Creep of Eccentrically Loaded Short Reinforced Concrete Columns

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Synopsis

Presents methods and results of an experimental and analytical investigation undertaken to clarify the creep behavior of reinforced concrete members subject to sustained combined bending and axial load.

A total of 32 column specimens were tested during 46 weeks of sustained loading, of which 8 were plain concrete specimens, and 24 were tied columns with 2.09 and 3.72 per cent reinforcement. Creep characteristics of concrete was determined by the plain concrete specimens. Three conditions of eceentricity, which varied from 0 to 0.5 times the depth of the column cross section, were investigated in each of two different amounts of reinforcement.

A general analysis was developed, by means of which the creep of the reinforced concrete member subject to sustained eccentric load with large eccentricity may be explained, and the redistribution of stresses in concrete and reinforcement, the variation in deformation of column may be predicted with satisfactory accuracy.

Introduction

Objects of this investigation were, as one series of our test program, to study creep behaviors of reinforced concrete columns subject to combined bending and axial load. Both analytical and experimental works were involved. In the analytical phase, theory was developed for computing the redistribution of stresses in concrete and reinforcement, the variation in deformation due to creep of concrete of eccentrically loaded reinforced concrete members. Three conditions of eccentricity were chosen in such a way that the concrete stress in the lower edge of the section might be compressive, slightly tensile and more or less the flectural strength of the concrete.

Outline of tests

The thirty-two test specimens were divided into three groups, one of axially loaded columns, two of eccentrically loaded columns. The specimens were tied columns with longitudinal reinforcement. An outline of the tests is given in Table 1, which indicates that the major variables were amount of reinforcement, and eccentricity of load.

Table 1

Group	Column No*	Column Size (cm)	Eccentricity e(cm)	Eccentricity $e_1 = e/h$	Longitudinal reinforcement	Sustained Load(ton)	Note
I	AD1, 2, 3, 4	12×18×120	0	0	0	6	
	A5, 6, 7, 8	〃	0		0	0	Controls
	BD9, 10	〃	0	0	4-φ 12 mm	6	
	B11, 12	〃	—	—	〃	0	Controls
	CD21, 22	〃	0	0	4-φ 16 mm	6	
	C23, 24	〃	—	—	〃	0	Controls
II	BE13, 14	12×18×140	4.25	0.235	4-φ 12 mm	4	
	B15, 16	〃	—	—	〃	0	Controls
	CE25, 26	〃	4.5	0.25	4-φ 16 mm	4	
	C27, 28	〃	—	—	〃	0	Controls
III	BF17, 18	12×18×140	9.0	0.5	4-φ 12 mm	2	
	B19, 20	〃	—	—	〃	0	Controls
	CF29, 30	〃	9.0	0.5	4-φ 16 mm	2.5	
	C31, 32	〃	—	—	〃	0	Controls

* A means 'plain concrete,' while B, C 'reinforcement of 4-φ 12 mm, 4-φ 16 mm' respectively. D indicates 'axial loading', and E, F 'eccentric loading with small and large eccentricity', respectively.

The axially loaded plain concrete specimens as well as those reinforced were all of 12×18 cm cross-section with a length of 120 cm, while the eccentrically loaded tied columns were all of the same sectional size as those axilly loaded but 140 cm long, because of their enlargement of the end section. The two conditions of reinforcement, consisting of four 12 and 16 mm diameter plain bars inserted symmetrically 2 cm inside from the sides in the section, are shown in Fig. 1.

All specimens were tested from the age of three months, and the loading

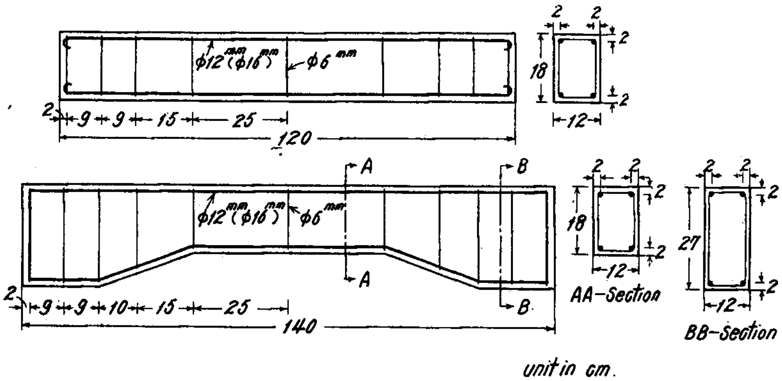


Fig. 1. Test column speciment.

period was about 46-week for the creep specimens. Eight plain concrete columns, four of which were casted as the controls for shrinkage, were used for determining the creep characteristics of the concrete for this tests. Four reinforced conrete specimens including two controls for shrinkage, each for two conditions of the reinforcement, were used for the tests of creep due to axial load $P=6$ tons. Sixteen reinforced concrete specimens of larger length of 140 cm, eight of which were casted as the controls for shrinkage, for the two different conditions of reinforcement and eccentricity, were used for determining creep due to bending and axial load.

The sustained loads for the creep specimens were so chosen as to give concrete compressive stresses of about 30 kg/cm^2. Strains on the concrete surfaces were measured on 10-in. gauge length along the longitudinal center line of column by using a Whittemore strain meter during the initial loading, and at certain intervals during the 46-week period of sustained loading.

Materials, Fabrication. and Test Methods

Materials

ONODA normal portland cement was used throughout the tests. The strength determined by the JIS Designation was 267 kg/cm^2 for compression and 54 kg/cm^2 for bending at 28 days.

The fine aggregate used was a Kizu River sand having an average fineness modulus of about 2.51. The coarse aggregate was a Katsura River gravel of 20 mm maximum size. Both aggregates passed the usual specification tests.

The concrete mix was designed to have 28-day cylinder strength of about 200 kg/cm^2, and a slump of about 12 cm. Proportion of concrete mix actually used was 1 : 2.3 : 3.7 by weight with water-cement ratio also by weight of 0.65. It was mixed and compacted by hand.

Plain bars of nominal 12 mm and 16 mm diameter meeting JIS Designation were used as longitudinal reinforcement.

Fabrication and curing

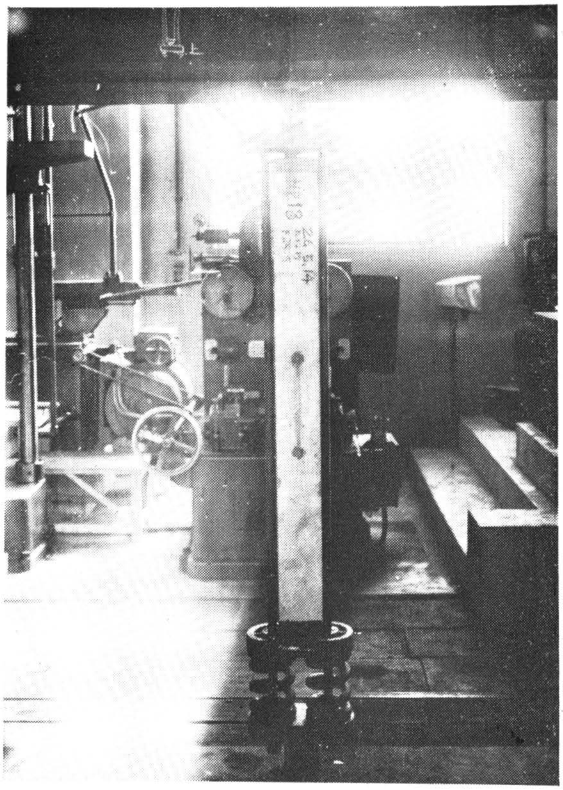
The columns having 120 cm length were cast in metal forms and the others having 140 cm length in wooden forms. The forms were removed the next day, and then all columns were stored in a fog room for three months. Plugs, as the gauges, were muonted, after casting, through cored holes on the two opposite surfaces of the columns. All specimens were removed from the fog room two days before loading, and stored in the laboratory. After the creep specimens were loaded, all columns inclusive of unloaded columns were stored in a storage room during the test period of 46 weeks. In the storage room the temperature ranged from 18°C to 22°C, and the relative humidity from 65 to 80%.

Control cylinders of standard size 15×30 cm were also cast, and stored in the same way as the column specimens.

Test methods

Each creep specimen was loaded together with two springs several times up to the specified load by the Amsler's Pipe Testing Machine of capacity of 20 tons. Under full-loading the deformation of the springs was settled by tightening two tie rods jointing upper and lower bearing plates by screwing the nuts. Then, the load on the testing machine was released, the specimen being loaded by the springs. (See Photograph). The loading methods and devices were quite the same as reported in our previous paper published in 1953. (1)

The eccentrically loaded columns were all tested with 'flat end.' The maximum concrete compressive stress maintained in the creep specimens was about 30 to 35 kg/cm² independently of the variable eccentricity of 0 to 0.5 times the depth of the column section.



Analysis of Creep

Analysis for computing redistribution of stresses between concrete and reinforcement, and variation in deformations due to creep of concrete in reinforced concrete member subject to sustained combined bending and axial load, is to be derived for two cases, one is derived for the case where the concrete tensile stress is zero or less than the flexural strength of concrete, and where it is very logical to assume that the whole concrete section resists to combined bending and axial load, and the other is derived for the case where the concrete tensile stress is larger than the flexural strength of concrete, and where the assumption should be made that the concrete in tension side is no more valid for computing the stresses of concrete and steel. The case where the eccentricity is fairly small, belongs to the former case, and the case of comparatively large eccentricity to the latter one.

The three fundamental assumptions are adopted for creep behavior of concrete in developing the analysis.

1. Proportionality of creep strain to stress.——It is assumed that creep is in direct proportion to stress and the proportion constant is equal both for tension and for compression. (Davis-Glanville's Law)

2. Constant rate of creep speed for unit stress for the same concrete loaded from different age.——It is assumed that the creep-time curve for unit stress of concrete loaded from any time $t=t_a$ is quite the same as the one of the same concrete loaded from time $t=0$ except the ordinate of creep strain being reduced by the creep strain at time $t=t_a$ from the latter. (Whitney's Law) (2)

3. Superposition of creep strains——It is assumed that the creep strains due to sustained stresses can be superposed together so far the total stress in concrete is less than the allowable stress.

The followiug assumptions were also made for concrete and steel in the analysis.

4. Linear stress and strain distribution——It is assumed that a linear distribution of strain and stress over the depth of the column exists.

5. No creep in reinforcement——It is assumed that the reinforcement causes no creep practically under such a low stress as the one given in ordinary reinforced concrete design.

Notation

The letter symbols used in the paper are generally defined by figures or when used for the first time. The most commom symbols are listed below for convenient reference.

A_c: area of concrete section

A_s, A'_s: area of tensile and compressive reinforcement, respectively

b: width of rectangular member

c, c': distance from mid-depth of section to centroid of tensile and compressive reinforcement

d, d': distance from tensile and compressive reinforcement to compression edge of member

D_c: compressive stiffness of concrete section ($=E_cA_c$)

D_s: compressive stiffness of reinforcing steel ($=E_sA_s$)

E_c, E_s: modulus of elasticity of concrete and steel

E_{co}, E_{ct}: modulus of elasticity of concrete at time $t=0$ and $t=t$

e, e': eccentricity of load with respect to mi-depth of section, and to center of gravity of transformed section

e_1: ratio e/h

h: height of rectangular section

K: experimental constant relating to axial shrinkage of member

K_c: bending stiffness of concrete section $(=I_cE_c)$

K_s: bending stiffness of reinforcing steel $(=I_sE_s)$

I_c, I_s: moment of inertia of concrete section and reinforcing steel with respect to center of gravity of transformed section

n: ratio E_s/E_c

$\bar{p}_0$: ratio $(A_s+A_s')/A_c$

$\bar{p}$: ratio $(A_s+A_s')/bd$

p, p' ratio A_s/bd and A_s'/bd

P: sustained load

q_t: variable relating to variation in modulus of elasticity of concrete with time after loading

q: ratio I_s/I_c

t: time after loading on member

x_0, x_t: distance from neutral axis to compression edge of member at time $t=0$ and $t=t$, respectively

α: ratio $\gamma/(1+\gamma)=\dfrac{D_s}{D_s+D_c}$

β: ratio $\lambda/(1+\lambda)=\dfrac{K_s}{K_s+K_c}$

γ: ratio $D_s/D_c=n\bar{p}_0$

λ: ratio $K_s/K_c=nq$

δ_{ct}: total compressive strain (sum of elastic and creep strain) of concrete, also of concrete at compression edge of section

δ'_{ct}: total tensile strain (sum of elastic and creep strain) of concrete at tension edge of section

ε_{co}, ε_{ct}: elastic compressive concrete strain at $t=0$ and $t=t$, respectively

ε'_{co}: elastic tensile concrete strain at $t=0$

ε_{so}, ε_{st}: strain in tensile reinforcement at $t=0$ and $t=t$, respectively

ε'_{so}, ε'_{st}: strain in comressive reinforcement at $t=0$ and $t=t$, respectively

σ_{co}, σ_{ct}: compressive stress in concrete at $t=0$ and $t=t$, respectively

σ'_{co}, σ'_{ct}: tensile stress in concrete at $t=0$ and $t=t$, respectively

σ_{cpo}, σ_{spo}: compressive stress in concret and in reinforcement respectively, due to only axial load P at $t=0$,

σ_{cMo}, σ'_{cMo}: stress in concrete at compression and tension edge of section respectively, due to only bending caused by eccentric load P at $t=0$

$\sigma_{sMo}, \sigma'_{sMo}$: stress in tensile and compressive reinforcement, respectively, due to only bending moment caused by eccentric load P at $t=0$

ρ_o, ρ_t: radius of curvature of section at $t=0$ and $t=t$, respectively

ε_o, θ_o: compressive and angular strain in reinforced concrete section at $t=0$, respectively

φ_t: creep characteristics of concrete, indicating ratio of net creep strain to elastic strain

φ_{rt}: creep characteristics of reinforced concrete

The other symbols are defined in the sentences when necessary.

Creep formula of concrete (1)

Total strain (δ_{ct}) inclusive of elastic strain of concrete per unit stress is expressed by the assumptions described before, as follows:

$$\delta_{ct}=\varepsilon_{ct}+\int_{t_a}^{t}\varepsilon_{ct}\frac{d\varphi_t}{dt}dt \quad \cdots\cdots(1)$$

where

ε_{ct}: elastic strain corresponding to time variable stress σ_{ct}

φ_t: creep characteristics of concrete

When considering the variation in modulus of elasticity, E_{ct}, with the age, δ_{ct} is evaluated by

$$\delta_{ct}=\varepsilon_{cto}(1-q_t)+\int_{t_a}^{t}\varepsilon_{cto}\frac{d\varphi_t}{dt}dt+\int_{t_a}^{t}\varepsilon_{cto}\frac{dq_t}{dt}dt \quad \cdots\cdots(2)$$

where ε_{cto} is a virtual strain obtained by dividing the variable stress σ_{ct} by modulus of elasticity E_{co} at time $t=0$, $\left(\varepsilon_{cto}=\frac{\sigma_{ct}}{E_{co}}\right)$, and E_{ct} is assumed

$$E_{ct}=E_{co}\frac{1}{1-q_t} \quad (q_t\text{: fuction of time}) \quad \cdots\cdots(3)$$

Theory and experiments show that the effect of variation in modulus of elasticity of concrete upon the creep behavior of reinforced concrete members is so small that it can be negligible practically in the calculation. Therefore Eq. (1) is used throughout in the following analysis as the fundamental equation.

Creep eqnations for Case 1 (Considering tension resisted by concrete)

For combined bending moment Pe and axial load P, calculation can be done separately for each of bending and axial load, and then be added together. In this case the elastic analysis for a homogeneous material are quite valid. Since it is assumed that a law of superposition of creep strain exists so far the total sum of stresses is less than the allowable stress of concrete, the creep strain due to P and Pe can be also obtained in the same way as in the elastic analysis.

Professor Fr. Dischinger (3) gives the following equations for creep of reinforced concrete members subject to sustained load P with small eccentricity e' as shown in Fig. 2.

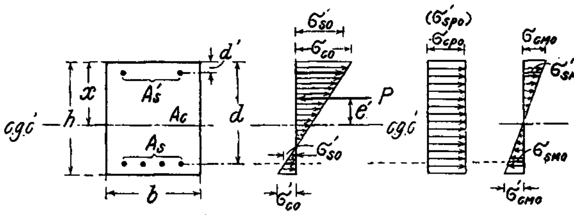


Fig. 2. Flexural analysis for small eccentricity.

When the column is loaded ($t=0$),

for P

$$\left.\begin{array}{ll}\text{compressive stress in concrete} & \sigma_{cpo}=\dfrac{P_c}{D_c} \\ \text{compressive stress in reinforcement} & \sigma_{spo}=\dfrac{P_s}{D_s}\end{array}\right\}\cdots\cdots\cdots\cdots\cdots(4a)$$

where

$$P_c=P(1-\alpha)=P\frac{D_c}{D_s+D_c}$$

$$P_s=P\alpha=P\frac{D_s}{D_s+D_c},\quad D_c=E_cA_c,\quad D_s=E_s(A_s+A_s')$$

for Pe'

$$\left.\begin{array}{lll}\text{stress in concrete:} & \text{compression} & \sigma_{cMo}=\dfrac{M_c}{I_c}x_o \\ & \text{tension} & \sigma'_{cMo}=\dfrac{M_c}{I_c}(h-x_o) \\ \text{stress in reinforcement:} & \text{compression} & \sigma'_{sMo}=\dfrac{M_s}{I_s}(x_o-d') \\ & \text{tension} & \sigma_{sMo}=\dfrac{M_s}{I_s}(d-x_o)\end{array}\right\}\cdots\cdots(4)$$

where $M_c=Pe'(1-\beta)=Pe'\dfrac{K_c}{K_s+K_c}$

$$M_s=Pe'\beta=Pe'\frac{K_s}{K_s+K_c},\quad K_c=I_cE_c,\quad K_s=I_sE_s$$

and it is noticed that I_c, I_s indicate the moment of inertias of concrete section and reinforcement with respect to **center of gravity of transformed section.**

Therefore, for P and Pe'

$$\left.\begin{aligned}&\text{stress in concrete: compression } \sigma_{co}=\sigma_{cpo}+\sigma_{cMo}\\&\qquad\text{tension } \sigma'_{co}=-\sigma_{cpo}+\sigma'_{cMo}\\&\text{stress in reinforcement: tension } \sigma_{so}=-\sigma_{spo}+\sigma_{sMo}\\&\qquad\text{compression } \sigma_{so}'=\sigma_{spo}+\sigma'_{sMo}\end{aligned}\right\}\cdots\cdots(4c)$$

The deformations of column are quite equal to those of reinforcing steel, and are given as follows:

$$\left.\begin{aligned}&\text{for axial compression: } \varepsilon_o=\frac{\sigma_{spo}}{E_s}=\frac{P_s}{D_s}=\frac{P_c}{D_c}\\&\text{for bending roation: } \theta_o=\frac{M_s}{K_s}=\frac{M_c}{K_c}\end{aligned}\right\}\cdots\cdots(5)$$

Hence the concrete compressive and tensile strains at upper and lower edges are shown by

$$\left.\begin{aligned}&\text{compression: } \varepsilon_{co}=\varepsilon_o+\theta_o x_o\\&\text{tension: } \varepsilon'_{co}=-\varepsilon_o+\theta_o(d-x_o)\end{aligned}\right\}\cdots\cdots(6)$$

When the concrete of the column creeps, condition that the deformations of concrete must be equal to those of reinforcement at all stages of creep process, causes the variation in stresses and strains of both constituents of reinforced concrete: thus P_c, P_s, and M_c, M_s must be changed to P_{ct}, P_{st}, and M_{ct} M_{st}, at time $t=t$, as follows.

$$P_{ct}=P_c\cdot exp(-\alpha\varphi_t)$$

$$P_{st}=P_s+P_c\{1-exp(-\alpha\varphi_t)\}$$

$$M_{ct}=M_c\cdot exp(-\beta\varphi_t), \qquad \text{and}$$

$$M_{st}=M_s+M_c\{1-exp(-\beta\varphi_t)\}$$

The streeses of concrete and reinforcement are, therefore, given by

$$\left.\begin{aligned}&\sigma_{ct}=\sigma_{cpo}\cdot exp(-\alpha\varphi_t)+\sigma_{cMo}exp(-\beta\varphi_t)\\&\sigma'_{ct}=-\sigma_{cpo}exp(-\alpha\varphi_t)+\sigma'_{cMo}exp(-\beta\varphi_t)\\&\sigma_{st}=-\sigma_{spo}(1+\varphi_{rpt})+\sigma_{sMo}(1+\varphi_{rMt})\\&\sigma_{st}'=\sigma_{spo}(1+\varphi_{rpt})+\sigma'_{sMo}(1+\varphi_{rMt})\end{aligned}\right\}\cdots\cdots(7)$$

and the concrete strains at the upper and lower edges are

$$\left.\begin{aligned}&\delta_{ct}=\varepsilon_o(1+\varphi_{rpt})+\theta_o x_o(1+\varphi_{rMt})\\&\delta'_{ct}=-\varepsilon_o(1+\varphi_{rpt})+\theta_o(d-x_o)(1+\varphi_{rMt})\end{aligned}\right\}\cdots\cdots(8)$$

where

$$\varphi_{rpt}=\frac{1}{\gamma}\{1-exp(-\alpha\varphi_t)\}, \quad \gamma=\frac{D_s}{D_c}=n\bar{p}_o$$

$$\varphi_{rMt}=\frac{1}{\lambda}\{1-exp(-\beta\varphi_t)\},\quad \lambda=\frac{K_s}{K_c}=nq$$

and these are nothing but the creep characteristics of reinforced concrete members for axial compression and for bending.

The creep of concrete at the upper and lower edges of section, and the variation in stresses in concrete are shown in Fig. 3 for the columns tested under the load and eccentricity listed in Table 1. (Group II).

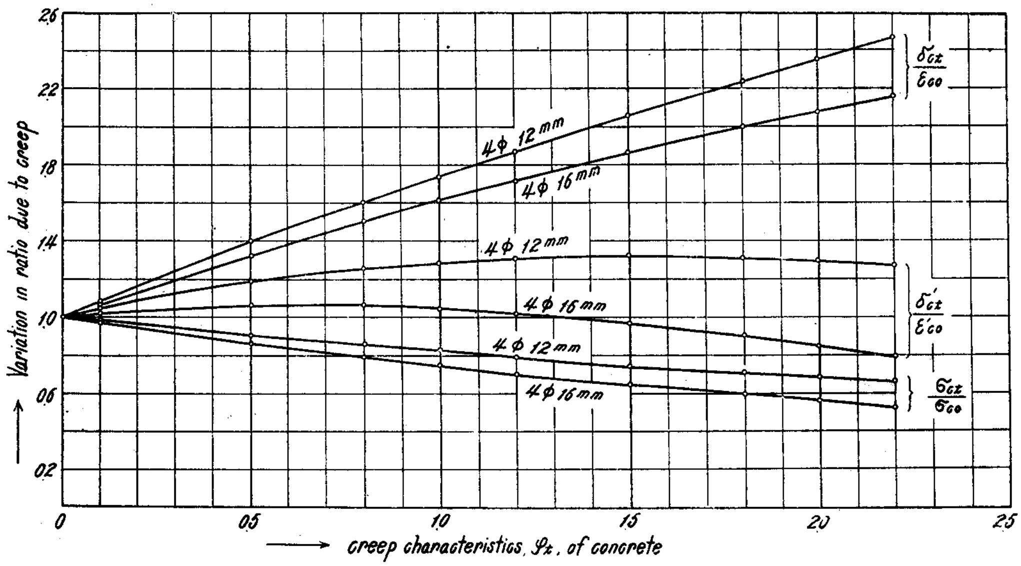


Fig. 3. Variation in stresses and strains for small eccentricity.

Creep eqnations for Case 11 (considering no tension resisted by concrete)

When the eccentricity is large enough to produce cracks in the tension side of column, both elastic and creep equations for the case must be derived under the condition that the concrete in the tension side must be ignored.

For simplicity, an analysis is developed, as follows, for the column of rectangular cross-section ($b\times h$) having reinforcements (A_s, A_s'), and for axial load P acting on the column with large eccentricity e with respect to the mid-depth of the cross-section, as shown in Fig. 4(a). Fig. 4(b) illustrates the distribution of stresses and strains in the column section at time $t=0$ when the column is loaded. Distance x_0 of neutral axis from the compressive edge of the section is easily obtained by the common elastic analysis using n and considering no tension resisted by concrete.

Expressing at that time $t=0$ the radius of curvature as ρ_0, the concrete

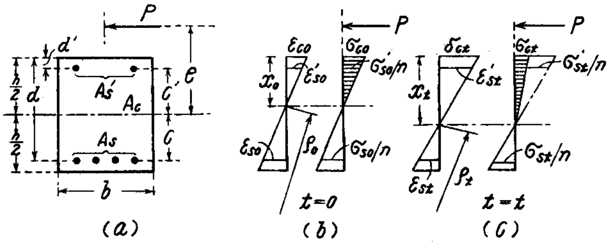


Fig. 4. Flexural analysis for large eccentricity.

compressive strain ε_{co}, stress σ_{co}, in the upper edge, and the steel strain ε_{so}, ε'_{so}, stress σ_{so}, σ'_{so} are shown as follows.

$$\left.\begin{aligned} \varepsilon_{co}&=\frac{x_o}{\rho_o}, & \sigma_{co}&=E_c\frac{x_o}{\rho_o} \\ \varepsilon_{so}&=\frac{d-x_o}{\rho_o}, & \sigma_{so}&=E_s\frac{d-x_o}{\rho_o} \\ \varepsilon'_{so}&=\frac{x_o-d'}{\rho_o}, & \sigma'_{so}&=E_s\frac{x_o-d'}{\rho_o} \end{aligned}\right\} \qquad (9)$$

The creep equations in the following analysis are established under the assumptions that the concrete in the compression side of the section begins to creep from the initial condition given by Eq. (9), and that the creep formula of concrete, Eq. (1), is satisfied at the upper edge fibre.

The creep of concrete, on one hand, increases ε_{co} to δ_{ct}, reduces σ_{co} to $\sigma_{ct}=\varepsilon_{ct}\cdot E_c$, and shifts down the neutral axis x_o to x_t, and, on the other hand, changes the stresses, σ_{so}, σ'_{so}, the strains, ε_{so}, ε'_{so} of reinforcement to σ_{st}, σ'_{st} and ε_{st}, ε'_{st}, respectively, so that the equilibrium between the internal forces and couple resisted by the concrete and reinforcement, and the external loads P and Pe, may exist at all stage of creep performance.

Denoting the radius of curvature of the column section ρ_t,

$$\left.\begin{aligned} \delta_{ct}&=\frac{x_t}{\rho_t}, & \sigma_{ct}&=\varepsilon_{ct}\cdot E_c \\ \varepsilon_{st}&=\frac{d-x_t}{\rho_t}, & \sigma_{st}&=E_s\frac{d-x_t}{\rho_t} \\ \varepsilon'_{st}&=\frac{x_t-d'}{\rho_t}, & \sigma'_{st}&=E_s\frac{x_t-d'}{\rho_t} \end{aligned}\right\} \qquad (10)$$

Fig. 4(c) shows the relations between strains and stresses in the column section at $t=t$.

Thus, the equations of equilibrium for the axial force and bending moment yields.

$$P=\frac{1}{2}bx_t\sigma_{ct}+A'_s\sigma'_{st}-A_s\sigma_{st} \qquad \cdots\cdots(11)$$

$$M=Pe=\frac{1}{2}bx_t\sigma_{ct}\left(\frac{h}{2}-\frac{x_t}{3}\right)+A_s'\sigma'_{st}c'+A_s\sigma_{st}\cdot c \qquad \cdots\cdots(12)$$

where c and c' mean the distances from the mid-depth of column section to the centroid of tensile and compressive reinforcement, respectively.

Substituting Eq. (10) into Eq. (11), and solving for x_t,

$$x_1=\frac{x_t}{d}=\frac{\frac{P}{E_cbd}\rho_t+ndA}{\frac{1}{2}\varepsilon_{ct}\rho_t+nd\bar{p}} \qquad \cdots\cdots(13)$$

where

$$x_1=\frac{x_t}{d}, \qquad A=p+p'd_1'$$

$$\bar{p}=p+p', \quad n=E_s/E_c, \quad p=\frac{A_s}{bd}, \quad p'=\frac{A'_s}{bd}, \quad d'_1=\frac{d'}{d}$$

From Eq, (11)

$$\frac{1}{2}bx_t\cdot\sigma_{ct}=P-A_s'\sigma_{st}'+A_s\sigma_{st}$$

Substituting the above and Eq. (10) into Eq. (12), and solving for x_t

$$x_1=\frac{1}{2\bar{p}}[E\rho_t+B-E\sqrt{f(\rho_t)}] \qquad \cdots\cdots(14)$$

where

$$B=3\left(\frac{A}{3}+\bar{p}i+pc_1-p'c_1'\right)$$

$$C=3(Ai+pc_1-p'c_1'd_1')$$

$$D=3\left(i-\frac{e}{d}\right)$$

$$E=P/E_sbd^2$$

$$F=\frac{1}{E}(2B-4\bar{p}D)$$

$$G=\frac{1}{E^2}(B^2-4\bar{p}C)$$

$$f(\rho_t)=\rho_t^2+F\rho_t+G$$

$$i=\frac{h}{2d}, \qquad c_1=\frac{c}{d}, \qquad c_1'=\frac{c'}{d}$$

A, p, p', $\bar{p}$, ……are given in Eq. (13)

Solving Eqs. (13) and (14) for ε_{ct}

$$\varepsilon_{ct}=\frac{2nd\bar{p}}{\rho_t}\left\{\frac{2(E\rho_t+A)}{E\rho_t+B-E\sqrt{f(\rho_t)}}-1\right\} \qquad \cdots\cdots(15)$$

Then, from Eq. (10)

$$\delta_{ct}=\frac{x_t}{\rho_t}=\frac{E\rho_t+B-E\sqrt{f(\rho_t)}}{2\bar{p}}\cdot\frac{1}{\rho_t} \qquad \cdots\cdots(16)$$

Eqs. (15) and (16) show that the compressive elastic strain ε_{ct} and the total strain (the sum of elastic and creep strain) δ_{ct} of the concrete in the compressive edge of the column section can be expressed as functions of ρ_t. Thus, considering that ρ_t is also a fuction of time, or creep characteristics, φ_t, of concrete, Eq. (1) can be transformed into

$$\int_{\rho_0}^{\rho_t}\left(\frac{\dot{\delta}_{ct}}{\varepsilon_{ct}}-\frac{\dot{\varepsilon}_{ct}}{\varepsilon_{ct}}\right)d\rho_t=\int_{0}^{\varphi_t}d\varphi_t \qquad \cdots\cdots(17)$$

where

$$\dot{\varepsilon}_{ct}=\frac{d\varepsilon_{ct}}{d\rho_t}, \qquad \dot{\delta}_{ct}=\frac{d\delta_{ct}}{d\rho_t}$$

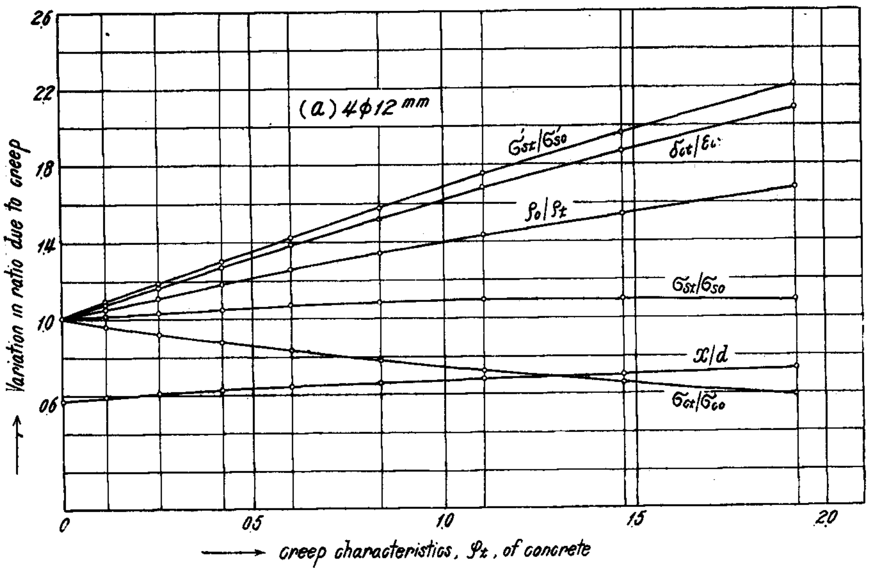
Substituting Eqs. (15) and (16) into Eq. (17),

$$\varphi_t=\int_{\rho_0}^{\rho_t}\frac{\dot{\delta}_{ct}}{\varepsilon_{ct}}d\rho_t-\Big|l_n\,\varepsilon_{ct}\Big|_{\rho_0}^{\rho_t} \qquad \cdots\cdots(18)$$

The first term of the right side of Eq. (18) can be computed by numerical integration. When the relation between φ_t and ρ_t is known by Eq. (18), the strains, stresses in concrete and reinforcement, and location of neutral axis are calculated by Eqs, (10) and (13) or (14). And the increase in lateral deflection of the column is also predicted by the ratio ρ_0/ρ_t.

Fig. 5 exhibits followed by the above analysis how the stresses, strains of concrete and reinforcement vary with the creep characteristics, φ_t, of concrete in the test column section, which is subjected to sustained combined bending and axial load with $e_1=0.5$ as listed in Table 1.

(I)



(II)

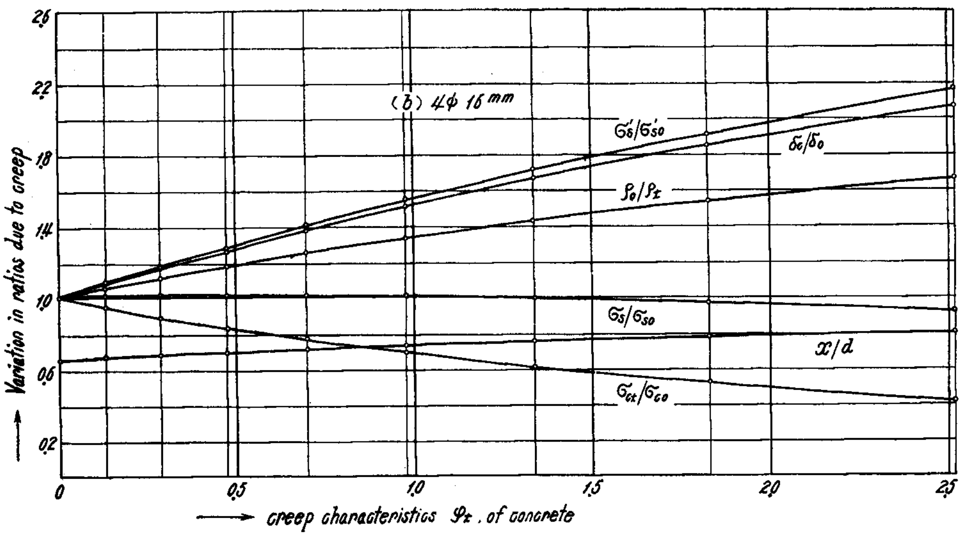


Fig. 5. Creep behavior of eccentrically loaded column in case of $e_1=0.5$

Results of Test

Compression tests of control cylinders

The results of the compression tests of control cylinders are shown in Table 2.

Table 2

Columns reprsented	Ultimate compressive strength (kg/cm^2)		Modulus of elasticity (10^4 kg/cm^2)*	
	12-week	58-week	12-week	58-week
1, 5, 9, 11, 13, 15, 17, 19,	215	225	29.8	31.5
2, 6, 10, 12, 14, 16, 18, 20,	214	224	28.7	29.7
3, 7, 21, 23, 25, 27, 29, 31,	211	231	25.7	27.5
4, 8, 22, 24, 26, 28, 30, 32	199	219	25.3	27.1
Mean	210	225	27.4	29.0

Each result is the average of two tests.
* Secant value at 28.3 kg/cm^2

The modulus of elasticity, E_c, of concrete was determined by the secant method at stress of 28.3 kg/cm^2. Between 12 weeks and 58 weeks, the concrete showed a slight increase in compressive strength, and in modulus of elasticity. Since the effect of increase in modulus of elasticity, E_{ct}, of concrete upon the creep behavior of reinforced concrete member is insignificant as described previously, the modulus of elasticity $E_c = 27.4 \times 10^4$ kg/cm^2 at 12-week age was used throughout in the theoretical analysis.

Creep of concrete

The test results for the creep of plain concrete column are shown in Fig. 6. The fine-solid line is gross creep strain curve, and the broken line is net creep

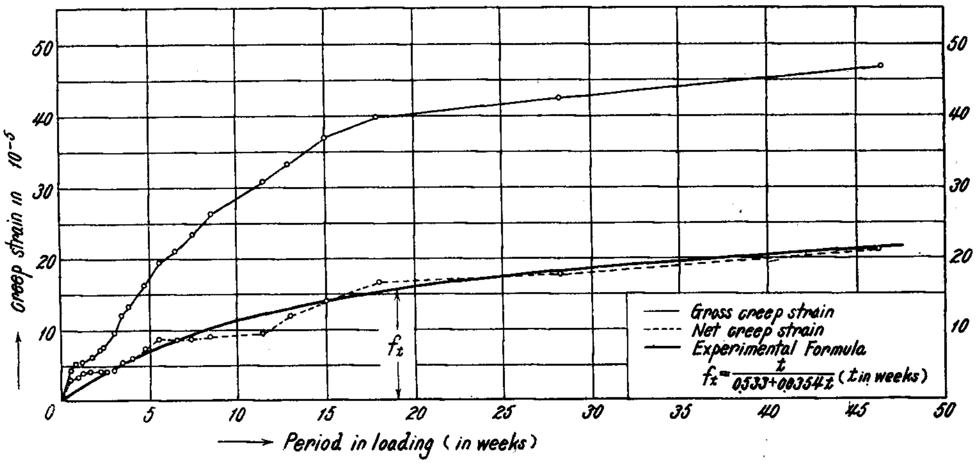


Fig. 6. Creep strain curves for plain concrete.

strain curve. The gross value is total strain exclusive of the immediate elastic strain accompanying loading. The net value, coreected for shrinkage and effects other than load, obtained by substracting the strains of unloaded companion columns from the gross value, and is expressed experimentally by a hyperbolic form (1), (5)

$$f_t = \frac{t}{a+bt} = \frac{t}{0.533+0.0354t} \qquad (t \text{ in weeks})$$

where

f_t = net creep strain (10^{-5})

t = time after loading in weeks

a, b = constants to be determined by tests

The curves are based on four loaded columns $AD1\sim4$, and four companion columns $A5\sim8$. Since the immediate elastic strain ε_{co} is 11.1×10^{-5}, the creep characteristics φ_t is given

$$\varphi_t = \frac{f_t}{\varepsilon_{co}} = \frac{t}{5.916+0.393t} \qquad (t \text{ in weeks})$$

These equations predict that the final values of creep strain, f_{tn}, and of creep characteristics, φ_{tn}, will be 28.2×10^{-5} and 2.54, respectively.

Elastic stress in columns

The computed stress and the measured stress values based on strain readings taken directly after loading are listed in Table 3. The measured concrete stresses σ_{co}, σ'_{co} in the top and bottom surfaces of the column section are based on strain readings between plug sets in the concrete at the two opposite surfaces, while the measured stresses in the tensile stress, σ_{so}, and in the compressive stress, σ'_{so}, is based on strain readings obtained by straightline interpolation from the compressive and tansile concrete surface readings.

Table 3

Column No.	Eccentricity (e_1)	Computed stresses*(1) (kg/cm²)				Measured stresses*(1) (kg/cm²)			
		σ_{co}	σ'_{co}	σ_{so}	σ'_{so}	σ_{co}	σ'_{co}	σ_{so}	σ'_{so}
AD1, 2, 3, 4,	0	27.8	—	—	—	30.5	—	—	—
BD9, 10	0	24.0	—	-184	-184	27.5	—	-211	-211
CD21, 22	0	21.7	—	-166	-166	27.4	—	-210	-210
BE13, 14	0.235	36.7	-4.7	1	-246	41.1	-11.0	42	-271
CE25, 26	0.25	32.9	-3.9	0	-221	32.9	-8.2	40	-200
BF17, 18	0.5	35.2	—	187	-212	38.1	(-30.9)*(2)	173	-242
CF29, 30	0.5	35.4	—	146	-219	38.4	(-29.0)*(2)	174	-200

* (1) Sign(+) means compression for concrete, and tension for steel.

* (2) Apparent tensile stress obtained by the use of E_c for compression.

The modulus of elasticity of the concrete was based on the tests of companion cylinders, and it was also assumed as to be equal to the modulus of elasticity for tension. And 2100000 kg/cm² was used as the modulus of elasticity of the steel.

The measured compressive stresses in both steel and concrete showed slight deviations from the computed values. It may be the reason for this deviation that the column specimens had been effected by shrinkage of the concrete before loading, and the slight inaccuracy in the eccentricity of load and in the alignment of reinforcement in the section.

Creep strain in tied column

Axially loaded column ($e=0$)

Fig. 7 shows how the ratios of the creep strain values to the elastic values

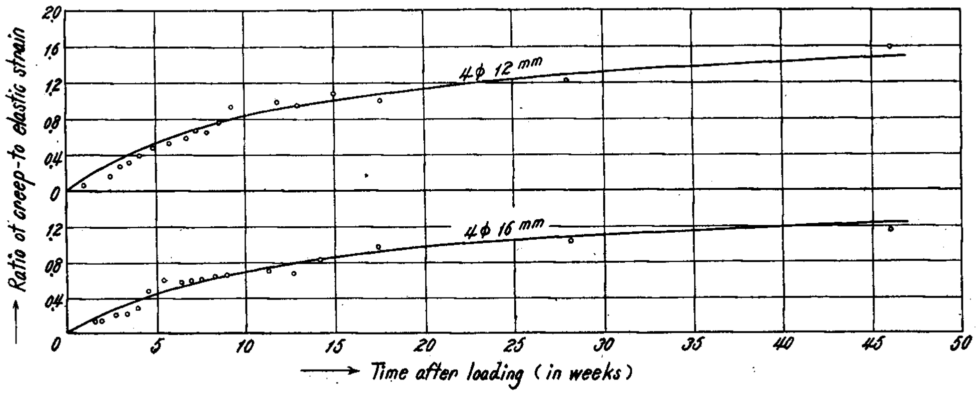


Fig. 7. Creep of compressive concrete strain in case of $e_1=0$

vary with the duration of load on the axially loaded columns. Each plotted value is the average of the readings on both side surfaces of each of two similar columns. Table 4 lists the immediate-elastic strain values, the 46-week creep strain values

Table 4

Column No.	Eccentricity (e_1)	Compression side			Tension side		
		A*	B*	A/B	A*	B*	A/B
AD1, 2, 3, 4,	0	11.1	21.3	1.92			
BD9, 10	0	10.1	16.5	1.61			
CD21, 22	0	10.0	11.5	1.15			
BE13, 14	0.235	15.0	20.2	1.35	4.0	1.7	0.425
CE25, 26	0.25	12.0	14.0	1.17	3.0	-1.6**	-0.533**
BF17, 12	0.5	14.2	15.7	1.10	11.7	-11.7**	
CF89, 30	0.5	14.0	12.3	0.88	11.0	-11.0**	

* A: immediate elastic strain (10^{-5})
* B: 46-week creep strain (10^{-5})
** Sign(−) means contraction

and the ratio of the two for each pair of columns. The silid line curves in Fig. 7 are obtained theoretically by Eq. (18), only the axial load being taken into consideration.

The test columns were subjected to sustained loading at the age of 12-week. If the time of loading were earlier, for example, 28-day, the creep strain values, i. e. creep characteristics would be far larger than the test results obtained here.

It is quite correct theoretically to use the creep characteristics, ϕ_t, of concrete subject to load at 12-week, instead of φ_t at $t=0$, (for instance 4-week), for computing the creep values of reinforced concrete columns made with the similar concrete and also subject to load at 12-week. Because the creep characteristics of reinforced concrete members ϕ_{rpt}, ϕ_{rMt} (as shown by Eq. (8)) subject to sustained load not at time $t=0$ but at time $t=t_a$, can be given as follows.

$$\phi_{rpt}=\frac{1}{\gamma}\{1-exp.(-\alpha\varphi_t+\alpha\varphi_{ta})\}=\frac{1}{\gamma}\{1-exp.(-\alpha\phi_t)\}$$

or

$$\phi_{rMt}=\frac{1}{\lambda}\{1-exp.(-\beta\varphi_t+\beta\varphi_{ta})\}=\frac{1}{\lambda}\{1-exp.(-\beta\phi_t)\}$$

where

φ_{ta} = creep characteristics at time $t=t_a$, of concrete loaded form $t=0$

ϕ_t = creep characteristics at time $t=t$, of concrete loaded from $t=t_a$

Therefore, in computing the creep values of reinforced concrete members subject to load at any time $t=t_a$, ϕ_{rt} is no more φ_t than φ_{rt} is φ_t.

It appears that a general agreement exists between measurements and theory. As the theory predicts, measurements of the column group B having four 12 mm dia. reinforcement show larger creep than group C having four 16 mm dia. bars.

Column loaded with small eccentricity

Fig. 8 illustrates the increase in compressive creep strain values of the column

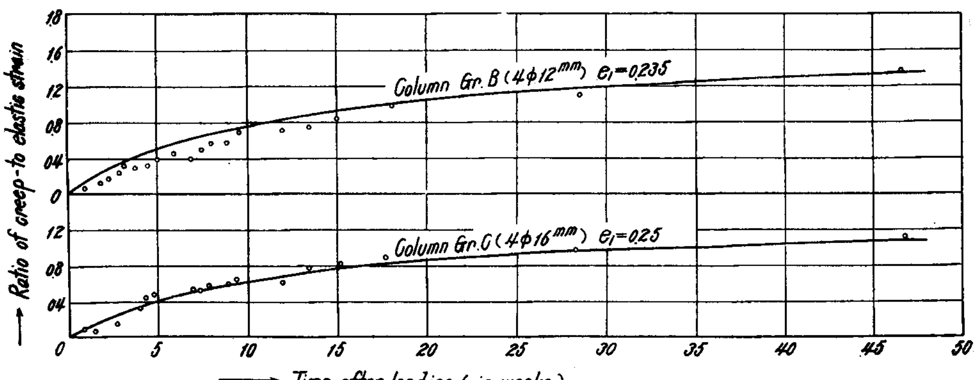


Fig. 8. Creep of compressive concrete strain in case of small eccentricity.

for each pair of group B and C. The solid-line curves show the thoeretical values calculated by Eq. (8). The immediate elastic strain values, the 46-week creep strain values, and the ratio of the two are also given in Table 4.

The theoretical creep values of tensile concrete strain for a pair of column groups B and C, calculated by Eq. (8), are shown in the upper set of curves of Fig. 9, and the measured strain values are also given in the lower set of the

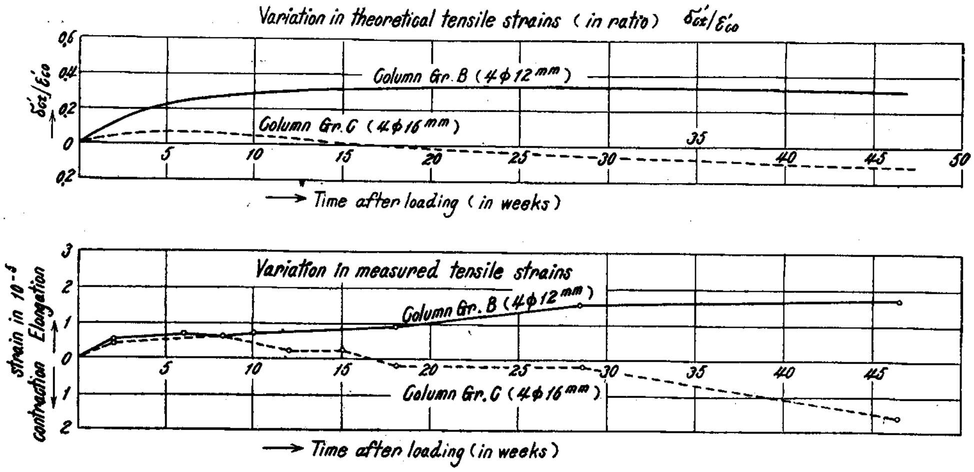


Fig. 9. Creep of tensile strain in case of small eccentricity.

figure. The tensile creep values were so small that the measurements were not completely accurate because of scattering of the measurement and of the variation in creep values sometimes being beyond the limiting accuracy of the Whittemore strain meter used for measuring strains. However, measurements show the growth of tensile creep strains, the general tendency of which can be predicted by the theory.

Column loaded with larger eccentricity

The eccentricity of load was 0.5h for both column groups B and C, and the sustained loads were 2 tons for group B and 2.5 tons for group C. Fig. 10 shows

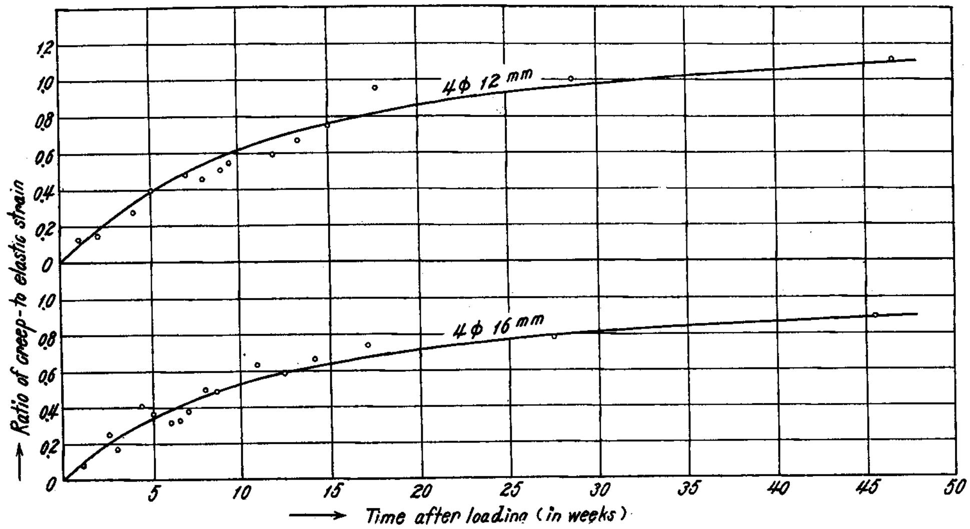


Fig. 10. Creep of compressive concrete strain in case of $e_1=0.5$

how the compressive concrete strains vary with the duration of sustained load. The solid-line curves drawn in the figure give the theoretical values computed by the analysis described before, and are also shown in Fig. 5. Measurements are in good agreement with the theory.

The elastic and creep strain values, and the ratio of each are shown in Table 4. The immediate elastic tensile strain values of concrete, as shown in Table 4, were fairly large, so that the tensile stress might be possibly more or less the flexural strength of the concrete, although no cracks were detected by neckedeye in the tension side of the column of each pair of groups B and C. Some of the colnmns, however, began to exhibit crackings in the tension side about two months after loading, and the numbers of the cracks appeared to increase with the duration of loading.

The tensile creep values of concrete, which were obtained also by correcting for shrinkage and effects other than load, are plotted in Fig. 11 for both groups B and C. The corrections based on control specimen readings are not completely

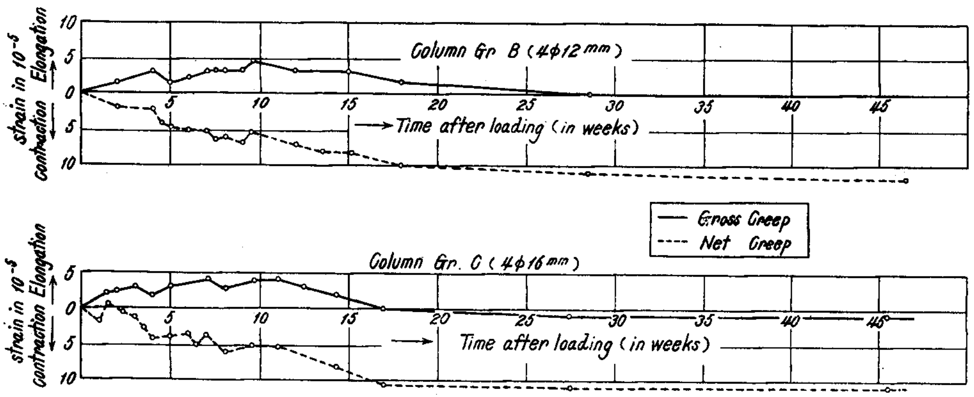


Fig. 11. Creep of tensile strain in case of $e_1=0.5$

valid, because of no cracks in tension side being detected in the control specimens, but it can be considered that the corrections provide at least a rough picture of the separation of deformations due to sustained loading and those due to shrinkage and other causes.

The results plotted in Fig. 11 show that a large part of strain in the tension side of column is due to causes other than sustained loading (principally shrinkage between cracks).

Shrinkage of plain and reinforced concrete columns

Deformations of unloaded plain and reinforced concrete specimens were measured as controls for the correction of strains of loaded specimens. The measurements do not exactly show the shrinkage values of the control specimens because of slight variations in temperature and humidity in the storage room. But it is believed that the deformations of unloaded columns are principally due to shrinkage.

As listed in Table 1, the measurements were done in the shorter size specimens of 120 cm long for plain concrete, and in both shorter (120 cm) and longer (140 cm) size specimens for reinforced concrete columns. The differences of measurements between in the shorter and longer controls were so small that the averages of both controls were adopted as the correction values for the loaded reinforced concrete specimens having the same amount of reinforcements.

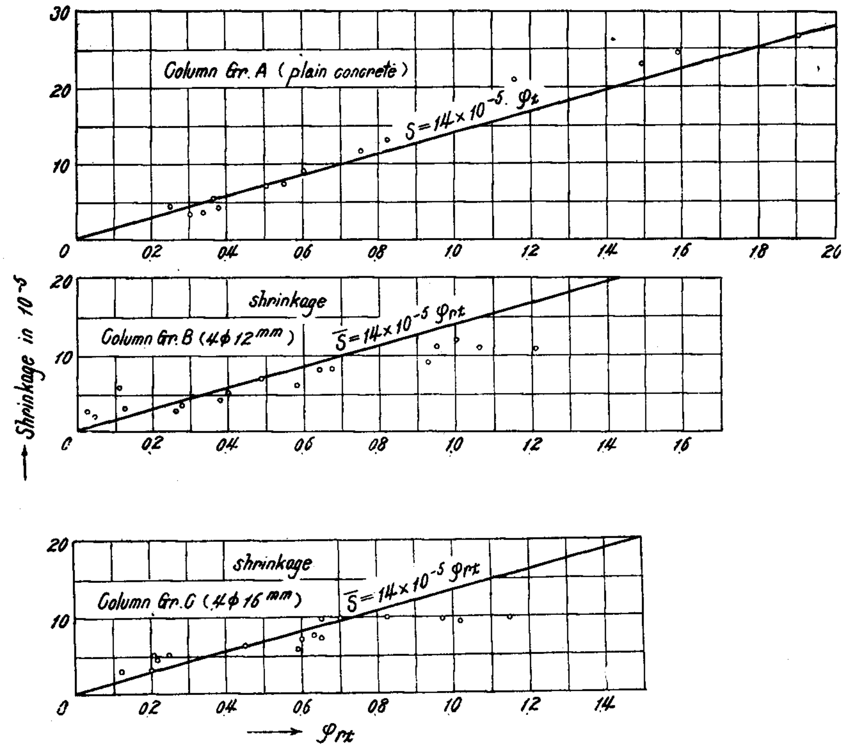


Fig. 12. Shrinkage of columns.

Since the time-shrinkage curve of concrete is very similar to the time-creep curve for concrete, it may be often assumed for convenience in mathematical analysis for creep, that the shrinkage S_t can be expressed as follows: (1) (3)

$$S_t = K \cdot \varphi_t$$

where K is a constant to be determined by tests, and φ_t is creep characteristics of concrete. In our present test K is about 14.0×10^{-5}.

Under this assumption the theory gives that the shrinkage, $\bar{S}_t$, of reinforced concrete column made with the similar concrete is

$$\bar{S}_t = K\varphi_{rpt}$$

where φ_{rpt} is creep characteristics of reinforced concrete as given by Eq. (8). The shrinkage values plotted in Fig. 12 show that the above assumption is fairly well but that the reinforced concrete specimens in our pressent test seemed to attain stable states earlier than the creep of concrete did.

Conclusion

On the basis of literature studies and of our test, a general creep theory was developed, by means of which the creep behavior of the test column subjected to sustained loading with any eccentricity could be predicted. Major conclusions are as follows:

(1) At the end of 1/2 year of sustained axial load, irrespective of a comparably older age (12-week) of concrete at the begining of loading, the plain concrete column had attained about 65 percent of the final creep strain, while the tied columns reinforced vertically with $p=2.08\%$ and $p=3.72\%$ had attained about 70 percent of their final creep strains.

(2) At the end of 46-week sustained loading, plain concrete had creep strain of about 1.9 times as large as the immediate elastic strain, although loading began at 12-week age.

(3) For eccentric load with small eccentricity of $e_1 \doteqdot 0.25$, the tensile concrete strains were so small and, in column group B reinforced with four 16 mm bars, varied even to compressive strain values at the end of 46-week of sustained loading, while the compressive concrete creep strains observed were 0.014 to 0.020 percent (about 1.35 to 1.17 times as large as the immediate elastic strain) at the end of 46-week duration of loading.

(4) So far the creep due to sustained axial load or eccentric load with small ecentricity is concerned, the creep theory developed by Prof. Fr. Dischinger is in good conformity with measurements, which predicts that a large ratio of reinforcement causes much reduction in compressive concrete creep and in compressive concrete stress, and less increase in steel compressive stress.

(5) After 46-week of sustained loading with eccentricity $e_1=0.5$, the compressive concrete creep of reinforced concrete columns was about 1.10 to 0.88 times as large as the immediate elastic strain, while the variation of strain in tension side should be attributed probably due to causes other than sustained loading

(principally shrinkage), and the tensile cracks increased in number with the duration.

(6) Agreement between measured and computed compressive concrete creep indicates that the analytical solution developed by the writers will satisfactorily predict, qualitatively as well as quantitatively, all the phenomena observed by the tests, such as increase in compressive concrete strain, in compressive steel stress, in deflection of column, and reduction in compressive stress of concrete. The analysis also gives a variation in location of neutral axis and in tensile steel stress with the duration of sustained loading.

(7) For analytical purpose, shrinkage of plain and reinforced concrete may be expressed by such a simple function of their compressive creep characteristics as $S_t = (10\sim15)10^{-5}\times(\varphi_t$ or $\varphi_{rpt})$. It is noticed, however, that the actual shrinkege may deviate from the analytically assumed value.

Acknowledgment

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