On the two-dimensional flow around the flap wing section composed of two circular arcs

On the two-dimensional flow around the flap wing section composed of two circular arcs.

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In this paper the author treats the problem of two-dimensional potential flow around flap wing section composed of two circular arcs by the method of conformal representation. And the results of approximate calculation by the method of vortex field are added.

The two-dimensional flow of a perfect fluid around a thin flap wing section composed of two segments of straight lines has already been investigated. In this paper the author investigates the same flow but the section is composed of two segments of circular arcs. In the following paragraphs, the conformal representations applied and the lift and moment acting on the section will be described.

I. Conformal Representation.

To simplify the problem but without losing the generality, we assume the two circular arcs, which form the flap wing section, have the same radius. We consider this flap wing section AHB in the Z-plane as Fig. 1. We take the y-axis

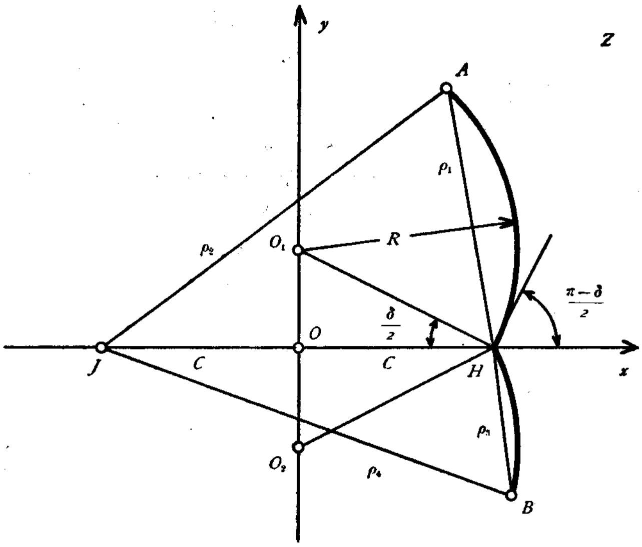

Fig. 1.

passing through the two centers O_1 and O_2 of the circular arcs and x-axis passing through hinge point H and perpendicular to the y-axis. Let the flap angle be δ we get

$$\overline{HO}=C=R\cos\frac{\delta}{2}.$$

Now we take a point J at $x=-C$ or make $\overline{OJ}=C$ and express

$$\overline{AH}=\rho_1,\ \overline{AJ}=\rho_2,\ \overline{BH}=\rho_3 \text{ and } \overline{BJ}=\rho_4.$$

Next we transform this Z-plane into the t-plane by the following relation

$$\frac{Z-C}{Z+C}=t \qquad (1)$$

Let the points in the t-plane corresponding to A and B in the Z-plane be $t=t_1$ and $t=t_2$, then

$$\log t_1=\log\frac{\rho_1}{\rho_2}+i\frac{\pi-\delta}{2},$$

$$\log t_2=\log\frac{\rho_3}{\rho_4}-i\frac{\pi-\delta}{2}.$$

If we denote $t=p^{iq}$ and $t_1=p_1e^{iq_1}$ etc., then

$$p_1=\frac{\rho_1}{\rho_2},\quad q_1=\frac{\pi-\delta}{2}, \quad \text{etc.}$$

The transformed flap wing section in the Z-plane is composed of two segments of straight lines as shown in Fig. 2. And moreover $Z=-C$ corresponds to $t=\infty$ and $Z=\infty$ corresponds to $t=1$.

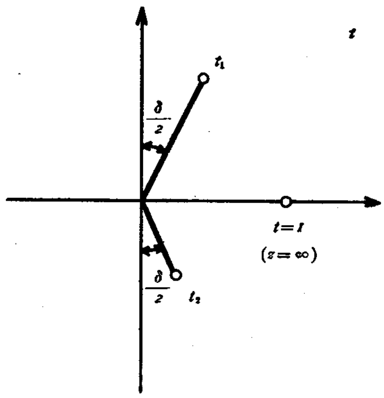

Fig. 2.

Next we transform this flap wing in the t-plane into a circle of radius one in the ζ-plane as shown in Fig. 3. The relation is as follows*

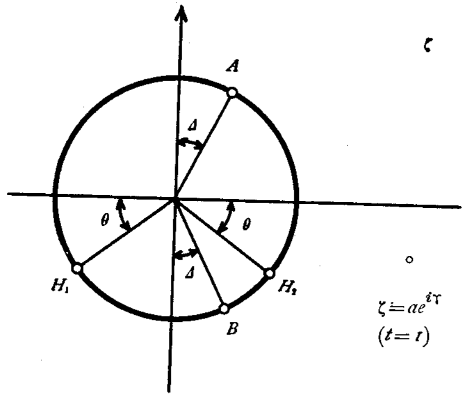

Fig. 3.

*The Memoirs of the College of Engineering Kyoto Imperial University Vol. VIII, No. 2.

$$t=k\frac{(\zeta+e^{i\theta})^{1+\varepsilon}(\zeta-e^{-i\varepsilon})^{1-\varepsilon}}{\zeta} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where k is a constant factor.

This relation can be derived by considering the flow due to sources of strengths $2\pi(1+\varepsilon)$, $2\pi(1-\varepsilon)$ and -2π placed at the points H_1, H_2 and origin respectively in the ζ-plane, which has the circle as one of the stream lines, and the flow due to a source of strengtn 2π at the origin of the t-plane. By this relation H_1 and H_2 correspond to the origin of the t-plane and so to the hinge point H in the Z-plane. We denote the points corresponding to t_1 and t_2 by A and B. The value of ε is determined from the flow in the t-plane i.e.

$$\frac{1+\varepsilon}{1-\varepsilon}=\frac{\pi+\delta}{\pi-\delta},$$

hence

$$\varepsilon=\frac{\delta}{\pi} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

Now we must determine Δ and θ in Fig. 3. The velocity at the points A and B due to the sources placed at H_1, H_2 and origin must be zero. This relation is expressed in the following equation

$$(1+\varepsilon)\sqrt{\frac{1+\sin(\theta-\Delta)}{1-\sin(\theta-\Delta)}} -(1-\varepsilon)\sqrt{\frac{1+\sin(\theta+\Delta)}{1-\sin(\theta+\Delta)}}=0 \quad\cdots\cdots\cdots (4)$$

And considering the stream lines spring out from H_1 and H_2, we can find the constant k as follows

$$k=k' e^{-i\varepsilon\theta} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

where k' is a real number.

Now let the potential function at A and B be ϕ_1 and ϕ_2 respectively, then

$$\left.\begin{aligned}
\phi_1&=\log 2(\cos\Delta+\sin\theta)+\varepsilon\log\frac{\sin\left(\frac{\pi}{4}+\frac{\theta}{2}+\frac{\Delta}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{\theta}{2}-\frac{\Delta}{2}\right)},\\
\phi_2&=\log 2(\cos\Delta-\sin\theta)+\varepsilon\log\frac{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}+\frac{\Delta}{2}\right)}{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}-\frac{\Delta}{2}\right)}.
\end{aligned}\right\}\quad\cdots\cdots (6)$$

But $\phi_1-\phi_2=\log\frac{p_1}{p_2}$, hence

$$\frac{p_1}{p_2}=\frac{\cos\Delta+\sin\theta}{\cos\Delta-\sin\theta}\left[\frac{\sin\left(\frac{\pi}{4}+\frac{\theta}{2}+\frac{\Delta}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{\theta}{2}-\frac{\Delta}{2}\right)}\cdot\frac{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}-\frac{\Delta}{2}\right)}{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}+\frac{\Delta}{2}\right)}\right]^{\varepsilon} \quad\cdots\cdots\cdots\cdots (7)$$

From eqs. (4) and (7), the values of Δ and θ can be determined. Then k' is given by

$$\log k'=\log p_1-\phi_1.$$

Summarizing the above transformations, the relotion between Z and ζ is expressed as follows

$$\frac{Z-C}{Z+C}=k\frac{(\zeta+e^{i\theta})^{1+\varepsilon}(\zeta-e^{-i\theta})^{1-\varepsilon}}{\zeta} \quad\cdots\cdots\cdots\cdots (8)$$

II. Potential Flow and Velocity.

Let the x and y-components of the velocity in the Z-plane far from the flap wing be denoted by u and $-v$ respectively and express

$$u+iv=we^{i\beta} \quad \text{(see Fig. 4)}$$

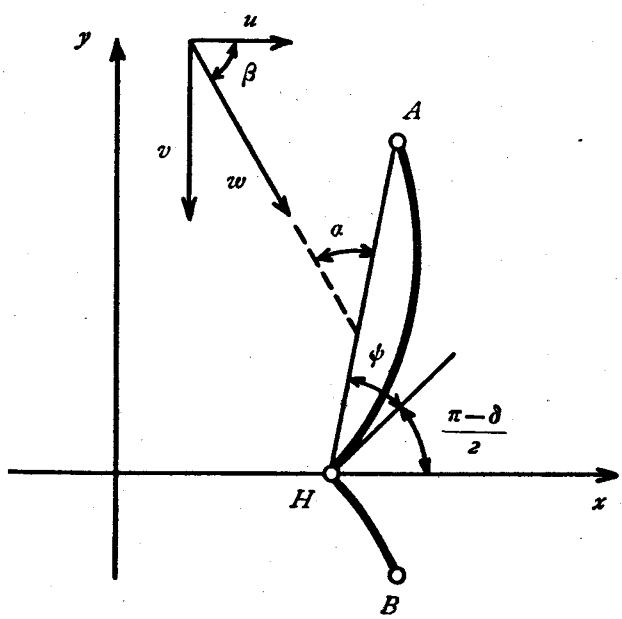

Fig. 4.

The corresponding flow in the t-plane is expressed by the flow due to a doublet placed at $t=1$, and the complex velocity potential of this flow is as follows,

$$W_1=-\frac{2c(u+iv)}{t-1}=-\frac{2c\,we^{i\beta}}{t-1}.$$

Let the point in the ζ-plane corresponding to $t=1$ be $\zeta=ae^{i\gamma}$ and the corresponding moment of doublet to be m, then

$$W_1=-\frac{m}{\zeta-ae^{i\gamma}}.$$

Comparing these two flows we get

$$m=2cw\sigma e^{i(\beta+\varphi)}$$

where $\sigma e^{i\varphi}=\left(\frac{d\zeta}{dt}\right)_{at\,\zeta=ae^{i\gamma}}$

The flow around the flap wing is the combined flow of this doublet and the circulation flow around the flap wing. To simplify the calculation, first we transform the ζ-plane into the S-plane by the following relation $S=\zeta e^{-i\tau}$. Then the conjugate complex of the velocity is expressed as follows.

$$\frac{dW}{ds}=\frac{2cw\sigma e^{i(\beta+\varphi-\tau)}}{(s-a)^2}-\frac{2cw\sigma e^{-i(\beta+\varphi-\tau)}}{a^2\left(s-\frac{1}{a}\right)^2}-\frac{i\Gamma}{2\pi}\frac{1}{s-a}+\frac{i\Gamma}{2\pi}\frac{1}{s-\frac{1}{a}}.$$

Returning to the ζ-plane

$$\frac{dW}{d\zeta}=\frac{2cw\sigma e^{i(\beta+\varphi)}}{(\zeta-ae^{i\tau})^2}-\frac{2cw\sigma e^{-i(\beta+\varphi-2\tau)}}{a^2\left(\zeta-\frac{e^{i\tau}}{a}\right)^2}-\frac{i\Gamma}{2\pi}\frac{1}{\zeta-ae^{i\tau}}+\frac{i\Gamma}{2\pi}\frac{1}{\zeta-\frac{e^{i\tau}}{a}}. \quad \cdots\cdots\cdots\cdots (9)$$

The magnitude of the circulation can be determined by putting $\frac{dW}{d\zeta}=0$ at the point corresponding to the trailing edge i.e. B.

In the case of a small flap angle i.e. when δ or ε is every small and also with small camber i.e. when a is very large, we have the magnitude of the circulation as follows,

$$\frac{\Gamma}{2\pi}=\frac{4cw\sigma}{a^2}\,\frac{\cos(\beta+\varphi+\Delta-2\gamma)+\frac{2}{a}\sin(\beta+\varphi+2\Delta-3\gamma)-\frac{3}{a^2}\cos(\beta+\varphi+3\Delta-4\gamma)}{1+\frac{2}{a}\sin(\Delta-\gamma)-\frac{2}{a^2}\cos 2(\Delta-\gamma)}. \quad \cdots (10)$$

III. Lift and Moment.

The force and moment acting on the aerofoil can be calculated by the Blasius' formula.

Let the x and y-components of the force be Px and Py respectively, then

$$Px-iPy=\frac{\rho i}{2}\oint\left(\frac{dW}{dz}\right)^2 dz=\frac{\rho i}{2}\oint\left(\frac{dW}{d\zeta}\right)^2\frac{d\zeta}{dz}\,dz,$$

but $$\frac{d\zeta}{dz}=\frac{(1-t)^2}{2c\frac{dt}{d\zeta}}.$$

Hence

$$Px-iPy=-\frac{\rho i}{2}\oint_{\zeta=ae^{i\tau}}\left(\frac{dW}{d\zeta}\right)^2\frac{(1-t)^2}{2c\frac{dt}{d\zeta}}d\zeta. \quad (11)$$

In the same way the moment around the point H or $Z=C$ is

$$\mathfrak{M}=-\frac{\rho}{2}\Re\oint\left(\frac{dW}{dz}\right)^2(z-c)\,dz$$

or $$\mathfrak{M}=+\frac{\rho}{2}\Re\oint_{\zeta=ae^{i\tau}}\left(\frac{dW}{dz}\right)^2\frac{t(1-t)}{\frac{dt}{d\zeta}}d\zeta. \quad \cdots\cdots (12)$$

where $\Re$ denotes the real part of the integral.

To calculate the above integrals we expand $\left(\frac{dW}{d\zeta}\right)^2$ in the power series in the neighbourhood of $\zeta=ae^{i\tau}$ and we get,

$$\left(\frac{dW}{d\zeta}\right)^2=\frac{4c^2w^2\sigma^2e^{2i(\beta+\varphi)}}{(\zeta-ae^{i\tau})^4}-\frac{i\Gamma}{2\pi}\frac{4cw\sigma e^{i(\beta+\varphi)}}{(\zeta-ae^{i\tau})^3}+\frac{1}{(\zeta-ae^{i\tau})^2}\left[\frac{i\Gamma}{2\pi}\frac{4acw\sigma e^{i(\beta+\varphi-\tau)}}{(a^2-1)}-\frac{\Gamma^2}{4\pi^2}-\frac{8c^2w^2\sigma^2}{(a^2-1)^2}\right]+\cdots\cdots\cdots\cdots (13)$$

Also we expand t in the neighbourhood of $\zeta=ae^{i\tau}$ and we get

$$\frac{t}{k}=a_0\left[b_0+\sum b_n(\zeta-ae^{i\tau})^n\right] \quad \cdots\cdots\cdots\cdots (14)$$

where $$a_0=\left[\frac{ae^{i\tau}+e^{i\theta}}{ae^{i\tau}-e^{-i\theta}}\right]^{\varepsilon},$$

$$b_0=ae^{i\tau}-\frac{1}{ae^{i\tau}}+2i\sin\theta,$$

$$b_1=1+\frac{1}{(ae^{i\tau})^2}-\frac{2\varepsilon\cos\theta}{ae^{i\tau}},$$

$$b_2=-\frac{1}{(ae^{i\tau})^3}-\frac{2\varepsilon\cos\theta}{(ae^{i\tau}+e^{i\theta})(ae^{i\tau}-e^{-i\theta})}\left[1+\frac{1}{(ae^{i\tau})^2}\right]+\frac{2\varepsilon\cos\theta}{ae^{i\tau}(ae^{i\tau}-e^{-i\theta})}\left[1-\frac{(1-\varepsilon)\cos\theta}{ae^{i\tau}+e^{i\theta}}\right],$$

$$b_3=\frac{1}{(ae^{i\tau})^4}+\frac{2\varepsilon\cos\theta}{(ae^{i\tau})^3(ae^{i\tau}+e^{i\theta})(ae^{i\tau}-e^{-i\theta})}+\frac{2\varepsilon\cos\theta}{(ae^{i\tau}+e^{i\theta})(ae^{i\tau}-e^{-i\theta})^2}\left[1+\frac{1}{(ae^{i\tau})^2}\right]\left[1-\frac{(1-\varepsilon)\cos\theta}{ae^{i\tau}+e^{i\theta}}\right]-\frac{2\varepsilon\cos\theta}{ae^{i\tau}(ae^{i\tau}-e^{-i\theta})^2}\left[1-\frac{2(1-\varepsilon)\cos\theta}{ae^{i\tau}+e^{i\theta}}+\frac{2(1-\varepsilon)(2-\varepsilon)\cos^2\theta}{3(ae^{i\tau}+e^{i\theta})^2}\right].$$

Hence at $\zeta=ae^{i\tau}$,

$$ka_0b_0=1 \quad\text{and}\quad \left(\frac{d\zeta}{dt}\right)_{\zeta=ae^{i\tau}}=\frac{1}{ka_0b_1}.$$

Using these values

$$\frac{(1-t)^2}{\frac{dt}{d\zeta}}=ka_0b_1(\zeta-ae^{i\tau})^2+2ka_0b_1(b_2-b_2)\times(\zeta-ae^{i\tau})^3+\cdots\cdots\cdots\cdots (15)$$

and

$$\frac{t(1-t)}{\frac{dt}{d\zeta}}=-k\frac{a_0}{b_1}\Big[b_1b_0(\zeta-ae^{i\tau})+(b_1^2-b_0b_2)(\zeta-ae^{i\tau})^2+\left(\frac{2b_2^2b_0}{b_1}-2b_0b_3\right)(\zeta-ae^{i\tau})^3+\cdots\Big] \quad (16)$$

Hence from eq. (11)

$$P_x - iP_y = -\frac{\rho i}{2}\oint\left[-ka_0b_1\frac{i\Gamma}{2\pi}\frac{2w\sigma e^{i(\beta+\varphi)}}{(\zeta-ae^{i\tau})}\right.$$
$$\left.-\ldots\right]d\zeta = -i\rho ka_0b_1\Gamma we^{i\beta}\sigma e^{i\varphi}.$$

Introducing $\sigma e^{i\varphi} = \dfrac{1}{ka_0b_1}$

$$P_x - iP_y = -i\rho\Gamma we^{i\beta}\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

which is the well-known result.

The moment can be calculated also by taking the coefficient of $(\zeta-ae^{i\tau})^{-1}$ in the expansion of $\left(\frac{dW}{d\zeta}\right)^2\frac{t(1-t)}{\frac{dt}{d\zeta}}$. Let this coefficient be denoted by M, then from eq. (12)

$$\mathfrak{M} = -\rho\pi\,\mathfrak{J}(M)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

where $\mathfrak{J}$ denotes the imaginary part of M. The coefficient of M is as follows.

$$M = \frac{\Gamma^2}{4\pi^2} + \frac{8c^2w^2\sigma^2}{(a^2-1)^2} - \frac{i\Gamma}{2\pi}\frac{4acw\sigma e^{i(\beta+\varphi+\tau)}}{(a^2-1)}$$
$$+\frac{i\Gamma}{2\pi}4cwe^{i\beta} - \frac{b_2}{b_1}\frac{i\Gamma}{2\pi}4cw\sigma e^{i(\beta+\varphi)}$$
$$-\left(\frac{b_2^2}{b_1^2}-\frac{b_3}{b_1}\right)8c^2w^2\sigma^2e^{2i(\beta+\varphi)}.\ \ldots\ldots (19)$$

In the case of a small flap angle and also a small camber i.e. when ε is very small and a is very large, we can express $\mathfrak{M}$ approximately in the follwing form.

$$\mathfrak{M} = -\frac{4\rho\Gamma cw}{a}\left[\sin\theta\sin(\beta-\gamma) - \varepsilon\cos\theta\cos(\beta-\gamma)\right.$$
$$+\frac{3\cos(\beta-2\gamma)-\cos\beta}{2a} + \frac{3\varepsilon\sin 2\theta\sin(\beta-2\gamma)}{2a}$$
$$\left.+\frac{\sin\theta\sin(\beta-\gamma)+2\sin\theta\sin(\beta-3\gamma)}{a^2}\right]$$
$$-\frac{8\rho\pi c^2w^2}{a^2}\left[\sin 2(\beta-\gamma) - \varepsilon\sin 2\theta\cos 2(\beta-\gamma)\right.$$
$$+\frac{4\sin\theta\cos(2\beta-3\gamma)}{a} - 2\varepsilon^2\cos^2\theta\sin 2(\beta-\gamma)$$
$$+\frac{4\left(\cdot-\frac{2}{3}\cos^2\theta\right)\varepsilon\cos\theta\sin(2\beta-3\gamma)}{a}$$
$$\left.-\frac{(6+4\sin^2\theta)\sin(2\beta-4\gamma)}{a^2}\right]\ldots\ldots\ldots\ldots (20)$$

IV. Approximate Relations between Z and ζ.

The approximate relations between Z, t and ζ-planes are summarized as follows. In the following we assume that ε is very small and a is very large

In the Z-plane

$$\frac{\rho_1}{\rho_2} = \frac{1}{\sqrt{4\left(\frac{c}{\rho_1}\right)^2 - 1 + \sin^2\frac{\delta}{2}} + \sin\frac{\delta}{2}}$$

$$\approx \frac{1}{\sqrt{4\left(\frac{c}{\rho_1}\right)^2 - 1} + \frac{\delta}{2}},$$

$$\frac{\rho_3}{\rho_4} \approx \frac{1}{\sqrt{4\left(\frac{c}{\rho_3}\right)^2 - 1} + \frac{\delta}{2}}$$

Hence

$$\frac{p_1}{p_2} = \frac{\rho_1}{\rho_2}\frac{\rho_4}{\rho_3} \approx \frac{\rho_1}{\rho_3}\frac{1+\frac{\rho_3}{4c}\left(\delta-\frac{\rho_3}{2c}\right)}{1+\frac{\rho_1}{4c}\left(\delta-\frac{\rho_1}{2c}\right)}$$

The value of Δ can be determined by eq. (4) which becomes approximately

$$\left(\varepsilon - \frac{\Delta}{\cos\theta}\right)\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \approx 0,$$

hence

$$\Delta \approx \varepsilon\cos\theta = \frac{\delta}{\pi}\cos\theta.$$

Eqs. (6) can be expressed approximately as follows

$$\phi_1 = \log 2(\cos\Delta + \sin\theta)\left[\frac{1+\frac{\cos\theta}{1+\sin\theta}\cdot\frac{\Delta}{2}}{1-\frac{\cos\theta}{1+\sin\theta}\cdot\frac{\Delta}{2}}\right]^{\varepsilon},$$

$$\phi_2 \approx \log 2(\cos\Delta - \sin\theta)\left[\frac{1+\frac{\cos\theta}{1-\sin\theta}\frac{\Delta}{2}}{1-\frac{\cos\theta}{1-\sin\theta}\frac{\Delta}{2}}\right]^{\varepsilon},$$

hence $\quad \dfrac{p_1}{p_2} \approx \dfrac{\cos\Delta+\sin\theta}{\cos\Delta-\sin\theta}\left[\dfrac{1-\varepsilon\sin\theta}{1+\varepsilon\sin\theta}\right]^{\varepsilon}.$

From above eq. θ can be determined.

In the special case of $\varepsilon=0$, let $\theta=\theta_0$ then

$$\sin\theta_0 = \frac{p_1-p_2}{p_1+p_2}.$$

And approximate value of θ is given $\theta=\theta_0+\bar{\theta}$. where $\bar{\theta}$ is determined from the following equation.

$$\bar{\theta} = \frac{(\cos\Delta-\sin\theta_0)^2}{2\cos\Delta\cos\theta_0}\frac{p_1}{p_2} - \frac{\cos^2\Delta-\sin^2\theta_0}{2\cos\Delta\cos\theta_0}\left[\frac{1-\varepsilon\sin\theta_0}{1+\varepsilon\sin\theta_0}\right]^{\varepsilon}.$$

Also from $\phi_1=\log\dfrac{p_1}{k'}$, k' can be determined and approximately as follows.

$$\frac{1}{k'} \approx 4(\cos\Delta+\sin\theta)\left[\frac{c}{\rho_1} - \frac{p_1}{8c} + \frac{\delta}{4}\right].$$

Next we must determine the value of $ae^{i\tau}$. Expanding t in the series assuming ζ to be very large we get,

$$t = k\left[\zeta + 2i\sin\theta + 2\varepsilon\cos\theta - \frac{1-i\varepsilon\sin 2\theta}{\zeta} - \ldots\ldots\right]$$

Hence the first approximation for $ae^{i\tau}$ is

$$\frac{e^{i\varepsilon\theta}}{k'} - 2i\sin\theta \approx \frac{1}{k'} - i2\sin\theta + i\frac{\sin\varepsilon\theta}{k'}$$

and $\delta\approx\frac{1}{k'+k'^3}$,

$$\varphi\approx\varepsilon\theta+\frac{k'^2(\varepsilon\sin 2\theta+2\sin\varepsilon\theta-4k'\sin\theta)}{1+k'^2}.$$

So the second approximation for $ae^{i\tau}$ is

$$ae^{i\tau}\approx\frac{1}{k'}+\frac{k'-2\varepsilon\cos\theta}{1+k'^2}-2i\sin\theta+\frac{i\sin\varepsilon\theta}{k'}+i\frac{k'(2k'\sin\theta-\varepsilon\sin 2\theta-\sin\varepsilon\theta)}{1+k'^2}.$$

Finally let the incidence angle α of the flap wing section be the angle between w and the chord AH, then the relation between β and α is as follows.

$$\alpha=\frac{\pi}{2}+\frac{\delta}{2}-\beta+\psi,\quad\text{where } \psi=\sin^{-1}\frac{\rho_1}{2R}.$$

V. Approximate Solution.

Since the solution developed above is investigated by the method of conformal representation, the solution is accurate but tedious for calculation. In this paragraph, therefore, we add the approximate solution by the method of vortex field.

At first we consider the flap wing section composed of two segments of straight lines. We transform this section into a circle in the ζ-plane by the following relation.

$$Z_1=\frac{(\zeta-e^{i\theta})^{1+\varepsilon}(\zeta-e^{-i\theta})^{1-\varepsilon}}{\zeta}.$$

The relation between Z_1- and ζ-planes is shown in Fig. 5 and

$$r_1\approx 2\,(1-\cos\theta),$$
$$r_2\approx 2\,(1+\cos\theta)$$

and $\Delta\approx\frac{\delta}{\pi}\sin\theta.$

When the flow far from the flap wing is parallel to the real axis of Z_1-plane and ζ-plane, then it flows smoothly both at the leading edge A and trailing edge B. In this case the magnitude of circulation is

$$\Gamma_f=4\pi u\sin\Delta.$$

Hence the lift is given by

$$A_f=\pi\rho u^2 t\sin\Delta$$

where t is the chord length.

And the moment around the hinge point H is

$$\mathfrak{M}_f=A_f\frac{t}{2}\cos\theta,$$

the moment being positive in a clockwise direction.

To solve the problem of thin aerofoil section of arbitrary shape by the method of vortex field, we usually apply the following three forms of circulation distribution along the chord.

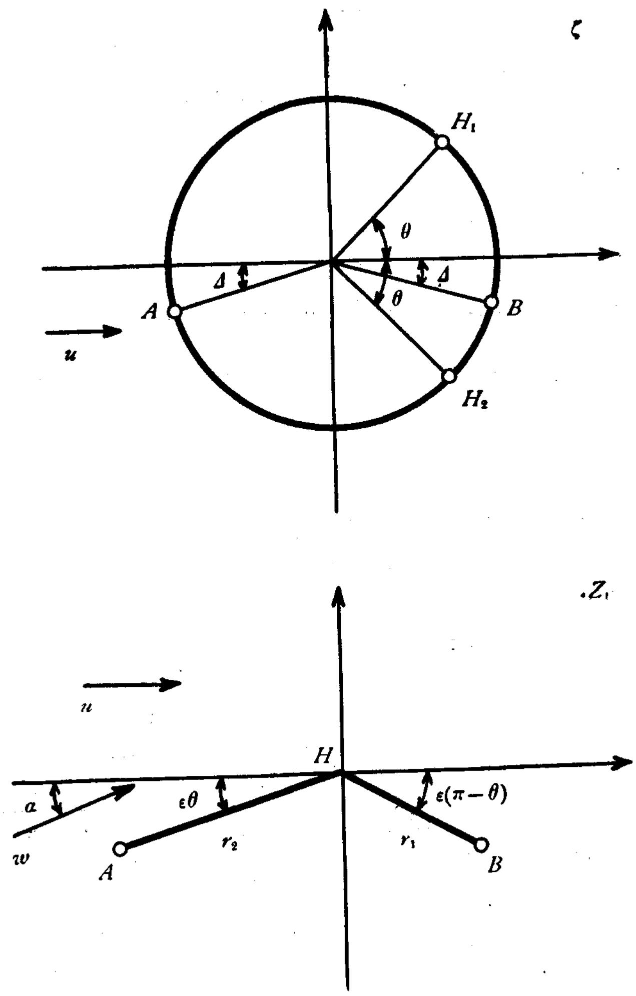

Fig. 5.

$$\sqrt{\frac{1-\frac{2x}{t}}{1+\frac{2x}{t}}},\quad\sqrt{1-\left(\frac{2x}{t}\right)^2},\quad\frac{2x}{t}\sqrt{1-\left(\frac{2x}{t}\right)^2}.$$

First we consider the flap wing section composed of straight lines but flow is not smooth at A. In this case we take $\sqrt{\frac{1-\frac{2x}{t}}{1+\frac{2x}{t}}}$. Let the flap wing be placed as in Fig. 5 and the flow far from the wing make an angle α with the real-axis and with magnitude w, then

lift $A_{sf}=\pi\rho w^2 t\left[\sin\alpha+\frac{\delta}{\pi}\sin\theta\cos\alpha\right]$,

and the moment around the point H is

$$\mathfrak{M}_{sf}=\frac{\pi\rho w^2 t^2}{8}\left[\sin 2\alpha\,(1+2\cos\theta)+2\frac{\delta}{\pi}\cos^2\alpha\sin 2\theta\right].$$

We proceed now to the case of the flap wing composed of two circular arcs. In this case we add $\sqrt{1-\left(\frac{2x}{t}\right)^2}$. This latter distribution gives a nearly circular arc section and the form of the section is

$$y_c = f\left[1-\left(\frac{2x}{t}\right)^2\right],$$

and the corresponding circulation when the velocity far from the wing is parallel to the real axis and magnitude of u, is

$$\Gamma_c = 2\pi f u.$$

Adding this type of distribution, the form of flap wing section is as shown in Fig. 6 and

for $\widehat{AH}$ $\quad y=\left(x-\frac{t\cos\theta}{2}\right)\left[\varepsilon\theta-\frac{4f}{t^2}\left(x+\frac{t\cos\theta}{2}\right)\right],$

for $\widehat{HB}$ $\quad y=-\left(x-\frac{t\cos\theta}{2}\right)\left[\varepsilon(\pi-\theta)\right.$

$$\left.+\frac{4f}{t^2}\left(x+\frac{t\cos\theta}{2}\right)\right],$$

and

$$\bar{\alpha}=\varepsilon\theta+\frac{2f}{t}(1-\cos\theta).$$

When the flow far from the flap wing is parallel to the real axis in Fig. 6, it flows smoothly at A and B.

The lift and moment around the hinge point H in the general case are as follows.

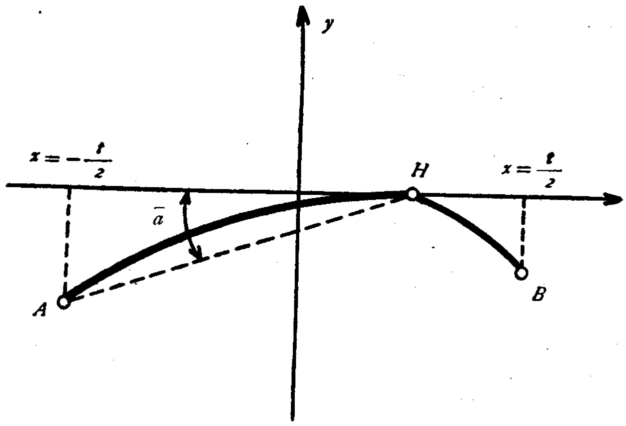

Fig. 6.

$$A=\pi\rho w^2 t\left[\sin\alpha+\frac{\delta}{\pi}\cos\alpha\sin\theta+\frac{2f}{t}\cos\alpha\right]$$

$$\approx\pi\rho w^2 t\left[\alpha+\frac{\delta}{\pi}\sin\theta+\frac{2f}{t}\right]$$

and

$$\mathfrak{M}=\frac{\pi\rho w^2 t^2}{8}\left[\sin 2\alpha(1+2\cos\theta)+2\frac{\delta}{\pi}\cos^2\alpha\sin 2\theta\right.$$

$$\left.+\frac{8f}{t}\cos\theta\cos^2\alpha\right]$$

$$\approx\frac{\pi\rho w^2 t^2}{4}\left[\alpha(1+2\cos\theta)+\frac{\delta}{\pi}\sin 2\theta+\frac{4f}{t}\cos\theta\right].$$

(Berlin, 9th. Feb. 1937.)