Researches on the Fatigue Deformation

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1. Introduction

In order to design machine parts taking fatigue into consideration, the diagram of endurance limit is generally needed. The diagram is the one that represents the relation between mean stress and stress amplitude at the endurance limit and it shows the yield limit caused by the maximum stress.

As an example, in case of the repeated tension-compression, those relations thus far are represented by the lines AT and RS in Fig. 1. Explaining further those relations in details, the line AT is a straight line drawn from A to T and the line RS is a straight line drawn diagonally from S with a inclination of 45 degree. The point A denotes the completely reversed fatigue limit and the point T, the breaking stress on final area while the point S, the yield point in static test.

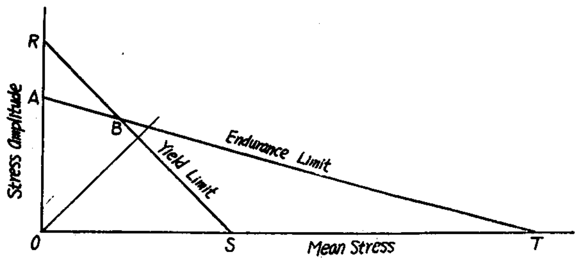


Fig. 1. Diagram of endurance limit.

The fact that the relation shown by the line AT coincides well with the one obtained from fatigue tests has been generally realized in the range where the maximum stress is less than the yield stress in static tests. Precisely speaking, the fact that the line AT shows a good agreement with experimental results is merely an empirical relation. And this relation is very convenient because, if the completely reversed fatigue limit is known, the approximate diagram of endurance limit can be obtained.

On the contrary, the experimental proof of validity of the line RS, although the line has been regarded as being valid, has hardly been given, besides, there are some doubts existing concerning its validity. For example, considering the point R which shows the case of completely reversed stress, the stress varies between the two equal values of positive and negative; consequently, it is not conceivable that the yielding phenomenon is likely to appear under such a condition. Therefore, it would be definitely erroneous to regard the point R as a critical stress at which the deformation increases drastically. Also, the straight line RS has been drawn from the mere static conception and no consideration has been given to the effect of fluctuation of stress. The fact that the fatigue strength is much lower than the static strength seems to be caused by the effct of fluctuation of stress.

Considering these facts, validity of the line RS drawn from the wholly static conception is quite incredible. In fact, A. Ono[1] once discovered that if the repeated tension between 0 and a certain stress just below the yield stress is applied to some materials, the yielding phenomenon occurs after a certain stress reversals. Also some machine parts in service, say, the vehicular springs, become unserviceable owing to the permanent deformation which gradually appears during their use. That is, the vehicular spring is subjected to fluctuating stress superimposed with the stress due to the body weight which is considered as a mean stress and the permanent deformation, caused by yielding, appears even under small stresses which cannot be predicted from a consideration of static cases.

Taking into account the above experimental and more or less empirical facts, it should be noted that the yield limit shown by the line RS is not always appropriate for all materials. And it appears that it is not proper to regard the region OABS in Fig. 1 as a permissible safety region in conventional design for all materials, but the region becomes much smaller, and it is considered necessary to clarify a yield limit based upon experimental results. In other words, efforts should be made to clarify the relation between the permanent deformation under the static stress and that under the reversed stress.

Here, a few experimental results associated with the present problem will be examined. L. Bairstow[2], had reported in his paper that, comparing experimentally the magnitudes of deformations under static and repeated stresses in case the material has a clear yield point, the magnitudes of both deformations under static and repeated stresses agree with each other when they are in the range where the maximum stress is beyond the yield stress; but the deformation under the repeated stress is larger than that under the static stress when they are between the elastic limit and the yield point. H. J. Gough[3] also found that the magnitudes of deformation of material having a clear yield point are the same in both cases of the static and repeated stresses over the whole range of magnitude of stress. While Ichihara's[4] results have shown that, for materials without a clear yield point, the magnitude of deformation under repeated stress is much larger than that under the static stress.

Studying these results so far, the results provided by Bairstow and Gough show no difference between the magnitudes of deformation in both cases of the static and repeated stresses in the range over the yield stress, but, on the contrary, Ichihara's result shows different aspect. As a reason for this variance, it may be considered due to the fact that the former used the materials having a clear yield point, but the latter used the ones without a clear yield point. However, in spite of using materials having the similar static characteristics concerning the yielding phenomenon, Bairstow and Gough had found different results at any optional stresses between the elastic limit and the yield point.

Observing the previous experimental results in detail as has been referred to above, it seems that no clear conclusion can be drawn at present for the deformation on account of the repeated stress. This investigation, therefore, was carried out to clarify the characteristics of steel for fatigue deformation—the term "fatigue deformation" will be used as meaning the deformation under the repeated stress as mentioned above—and the theoretical consideration was developed in order to discuss fatigue deformation more accurately.

2. Materials tested and specimens

The materials used in this experiments were four kinds of carbon steels and three kinds of spring steels, and their chemical compositions are shown in Table 1. The mechanical properties and heat treatment are shown in Table 2. The materials SUP 3 (A) and SUP 3 (B) had the same chemical compositions, but were received at the different times. Since the carbon steels 2245, 1870 FA and spring steels did not show their yield points, the yield stresses shown in Table 2 were indicated by the stress which produced 0.2% of permanent strain. The experiments were made with the Nishihara's fatigue testing machines, one for plain bending and the other for torsion. The number of cycles per minute of these machines was about 2000 for the former and about 2600 for the latter.

Table 1. Chemical composition of materials (%).

Symbol	C	Mn	Si	S	P	Cu	Cr
5637	0·10	0·62	0·16	0·040	0·044	—	—
982 FA	0·51	0·38	0·27	0·010	0·023	—	—
2245	0·61	0·61	0·25	0·026	0·044	—	—
1870 FA	0·90	0·22	0·29	0·007	0·024	—	—
SUP 3 (A)	0·85	0·47	0.33	0·015	0·022	0·20	0·32
SUP 3 (B)	”	”	”	”	”	”	”
SUP 4	1·03	0·51	0·32	0·046	0·040	0·19	0·23

Table 2. Heat treatment and mechanical properties of materials.

Symbol	Heat Treatment	Tensile Test		Bending Test		Torsion Test		Shore Hard-ness Number
		Yield Point kg/mm²	Ultimate Strength kg/mm²	Elastic Limit kg/mm²	Yield Point kg/mm²	Elastic Limit kg/mm²	Yield Point kg/mm²	
5637	As-received	—	—	—	—	13·8	19·5	—
982 FA	〃	41·9	69·5	—	—	21·5	24·5	28·0
2245	Heated at 830°C in 10 min., then oil quenched, and tempered at 550°C in 1hr.	49·6	73·6	45·0	90·0	26·0	44·2	30·7
1870 FA	As-received	—	—	—	—	25·0	31·0	—
SUP 3 (A)	Heated at 850°C in 1hr, then oil quenched, and tempered at 475°C in 1hr.	130·5	145·0	65·0	162·0	—	—	38.4
SUP 3 (B)	〃	106·0	126·5	50·0	147·0	—	—	34.1
SUP 4	〃	—	—	—	—	65·2	77.2	—

The form and dimensions of fatigue specimens used are shown in Fig. 2.

Observation of permament strain due to the fatigue deformation was conducted by the following methods: as to the bending perment strain, it was calculated from deflection of the specimen which was measured by a dial gage, while the torsional permanent strain was measured by the optical-lever method.

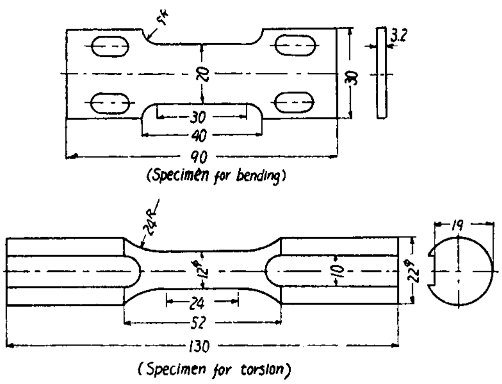


Fig. 2. Forms of fatigue specimens.

3. Experimental Results

(1) Bending fatigue deformation

Fatigue tests were performed on the carbon steel 2245 and spring steels SUP 3 (A) and SUP 3 (B), shown in Table 2, and the stress was shown by the ratio between the mean stress, σ_m, and the stress amplitude, σ_a. The ratio of σ_a to σ_m was taken as 1.0 and 0.4 for 2245, and 1.0, 0.5, 0.4, 0.3, 0.2 and 0.1 for SUP 3 (A) and 0.2 for SUP 3 (B).

Deflections of the specimens during the fatigue test were measured at various stages of stress repetitions, and the relations between the nominal values of permanent strain calculated from them and the number of stress repetitions are shown in

Figs. 3, 4 and 5.

As seen in three figures, the permanent strain increases rapidly immediately after the beginning of tests, and then the rate of increase becomes gradually small. And the aspect of augment of the permanent strain may be divided into two groups.

In the 1st group the permanent strain increases continuously until the specimen fractures; on the contrary, in the 2nd group such increase cannot be seen after some stress repetitions have taken place. The former corresponds to the case of respectively large stress while the latter, to the small stress. Most specimens of the latter group did not fracture regardless of increase of the number of stress repetitions, although some specimens resulted in fracture at the fillets because of the stress concentration. In other words, it may well be concluded that the 1st group corresponds to the case of stress beyond the fatigue limit while the 2nd group, to the case of stress below

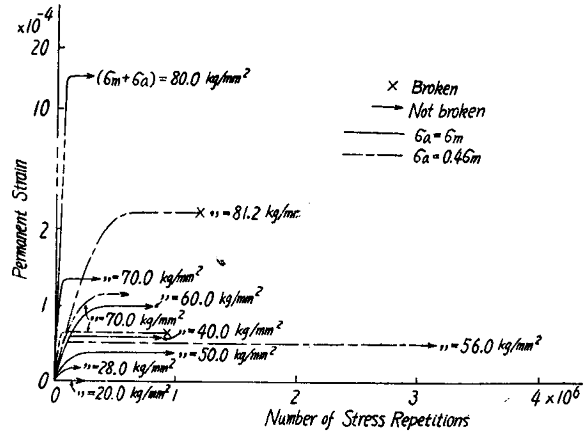


Fig. 3. Relation between the permanent strain and the number of stress repetitions (2245).

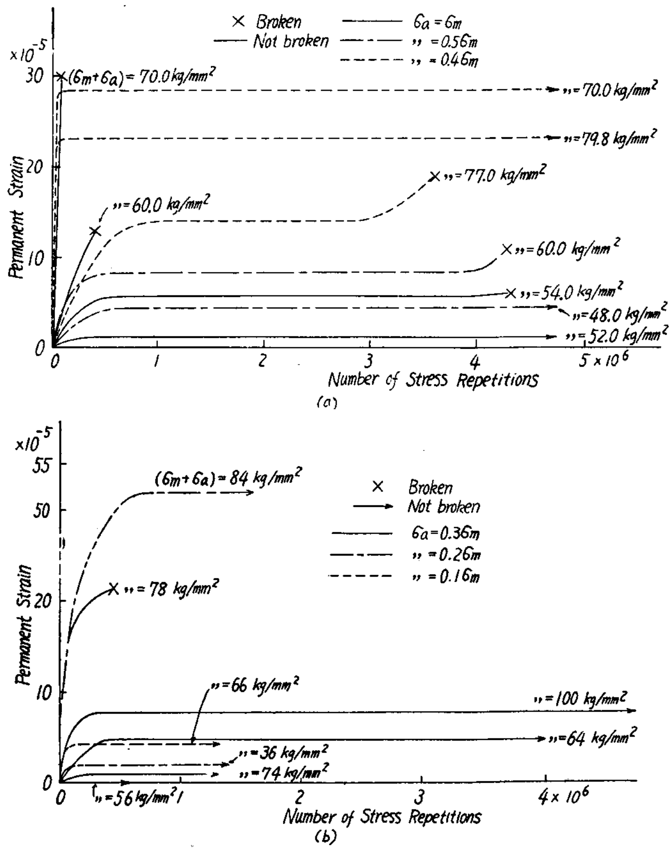


Fig. 4. Relation between the permanent strain and the number of stress repetitions (SUP 3 (A)).

the fatigue limit.

(2) Torsional fatigue deformation

Fatigue tests of this series were performed on the carbon steels 5637, 982 FA, 2245, 1870 FA and the spring steel SUP 4 under the mean stress, τ_m, and the stress amplitude, τ_a. The ratio of τ_a to τ_m was selected as 1.0 for the carbon steels 5637, 982 FA and 1870 FA, 1.0 and 0.4 for the carbon steel 2245, and 1.0, 0.4 and 0.3 for the spring steel SUP 4.

The specific torsional angle was measured at various stages of stress repetitions, thereby the nominal value of permanent strain was calculated. The relations between the permanent strain and the number of stress repetitions are shown in Figs. 6~10.

As seen in these figures, the aspect of increase of permanent strain with stress repetitions is quite similar to the case of bending fatigue deformation mentioned above. Only in the mild steel, however, the increase of permanent strain was sometimes stepwise as shown in Fig. 6. And it is considered that this is peculiar to mild steel.

In addition, the stress-endurance curves obtained

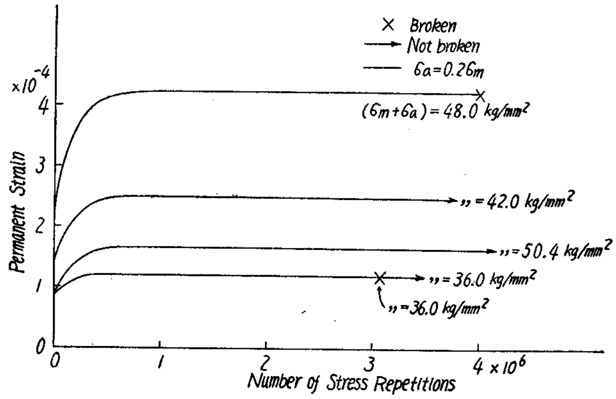


Fig. 5. Relation between the permanent strain and the number of stress repetitions (SUP 3 (B)).

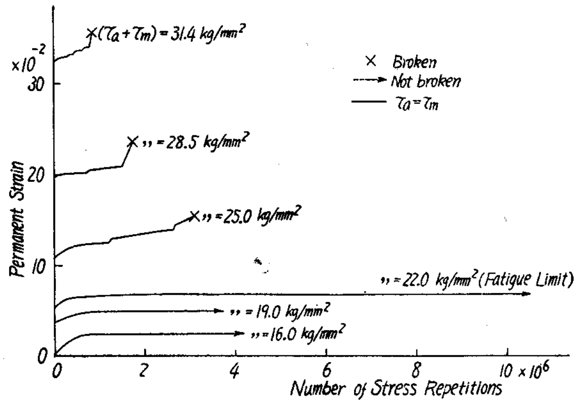


Fig. 6. Relation between the permanent strain and the number of stress repetitions (5637).

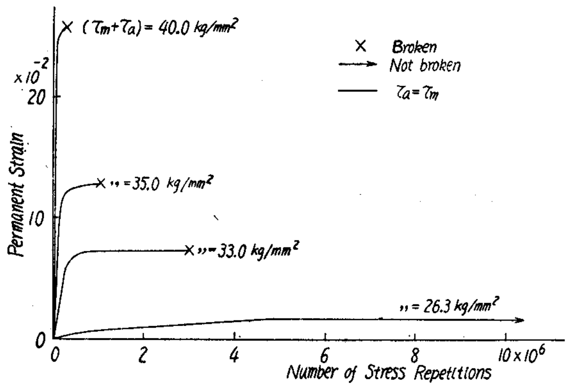


Fig. 7. Relation between the permanent strain and the number of stress repetitions (982 FA).

from the above experiments of 5637, 982 FA and 1870 FA are shown in Fig. 11. However, the curves for 2245 and SUP 4 could not be drawn because all the tests were conducted at stresses below the fatigue limit due to lack of capacity of the testing machine.

(3) Fatigue deformation on the pre-loaded material

The bending fatigue tests on the spring steel SUP 3 (B) were made under $\sigma_a/\sigma_m=0.2$. And the nominal values of permanent strain caused by the pre-loading were 0.001 and 0.002.

The relation between the permanent strain and the number of stress repetitions obtained from the experimental results is shown in Fig. 12.

The increasing manner of the permanent strain in the figure was also similar to that of the case without pre-loading.

4. Analytical consideration

(1) Fatigue deformation under uniform stress distribution

The fatigue deformation under uniform stress distribution appears in alternating

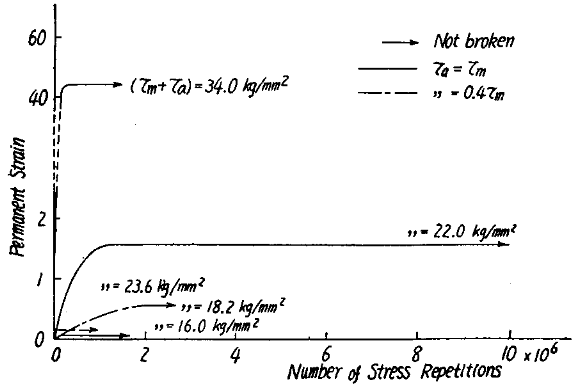


Fig. 8. Relation between the permanent strain and the number of stress repetitions (2245).

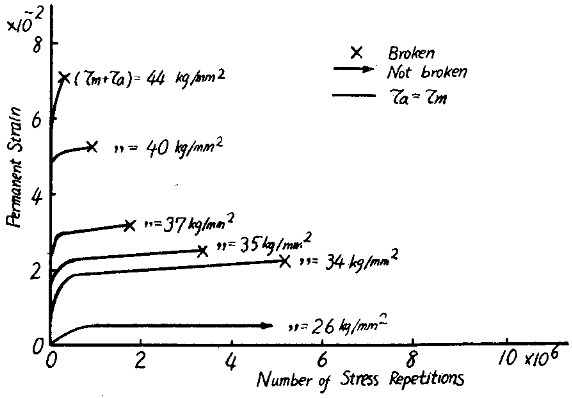
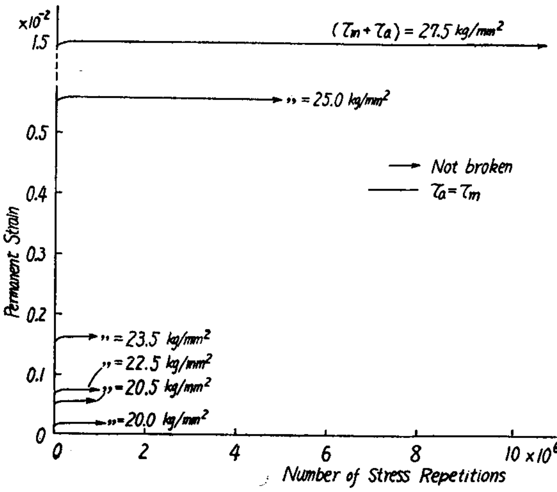


Fig. 9. Relation between the permanent strain and the number of stress repetitions (1870 FA).



(a)

Fig. 10a. Relation between the permanent strain and the number of stress repetitions (SUP 4).

tension-compression or torsion tests in case of the thin-walled tube specimens, and in the forthcoming treatment the authors will discuss on the former case because the principle on the latter case is the same as that on the former.

It has been clarified by the experiments that the rate of increase of fatigue deformation in alternating tension-compression is very large at the beginning of stress repetitions, but it diminishes gradually and finally vanishes so long as the magnitude of stress is not very large. Also in the fatigue deformation under alternating bending or torsion, the increasing rate, as shown in these experiments, is in like manner as the above.

Then we shall represent the magnitude of fatigue deformation at a given mean stress and stress amplitude by the final value reached at a certain number of stress repetitions after which the deformation does not increase any more, and denote it by the strain. In the subsequent treatment the symbol ε_{fp} will be adopted as the permanent strain in tension-compression at the final value and the symbol γ_{fp} as that in case of torsion.

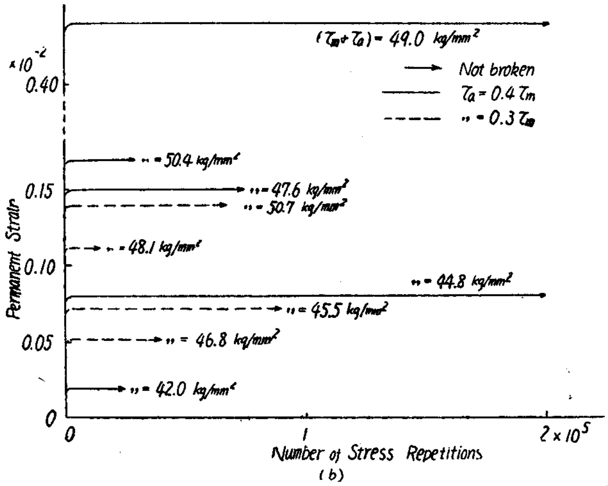


Fig. 10b. Relation between the permanent strain and the number of stress repetitions (SUP4).

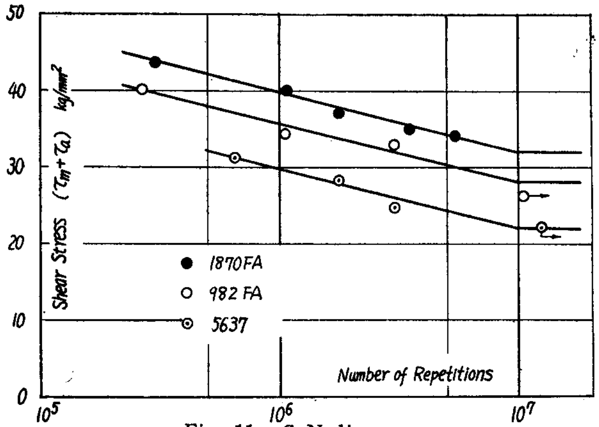


Fig. 11. S-N diagram.

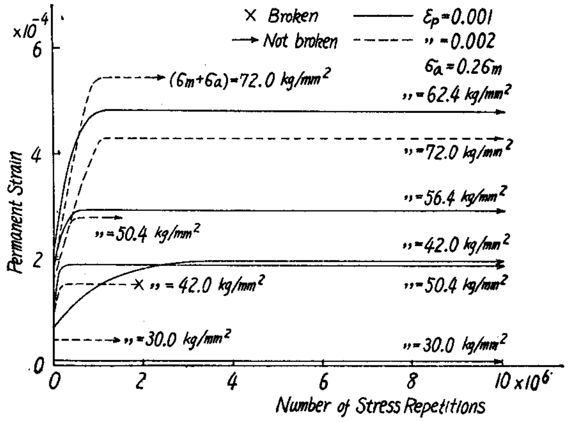


Fig. 12. Relation between the permanent strain and the number of stress repetitions (SUP3(B)).

Next, let us consider an expression which represents the relation between the mean stress σ_m, the stress amplitude σ_a, and the permanent strain ε_{fp} obtained from the results of fatigue tests. In a special case of very small stress amplitude, that is, $\sigma_a=0$, the relation between σ_m and ε_{fp} should coincide with that between the static permanent strain ε_p and the static stress σ in tension test. Thus we can obtain the relation between σ_m, σ_a and ε_{fp} in alternating tension-compression on the base of the relation between σ and ε_p in static tension.

Generally, the relation between σ and ε_p in the static test of metals can be divided into the following two cases according to whether or not a clear yield point exists. In other words, it is expressed by the following equation when the materials have the clear yield stress σ_s:

$$\varepsilon_p = \varepsilon_s + A_t(\sigma - \sigma_s)^{n_t}, \quad \text{for} \quad \sigma \geq \sigma_s, \tag{1}$$

where ε_s is the permanent strain at the end of yielding. While in the case of materials which have no clear yield stress, it is expressed by:

$$\varepsilon_p = A_t(\sigma - \sigma_e)^{n_t}, \quad \sigma \geq \sigma_e, \tag{1'}$$

where σ_e is the elastic limit. And A_t and n_t in these expressions are the constants determined from the experimental result of static tension or compression test. For modification of these relations to the case of fatigue, we substitute $(\sigma_m+\sigma_a)$ for σ in the equation (1) or (1′), namely,

$$\sigma = \sigma_m + K_t\sigma_a. \tag{2}$$

The equation (2) agrees with that of static test, when $\sigma_a=0$. Also if $K_t=1$ in Eq. (2), σ becomes equal to $(\sigma_m+\sigma_a)$ which denotes the maximum stress, and it means that the fatigue deformation is determined only from the magnitude of maximum stress σ_{max} and this explanation has been generally applied for all materials. The fatigue deformation, however, depends not only on the maximum stress but also on the magnitude of stress amplitude, and may be influenced more strongly by the latter. In other words, the deformation will be perhaps larger by the cyclic action of σ_a than the one when the maximum stress $(\sigma_m+\sigma_a)$ acts statically. Thus, K_t may be supposed as a dimensionless number greater than 1. Further, since the effect of σ_a on the fatigue deformation presumably becomes greater as the mean stress σ_m becomes larger, K_t may be considered as the following equation,

$$K_t = \alpha\sigma_m{}^q,$$

where α and q are constants peculiar to materials. To make α dimensionless, we put

$$K_t = \alpha_t\left(\frac{\sigma_m}{\sigma_s}\right)^{q_t} \quad (3) \qquad \text{or} \qquad K_t = \alpha_t\left(\frac{\sigma_m}{\sigma_e}\right)^{q_t} \quad (3'),$$

where α_t and q_t are constants, and the Eq. (3) is for the materials having the clear yield point while the latter Eq. (3′) is for materials without clear yield point. Putting Eqs. (2), (3) and (3′) into Eqs. (1) and (1′), we have

$$\varepsilon_{fp} = \varepsilon_s + A_t \left\{ \sigma_m + \alpha_t \left(\frac{\sigma_m}{\sigma_s} \right)^{q_t} \sigma_a - o_s \right\}^{n_t} , \tag{4}$$

or

$$\varepsilon_{fp} = A_t \left\{ \sigma_m + \alpha_t \left(\frac{\sigma_m}{\sigma_e} \right)^{q_t} \sigma_a - \sigma_e \right\}^{n_t} . \tag{4'}$$

These Eqs. (4) or (4′) represents the permanent strain due to the fatigue in repeated tension-compression. The constants A_t and n_t, mentioned above, should be determined from the static tension or compression test while the constants α_t and q_t, from the fatigue test.

As seen in the said experimental results, for the materials with the clear yield point, the plastic strain due to fatigue do not occur when the maximum stress is less than σ_s. Accordingly, Eq. (4) is applicable only under the condition of $o_m + \sigma_a > \sigma_s$. While, for the materials without the clear yield point, Eq. (4′) is valid even under the condition of $\sigma_m + \sigma_a < \sigma_e$. Then putting $\varepsilon_{fp} = 0$ in Eq. (4′), the condition under which the fatigue deformation begins to occur can be obtained from the following equation,

$$\sigma_m + \alpha_t \left(\frac{\sigma_m}{\sigma_e} \right)^{q_t} \sigma_a - \sigma_e = 0 . \tag{5}$$

Similarly to Eq. (4′), we can express the relation between the mean shear stress τ_m, the shear stress amplitude τ_a, and the permanent shear strain γ_{fp}, by:

$$\gamma_{fp} = A_s \left\{ \tau_m + \alpha_s \left(\frac{\tau_m}{\tau_e} \right)^{q_s} \tau_a - \tau_e \right\}^{n_s} , \tag{6}$$

in which constants A_s and n_s are obtained from the static torsion test on thin-walled tubes, while the constants α_s and n_s, from the alternating torsion test on similar specimens.

(2) Fatigue deformation under non-uniform stress distribution

Similarly as in the foregoing expression, the relation under uneven stress distribution for bending or torsion of solid bar can be obtained from the corresponding static test. That is, the nominal value of permanent strain at the surface under repeated bending stress (the mean stress σ_m and the stress amplitude σ_a), ε^*_{fp}, may be obtained by the following equation for the materials having no clear yield point,

$$\varepsilon^*_{fp} = A_t^* \left\{ \sigma_m + \alpha_t^* \left(\frac{\sigma_m}{\sigma_e} \right)^{q_t^*} \sigma_a - \sigma_e \right\}^{n_t^*} , \tag{7}$$

where the constants A_t^* and n_t^* are determined from the static bending test, the constants α_t^*, q_t^* from the fatigue test, while σ_e is the elastic limit under static bending. The same relation exists for the permanent shear strain at the surface, γ_{fp}^{*}, under torsion on the specimen of solid bar or thick-walled tube and it may be expressed by the following equation,

$$\gamma_{fp}^{*} = A_s^{*}\left\{\tau_m + \alpha_s^{*}\left(\frac{\tau_m}{\tau_e}\right)^{q_s^{*}}\tau_a - \tau_e\right\}^{n_s^{*}}, \tag{8}$$

in which the constants A_s^{*}, n_s^{*} and α_s^{*}, q_s^{*} must be obtained from the static and fatigue torsion tests respectively.

(3) Variation of stress distribution due to fatigue deformation

When the fatigue deformation is brought about under the uneven stress distribution (i.e. under alternating bending or torsion of solid specimen), the distribution of mean stress in the specimen necessarily varies. The authors will now consider the method that gives the distribution of mean stress by the alternating bending test on a bar of rectangular section (thickness 2h, breadth b).

It has been assumed in many cases that the stress-strain relation at any layer of specimen subjected to bending in static test, when the materials have no clear yield point, is identical with that under axial loading[5]. In the forthcoming treatment the authors assume that the relation between σ_m, σ_a and ε_{fp} in Eq. (4') under alternating tension-compression holds also for all the layers in the bending fatigue specimen.

Now, if the bending fatigue test is performed under a static bending moment M_m and a moment amplitude M_a with a bar of rectangular section, the distribution of mean stress due to M_m at the beginning of the test and the coressponding strain distribution will be linear throughout as shown with a full line in Fig. 13. Similarly these distribution due to M_a are also linear. Consequently the magnitude of mean stress σ_m and stress amplitude σ_a at the surface are then obtained from the following equation as generally calculated.

$$\sigma_m = \frac{6M_m}{bh^2}, \quad \sigma_a = \frac{6M_a}{bh^2}.$$

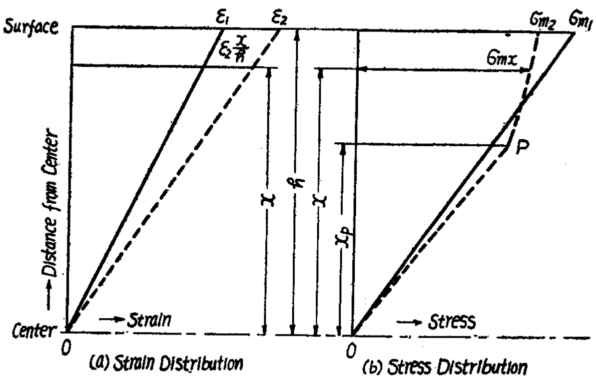


Fig. 13. Distributions of strain and stress in a specimen under a static bending moment.

The bar, however, will bent gradually owing to the fatigue deformation. Then let us assume that the increase of fatigue deformation is stopped when the strain of surface due to the mean stress at the beginning of test, ε_1, has increased to ε_2. Even in this state, the strain distribution may be considered to be still linear from the Bernoulli's assumption on the maintenance of plane after deformation, as shown with a broken line in Fig. 13, but the distribution of mean stress, however, is no longer linear as shown with a broken line in the figure. This is because the stress in the inner region increases elastically in order to supplement the relaxation of mean stress near the surface.

So, we will primarily consider the distribution of mean stress. Since the distribution of stress amplitude can be considered to be linear no matter how the fatigue deformation occurs, ε_{fpx} can be obtained, from consideration of the assumption mentioned above. In other words, from the Eq. (4) by substituting σ_m and σ_a for σ_{mx} and $\sigma_a \frac{x}{h}$,

$$\varepsilon_{fpx} = \mathrm{A}_t \left\{ \sigma_{mx} + \alpha_t \left(\frac{\sigma_{mx}}{\sigma_e} \right)^{q_t} \sigma_a \frac{x}{h} - \sigma_e \right\}^{n_t} , \tag{9}$$

where σ_{mx} and ε_{fpx} denote the values of mean stress and permanent strain at the distance x from the center axis. And the sum of ε_{fpx} and the elastic strain due to σ_{mx} should be equal to $\varepsilon_2 \frac{x}{h}$. Therefore, we have

$$\frac{\sigma_{mx}}{E} + \varepsilon_{fpx} = \varepsilon_2 \frac{x}{h} , \tag{10}$$

where E is the modulus of elasticity in static test. Eliminating ε_{fpx} from these equations, σ_{mx} can be obtained as a function of x and ε_2. Consequently, when ε_2 is known, the distribution of mean stress in the inner region becomes clear. Such a distribution of σ_{mx}, however, exists only in the outer region of the point P. In other words, the point P is a critical point of the linear stress distribution, and x_e, which denotes a distance from the center axis to the point P, can be obtained from the condition that the value of ε_{fpx} is zero at P. By substituting $E\varepsilon_2 \frac{x_e}{h}$ for σ_{mx} in Eq. (9), we get

$$E\varepsilon_2 \frac{x_e}{h} + \alpha_t \left(\frac{E\varepsilon_2 \frac{x_e}{h}}{\sigma_e} \right)^{q_t} \sigma_a \frac{x_e}{h} - \sigma_e = 0 . \tag{11}$$

Thus the distribution of σ_{mx} is obtained, provided ε_2 is found by the test, and the distribution of residual stress also can be found as the difference between the initial linear distribution of mean stress and the above distribution of σ_m.

In the above-mentioned procedure, the fatigue test must be carried out to obtain the distribution of mean or residual stress, but ε_2, in the procedure, can be obtained not by tests but by calculation. That is, σ_{mx} is found as the function of x and ε_2 in the above method, and the bending moment produced by σ_{mx} must be equal to the initial static bending moment M_m. Therefore the following equation is led,

$$2\int_0^h \sigma_{mx} \cdot xbdx = M_m . \tag{12}$$

So putting σ_{mx} obtained in the above into this equation, ε_2 can be found. Thus if ε_2 is obtained by the calculation, the nominal value of permanent strain, ε_{fp}^*, in alternating bending can be calculated from the following equation because ε_1 is an initial strain and is known,

$$\varepsilon_{fp}^* = \varepsilon_2 - \varepsilon_1 . \tag{13}$$

As has been mentioned above, the relation between σ_m, σ_a and ε_{fp}^* in alternating bending can be obtained from calculation, provided the relation between σ_m, σ_a and ε_{fp} in alternating tension-compression is known.

Similarily, from the torsional fatigue deformation of thin-walled tube, the mean or residual stress in the fatigue deformation or the fatigue deformation in alternating torsion tests on solid specimens can be found.

(4) Fatigue deformation of the pre-loaded materials

In this section, a procedure to find the fatigue deformation of pre-loaded material from that of the material without pre-loading will be suggested. As previously mentioned, each relation between the stress and the permanent strain in the static and fatigue tests can be represented, respectively, with the following equations,

$$\varepsilon_p = A_t(\sigma - \sigma_e)^{n_t}, \quad \sigma \geq \sigma_e , \tag{14}$$

$$\varepsilon_{fp} = A_t(\sigma_m + K_t\sigma_a - \sigma_e)^{n_t} . \tag{15}$$

Fig. 14 illustrates these relations, in which the abscissa designates the total strain for both cases and the ordinate designates the stress for static test or the maximum stress $(\sigma_m + \sigma_a)$ for fatigue test. Then consider a material, the stage of which is reached after removing the load from the condition given by A in Fig. 14. It has been known that, when the material at B is again loaded gradually, the stress-strain diagram becomes approximately like the line BAC. Similarly, it would be regarded as likely that the relation between the maximum stress and the strain due to alternating load in fatigue test is represented with the line BA′C′. The magnitude of the permanent strain due to fatigue in this case may be shown by the following equation,

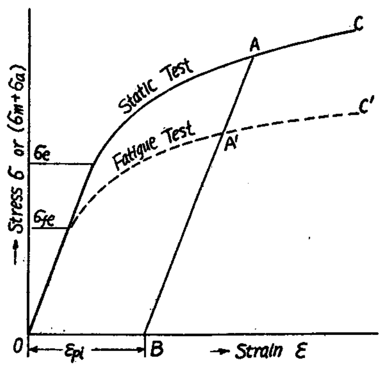


Fig. 14. Relation between stress and strain in static and fatigue tests.

$$\varepsilon_{fp} = A_t(\sigma_m + K_t\sigma_a - \sigma_e)^{n_t} - \varepsilon_{pi} , \tag{16}$$

where ε_{pi} is the strain at the state B.

The above refers to the cases in which the pre-loading and alternating loading are given under a uniform stress distribution, however, in the other cases in which the pre-loading is subjected under uneven stress distribution, such as bending or torsion, the specimens develope into a state of work-hardening and produce simultaneously the residual stress. Therefore, in order to find the fatigue deformation caused by the alternating load, both of these effects must be taken into consideration. In the subsequent discussion, attempts will be made to explain only on the specimen of rectangular cross-section subjected to the bending pre-loading and the alternating bending load, and the other cases can be treated in the similar way.

Now Fig. 15 (a) illustrates the stress-strain diagram in the static bending test (the stress at the ordinate designates the value obtained by dividing bending moment by sectional coefficient), and supposing it is subjected to the pre-load equal to σ_i^*, the strain at the surface will be ε_i^*. The true stress necessary to produce the strain ε_i^* is σ_i from the diagram in Fig. 15 (b), which represents the result of static tension test. So, observed only from the work-hardening caused by pre-loading, the outermost layer of specimen takes the state of B which leaves the permanent strain ε_{pi}. And the characteristics of fatigue deformation of the outermost layer due to the subsequent alternating load can be shown with the line BA′C′ as explained in Fig. 14, and when the maximum stress exceeding a certain stress shown at A′ functions as an alternative stress, the fatigue deformation appears for the first time. That is, permanent strain due to fatigue deformation in this case is found from Eq. (16). Therefore, if the stress amplitude σ_a is known, a magnitude of mean stress that makes ε_{fp} zero can be obtained from Eq. (16). Thus, a critical mean stress at which the fatigue deformation begins to appear at the outermost layer can be obtained.

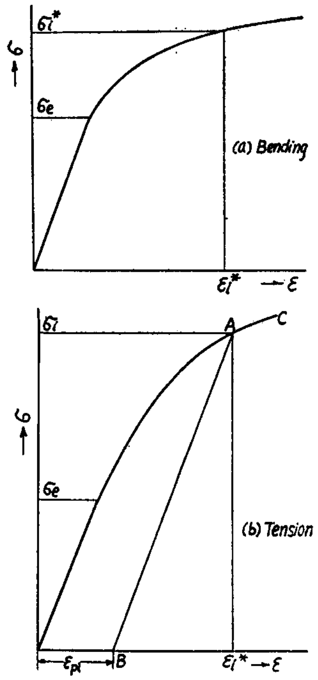


Fig. 15. Relation between stress and strain in static tension and bending tests.

Then applying the similar procedure to any layer successively in the specimen and finding the mean stress to make $\varepsilon_{fp}=0$, a distribution of critical mean stresses becomes like the one which is shown by a chain line in Fig. 16, because σ_a is smaller at the inner layers than at the outer layers. While the residual stress is developed in the specimen by removing pre-load and this distribution may be easily calculated by the Nadai-thought method and is illustrated with a two point chain line in Fig. 16.

Since the residual stress can be regarded generally as the mean stress for the fatigue test, the total stress obtained by adding the residual stress to the mean stress caused by the external load can be considered as the mean stress in this case. The full line in the figure illustrates the distribution of mean stress due to external load and the broken line obtained by adding the full line to the two point chain line represents the actual distribution of mean stress. And the fatigue deformation begins to appear when the broken line comes into contact with the chain line. Fig. 16 illustrates this instance, and the mean stress σ_{ml} due to external load is the critical mean stress at which the fatigue deformation begins to appear under the given stress amplitude σ_a.

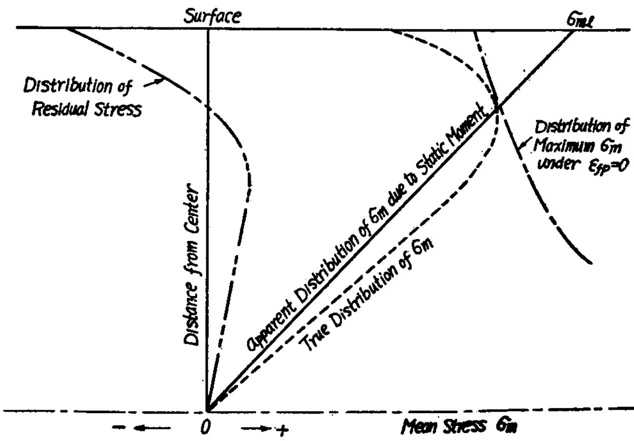


Fig. 16.

5. Consideration on the experimental results

(1) Fatigue yield point

Let us consider the relation between the static yield point and the fatigue yield point from the experiments of a torsional fatigue deformation. Showing the results of fatigue tests, described in chapter 3, together with those of static tests for various materials, we get the stress-strain diagrams as in Figs. 17~21. In these figures the coordinates denote the shear stress and the shear strain for the results of static tests, and the maximum shear stress $(\tau_m+\tau_a)$ and the shear strain are obtained as follows, for the results of fatigue tests; the shear strain just before the fracture is shown for

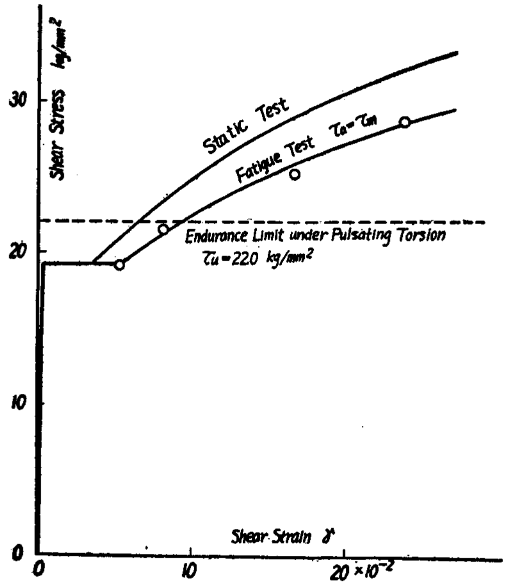


Fig. 17. Relation between the shear stress and strain in static and fatigue tests (5637).

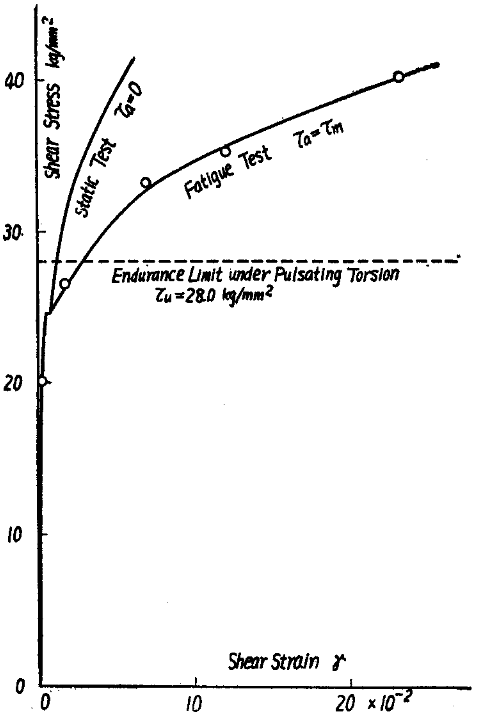


Fig. 18. Relation between the shear stress and strain in static and fatigue tests (982FA).

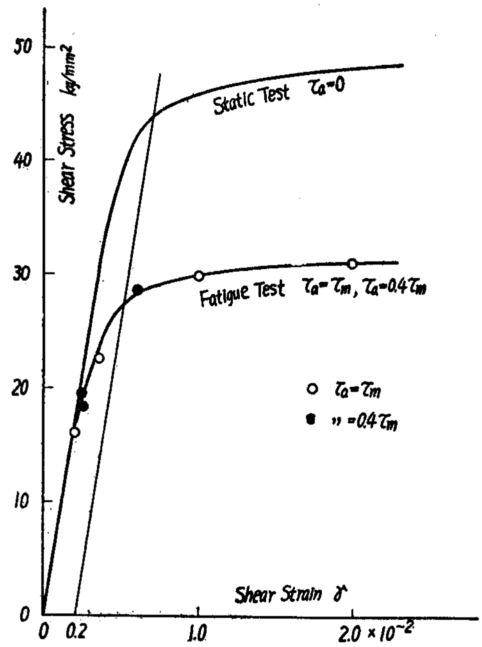


Fig. 19. Relation between the shear stress and strain in static and fatigue tests (2245).

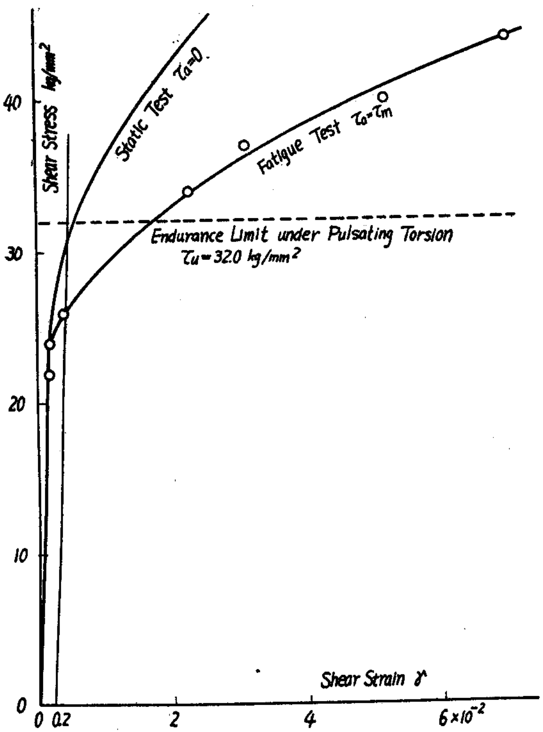


Fig. 20. Relation between the shear stress and strain in static and fatigue tests (1870FA).

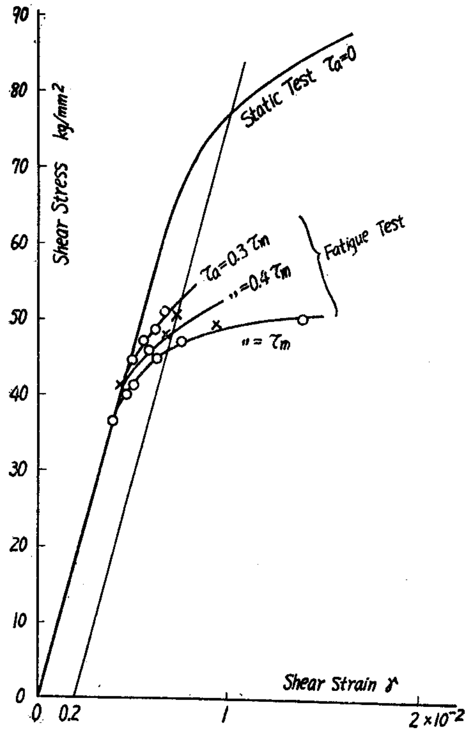


Fig. 21. Relation between the shear stress and strain in static and fatigue tests (SUP4).

the specimens that fractured and the shear strain at the time the fatigue deformation ceases is shown for the specimens that did not fracture.

As seen by these figures, on the materials 5637 and 982 FA having the clear yield point, both strains which take place in static and fatigue tests coincide each other when the maximum torsional moment is below the static yield point. However, if the maximum torsional moment exceeds the static yield moment, remarkable fatigue deformation appears and the strain in fatigue test becomes considerably greater than that in static test. While for the materials 2245, 1870 FA and SUP 4, without the clear yield point, the fatigue deformation appears even at the maximum torsional moment lower than the static torsional yield moment, and the strain is greater in fatigue than in static.

Table 3. Comparison between the static test and fatigue test concerning the elastic limit and yield limit.

Symbol	Static elastic limit kg/mm²	Static yield limit kg/mm²	Stress Condition in fatigue tests	Fatigue elastic limit kg/mm²	Fatigue yield limit kg/mm²
2245	26·0	44.2	$\tau_a=0{\cdot}4\tau_m$	16·8	28·0
			" $= \tau_m$	15·0	28·0
1870 FA	25·5	31·0	$\tau_a= \tau_m$	24·0	26·0
SUP 4	65·2	77·0	$\tau_a=0{\cdot}3\tau_m$	44·5	51·5
			" $=0{\cdot}4\tau_m$	38·4	48·0
			" $= \tau_m$	38·4	45·2

Table 3, then, shows the comparisons between the static and fatigue yield points, which are defined as stresses producing 0.2% permanent set. As it is known from this, the fatigue yield points are fairly lower than the static yield points in all materials. Although the comparison of elastic limit is cited in the table also, the fatigue elastic limit is also lower than that in the static case. Thus it would be concluded as follows: the fatigue yield point coincides with the static yield point in materials having the clear yield points, but it does not and is lower in materials having no clear yield point.

(2) Yield and elastic limits on the diagram of endurance limit

Since the static and fatigue yield points coincided one another for 5637 and 982 FA, the line which indicates the yield limit is represented by a straight line drawn from the static yield point, indicated by 45 deg., and it coincides with the one which have been drawn in the past. Such relation, however, does not hold in other materials.

Fig. 22 is the result on 2245, and it illustrates the results when the permanent strain ε_{fp}^* are 0.001% (elastic limit) and 0.01% in alternating bending and the one γ_{fp}^* are 0.001% (elastic limit) and 0.2% (yield limit) in alternating torsion.

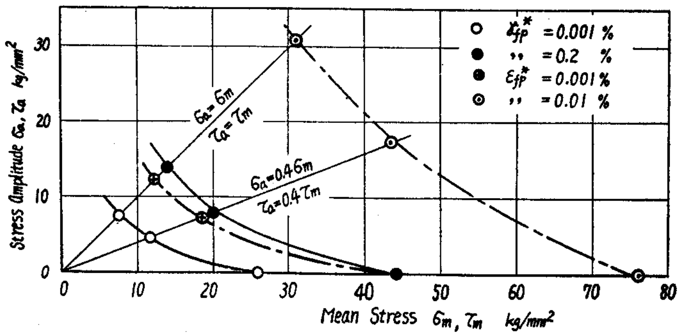


Fig. 22. Diagram of fatigue deformation (2245).

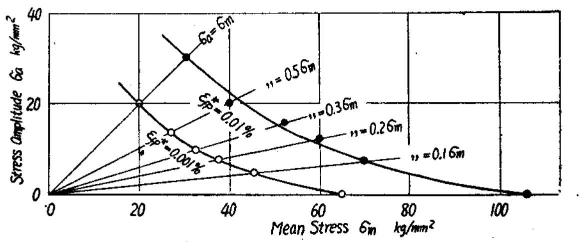


Fig. 23, Diagram of fatigue deformation (SUP3 (A)).

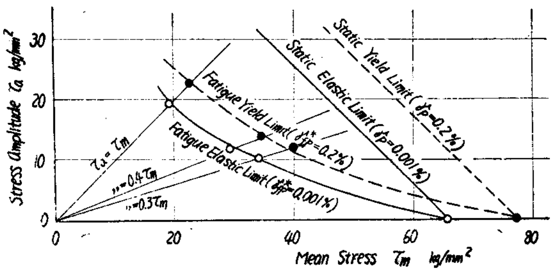


Fig. 24. Diagram of fatigue deformation (SUP4).

Fig. 23 shows the results on SUP 3 (A) when the permanent strains are 0.001% (elastic limit) and 0.2% (yield limit).

Also Fig. 24 shows the case of torsion on SUP 4 and, for the comparison between fatigue and static tests, elastic and yield limits in static test are concurrently represented with the fatigue elastic and yield limits.

As is seen in these figures, especially in Fig. 24, the behaviour of materials on the plastic deformation is inconsistent depending upon whether the stress applied is static or dynamic. In other words, the yield limit, which has been indicated on the diagram of endurance limit of materials having no clear yield point, is not proper and must be obtained from experiments. And the fact that the fatigue yield limit lay in the left-hand range of the static yield limit means that the safe stress range for fracture and deformation is much smaller, and the authors wish to stress the point that the determination of fatigue yield limit is very important and valuable for practical design.

Moreover we will apply the treatment described in 2 of chapter 4 to the experimental results.

The permanent strain in bending or torsional fatigue tests has been represented with Eq. (7) or (8), and the 1st and 2nd terms in the bracket of those equations denote a converted static stress obtained from a suitable combination of the mean stress and stress amplitude in the fatigue tests. Therefore the relations between

σ_m and σ_a or between τ_m and τ_a producing the same permanent strain ε^*_{fp} or γ^*_{fp} as the permanent strain which appears under a static stress σ_{st} or τ_{st} are obtained from the following equations,

$$\sigma_m+\alpha_t^*\left(\frac{\sigma_m}{\sigma_e}\right)^{q_t^*}\sigma_a=\sigma_{st}\,, \tag{17}$$

or

$$\tau_m+\alpha_s^*\left(\frac{\tau_m}{\tau_e}\right)^{q_s^*}\tau_a=\tau_{st}\,. \tag{18}$$

So let us consider the experimental results on SUP 3 (A) in Fig. 27. The chain lines in the figure have been calculated from Eq. (17) by substituting 4 and 0.5 for α_t^* and q_t^*. As seen in the figure, the computed results have good coincidence with the experimental results.

Further for SUP 4 in Fig. 24, the full and dotted lines have been computed from Eq. (18) by substituting 4.3 and 5 for α_s^* and q_s^*, and they also show good agreements.

(3) Fatigue deformation of the material subjected to the preliminary cold-working

The results of fatigue test on the material SUP 3 (B) on which the preliminary cold-working was applied are shown in Fig. 12. Fig. 25 represents the relation between the permanent strain and the mean stress obtained from the tests. As seen in the figure, the effect of the permanent strain on the stress limit, at which the fatigue deformation begins to appear, is very small. However, at the considerably large stress, ε^*_{fp} becomes smaller as the permanent strain applied preliminary becomes larger. Therefore the procedure of "Setting"—this is a kind of plastic working, that is, over-loading before the service and usually applied to springs—seems to be effectual to prevent the fatigue deformation.

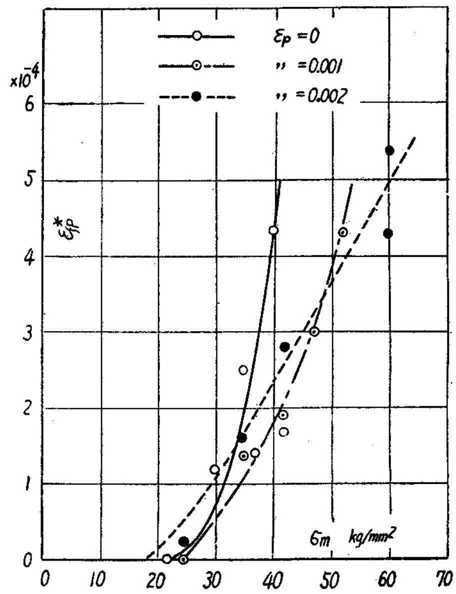


Fig. 25. Bending fatigue deformation when the specimen is subjected to preliminary plastic-working under bending.

(4) Comparison between experimental and analytical results

The analytical procedure to calculate the mean stress distribution in the specimen due to fatigue deformation under uneven stress distribution has been introduced in Chapter 4. In order to ascertain whether the distribution of mean stress obtained is appropriate or not, let us compare them with the experimental results mentioned above. However, the direct comparison is impossible because the distribution of mean stress cannot be observed directly. The comparison, therefore, will be proceeded on the magnitude of fatigue deformation which can be calculated from the distribution of mean stress by using Eq. (13).

First, it will be compared on the experimental result of SUP 3 (A) in Fig. 23 for the alternating bending. To calculate ε_{fp}^{*}, the relation under alternating tension-compression as shown in Eq. (4′) must be found. Obtaining A_t, n_t, σ_e from the static tension test, their values are as follows:

$$A_t = 19.5\times 10^{-8}\,\mathrm{mm^2/kg}\,,\quad n_t = 1.84\,,\quad \sigma_e = 65\,\mathrm{kg/mm^2}\,.$$

On the other hand, α_t and q_t are the ones that are not obtained unless the alternating tension-compression tests are made, but, since such tests are not performed in this experiments, they will be obtained from the results of alternating bending. That is, assuming the elastic limits are the same in both cases of alternating tension-compression and bending tests, the values of α_t and q_t sought from the experimental results of elastic limit in Fig. 23 will be as follows:

$$\alpha_t = 6.5\,,\quad q_t = 1.0\,.$$

Then, Eq. (4′) becomes

$$\varepsilon_{fp} = 19.5\times 10^{-8}\left\{\sigma_m + 6.5\left(\frac{\sigma_m}{65}\right)\sigma_a - 65\right\}^{1.84}. \tag{19}$$

Basing upon this equation and following the above-mentioned procedure, we can obtain the distribution of mean stress in the specimen for many kinds of mean stresses and stress amplitudes. Fig. 26 shows the distribution of mean stress in the interior after fatigue deformation for five kinds of mean stresses (these stresses are the values obtained from conventionally elastic calculation) under $\sigma_a = 20\,\mathrm{kg/mm^2}$, and the magnitude of bending permanent strain ε_{fp}^{*} which is produced in each case is concurrently shown.

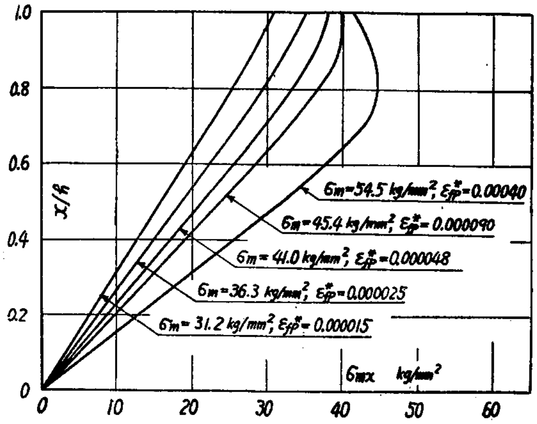


Fig. 26. Distribution of mean stress in the specimen when the fatigue deformation has appeared (σ_a=20 kg/mm²).

As seen in the figure, when the stress amplitude is the same, the relaxation of mean stress near the surface becomes remarkable as the mean stress grows larger. Since the magnitude of fatigue deformation in alternating bending can be obtained in this manner, comparing the calculated value in the case of $\varepsilon_{fp}=0.01\%$ with the experimental results, the full line shown in Fig. 27 is obtained and it shows a fairly good agreement. Further, Fig. 28 shows the distribution of residual stress produced in the interior by the fatigue deformation, which is found from Fig. 26. A similar comparison will be made next on 2245 under torsion. In this case, the relation similar to Eq. (6) must be found by using the thin-walled tube specimens for the calculation, but, since it is not done, the experimental results on the elastic limit for the alternating torsion tests on solid specimens will be applied for α_s, q_s. Also, τ_e is obtained from static torsion test on solid specimen, and A_s, n_s from stress-shear strain diagram of thin-walled tube specimen, prepared by applying the Prandtle's method for static torsion test on solid specimen. Thus we have

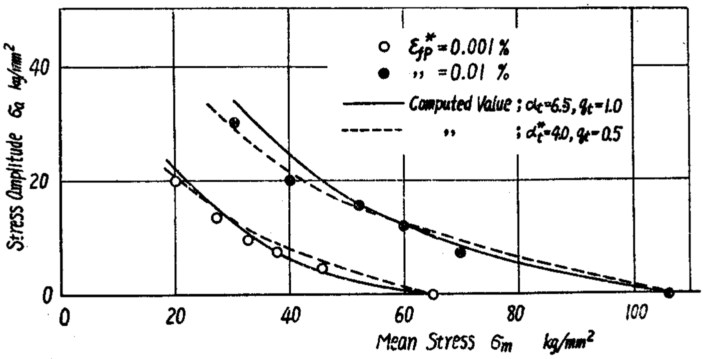


Fig. 27. Comparison between experimental results and computed ones (SUP 3 (A)).

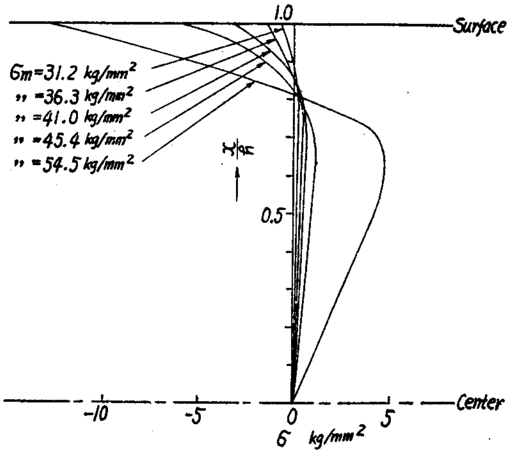


Fig. 28. Distribution of residual stress in the specimen (SUP 3 (A)).

$$A_s = 14.1\times 10^{-7}\ \mathrm{mm^2/kg}\,,\quad n_s = 1.46\,,\quad \tau_e = 26\ \mathrm{kg/mm^2}\,,\quad \alpha_s = 7.8\,,\quad q_s = 1.0\,.$$

Consequently, similarly as in the case of alternating bending, the distribution of mean stress in the specimen can be obtained for many kinds of mean stresses and stress amplitudes. Fig. 29 illustrates the distribution after the fatigue deformation under many kinds of nominal mean stress as in which τ_a is equal to 5, 10 and 15 kg/mm² respectively.

A comparison of a calculated result of $\gamma^*_{fp}=0.2\%$ with the experimental results, as seen in Fig. 30, shows a good agreement between both results.

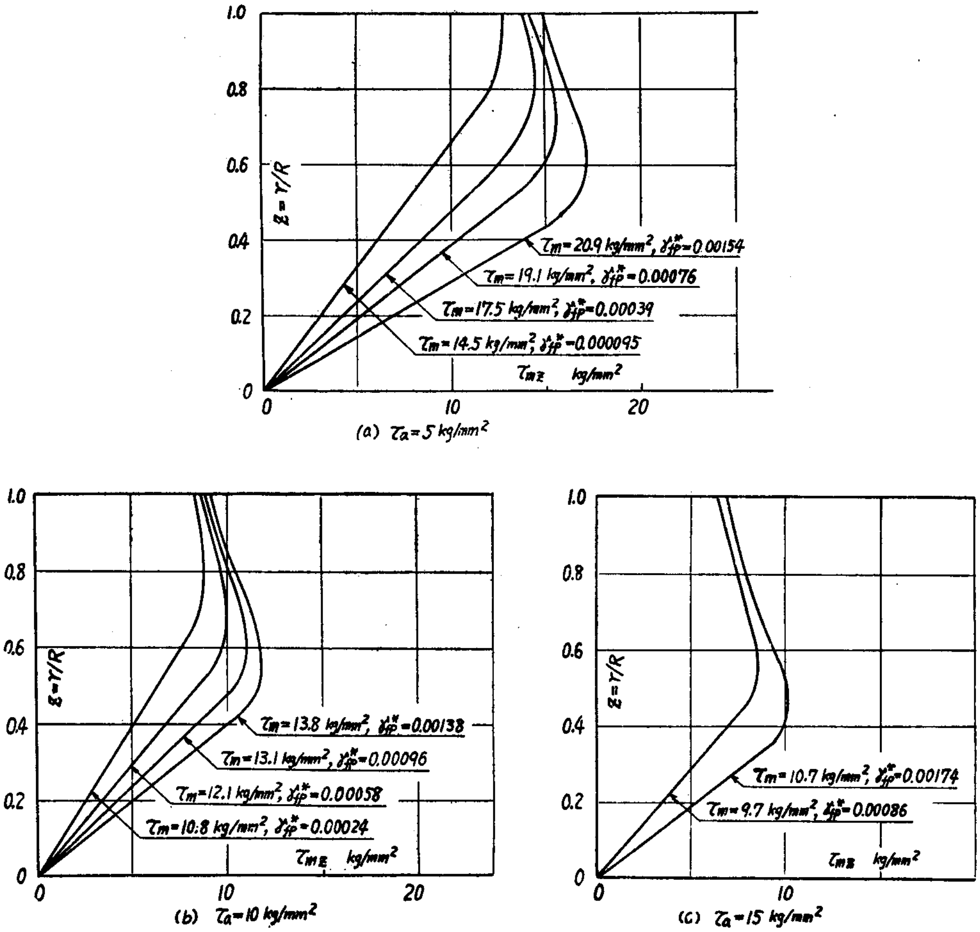


Fig. 29. Distribution of mean stress in the specimen when the fatigue deformation has appeared (2245).

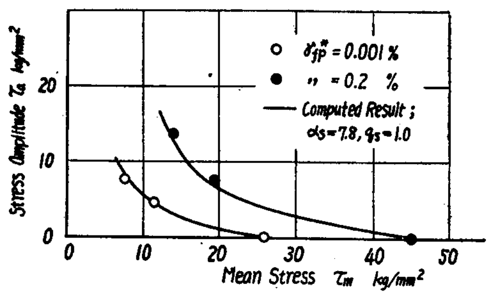


Fig. 30. Comparison between experimental results and computed ones (2245).

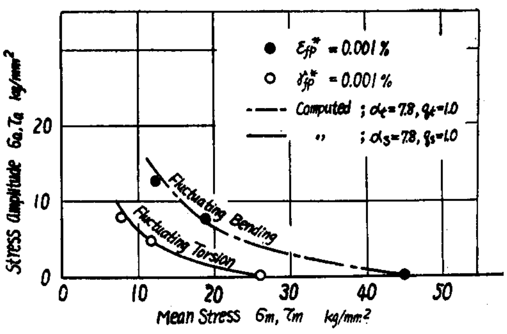


Fig. 31. Comparison between fluctuating bending and torsion (2245).

Let us consider, here, what relation between α_t, q_t in Eq. (4′) and α_s, q_s in Eq. (6) exists in case of the same material.

The experimental results on 2245 in Fig. 22 of elastic limits in case of bending and torsion, i.e. when ε_{fp}^* and γ_{fp}^* are equal to 0.001% respectively, coincide well with the experimental results, as seen in Fig. 31, by putting the following constants in Eq. (4′) and (6):

$$\alpha_t = 7.8\,, \quad q_t = 1.0$$
$$\alpha_s = 7.8\,, \quad q_s = 1.0\,.$$

In other words, when the relations between $\varepsilon_{fp}, \sigma_m$ and σ_a or γ_{fp}, τ_m and τ_a are represented in (4′) or (6), it seems that the relation of

$$\alpha_t = \alpha_s\,, \quad q_t = q_s$$

is established.

It shows that if those constants are found in one alternating test, the constants for the other test are instantly known, and it is very convenient for practical application.

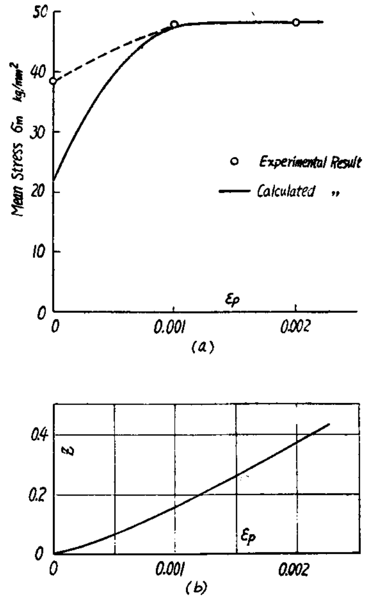


Fig. 32. Effect of plastic-working under bending on bending fatigue deformation.

Lastly, we will consider the experimental results on SUP 3 (B) shown in Fig. 25, which represent the fatigue deformation on the materials which are cold-worked prior to alternating stress tests. The magnitude of mean stress at which the fatigue deformation begins to appear on the material which is plastically worked can be found from the procedure described in chapter 4. On SUP 3 (B), of which experimental results are represented in Fig. 25, we calculated the magnitude of mean stress at which the fatigue deformation begins to appear, and it varies with the plastic working applied under bending. A full line in Fig. 32 (a) shows the result. And the result means that the larger the bending permanent strain ε_p becomes, the larger the mean stress at which the fatigue deformation begins to appear, but that no difference exists for ε_p beyond about 0.001. However, this result does not coincide with experimental results in Fig. 25 at all. That is, in experimental results there is little effect between mean stresses at which the fatigue deformation begins to appear, no matter how the plastic working is carried out. Such a striking difference seems to be caused by the assumption that the relation between stress $(\sigma_m+\sigma_a)$ and strain ε of materials plastically worked becomes like a line BA′C′ in Fig. 14. In fact, BA′ may be not be perfectly linear, and, as a matter of course, will bent gradually to right-hand side near A′. Then in order to compare the calculated result with the test result, it must be carried out in a state under which some magnitude of fatigue deformation is produced. Further Fig. 32 (b) represents the aspect that the point at which the fatigue deformation begins to appear moves to the inner region with ε_p, where z shows the value obtained by dividing the distance from the surface to the point by half of the specimen.

6. Conclusions

The following is the summary of conclusion reached as the result of this investigation:

(1) In bending and torsional fatigue tests with the mean stress, the fatigue deformation appears rapidly after the beginning of tests and then the increasing rate decreases gradually. And the deformation continues until the specimen fractures when the stress is beyond the fatigue limit but, it completely stops after certain number of repetitions when the stress is below the fatigue limit.

(2) On the materials showing the obvious yield point in the static tests, the fatigue deformation does not appear in case the maximum stress is below the static yield stresses but is remarkably large at the maximum stress beyond the static yield stress. On the other hand, on the materials without the obvious yield point, the fatigue deformation appears even at the maximum stress below the static yield stress.

(3) A peculiar aspect in an increase of fatigue deformation in mild steel that the deformation increases stepwise at some intervals has been observed.

(4) To give a permanent deformation by the preliminary plastic-working is effective because it decreases the degree of fatigue deformation caused by the subsequent repeated stress.

(5) The equation which is convenient for the expression of fatigue deformation under uniform stress distribution have been obtained. They include two constants determined from the fatigue test and the constants are the same in value in both cases of normal stress and shear stress.

(6) The convenient expression has been obtained to calculate the fatigue deformation under uneven stress distribution from the corresponding static test.

(7) The relation between the mean stress, the stress amplitude and the permanent strain in the alternating bending or torsion fatigue tests have been obtained analytically by the above expression for the uniform stress distribution, and the test results for carbon steels and spring steels have been well explained.

(8) The distribution of mean stress and residual stress due to fatigue deformation have been obtained.

(9) The fatigue deformation of the material with the pre-loading is predicted from the above analytical treatment.

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