Stresses in Bolt Head

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The stress in bolt head is expected to be very high at the transition part of the cross section, which means the bolt connection has weak point in the part. We have tried to investigate the stress distribution by analytical and experimental methods as a case of two dimensional problem in a meridian plane. In the analytical consideration, we separate the domain into two parts, bolt head and shunk, and choose the stress functions suitable for the boundary conditions in each domain. The results of calculation show a little higher value compared with those found by experiments.

1. Co-ordinate

As shown in Fig. 1, we separate the bolt into two parts, bolt head A and shunk B. The domain A is limited by straight lines, while the domain B contains curved boundary. We use the Cartesian co-ordinate (x, y) to A and the curvilinear co-ordinate to B represented by the transformation

$$z = w + e^{w}, \qquad \cdots\cdots(1)$$

where

$$\left.\begin{aligned} z &= x + iy, \\ w &= u + iv, \end{aligned}\right\} \qquad \cdots\cdots(2)$$

or

$$\left.\begin{aligned} x &= u + e^{u}\cos v, \\ y &= v + e^{u}\sin v. \end{aligned}\right\} \qquad \cdots\cdots(3)$$

Fig. 2 shows the curvilinear co-ordinate and the curve v=const. gives the boundary of bolt shunk.

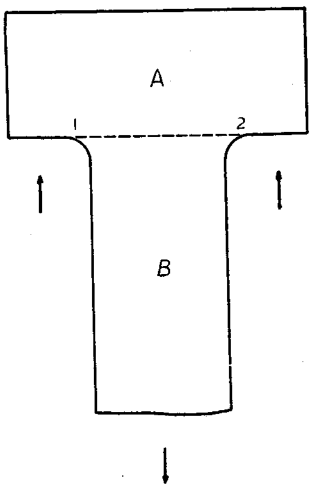

Fig. 1

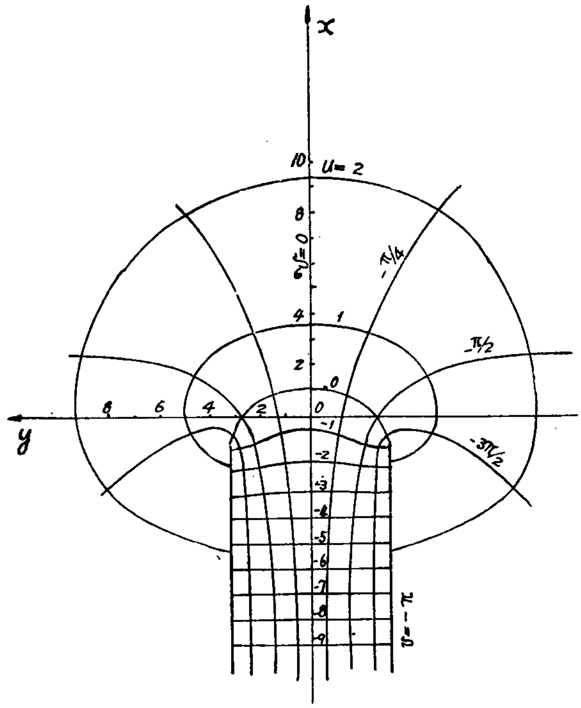

Fig. 2 Curvilinear Co-ordinate expressed by $z=w+e^{w}$.

2. Stress Functions and Stresses

In the two dimensional stress problem, various types of stress function are introduced. We consider the following stress functions according to Neuber's stress function[1],

$$\left.\begin{aligned} F_A =& \sum_{n=1}^{\infty} A_n e^{-\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \sum_{n=1}^{\infty} B_n x e^{-\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \sum_{n=1}^{\infty} C_n y e^{-\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y \\ &+ \sum_{n=1}^{\infty} D_n e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \sum_{n=1}^{\infty} H_n x e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \sum_{n=1}^{\infty} I_n y e^{\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y\,, \\ F_B =& \frac{p}{2}y^2 + \sum_{n=1}^{\infty} A_n' e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \sum_{n=1}^{\infty} B_n' x e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \sum_{n=1}^{\infty} C_n' y e^{\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y\,. \end{aligned}\right\} \cdots(4)$$

These functions correspond to the plane stress state. The function F_B shows the uniform tension when x tends to $-\infty$. The functions F_A and F_B satisfy evidently the condition $(\partial^2/\partial x^2+\partial^2/\partial y^2)^2F=0$. As will be mentioned in chapter 3, we considered the simplified boundary conditions. By this simplification, we adopt the first two series terms in equation (4). Hence we obtain

$$\left.\begin{aligned}
\sigma_{xA}&=\frac{\partial^2 F_A}{\partial y^2}=-\sum A_n\left(\frac{n\pi}{b}\right)^2 e^{-\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y-\sum B_n\left(\frac{n\pi}{b}\right)^2 xe^{-\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y,\\
\sigma_{yA}&=\frac{\partial^2 F_A}{\partial x^2}=\sum A_n\left(\frac{n\pi}{b}\right)^2 e^{-\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y+\sum B_n\frac{n\pi}{b}\left(-2+\frac{n\pi}{b}x\right)e^{-\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y,\\
\tau_A&=-\frac{\partial^2 F_A}{\partial x\partial y}=-\sum A_n\left(\frac{n\pi}{b}\right)^2 e^{-\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y+\sum B_n\frac{n\pi}{b}\left(1-\frac{n\pi}{b}x\right)e^{-\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y,\\
\sigma_{xB}&=\frac{\partial^2 F_B}{\partial y^2}=p-\sum A_n'\left(\frac{n\pi}{b}\right)^2 e^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y-\sum B_n'\left(\frac{n\pi}{b}\right)^2 xe^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y,\\
\sigma_{yB}&=\frac{\partial^2 F_B}{\partial x^2}=\sum A_n'\left(\frac{n\pi}{b}\right)^2 e^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y+\sum B_n'\frac{n\pi}{b}\left(2+\frac{n\pi}{b}x\right)e^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y,\\
\tau_B&=-\frac{\partial^2 F_B}{\partial x\partial y}=\sum A_n'\left(\frac{n\pi}{b}\right)^2 e^{\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y+\sum B_n'\frac{n\pi}{b}\left(1+\frac{n\pi}{b}x\right)e^{\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y.
\end{aligned}\right\}\cdots(5)$$

The displacements ξ, η, ζ in the directions of x, y, z are calculated by the following formulas,

$$\left.\begin{aligned}
2G\xi &= -\frac{\partial F''}{\partial x}+2\gamma\phi_1''\\
&= -\frac{\partial}{\partial x}\left[F-\frac{4}{4-\gamma}\phi_1'+\frac{4}{4-\gamma}\phi_2'\right],\\
2G\eta &= -\frac{\partial F''}{\partial y}+2\gamma\phi_2''\\
&= -\frac{\partial}{\partial y}\left[F+\frac{4}{4-\gamma}\phi_1'-\frac{4}{4-\gamma}\phi_2'\right],\\
2G\zeta &= -\frac{\partial F''}{\partial z}+2\gamma\phi_3''\\
&= -\frac{2-\gamma}{4-\gamma}\Delta F\cdot z,
\end{aligned}\right\}\cdots\cdots\cdots\cdots(6)$$

where

$$\left.\begin{aligned}
F'' &= \phi_0''+x\phi_1''+y\phi_2''+z\phi_3'',\\
\phi_0'' &= \frac{4-4\gamma+\gamma^2}{\gamma(4-\gamma)}\left\{z\left(\frac{\partial^2\phi_1'}{\partial x^2}+\frac{\partial^2\phi_2'}{\partial y^2}\right)-\left(x\frac{\partial\phi_1'}{\partial x}+y\frac{\partial\phi_2'}{\partial y}\right)\right\}+\frac{2-\alpha}{4-\gamma}\frac{d^2}{12}\left(\frac{\partial^2\phi_1'}{\partial x^2}+\frac{\partial^2\phi_2'}{\partial y^2}\right)\\
&\quad+\frac{4}{4-\gamma}(\phi_1'+\phi_2')+2\phi_0',\\
\phi_1'' &= \frac{4}{\gamma(4-\gamma)}\frac{\partial\phi_1'}{\partial y},\\
\phi_2'' &= \frac{4}{\gamma(4-\gamma)}\frac{\partial\phi_2'}{\partial y},\\
\phi_3'' &= \frac{-4+2\gamma}{\gamma(4-\gamma)}z\left(\frac{\partial^2\phi_1'}{\partial x^2}+\frac{\partial^2\phi_2'}{\partial y^2}\right),\\
&\phi_1',\ \phi_2'\text{; harmonic functions,}\\
\phi_1 &= \frac{\partial\phi_1'}{\partial x},
\end{aligned}\right\}(7)$$

$$\phi_2 = \frac{\partial \phi_2'}{\partial y},$$

$$F = \phi_0 + x\phi_1 + y\phi_2, \quad \left(\varDelta\phi_0 = \varDelta\phi_1 = \varDelta\phi_2 = 0, \quad \varDelta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right),$$

G = modulus of rigidity,

$$\gamma = 2\left(1 - \frac{1}{m}\right),$$

m = Poisson's constant.

Hence we get

$$\left.\begin{aligned}
2G\xi_A &= \sum A_n \frac{n\pi}{b} e^{-\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y - \sum B_n\left(1 - \frac{n\pi}{b}x\right) e^{-\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \frac{4}{4-\gamma}\sum B_n e^{-\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y, \\
2G\eta_A &= \sum A_n \frac{n\pi}{b} e^{-\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y + \sum B_n x e^{-\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y - \frac{4}{4-\gamma}\sum B_n e^{-\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y, \\
2G\zeta_A &= -\frac{2-\gamma}{4-\gamma} z\left[\sum B_n\left\{-\left(\frac{n\pi}{b}\right)^2 x + \frac{n\pi}{b}\left(-2 + \frac{n\pi}{b}\right)\right\}\right] e^{-\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y, \\
2G\xi_B &= -\sum A_n' \frac{n\pi}{b} e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y - \sum B_n'\left(1 + \frac{n\pi}{b}x\right) e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \frac{4}{4-\gamma}\frac{p}{2}x, \\
2G\eta_B &= -py + \sum A_n' \frac{n\pi}{b} e^{\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y + \sum B_n' \frac{n\pi}{b} x e^{\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y, \\
&\qquad + \frac{4}{4-\gamma}\sum B_n' e^{\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y + \frac{4}{4-\gamma}\frac{p}{2}y, \\
2G\zeta_B &= -\frac{2-\gamma}{4-\gamma} z\left[p + \sum B_n'\left\{-\left(\frac{n\pi}{b}\right)^2 x e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y + \frac{n\pi}{b}\left(2 + \frac{n\pi}{b}x\right) e^{\frac{n\pi}{b}x} \cos\frac{n\pi}{b}y\right\}\right].
\end{aligned}\right\} \quad (8)$$

3. Boundary Conditions

Refering to Fig. 3, the boundary conditions are represented by next items.

(i) $x = h_1$: $\sigma_x = 0$, $\tau = 0$,

(ii) $x = -h_2$: $\sigma_x = f_1(y)$, $\tau = f_2(y)$,

(iii) $y = \pm b$: $\sigma_y = 0$, $\tau = 0$,

(iv) $v = \pm v_0$: $-\sigma_x \dfrac{\partial y}{\partial u} + \tau \dfrac{\partial x}{\partial u} = 0$, $\sigma_y \dfrac{\partial x}{\partial u} - \tau \dfrac{\partial y}{\partial u} = 0$,

(v) $u = -\infty$: $\sigma_x = p$, $\sigma_y = 0$, $\tau = 0$,

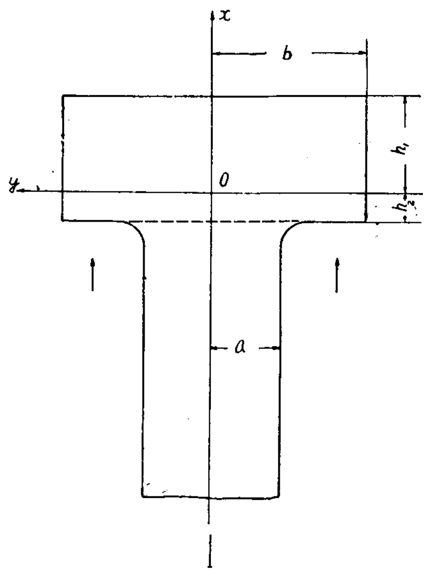

Fig. 3

$$\text{(vi)}\quad x=-h_2\ :\ \begin{array}{ll}\xi_A=\xi_B\,, & \sigma_{xA}=\sigma_{xB}\,,\\ \underline{\eta_A=\eta_B}\,, & \underline{\sigma_{yA}=\sigma_{yB}}\,,\\ \underline{\zeta_A=\zeta_B}\,, & \underline{\tau_A=\tau_B}\,,\end{array}\qquad\cdots\cdots(9)$$

where h_1, h_2 and b are shown in Fig. 3. $f_1(y)$ and $f_2(y)$ are determined by the reaction on the under surface of bolt head and the unknown stress along the separating line of 1~2. Then $f_1(y)$ and $f_2(y)$ may be expanded in the Fourier series,

$$\left.\begin{aligned} f_1(y)&=\sum(a_n+a_n')\cos\frac{n\pi}{b}y\,,\\ f_2(y)&=\sum b_n\sin\frac{n\pi}{b}y\,,\\ a_n+a_n'&=\frac{2}{b}\int_0^b f_1(y)\cos\frac{n\pi}{b}y\,dy\,,\\ b_n&=\frac{2}{b}\int_0^b f_2(y)\sin\frac{n\pi}{b}y\,dy\,,\end{aligned}\right\}\qquad\cdots\cdots(10)$$

a_n is the coefficient concerning to the given stress distribution on the supported surface of the bolt head and a_n', b_n are to the unknown stress acting on the separating boundary 1~2.

4. Determination of Coefficients

The unknown coefficients A_n, B_n, A_n' and B_n' in (4) are determined by the boundary condition (9). To symplify the treatise, we neglect the conditions shown by under lines. Then the condition of (9) (ii) gives

$$-\left(\frac{n\pi}{b}\right)^2 A_n e^{\frac{n\pi}{b}h_2}+\left(\frac{n\pi}{b}\right)^2 B_n h_2 e^{\frac{n\pi}{b}h_2}=a_n+a_n'\,,\qquad\cdots\cdots\cdots\cdots(11)$$

and from the first condition (9) (iii), we get the relation,

$$\left(\frac{n\pi}{b}\right)^2 A_n\cos n\pi+B_n\frac{n\pi}{b}\left(-2-\frac{n\pi}{b}h_0\right)\cos n\pi=0\,,$$

where we assume

$$xe^{-\frac{n\pi}{b}x}\approx -h_0e^{-\frac{n\pi}{b}x}\,.\qquad\cdots\cdots\cdots\cdots(12)$$

Then we obtain from (11) and (12)

$$A_n=-\left(\frac{b}{n\pi}\right)^2 e^{-\frac{n\pi}{b}h_2}\frac{a_n+a_n'}{1+\dfrac{h_2}{h_0+\dfrac{2b}{n\pi}}}\,,\qquad B_n=-\left(\frac{b}{n\pi}\right)^2 e^{-\frac{n\pi}{b}h_2}\frac{a_n+a_n'}{h_0+\dfrac{2b}{n\pi}+h_2}\,.\qquad\cdots\cdots(13)$$

The conditions of (9) (ii), (iv) give

$$-p\sin v_0+A_1'\left(\frac{\pi}{b}\right)^2\alpha_1+B_1'\left(\frac{\pi}{b}\right)^2\beta_1=0\,,$$

$$\left.\begin{aligned} &A'-h_2B_1' = -(a_1+a_1'-c_1)\left(\frac{b}{\pi}\right)^2 e^{\frac{\pi}{b}h_2}, \qquad (n=1) \\ &-A_n'+h_2B_n' = (a_n+a_n'-c_n)\left(\frac{b}{n\pi}\right)^2 e^{\frac{n\pi}{b}h_2}, \\ &A_n'\alpha_n+B_n'\beta_n = 0, \qquad (n\geqq 2) \end{aligned}\right\} \cdots\cdots(14')$$

where c_n are the coefficients in next series,

$$p = \sum_{n=1}^{\infty} c_n \cos\frac{n\pi}{b}y$$

$$\left(c_n = \frac{2p}{n\pi}\sin\frac{n\pi}{2}\right).$$

Hence we get,

$$\left.\begin{aligned} A_1' &= \frac{p\sin v_0\cdot h_2\cdot e^{-\frac{\pi}{b}h_2}\beta_1(a_1+a_1'-c_1)}{\left(\frac{\pi}{b}\right)^2(\alpha_1h_2+\beta_1)}, \quad (n=1), \\ A_n' &= -\frac{\beta_n\left(\frac{b}{n\pi}\right)^2 e^{\frac{n\pi}{b}h_2}(a_n+a_n'-c_n)}{\alpha_nh_2+\beta_n}, \quad (n\geqq 2), \\ B_1' &= \frac{\alpha_1e^{\frac{\pi}{b}h_2}(a_1+a_1'-c_1)+p\sin v_0}{\left(\frac{\pi}{b}\right)^2(\alpha_1h_2+\beta_1)}, \quad (n=1), \\ B_n' &= \frac{\alpha_n\left(\frac{b}{n\pi}\right)^2 e^{\frac{n\pi}{b}h_2}(a_n+a_n'-c_n)}{\alpha_nh_2+\beta_n}, \quad (n\geqq 2), \end{aligned}\right\} \cdots\cdots(15)$$

where we introduce the next relations,

$$\begin{aligned} &\sum A_n'\left(\frac{n\pi}{b}\right)^2\left\{e^u\sin v_0e^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y+(1+e^u\cos v_0)e^{\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y\right\} \\ &=\sum A_n'\left(\frac{n\pi}{b}\right)^2e^u\left\{\sin v_0e^{\frac{n\pi}{b}(u+e^u\cos v_0)}\cos\frac{n\pi}{b}(v_0+e^u\sin v_0)\right. \\ &\qquad\left.+(1+e^u\cos v_0)e^{\frac{n\pi}{b}(u+e^u\cos v_0)-1}\sin\frac{n\pi}{b}(v_0+e^u\sin v_0)\right\} \\ &=\sum_n A_n'\left(\frac{n\pi}{b}\right)^2e^u\sum_l \alpha_l^n(e^u-1)^l, \end{aligned}$$

and

$$\begin{aligned} &\sum B_n'\left(\frac{n\pi}{b}\right)^2\left\{e^u\sin v_0\cdot xe^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y+(1+e^u\cos v_0)\frac{b}{n\pi}\left(1+\frac{n\pi}{b}x\right)e^{\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y\right. \\ &=\sum B_n'\left(\frac{n\pi}{b}\right)^2e^u\left[\sin v_0(u+e^u\cos v_0)e^{\frac{n\pi}{b}(u+e^u\cos v_0)}\cos\frac{n\pi}{b}(v_0+e^u\sin v_0)\right. \\ &\qquad\left.+(1+e^u\cos v_0)\frac{b}{n\pi}\left\{1+\frac{n\pi}{b}(u+e^u\cos v_0)\right\}e^{\frac{n\pi}{b}(u+e^u\cos v_0)-1}\sin\frac{n\pi}{b}(v_0+e^u\sin v_0)\right] \\ &=\sum_n B_n'\left(\frac{n\pi}{b}\right)^2e^u\sum_l \beta_l^n(e^u-1)^l, \end{aligned}$$

α_l^n and β_l^n are the coefficients obtained by applying the Taylor expansion to the terms

$$\left.\begin{aligned}
&e^{-u}\left\{e^{u}\sin v_0 e^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y+(1+e^{u}\cos v_0)\,e^{\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y\right\},\\
&\text{and}\\
&e^{-u}\left\{e^{u}\sin v_0\cdot xe^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y+(1+e^{u}\cos v_0)\frac{b}{n\pi}(1+\frac{n\pi}{b}x)e^{\frac{n\pi}{b}x}\sin\frac{n\pi}{b}y\right\}
\end{aligned}\right\}\quad\cdots(16)$$

In the above consideration, we adopt the first term of expansion. Referring to (6), (13) and (15), we obtain the next conditions of displacement,

$$\left.\begin{aligned}
&-\frac{1}{2}\,\frac{8-\gamma}{4-\gamma}(a_1+a_1')+\left\{\frac{\beta_1}{\alpha_1h_2+\beta_1}\,a_1'-\frac{\beta_1c_1+h_2e^{-\frac{\pi}{b}h_2}\sin v_0}{\alpha_1h_2+\beta_1}\right\}\\
&\quad+\left\{\frac{\gamma}{4-\gamma}+\frac{\pi}{b}\,h_2\right\}\left\{\frac{\alpha_1}{\alpha_1h_2+\beta_1}\,a_1'+\frac{e^{-\frac{\pi}{b}h_2}\sin v_0-\alpha_1c_1}{\alpha_1h_2+\beta_1}\right\}=0\,,\quad(n=1),\\
&\frac{1}{2}\,\frac{8-\gamma}{4-\gamma}(a_n+a_n')+\left\{\frac{\beta_n}{\alpha_nh_2+\beta_n}\,a_n'-\frac{\beta_nc_n}{\alpha_nh_2+\beta_n}\right\}\\
&\quad+\left\{\frac{\gamma}{4-\gamma}+\frac{n\pi}{b}\,h_2\right\}\left\{\frac{\alpha_n}{\alpha_nh_2+\beta_n}\,a_n'-\frac{a_nc_n}{\alpha_nh_2+\beta_n}\right\}=0\,,\qquad(n\geqq 2).
\end{aligned}\right\}\quad\cdots\cdots(17)$$

From these we get

$$\left.\begin{aligned}
a_1'&=\frac{-\frac{8-\gamma}{8-2\gamma}a_1(\alpha_1h^2+\beta_1)+\beta_1c_1+h_2e^{-\frac{\pi}{b}h_2}\sin v_0+\left(\frac{\gamma}{4-\gamma}+\frac{\pi}{b}\,h_2\right)\left(a_1c_1-e^{-\frac{\pi}{b}h_2}\sin v_0\right)}{\frac{8-\gamma}{8-2\gamma}\,(\alpha_1h_2+\beta_1)+\beta_1+\left(\frac{\gamma}{4-\gamma}+\frac{\pi}{b}\,h_2\right)\alpha_1},\\
a_n'&=\frac{-\frac{8-\gamma}{8-2\gamma}\,a_n(\alpha_nh_2+\beta_n)+\beta_nc_n+\left(\frac{\gamma}{4-\gamma}+\frac{n\pi}{b}\,h_2\right)\alpha_nc_n}{\frac{8-\gamma}{8-2\gamma}\,(\alpha_nh_2+\beta_n)+\beta_n+\left(\frac{\gamma}{4-\gamma}+\frac{n\pi}{b}\,h_2\right)\alpha_n}.
\end{aligned}\right\}(18)$$

5. Examples

For an example of calculation, we consider a bolt head with the diameter of 4π supported uniformly and a bolt shunk with the diameter of $2a$ stressed by uniform tension, and the junction of two parts being rounded off by a curve expressed by v=const. The curvature $(1/\rho)$ of the curve is calculated by the equation

$$\begin{aligned}
\frac{1}{\rho}&=\frac{1}{2h^3}\,\frac{\partial h^2}{\partial v}\\
&=\frac{e^{u}\sin v}{(1+2e^{u}\cos v+e^{2u})^{3/2}},\quad\left[h^2=\left(\frac{\partial x}{\partial u}\right)^2+\left(\frac{\partial y}{\partial u}\right)^2=\left(\frac{\partial x}{\partial v}\right)^2+\left(\frac{\partial y}{\partial v}\right)^2\right]
\end{aligned}\quad\cdots\cdots(19)$$

and we get

$$\left(\frac{a}{\rho}\right)_{\substack{max.\\ v=\text{const.}}}=\left(\frac{y_{u=-\infty}}{\rho}\right)_{\substack{max.\\ v=\text{const.}}}=\left(\frac{v}{\rho}\right)_{\substack{max.\\ v=\text{const.}}}.\qquad\cdots\cdots\cdots\cdots\cdots(20)$$

The coefficients determined by (15), (16) and (18) are shown in Table 1. By these coefficients, we can calculate the stress distribution. And the expression for the principal stress along the curved boundary is reduced to be as follows.

$$(\sigma_u)_{v=v_0} = \left[p+\sum B_n'2\frac{n\pi}{b}e^{\frac{n\pi}{b}x}\cos\frac{n\pi}{b}y\right]_{v=v_0}. \qquad \cdots\cdots\cdots\cdots\cdots(21)$$

Table 1 Coefficients.

v_0	a/ρ	n	α_n	β_n	a_n+a_n'	A_n'	B_n'
2.85	15	1	0.0553	0.0575	0.9670	14.11	14.93
		2	−0.1615	0.1137	−0.3975	−4.415	−6.03
		3	−0.0265	0.0072	−0.6420	−0.0087	−0.0316
		4	0.0706	−0.0602	0.1137	−2.2800	−2.6750
		5	0.0139	−0.0076	0.4822	0.4360	0.6590
		6	−0.0266	0.0226	−0.0473	1.0380	1.1530
		7	−0.0064	0.0040	−0.3695	−0.7270	1.0890
		8	0.0088	−0.0080	0.0262	−0.0593	0.0653
		9	−0.0020	−0.0015	0.1533	−0.1003	0.1329
		10	−0.0041	0.0037	−0.0212	0.0768	−0.0845
		11	−0.0008	0.0006	−0.0627	0.1342	−0.1708
2.7	7.1	1	0.0613	0.0638	0.880	12.84	13.60
		2	−0.1792	0.1260	−0.362	−4.02	−5.49
		3	−0.0294	0.0080	−0.5842	−0.0079	−0.029
		4	0.0783	−0.0667	0.1034	−2.0750	−2.435
		5	0.0154	−0.0840	0.4380	0.3968	0.600
		6	−0.0294	0.0251	−0.0430	0.9450	1.050
		7	−0.0071	0.0044	−0.3362	−0.6620	0.992
		8	0.0098	−0.0089	0.0238	−0.0540	0.0595
		9	0.0022	−0.0089	0.1414	−0.0913	0.1210
		10	−0.0045	0.0041	−0.0193	0.0698	−0.0770
		11	−0.0009	0.0007	−0.0571	0.1222	−0.1554
2.575	5.1	1	0.1077	0.1140	0.4873	9.86	9.95
		2	−0.2286	0.1712	−0.1519	−2.99	−3.828
		3	−0.0505	0.0132	−0.3039	−0.048	−0.1729
		4	0.0961	−0.0822	0.0172	−0.3565	−0.9130
		5	0.0224	−0.0114	0.2192	0.0017	0.0754
		6	−0.0408	0.0358	0.0823	0.1785	0.2028
		7	−0.0167	0.0106	−0.1669	0.0326	0.5150
		8	0.0174	−0.0149	0.0822	0.2088	0.2452
		9	0.0047	−0.0032	0.1365	−0.5700	−0.0837
		10	−0.0075	0.0064	−0.0703	−0.2480	−0.2910
		11	−0.0023	0.0017	−0.1135	0.1249	0.1770

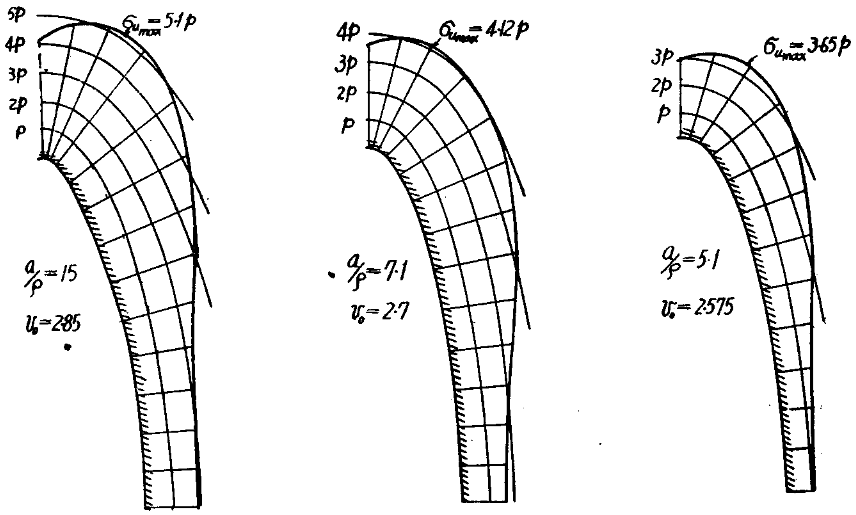

Fig. 4 Stress Distribution along the Curved Boundary of Bolt Shunk.

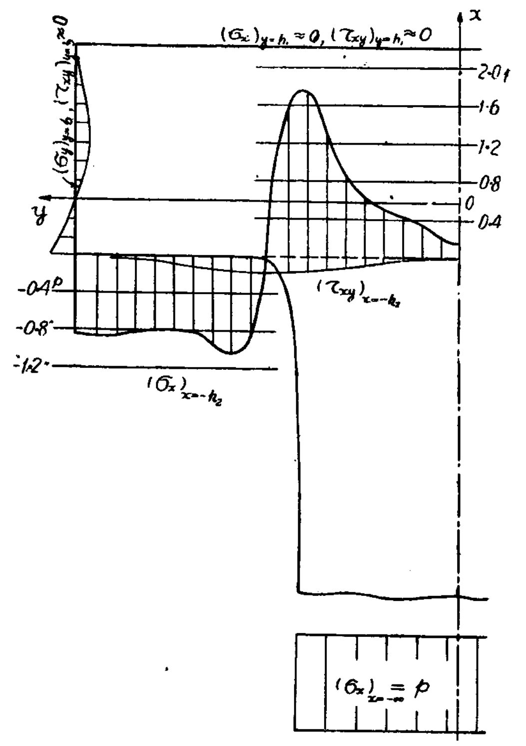

Fig. 5 Stress Distribution along the Connecting Line and Inadequate Stresses occured by Neglected Bundary Conditions. (v_0=2.7)

Fig. 4 are the stress distributions calculated by (21). The results of calculation on other boundaries show that the stresses to the neglected conditions are confined within small values and Fig. 5 shows the distribution of neglected stress. The distribution of stress on the separating line becomes as shown in Fig. 5 and Table 2.

Table 2 Distribution of Stress σ_x along the Separating Line and Supporting Surface. ($a=v_0=2.7$)

y/b	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$(\sigma_x)_{x=-h_2}/p$ $v=2.7$	0.16	0.36	0.52	0.85	1.75	0.26	-1.03	-0.80	-0.79	-0.86	-0.83

5. Stress Concentration in Bolt Head

The preceding results on the stress in the bolt head show the maximum stress occures along the curved boundary. So the value

$$\alpha = \frac{(\sigma_x)_{v=v_0 \, max.}}{p} \quad \cdots\cdots(22)$$

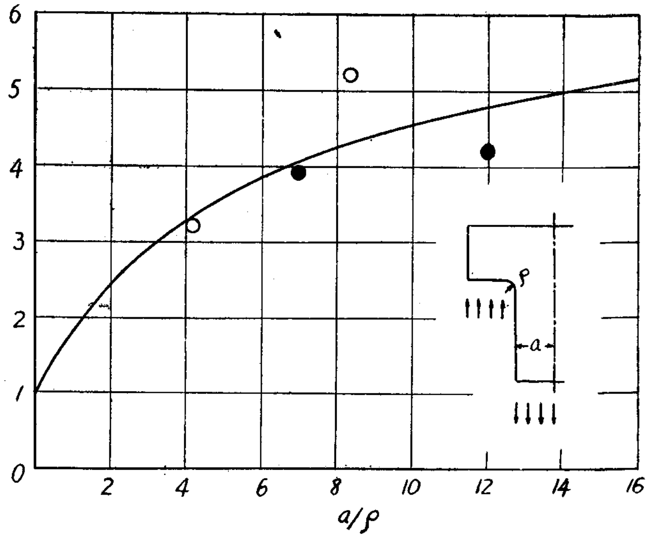

Fig. 6 Form Factor of Bolt Head.
(○: Brittle Coating ●: Photo-elasticity)

leads to the stress concentration factor or form factor in bolt head. Fig. 6 and Table 3 are the value of α to various a/ρ, where a/ρ is the value at the point of maximum stress $\sigma_{u_{max.}}$. In the figure the points shown by small circles and black points are results of the experiments by brittle coating[2] and photo-elasticity.[3][4]

Table 3 Form Factor.

a/ρ	0	4.17	5.1	7.0	7.1	8.33	12	15
α_I	1	—	3.65	—	4.12	—	—	5.1
α_{II}	—	3.2	—	—	—	5.2	—	—
α_{III}	—	—	—	3.9(4)	—	—	4.2(3)	—

α_I : theoretical value
α_{II} : experimental value by brittle coating
α_{III} : experimental value by photo-elasticity

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