σ_w and the absolute value of the correlation coefficient r, and Fig. 7 shows the linear correlation between x and y, that is the S–N relation plotted in log-log. scale.

Next, the values of endurance limit corresponding to the experimental values are calculated (where the values of parameters A and m are kept constant), and by means of Eq. (14) the probabilities of failure P corresponding to the values of σ_w are determined as shown in Table 2 and graphically in Fig. 8. The calculated results by the least square method of the most fitted curve as the distribution function of σ_w for these computed points are as follows:

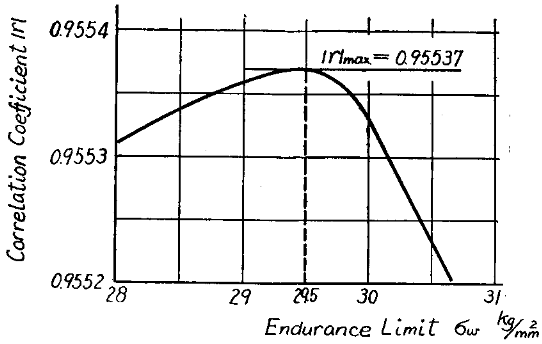

Fig. 6. Correlation Coefficient vs. Endurance Limit for Rail Steel.

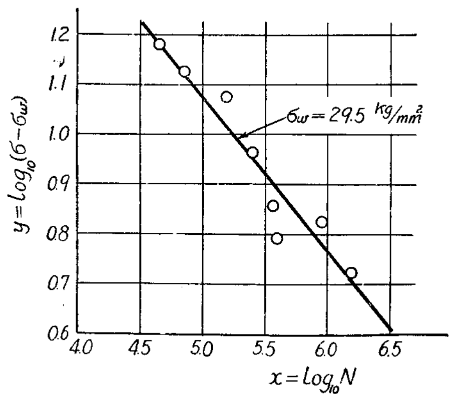

Fig. 7. S–N Curve of Rail Steel plotted by Log-Log Scale.

Table 2. Endurance Limit & its Probability of Failure corresponding to Experimental Value.

No.	Endurance Limit σ_w (kg/mm²)	Order ν	Probability of Failure $P=\frac{\nu}{n+1}$
30	27.95	1	0.111
37	28.74	2	0.222
21	29.44	3	0.333
77	29.69	4	0.444
78	29.73	5	0.556
27	29.76	6	0.667
3	30.19	7	0.778
4	31.02	8	0.889

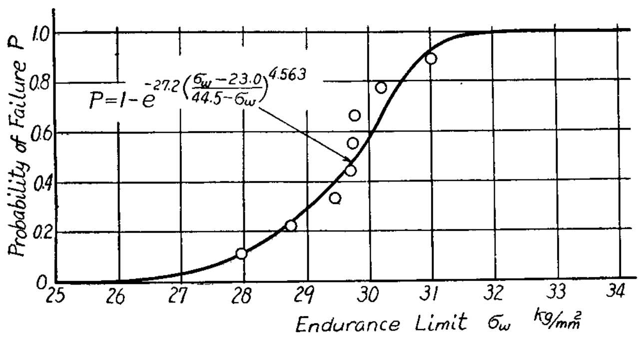

Fig. 8. Cumulative Distribution Curve of Endurance Limit for Rail Steel.

Cumulative distribution function: $P=1-e^{-27.2\left(\frac{\sigma_w-23.0}{44.5-\sigma_w}\right)^{4.563}}$

Here, the correlation coefficient between X and Y becomes:

$$r=0.97164$$

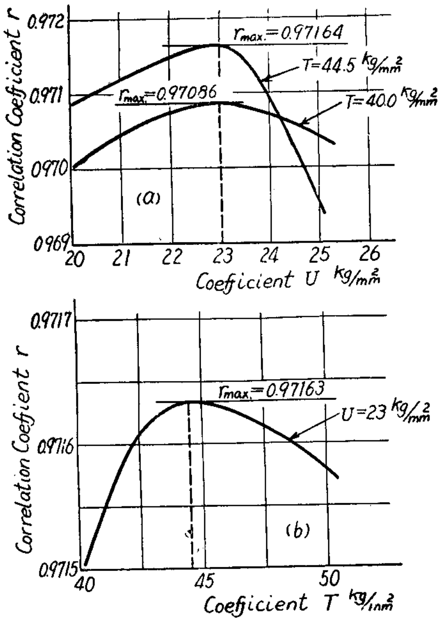

Fig. 9. Correlation Coefficient vs. Coefficients U and T for Rail Steel.

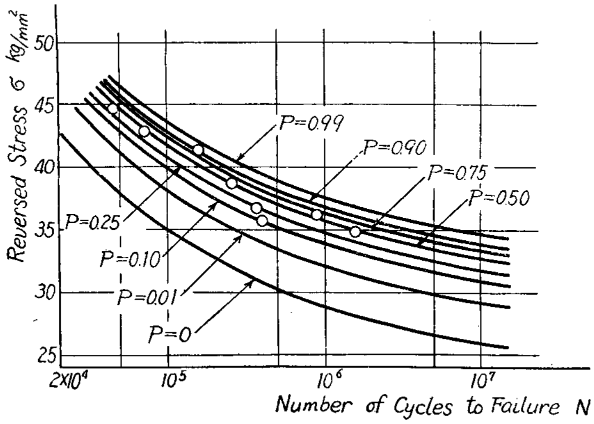

Fig. 10. S-N Curves of Rail Steel with Parameter P.

Table 3. Fatigue Test Results of 0·22% Carbon Steel.

No.	Stress σ (kg/mm²)	Number of Cycles to Failure N
1	33.0	1086400
2	33.0	643460
3	32.5	1056700
4	32.5	1175300
5	32.0	1644400
6	32.0	1334160
7	31.5	1034700
8	31.5	920810
9	31.0	1339800
10	31.0	2051300
11	30.5	1516200
12	30.5	1177040
13	30.0	4883200
14	30.0	4633980
15	29.5	8471000
16	29.5	5024150
17	29.2	3442400
18	29.2	4146600
19	28.5	6407700

The cumulative distribution curve of σ_w thus obtained are shown by a solid line in Fig. 8. Fig. 9 shows the relation between the correlation coefficient r and the values of T and U. The S-N curves with the probability of failure P as a parameter are obtained as shown in Fig. 10.

(2) Method in which the Parameter A is varied

Secondly, the similar numerical example is explained in which parameters σ_w and m are kept constant, another parameter A varies with P, and the scatter of test values is represented as the scatter of the parameter A. This computation process is applied to the rotating bending fatigue test results of the 0.22% carbon steel. Nineteen experimental values are shown in Table 3 and S-N plots of these values are shown in Fig. 11. The computation results by means of the least square method are as follows:

S-N equation: $\sigma - 25.0 = 316.9\, N^{-0.2762}$

Endurance limit: $\sigma_w = 25.0$ kg/mm²

Correlation coefficient: $r = -0.87103$

The solid line in Fig. 11 is the *S-N* curve computed as above.

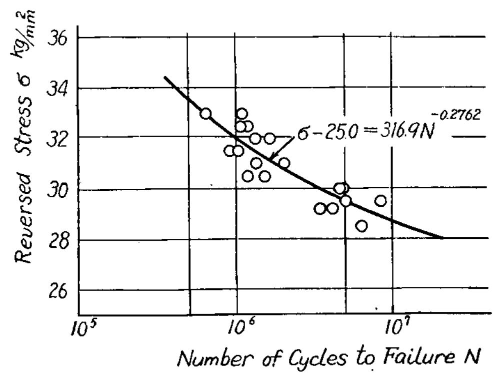

Fig. 11. *S-N* Curve of 0.22% Carbon Steel computed by the Least Square Method.

The values of the parameter A corresponding to the experimental values are calculated (where the values of the parameters σ_w and m are kept constant), and by means of Eq. (14), the probabilities of failure P corresponding to the values of A are determined and they are shown in Table 4 and graphically in Fig. 12. For these calculated values, the most fitted distribution function of A is computed by the least square method as follows:

Cumulative distribution function: $P=1-e^{-1.11\left(\frac{A-250}{480-A}\right)^{0.998}}$

Correlation coefficient: $r=0.98678$

The cumulative distribution curve of A is represented by a solid line in Fig. 12.

Table 4. Coefficient A & its Probability of Failure corresponding to Experimental Value.

No.	Coefficient A	Order ν	Probability of Failure $P=\frac{\nu}{n+1}$
12	261.2	1	0.05
19	265.4	2	0.10
17	268.3	3	0.15
11	280.1	4	0.20
18	282.4	5	0.25
8	288.4	6	0.30
9	295.3	7	0.35
7	297.9	8	0.40
16	319.1	9	0.45
2	321.6	10	0.50
10	332.2	11	0.55
6	344.2	12	0.60
3	345.7	13	0.65
14	346.7	14	0.70
13	351.8	15	0.75
4	356.1	16	0.80
5	364.6	17	0.85
15	368.6	18	0.90
1	371.6	19	0.95

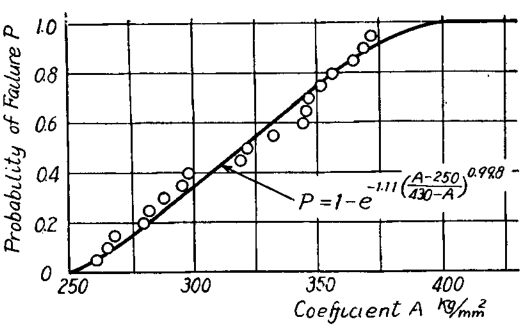

Fig. 12. Cumulative Distribution Curve of Coefficient A for 0.22% Carbon Steel.

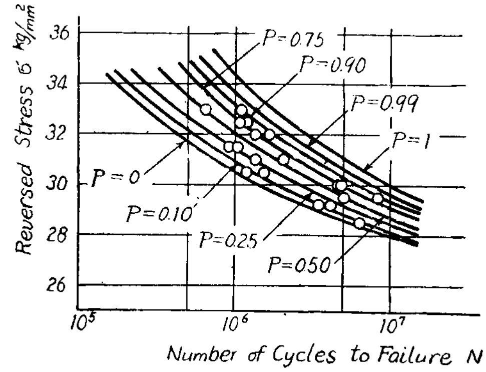

Fig. 13. *S-N* Curves of 0.22% Carbon Steel with Parameter P.

The S–N curves with a parameter P are shown in Fig. 13.

(3) Method in which the Parameters σ_w and A are varied

Thirdly, the numerical example is indicated in which the parameter m is kept constant, the other parameters σ_w and A vary with P, while maintaining a definite relation between them, and the scatter of test results is reduced to the scatter of the parameter σ_w. This computation process is applied to the rotating bending fatigue test results of the 0.61% carbon steel. Seventeen test results are given in Table 5 and S–N plots of these test values are shown in Fig. 14. The results computed by means of the least square method are as follows:

Table 5. Fatigue Test Results of 0·61% Carbon Steel.

No.	Stress σ (kg/mm²)	Number of Cycles to Failure N
1	35.0	108200
2	35.0	118080
3	33.0	297100
4	33.0	291770
5	32.0	402500
6	32.0	335900
7	31.0	616900
8	31.0	602520
9	31.0	803600
10	30.0	2971700
11	30.0	1166840
12	30.0	739800
13	29.0	4821600
14	29.0	2911120
15	29.0	1659200
16	28.0	3482000
17	28.0	7602100

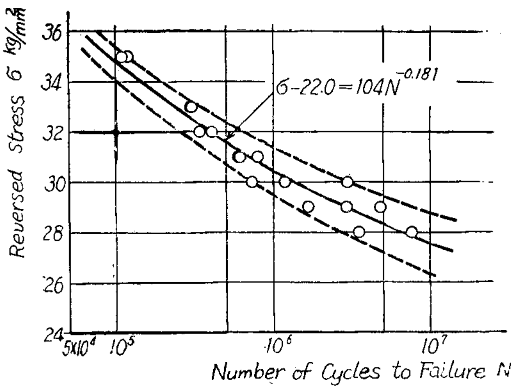

Fig. 14. S–N Curve of 0.61% Carbon Steel.

S–N equation: $\sigma-22.0=103.75\,N^{-0.181}$

Endurance limit: $\sigma_w=22.0\,\text{kg/mm}^2$

Correlation coefficient: $r=-0.95812$

The solid line given in Fig. 14 is the S–N curve calculated as above.

Now, the scatter band in the S–N diagram is drawn adequately as mentioned in Chapter 3. These two boundary lines of the scatter band are indicated by the dotted lines in Fig. 14. Taking the following two points on the lower boundary of the scatter band,

$$\left.\begin{aligned}\sigma_1&=35\,\text{kg/mm}^2, & N_1&=7.00\times10^4\\ \sigma_2&=30\,\text{kg/mm}^2, & N_2&=7.40\times10^5\end{aligned}\right\},$$

and calculating the coefficients σ_w' and A' of the lower boundary from these values by means of Eq. (10), the following values are obtained:

$$\sigma_w'=20.61\,\text{kg/mm}^2$$
$$A'=108.41\,\text{kg/mm}^2$$

In a similar manner, the coefficients σ_w'' and A'' of the upper boundary are calculated by taking the following two points,

$$\left.\begin{array}{ll}\sigma_3=35\,\text{kg/mm}^2, & N_3=1.18\times10^5\\ \sigma_4=30\,\text{kg/mm}^2, & N_4=2.97\times10^6\end{array}\right\},$$

on its boundary line, the following values are obtained:

$$\sigma_w''=23.69\,\text{kg/mm}^2$$
$$A''=93.63\,\text{kg/mm}^2$$

From the *S–N* equation, the values of σ_{w_0} and A_0 are as follows:

$$\sigma_{w_0}=\ 22.00\,\text{kg/mm}^2$$
$$A_0=103.75\,\text{kg/mm}^2$$

These three sets of values $(\sigma_{w_0},\ A_0)$, $(\sigma_w',\ A')$, and $(\sigma_w'',\ A'')$ are plotted in the $A-\sigma_w$ diagram as shown in Fig. 15 and the relation between A and σ_w is assumed to be expressed by the two straight lines passing through these three points in Fig. 15: that is, the relation between A and σ_w is assumed to be expressed by the following equations:

$$A=103.75-3.353\ (\sigma_w-22)\quad \text{for}\ \ \sigma_w\leqq22.0\,\text{kg/mm}^2$$
$$A=103.75-5.988\ (\sigma_w-22)\quad \text{for}\ \ \sigma_w\geqq22.0\,\text{kg/mm}^2$$

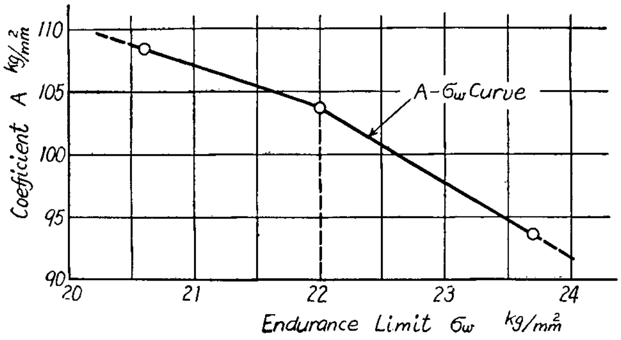

Fig. 15. Assumed Relation between Coefficient A and Eudurance Limit σ_w for 0.61% Carbon Steel.

The values of σ_w and A correspnding to each experimental value are calculated using the relations described above and the probabilities of failure P corresponding to the values of σ_w is determined respectively by Eq. (14). The results obtained are shown in Table 6 and are plotted in Fig. 16.

The cumulative distribution curve of σ_w, which is the most fitted curve for these computed points in Fig. 16, is computed by the least square method. As the results, the

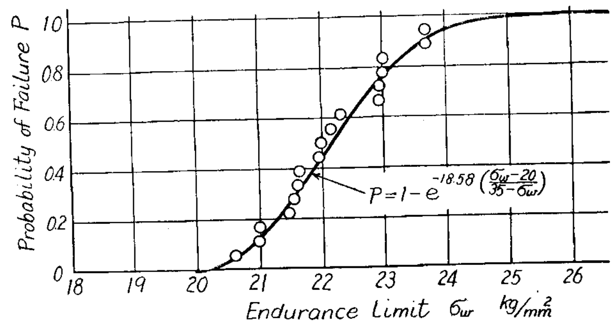

Fig. 16. Cumulative Distribution Curve of Endurance Limit for 0.61% Carbon Steel.

Table 6. Endurance Limit σ_w, Coefficient A and its Probability of Failure corresponding to Experimental Values.

No.	Endurance Limit σ_w (kg/mm²)	Coefficient A	Order ν	Probability of Failure $P=\frac{\nu}{n+1}$
12	20.61	108.41	1	0.0556
15	20.98	107.18	2	0.111
16	20.99	107.13	3	0.167
6	21.46	105.50	4	0.222
8	21.54	105.22	5	0.278
7	21.60	105.04	6	0.333
11	21.63	104.92	7	0.389
5	21.96	103.82	8	0.444
14	21.98	103.80	9	0.500
17	22.16	102.80	10	0.556
9	22.30	101.95	11	0.611
4	22.93	98.20	12	0.667
13	22.94	98.15	13	0.722
1	23.00	97.80	14	0.778
3	23.01	97.76	15	0.833
2	23.69	93.63	16	0.889
10	23.69	93.63	17	0.944

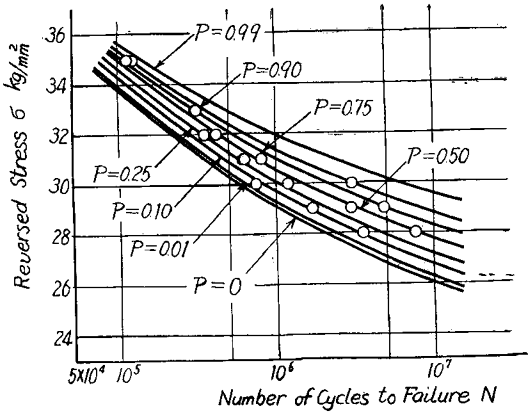

Fig. 17. *S–N* Curves of 0.61% Carbon Steel with Parameter *P*.

following distribution function of σ_w is obtained:

Cumulative distribution function:

$$P=1-e^{-18.58\left(\frac{\sigma_w-20}{35-\sigma_w}\right)^{1.847}}$$

Correlation coefficient:

$$r=0.98925$$

The solid line in Fig. 16 shows the distribution curve computed by the above method. From this relation, the values of the parameters σ_w and A corresponding to a given probability of failure P are calculated and then the *S–N–P* curves are obtained as shown in Fig. 17.

In these processes, the computation of the correlation coeffiicient is very complicated when the number of experimental values are more than twenty because the computed error of the correlation coefficient r must be less than 0.001 (about 0.1% of the value of r) and, consequently, computations must be done with the seven-figure logarithmic tables. For such a purpose, it will be convenient to use a proper statistical computor.

Conclusion

The computation process proposed in this report is a method which, without testing many specimens on a stress level, enables to compute the probability of failure P from the whole fatigue test data carried out on different stress levels. A summary of this calculating method is given below:

The *S–N* equation is assumed as

$$\sigma-\sigma_w=AN^m \quad (m>0)$$

and the probability of failure P corresponding to each experimental value is computed by representing the scatter of test results by the scatter of any one (denoted as z) of the three parameters σ_w, A and m in the S-N equation. The probability of failure P is plotted by the following equation:

$$P=\frac{\nu}{n+1}$$

By applying the following equation

$$P=1-e^{-k\left(\frac{z-U}{T-z}\right)^b}$$

as the most fitted distribution function of z, the S-N-P curves are obtained.

This process has been applied to the fatigue test results of three kinds of carbon steels as examples.

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