On the Resistance of Structures to Earthquake Shocks.

By Rio Tanabashi.

Introduction.

In this paper the author discusses the resistance of structures against earthquake shocks, especially buildings.

It is not necessary for building structures to be so strong that no part of the building is damaged by the macro-earthquake that happens say once a century.

But we must endeavour to avoid their falling down or complete destruction which causes loss of many lives. We need not be over nervous about restorable damage by big earthquakes, because all buildings require some alteration from time to time and besides the damage done to subordinate parts sometimes saves the main structure from fatal destruction.

The author, taking into consideration the recently revealed facts about earthquakes, shows a new way to design quake-resisting structures.

I. Some important results learned from the recent development of earthquake research in Japan.

We structural engineers were taught some new facts regarding the nature of earthquake from the recent development of earthquake research in Japan. Those are the facts which were brought to light by the seismological records got by the acceleration-seismograph of Dr. Ishimoto who is the head of the Earthquake Research Institute of the Tokyo Imperial University.

We had very little knowledge about the true value of acceleration of earthquake shocks, though our earthquake-resisting calculation of structures is based upon the value of acceleration of the earth's motion.

Our old records of earthquakes were all got from the displacement-seismograph, and so we calculated the max. acceleration of the seismic motion assuming that the earth's motion is in simple harmonic manner.

Thus the calculated max. acceleration of the 1923 earthquake at Hongo, where the displacement-seismograph of Tokyo Imperial University succeeded in recording the seismic motion, is as follows:

max. displacement	88.6 mm
main period	1.35 sec.
max. acceleration	95.8 mm/sec^2 (nearly equal to 0.1 g, where g is the gravitational acceleration.)

Thus the computed value of the acceleration must be considered to be always smaller than the true value. Now let us consider two different waves (i) and (ii), both having max. displacement about 2a and the period of main wave $2\pi/p$, as follows:

(i) $d = a \sin pt$

(ii) $d' = a \sin pt + \frac{1}{3} \sin 3pt.$

Then the accelerations are as follows in waves (i) and (ii) respectively;

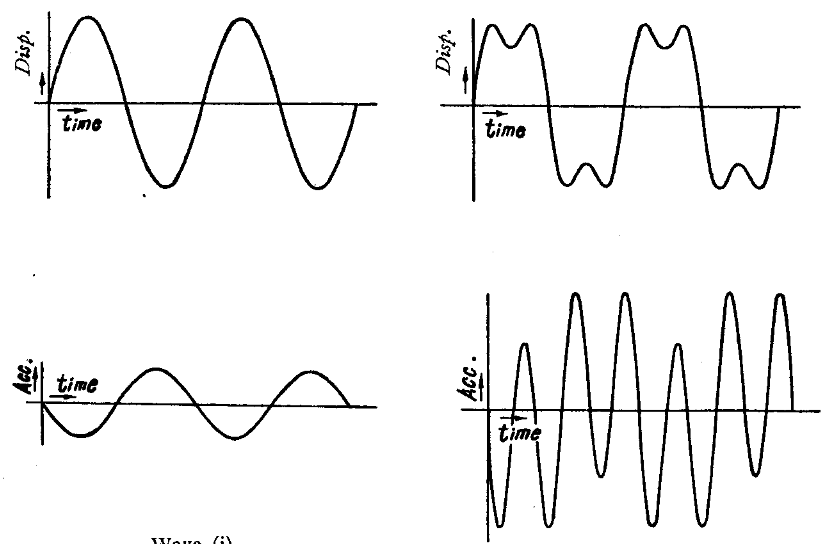
(i)' $\alpha = ap^2 \sin pt$

(ii)' $\alpha' = ap^2 \sin pt + 3ap^2 \sin 3pt.$

The displacement diagram of waves (i) and (ii) and the acceleration diagrams of those waves are shown in fig. 1.

The complicated form of the displacement records of the earthquake inevitably indicates that the true value of the max. acceleration must be far larger than the value computed upon the assumption of seismic motion as a simple harmonic movement of the earth.

The acceleration-seismograph of Dr. Ishimoto succeeded in recording the acceleration of seismic



Wave (i).

Wave (ii).

Fig. 1.

motion directly, and at the same time discovered experimentally that the waves of shorter period which are hidden by the waves of longer period in the displacement-seismograph predominate in the acceleration-records. (see Fig. 2)

At the time of the earthquake on 10th, April, 1933 in the neighbourhood of Long Beach U.S.A. American seismologists got the acceleration records at Long Beach 17 miles distant from the epicenter, and succeeded in recording waves with 0.23 g acceleration and 0.3 sec period in horizontal direction, and a wave with acceleration of 0.2 g and 0.2 sec period in vertical direction. It seems peculiar that the American engineers considered that such quick waves are harmless to structures.

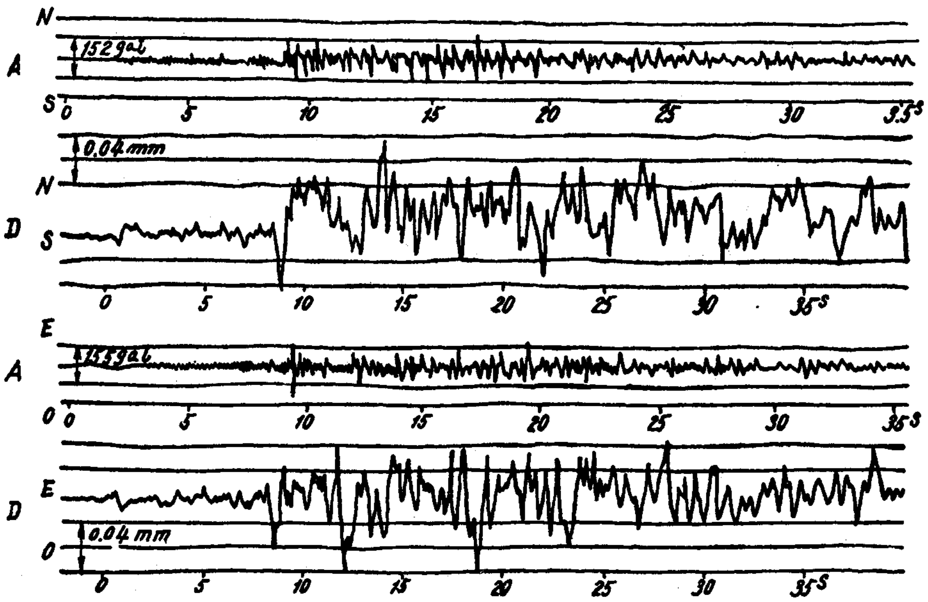


Fig. 2.
(A: Accerelation records. D: Displacement records.)

In Japan we have no record of acceleration-seismograph at the time of any recent big earthquake, but recently re-examined and analyzed records of displacement seismogram at the time of the great Mino-Owari earthquake of Oct. 28th 1891 recorded in Nagoya (the city lies at the end of the damaged region and 3% of the wooden houses collapsed) taught us that the waves having largest acceleration are those with 0.3 sec period

Table 1.

	Name of Building	No. of Stories	Date of Observation		Period of Horizontal Vibration E-W	Period of Horizontal Vibration S-N	Name of Observer
Steel and Steel with Reinforced Concrete Building	Marunouchi-Build.	9	1921 Dec.	under const.	1.11	1.14	
			1922 Feb.	almost completed	0.98	0.94	Omori
			1922 May	after earthquake	1.01	1.09	Omori
			1922 Oct.	after repair	0.67	0.71	Omori
			1923 Dec.	after earthquake	1.11	1.18	Horikoshi
			1926 May	after repair	0.50	0.48	
				after repair	0.23	0.25	Saita
	N.Y.K. Build.	8	1923 March	before earthquake	0.69	0.77	Omori
			1924 Nov.	after earthquake	0.90	0.80	Saita
	Kaijio-Build.	7	1922 June	before earthquake	—	0.45	Omori
	Nihon-kogyo-Ginko	8	1923 June	before earthquake	0.65	0.61	Omori
	Yurakkan	8	1922 Nov.	before earthquake	0.60	0.61	Omori
			1927 June	after earthquake	0.89	0.90	Omori
				after repair	0.45	0.55	Taniguchi
	Tokyo-Kaikan	6	1922 June	before earthquake	0.72	0.54	Omori
			1923 Sep.	after earthquake	1.30	1.20	Horikoshi
	Marunouchi-Hotel	9	1924 Dec.	after completion	0.50	0.60	Saita
	O.S.K. Build. (Kobe)	8	1923 April		0.53	0.57	Omori
	Kokkoh-seimei-Build.		1923 July		0.65	—	Omori
	Ginza-Build.	8	1923 July	skeleton completed	0.52	0.52	Omori
			1924 Dec.	completed	0.70	0.70	Saita
	Shokin-Ginko		1923 April	almost completed	1.19	1.09	Omori
	Todai Butsuri Kyoshitsu	4	1925 Feb.	completed	0.25	0.25	Saita
	Parliament Build.	3	1925 Aug.	steel skeleton completed	0.66	0.66	Imamura
Reinforced concrete Building	Earthquake research Institute	2			0.30	0.30	Ishimoto
	Naigai Build.	9	1923 July	before earthquake	0.65	0.65	Nagata
	Nippon-Ginko	8	1923 May	before earthquake	0.48	0.43	Nagata
	Meiji-Kaiun	8	1924 Oct.		0.50	0.50	Omori
	Mitsubishi's Laboratory	2	1924 Oct.			0.15	Suehiro
						0.35	Ishimoto
	Aviation Institute	2	1924 Oct.			0.17	Suehiro
						0.50	Ishimoto

and the value of max. acceleration more than 0.2 g.

Some Japanese authorities say that the value of acceleration must be greater than 0.4 g when 50% of the wooden houses collapse.

The reports from places damaged by heavy earthquake shocks in Japan also agree with this opinion, for it can be said that the max. acceleration of the earthquake motion must be greater than 0.4 or 0.5 g from the size of overturned tombstones. None of our engineers believe that the value of max. acceleration of the seismic motion during heavy earthquakes is as small as 0 1 g, which is the basis of our earthquake-resisting calculation of buildings. Thus the true resistance of buildings against earthquake shocks is discussed by the author in the following section.

The records of the acceleration seismograph emphasize the opinion that we must consider now that our buildings are to be designed to resist 0.4 g horizontal acceleration at least. But on the other hand the shorter period of the wave with large acceleration, which is now considered to be the natural frequency of the ground, threatens the basis of the earthquake resisting calculation. It is because our old assumptions are as follows:

"The period of the earth's motion at the time of an earthquake, especially a destructive earthquake, may be about one second, so we must build rigid structures having a shorter natural period of vibration to resist safely a horizontal forces of 0.1 g of its own mass, then taking safety-factor into consideration it may resist 0.3 g at least and probably about 0.4 g"

In this direction we designed buildings having a natural period of vibration lying between 0.2 sec to 0.6 sec as shown in table 1.

But the predominant period of seismic motion of the ground is not so long as 1 sec generally, and Dr. Ishimoto's analysis of the seismological records shows that they lie also between 0.2 sec—0.8 sec. (fig. 3)

First we lose the basis of our statical calculation of the quake-proof construction which is based upon the assumption that the natural period of vibration of buildings is shorter than the period of earthquake motion.

And at the same time we are taught that our rigidly constructed buildings inevitably fall into synchronization with the earthquake motion, and if this synchronous vibration is most dangerous to the structure, as is commonly considered by the leading structural engineers, none of our buildings is safe against earthquake shocks.

But there is no reason to fear the synchronous vibration, as the author mentions in the following section. Some important investigations on this problem made by other authorities must be mentioned here.

The Kyobashi Daiichi Sohgokan (The

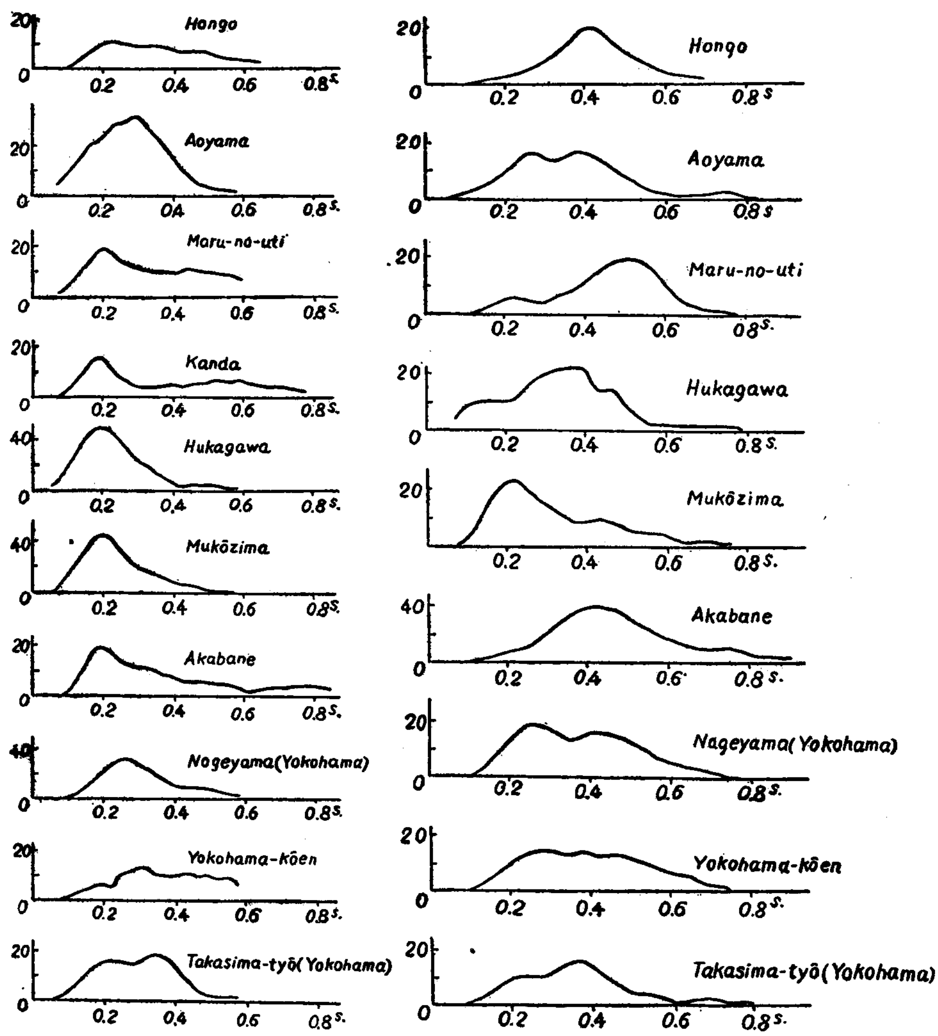


Fig. 3.

The predominating period of seismic motion for several places in Tokyo and Yokohama in the earthquake 6th Aug. 1933 (left) and 9th Oct. 1933 (right). Number of waves in ordinate period of wave in abscissa.

Office Building of the First Life Insurance Co. at Kyobashi, Tokyo) has a natural vibration period of 0.6 sec and stands upon ground having also a predominant period of 0.6 sec.

So if the synchronous vibration of the building is fatal, this building should have been damaged seriously at the time of the great disaster of 1923, but it did not suffer any serious damage by the earthquake.

The recent investigation of Dr. Sezawa of the Earthquake Research Institute of Tokyo Imperial University about "the dissipation of the energy of vibration into the ground" teaches us that such violent sinchronous vibration as ordinary dynamics says does not take place. It is a very important matter that such large dissipation into the ground takes place according to the nature of the ground and building, but it is too much of a mathematical problem for engineers.

The synchronous vibration takes place only in the case when the deformation of the structure is lineally proportional to the applied forces and the disturbing force maintains an equal period and acts continuously.

But neither structure nor earthquake have not such properties.

Before we consider synchronous vibration to be very dangerous, we must examine the properties of the destructive earthquake. It is difficult to discuss this problem throughly so long as we have not enough seismological records of a destructive earthquake at the severely damaged place. But there is no doubt that in the earthquakes at Tajima 1925, Tango 1927, and Idzu 1930, the destructive shocks were only one or two, and even during the earthquake of Kwanto Districts 1923 whose epicenter lay in the Ocean, far away from Tokyo and Yokohama, the severest shocks that are considered to have done damage to buildings were five or six, and it is said that the buildings which withstood the first several shocks withstood

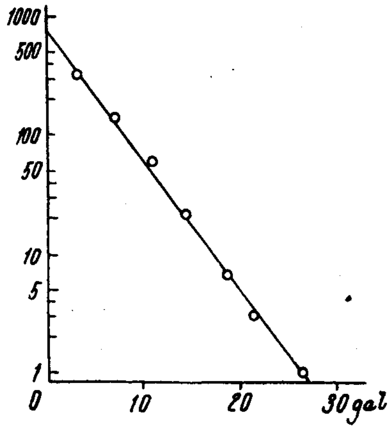


Fig. 4.

The earthquake 17th June 1931 observed at Hongo Number of waves in ordinate, value of acceleration in abscissa.

the earthquake throughout. Recent seismological study in Japan teaches us that the waves of earthquakes which come successively in equal period are five at most.

Dr. Ishimoto's analysis of the waves of earthquakes is very important to this problem. He has analyzed several earthquake records got by his acceleration-seismograph and got always such diagram as shown in fig. 4. This diagram shows clearly that only one wave has the largest acceleration and the smaller the value of acceleration is, the larger the number of waves is.

II. Are buildings designed according to the old theory safe against heavy earthquakes in future?

Excluding the synchronous problem from matters fatal to the safety of buildings, we are to discuss the safety-factor against earthquake shocks of buildings designed to resist 0.1 g horizontal force.

The safety-factor of the constructive materials can be considered to be 3, because the allowable stress of concrete is 1/3 of the 4 weeks-strength, and the allowable stress of steel is 1200 kg/cm^2, that is also 1/3 of the ultimate strength. (It must be mentioned here that to take ultimate stress of steel as standard is not reasonable and the stress at yield point is to be taken).

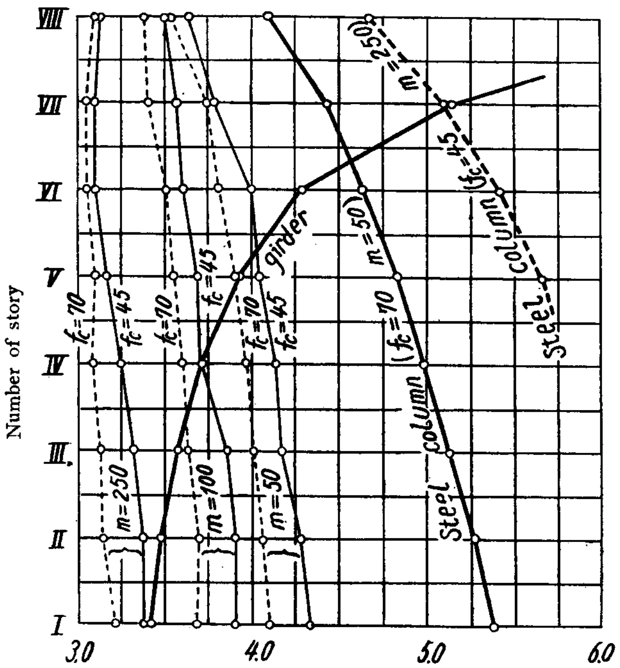
Now let us consider the safety-factor of the constructive material as v, and the stress owing to vertical loads as Sg, and the stress owing to earthquake forces as Sh.

We are to examine to what value of acceleration or "quakefactor" our building can stand safely when designed to resist 0.1 g or "quakefactor" 1. If the building can stand to 0.k g then the safety-factor for earthquake is k. The value k is commonly determined by the following relations:

$$Sg+Sh=S'$$
$$Sg+kSh=vS'$$
$$kSh=(v-1)Sg+vSh$$
$$k=(v-1)Sg/Sh+v \quad \text{.......................(1)}$$

Safety-factor to earthquake, k, is always larger than the safety-factor of material, v, according to the equation (1). And the larger the stress Sg compared with the stress Sh is, the larger the safety-factor to earthquake, k, compared with the safety-factor v is also.

Thus examined the safety-factor of a moderately reinforced concrete building is widely variable according to the parts of the building. It varies from a value a little larger than 3 to such a high value as 20. (see fig. 5)



Safety-factor for earthquake

Fig. 5.

Safety-factor of moderately reinforced concrete Building with rectangular frame calculated for 0.1 g earthquake. Thin lines for reinforced concrete column with the allowable stress *fc* in kg/cm^2 and the ratio of cross-sectional area to the reinforcement *m*. Steel column with equal dimensions as reinforced concrete column with written *fc* and *m*.

The author thinks it is not rational. So long as we are to construct buildings to resist large horizontal force due to max. acceleration of the earthquake, we must design each member to resist safely the earthquake of quakefactor-x considering the ultimate strength of member, and quakefactor-x may be 4 at least. To design buildings to resist earthquake of quakefactor 4 or even 5 is not so difficult and at the same time not so expensive in the case of reinforced concrete buildings of 3 or 4 stories. In fiig. 5 we show the safety-factors to earthquake of columns and girders of a moderately reinforced concrete building with rectangular frame.

Fig. 5 shows very clearly that the safety-factor to earthquake with regard to a reinforced concrete column always lies between 3 to 4, on the other hand that of girders very large in upper stories.

I must mention here that the safety-foctor to earthquake of a reinforced concrete column can not be determined by the relation (I) and is always smaller than the value given by the equation (I), because the increase of the eccentricity of compressive force by the increase of quakefactor inevitably causes moving of the neutral axis and decrease of compressive zone of the concrete and effectiveness of column.

But it must also be taken into account that the formula of reinforced concrete based upon the ratio of elasticities of concrete and steel may not hold to calculate the ultimete strength of reinforce concrete column and girder. The author is now investigating theoretically and experimentally the ultimate strength of reinforced concrete members from this point of view, and though it is not completed, I think it can be said fortunately that the ultimate strength of the column is higher than the computed value with ordinaly formula. My research upon reinforced concrete has an intimate relation in its direction with Austrian research into the plastic deformation of concrete.

The safety-factor of a steel member is always higher than that of reinforced concrete, computing the ultimate by ordinary formula. If we take into account its plastic property as in reinforced concrete it will be raised much more.

The author is of the opinion that the system of computing the building of nowaday is not rational especially for such a country as Japan where the aim of computation is to make a building resist safely a big earthquake that may happen say once a century and the limit of violence of which is hardly known, and we have to expect the building to resist the severest earthquake in its ultimate strength. The author proposes a plan of disigning buildings to resist earthquakes in the following form, that is considered to be more rational.

The safety-factor of concrete itself 1.2—1.5 (for 4 weeks strength).

The safety-factor of steel itself 1.0 (for the stress at yield point).

Live load taken 2.5—3.5 times as large as the value expected. (in this case the safety of the building is to be checked without taking the earthquake into consideration).

Quakefactor 6—8 (at least 4) live load to be taken of the value expected in this case.

This plan of computation will give a more rational proportion to the reinforced concrete building lower than 50 feet, I think. But in case the proportions of the building becomes slender, then the statical computation becomes irrational so long as the building must overturn by the horizonal forces. The profile of the building gives the limit of posibility of the computation in this system, and tall slender buildings must be given another consideration as in the following section.

III. Is the acceleration of the earthquake proportional to its damage?

We have recognized by the recent development of earthquake research that the period of natural vibration of buildings lies very near to the period of earthquake motion. So the practice of building to resist statical horizontal force proportional to the max. acceleration has lost its theoretical basis that the period of natural vibration of buildings is far shorter than the period of the earthquake. And the knowledge of the true value of the instantaneous acceleration of the earthpuake fixes the limit by which we can calculate the building statically to resist the large horizontal force caused by the acceleration. I think that the limit for constructing an ordinary building to resist statically the horizontal force of earthquakes with quakefactor about 6 may be lower than 50 feet.

But is a skyscraper very dangerous during earthquake? Our experience of earthquakes causes us to reply "No". Five-storied buddhist pagodas, about 100 feet high, usually suffered no serious damage by earthquakes, though their construction compared with other Buddhist temples has not such peculiarity as some building engineers believe except that they are built with very many wooden members of large size.

In our sad experience of the disaster 1923 we have no evidence that tall buildings were more dangerous even in reinforced concrete construction, though there is no evidence that tall buildings were safer.

The author thinks that men have a prejudice that tall bodies are dangerous during earthquakes, from the fact that slender bodies such as bins and Japanese tombstones overturn in every earthquake. But it must be noticed that there we forget the concept of dimension. A small boat overturns in a rough sea, but a big ship never.

Every tombstone overturns during an earthquake, but the Empire State Building never. It is common sense.

Dynamics teaches us that when the period of the natural vibration of a building is far shorter than that of an earthquake, then horizontal force proportional to the max. acceleration acts upon the structure statically, and when the period of natural vibration of buildings is far longer than that of earthquakes, then the structure is deformed proportional to the amplitude of earthquake. The facts brought to light are not such as the above mentioned. The natural frequency of buildings is very near to that of earthquakes, and so we must design ordinary buildings under the assumption that the natural frequency of the building may coincide with that of the earthquake, that is the worst case of synchronization.

In our case, the destructive force of the earthquake is proportional to the square of the max. velocity not the max. acceleration and the resistance of the structure against it is proportional to the potential energy conserved in the structure until it falls down or crashes and is not the safely applicable statical force in the horizontal direction. These hypotheses were proposed by the author first on July 1934 at a meeting of graduates of the Architectural Department of Kyoto Imperial University, and were published in the Journal of Japanese Architects May 1935.

If the period of the natural vibration of a system coincide with the period of simple harmonic earthquake motion in the form $a \sin pt$, then the deformation y of the sysiem is given by the following equation.

$$y=\frac{1}{2}apt \sin pt$$

y is proportional to a and the force acting upon the system is proportional to p^2y, that is p^2a. So the potential energy conserved in the deformed system is proportional to p^2a^2, that is the squrre of max. velocity. This consideration applies to an elastic system, and some systems having non-elastic property are investigated in the following section.

We are now to get some idea upon the limit of violence of earthquake. It may be said that we have no reason to attempt to estimate how violent earthquakes of the future may be. But we are considering that there may be some limit to the violence of earthquake. Our experience teaches us that the most violent damage was 80% of wooden houses collapsed in the earthquakes of Tajima, Tango, Idzu and Kwanto, though the areas of damaged regions are not equal.

And it must be very important to us that the limit of violence of any earthquake conceivable in the present stage of the science of earthquakes is the limit of max. velocity.

If we consider that a harmonic motion is produced by the earthquake, then the max. velocity of the ground may be given by the following equation.

$$V=KS/D \quad \text{......................................}(2)$$

where V=the max. velocity, S=the propagation velocity of the wave, D=the elastic constant of the ground and K is the limit of the strength of the ground. In present we have no determined valve for V, but in the future it may be determined by the aid of geophisists and seismologists.

What meaning has the idea of designing buildings to resist the the velocity of earthquakes to the constructive engineer? It means that we must determine the quakefactor on a sliding scale according to the period of the natural vibration of a building in following manner. Rigid or low buildings with short period must be designed with

Table 2.

Vibration Periods of San Francisco Buildings. (E.N.R. 1932. S. 886)
measured by Perry Byerly, James Hester, and Kenneth Marshall with E. E. Hall's Seismograph.

Building	Horizontal dimensions	Height above Ground (feet)	Stories	Period of Vibration Horizontal N-S	Period of Vibration Horizontal E-W	Vertical
450. Sutter. St.	138×160	340	26	1.2	1.4	...
Sheel Building, Bush & Battery sts.	117×137	389	29	1.80	1.85	...
Russ Bldg, Montegomery, Pine & Bush sts.	275×166	435	35	1.71 1.78 0.08*	1.89 0.85 0.17 0.06*	0.23
Hunter Dulin Bldg.	100×160	309	24	1.48 0.49 0.05	1.35 0.15 0.09*	... ...
Mark Hodgkins Hotel.	193×171	257½	19	1.27 0.14	0.95 ...	0.21 0.05
Williams Taylor Hotel.	137½×137½	325	28	1.34	1.32 0.57	0.04
Alexander. Bldg.	69×60	204½	15	1.23 0.39	1.32 0.41	
Sir Francis Drake Hotel.	116×138	282	23	1.49	1.82 1.29	0.51
Bank of America.	82×48	181	15	1.64 0.90 0.07*	1.41 0.90 0.50	0.52 0.07* 0.43
Pacific Gas & Electric Co. Bldg.	137½×137½	262	17	1.50+ 0.90	1.28+ 0.55	... ...
Maston Bldg.	133×133	320	17	1.44+ 1.21	1.26+	0.53
Insurance Center Bldg.	206×84	206	16	1.41 0.20	1.07 0.48	
De Young Bldg.	...	...	16	1.53 0.59 0.17	1.36 0.90 0.15	0.55 0.08*
Western States Life Bldg.	90×63	207	15	1.03 0.18	1.33 0.16	
City Hall.	—	—	—	—	0.65	

* Vibration believed due to Traffic.
+ Vibration of P.G.E. and Maston Bldg. are considered to be a coincident vibration.
(Notice that the second mode of vibration is c'early observed.)

high quakefactor, and on the contrary flexible or tall building with long period can be designed to resist a small quakefactor. It may be some what as follows:

Period of nat. vib.	Quakefactor
0.2 sec	5~7.5
0.5	2~3.0
1.0	1~1.5

The period of natural vibration lengthens with the height of the building. The period of tall buildings in America is shown in table 2. They are between 1.0 sec and about 2 sec, and if the predominant period of seismic motion of the ground is 0.2—0.8 sec, then these buildings must be safer than our old statical assumption says.

In this case the period of the second mode of vibration comes near to the period of the earthquake, but in the second mode of vibration the horizontal forces acting in upper stories are different in direction from those acting in lower stories. Therefore the stress due to horizontal forces does not become so large as ordinary statical computation makes it in lower stories, and there is no danger of overturning.

Now is it contrary to our experience that we consider the destructive force of an earthquake lies in its velocity? It is clearly shown in the following table 3:

A, B, C, D, E, and F are the earthquakes recorded in Tokyo during the era of Meiji, having the max. acceleration about 0.1 g or more. Among these the earthquake B is remembered as a big earthquake in Tokyo of Meiji era with a record of considerable damage to houses, but earthquakes other than B were quite harmless though the earthquake D had max. acceleration as big as quakefactor 3. G, H, I, and J are the great earthquake of Kwanto Districts and its after earthquakes: G, was a heavy earthquake causing tremendous damage but others were rather harmless though I seems to have had a larger acceleration than G. K is the earthquake which was recorded in southern California recently.

There we find that the value of acceleration never gives the degree of severity of an earthquake, though the values of acceleration described are very doubtful because they are calculated from the records of displacement seismograph.

Table 3.

	Max. Dis-placement	Main Period in sec.	Max. acce. mm/ sec²	Prop. to vel.	Prop. to vel.²
A) 4th Jan. 1893	3.5	0.3	777.1	11.7	136
B) 20th June 1894	76.5	1.3	887.7	59.0	3490
C) 18th Jan. 1895	41.5	0.9	997.2	45.5	2060
D) 11th Oct. 1895	6.1	0.2	3004.0	31.4	990
E) 20th Aug. 1896	2.9	0.3	636.0	9.6	92
F) 29 Dec. 1896	1.9	0.2	938.0	9.5	90
G) 1st Sept. 1923	88.6	1.35	959.62	66.0	4350
H) 2nd Oct. 1923	69.7	2.3	260.09	30.2	900
I) 5th Nov. 1923	15	0.5	1184.35	30	900
J) 15th Jan. 1924	59	1.8	359.45	32.8	1080
K) 10th March 1933	60	1.5	0.11 g	40.0	1600

Dr. Majima said that the max. displacement rather makes the standard to the damage and this table is given in his book. But the values proportional to the square of the velocity calculated by the author are the best values that give the severity of an earthquake, I think.

We are now going to explain that if the period of natural vibration is very near to that of the earthquake, then the high value of instantaneous acceleration included in the wave is of no meaning to the safety of structure.

Suppose that a structure like that shown in fig. 6 has a degree of freedom as a system of a mass point.

If the horizontal displacement of the system caused by the action of unit horizontal force to the mass point is k and the mass of the mass point is m, then the vibration of the system caused by a disturbing force q is given by the following equation.

$$m\frac{dx^2}{dt^2}+kx=q$$

where q is a function of time, and may be very irregular when we consider earthquake shocks as disturbing forces.

If the system starts to move from the rest position, then the motion will be given by the following equation, where the displacement x at the time t' is given :

$$x=\frac{1}{p}\int_0^{t'} q \sin p(t'-t)dt$$

where p means $\sqrt{k/m}$.

i) As the first case we consider the reciprocating motion of the ground shown in fig. 7a) as the disturbance, then the motion will be given in following relation.

$$x=\left|\frac{Q}{P}\sin p(t'-t)\right|_0^{t'}=-\frac{Q}{P}\sin pt'$$

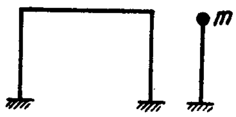


Fig. 6.

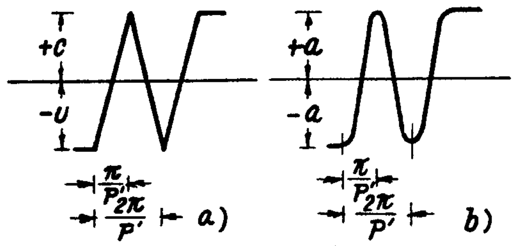


Fig. 7.

for $0\leqq t'\leqq \pi/p'$

$$x=-\frac{Q}{P}\sin pt'+\frac{2Q}{P}\sin p(t'-\pi/p')$$

for $\pi/p'\leqq t'\leqq 2\pi/p'$

$$x=-\frac{Q}{P}\sin pt'+\frac{2Q}{P}\sin p(t'-\pi/p')$$

$$-\frac{2Q}{P}\sin p(t'-2\pi/p')$$

for $2\pi/p'\leqq t'\leqq 3\pi/p'$

where Q is $\int_0^{\tau} q dt=2ap'/\pi$.

τ means the time in which the acceleration acts, and at the time π/p' and $2\pi/p'$ and s.f. twice as large as Q acts as a shock.

ii) As the second case we consider a simple harmonic motion shown in fig. 7 b) as the disturbance. Then :

$$q=ap'^2\cos p't$$

$$x=\frac{ap'^2}{p}\int^{t_1}\cos p't \sin p(t'-t)dt.$$

If $p\neq p'$, then

$$x=\frac{ap'^2}{p^2-p'^2}(\cos p't-\cos pt)$$

And If $p=p'$, then

$$x=\frac{a}{2}pt\sin pt.$$

In case ii) the max. velocity is $\int_0^{\frac{\pi}{2p}}qdt=ap'$. To make the disturbance of type i) to have same max. velocity as ii) we take the amplitude a' as $\frac{\pi}{2}a$ instead of a. Denote this case as i)′ and compare the max. displacement of the system by the disturbance i′) ii) and i) all in the case of resonance $p=p'$.

	$pt=\pi/2$	π	π
i′)	1.000a	0	3.000a
ii)	0.785a	0	2.350a
i)	0.635a	0	1.910a

If we take as the disturbance a reciprocating motion having the same velocity instead of a simple harmonic motion, then the deformation thus computed and at the same time the stress of the structure will be slightly larger than the latter, and if we take as a disturbance a reciprocating motion having equal amplitude and equal period with the simple harmonic motion then the deformation of the structure will be a little less. Though the instantaneous acceleration of the reciprocating motion is considered to be infinitely large, no fundamental difference from the disturbance in simple harmonic motion can be seen as regards to the behaviour of the structure.

Though such waves having sharp peaks as the author assumed may never happen, it is very convenient to note the irregular properties of earthquake shocks as the author explains the resisting quality of the structure in the following section so long as there is no fundamental difference between such waves and simple harmonic waves in regard to the vibration of the structure.

IV. The potential energy as the resistance-measure of structures.

Ductile and Brittle Material.

Commonly ductile materials mean those which make large deformation before destruction, and brittle materials mean those which make small deformation before destruction. The meaning is somewhat ambiguous and there can be proposed the following definitions about them.

We call those materials which can conserve large potential energy in themselves before their destruction "ductile" and those which can conserve small amount of potential energy as "brittle".

The definition is "quantitative" not "qualitative", as the definition of "elastic" or "plastic". So we may call those materials which can conserve potential energy before their plastic deformation takes place as elastic-ductile, and the material which can conserve the potential energy in itself by its plastic deformation as "plastic-ductile".

Now we examine the quake-resisting qualities of structures, one constructed of plastic-ductile material and the other of plastic-brittle. As plasticity we assumed ideal-plasticity, which can safely be assumed as being possessed by steel.

Increase of safety of statically indeterminate structure by plastic-ductility.

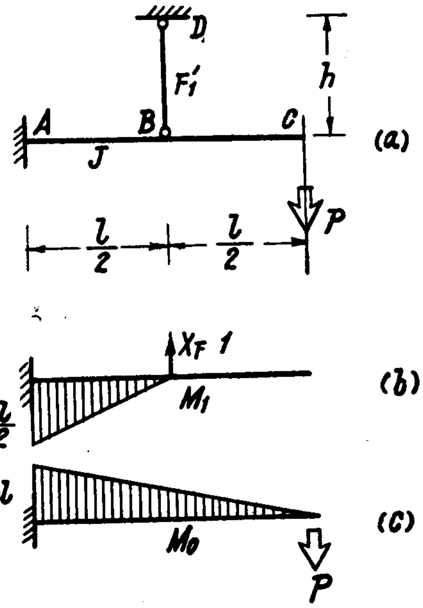


Fig. 8.

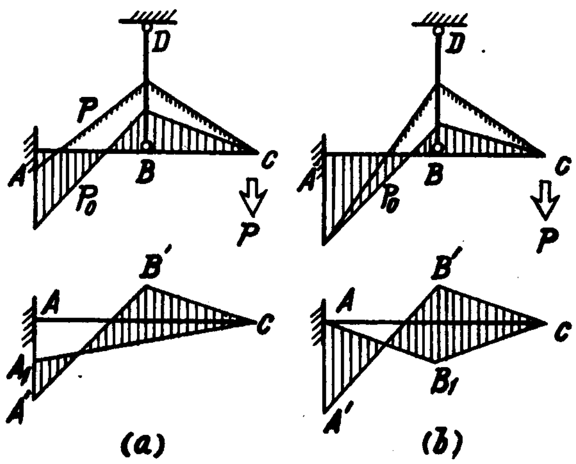


Fig. 9.

Dealing with n-th statically indeterminate structure it is apparent that the final destruction takes place after n parts of the structure have yielded. We may explain this fact in an example of statically indeterminate structure of 1st order in fig. 8. That is a cantilever girder fixed at the end A and loaded by P at the other end C with a suspending member BD at the middle point B of the girder. Denote the moment of intertia of the girder as J the cross-sectional area of the suspending member as F, then the normal force X acting upon the member BD is given by the following equation.

$$X=\frac{\delta_{01}}{\delta_{11}}=\frac{\int\frac{M_0M_1}{EJ}dx+\int\frac{N_1N_0}{EF}ds}{\int\frac{M_1^2}{EJ}dx+\int\frac{N_1^2}{EF}ds}$$

computed as follows,

$$X=\left\{\frac{1}{24}\frac{J_c}{J}\frac{5}{2}Pl^3\right\}\Big/\left\{\frac{J_c}{J}\frac{l^3}{24}+\frac{J_c}{F}h\right\}=bP.$$

Suppose the load P reaches the critical load P_0 given by the ordinary elastic-stastics, any one part of the structure should yield under this load. We exclude the case in which the bending moment of the middle point B reaches first to the critical value because the destruction concerns the statically determinate part. We can consider two cases of the failure of the structure under the critical load P_0, one as shown in fig. 9a the yielding of the suspending member the other as shown in fig. 9b the yielding of the girder at some section between A and B, say at A.

In the former case, if the girder does not yield at the same time then the structure can be loaded safely by a larger load than P_0: so long as the suspending member BD is able to elongate while suspending load bP_0, the girder acts as an cantilaver for a larger load than P_0, and if the moment diagram for the critical load P_0 is the area between line AC and line $A'B'C$, then the moment-diagram for a larger load than P_0 should be the area between Line A_1C and the line $A'B'C$. The structure safely stands until the load reaches the value under which some part of the girder yields.

In the later case, as shown in fig. 9, so long as the girder is able to deform while still resisting the critical bending moment we can assume the structure as a girder supported by two points A and B for a load larger than P_0.

If the moment-diagram for the critical load P_0 as the area contained by two lines AC and $A'B'C$, then the moment-diagram for the load larger than P should be the area contained by two lines $A'B'C$ and AB_1C. The structure can safely stand until the suspending member yields.

Generally speaking, for statically indeterminate structures until parts as many as the number of statically indeterminateness of any one part or of the whole yield, the final destructive yield does not take place, concerning respectively the stability of any one part and the whole of the structures.

As an extreme case, we can suppose that $n+1$ parts of the n-th statically in- determinate structure yield at the same time, but it should be very rare and so the ultimate strength and the safety-factor of the statically indeterminate structure may be higher than ordinary elastic-statics says.

It must be noticed that this raised safety limit of the statically indeterminate structure only concerns structures built of plastic-ductile material, and in structures built of plastic-brittle material this property is entirely lost. In the former example, if the suspending member BD is brittle, them it must be pulled off by the first critical load P_0 and the structure must be destroyed. If the structure can stand a larger load than P_0 without the co-operation of the suspending member, that means the suspending member is useless.

Built of plastic-ductile or brittle material the final destruction of externally n-th statically indeterminate structures takes place after n parts yield or lose their resistance, but the raising of the safety limit is considerable only in the case when the constructive material is plastic-ductile. This relation is shown schematically in fig. 10, (a) resp. (b) shows the case when ductile resp. brittle material is used in the construction.

In the case (a) the first yield A_1, the second yield A_2 higher than A_1, A_3 higher than A_2 and then the final yield A_4 higher than A_3 are generally considered, but in the case (b) the first

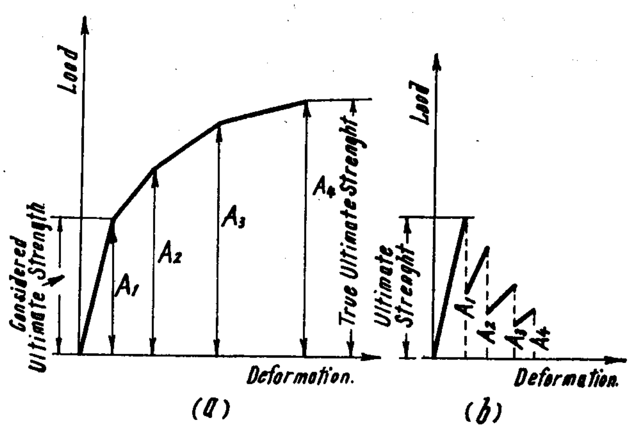


Fig. 10.

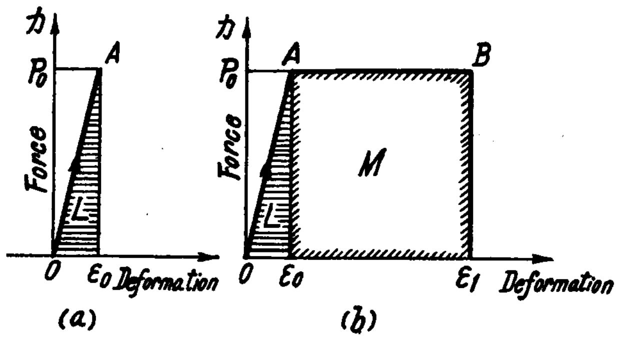


Fig. 11.

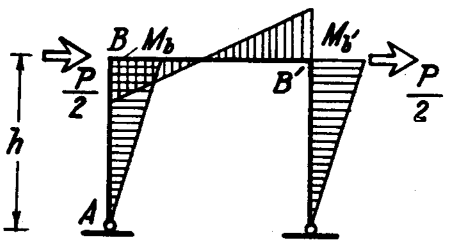


Fig. 12.

destruction A_1 followed by the second destruction A_2 lower than A_1, A_3 lower than A_2 and the final destruction A_4 lower than A_3 are generally considered; and so the first critical load, A_1 means the final critical load at the same time.

In our ordinary constructive design we are considering that the strength of the structure is A_1 shown in fig. 10 not taking into account the ductility or brittleness of the constructive material. It does not apply to the true safety of structures constructed of plastic-ductile material.

Considerations with regard to the dynamical force.

If the constructive design is quite satisfactory (in ordinally direction) and n parts yield under the critical load at the same time, the safety of the structure cannot be raised by the ductility of the constructive material as far as the statical force is concerned. But even in such an extreme case structures constructed of plastic-ductile material have considerably high safety against dynamically acting forces.

The resistance of material or structure to statical forces is given by the largest force safely applicable to it, P_0 in fig. 11 but the resistance of material or structure to dynamic force is given by the potential energy stored before its final destruction. So structures having the same resistance to statical force, are quite different in regard to resistance to dynamical force as earthquake-shock.

As an example we may take two structures having the same statical properties as shown in fig. 11 one built of plastic-ductile the other brittle material.

Two hinged portals as shown in fig. 12 are taken as an example, the end of the girder yield for the moment $Mb=Mb'=\pm\mathfrak{M}_0$, and so at the critical load $P_0=2\mathfrak{M}_0/h$ both ends of the girder yield and the structure sways horizontally to the amount of ε_1 without increase of resistance.

Suppose a shock such as the sudden movement of the ground with a finite velocity acts upon the structure, then the structure deforms as follows: Taking the structure's own weight as m and concentrated to the points B and B' as mass-points, then the movement of the structure from its equilibrium state is given by $m\dot{y}_0=K_0$ at the

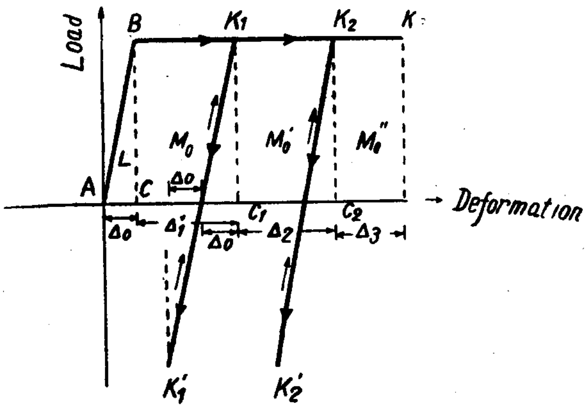


Fig. 13.

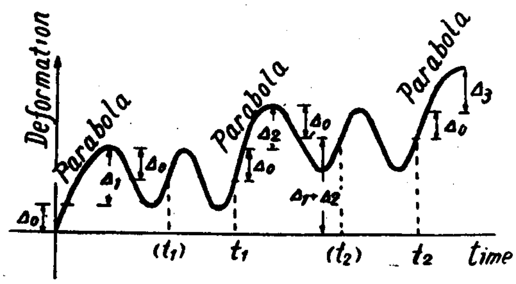


Fig. 14.

time $t=0$. where $\dot{y}_0$ means the horizontal velocity of points B and B' from the equilibrium state and K_0 means the amounr of shock.

The deformation of the structure must be the largest when the kinetic energy given by the shock is fully changed to the potential energy of the deformation. This relation is given in following equation:

$$T_0=\frac{1}{2}m\dot{y}_0^2=\frac{K_0^2}{2m}$$

for the case when the structure is brittle and the potential energy that can be stored before its destruction may be L, if the kinetic energy T_0 is larger than the potential energy L, then the structure collapses, but if the structure is plastic-ductile and the potential energy M in plastic domain besides L can be stored before its destruction, then a larger value of T_0 compared with L does not mean the destruction of the structure. The deformation of the structure in this case may be determined by the point in fig. 13 which satisfies the relation $L+M_0=T_0$. The displacement of the structure may be written in the form $\Delta_0+\Delta_1$, where Δ_0 is the elastic displacement and Δ_1 is the plastic. In ordinary structures we can suppose the load-deformation curve for the decreasing load as line K_1-K_1' in fig. 13, and so the deformation-velocity of the structure at the point of no load should be $\dot{y}_1$ satisfying the following relation:

$$T_1=\frac{1}{2}m\dot{y}_1^2=L \quad \therefore \ \dot{y}_1^2=2\frac{L}{m}$$

T_1 is the kinetic energy at that state.

In the case of non-dissipation of energy, the structure vibrates in simple harmonic motion with the maximum velocity of $\dot{y}_1$ and the amplitude of $e_0=\Delta_0$. As shown in fig. 14. Now let us consider that the next shock is given at the worst time, that is the time when the structure has its maximum velocity, shown as t_1 in fig. 14. Let the amount of the shock be K_0' and the deformation velocity of the structure change from $\dot{y}_1$ to $\dot{y}_1'$, there is following relation:

$$m(\dot{y}_1'-\dot{y}_1)=K_0' \qquad m\dot{y}_0'=K_0'+m\dot{y}_1$$

$$\frac{1}{2}m\dot{y}_1'^2=\frac{1}{2m}(K_0'+m\dot{y}_1)^2=T_1+T_1'$$

$$\therefore\quad T_1'=\frac{K_0'^2}{2m}+K_0'\dot{y}_1$$

The kinetic energy of the structure T_1' is changed to the potential energy M_0' and the maximum displacement of the structure should be $\Delta_0+\Delta_1+\Delta_2$ as shown in fig. 13. After that the structure vibrates in simple harmonic motion as above.

If the third shock K_0'' is given at the worst moment, then the following relations show the deformation of the structure after that.

$$T_0'+T_0''=\frac{1}{2m}(K_0''+m\dot{y}_1)^2. \qquad T_0''=\frac{K_0''}{2m}\,K_0''\dot{y}_1$$

$$T_0'+T_0''=L+M_0'' \qquad \therefore\quad T_0''=M_0''$$

The whole kinetic energy necessary to destroy the structure is given in the following relation:

$$T_0+T_0'+T_0''+\dots=\frac{1}{2m}(K_0^2+K_0'^2+K''^2+\dots)$$
$$+\dot{y}_1(K_1'+K_0''+\dots\dots)=L+M_0'+M_0''+\dots\dots=E$$

where E means the potential energy that can be stored before the destruction of the structure.

In regard to the earthquake shocks in which direction of action is alternating, the equation may be written in the following form:

$$T_0\pm T_0'\pm T_0''\pm\dots\dots=E$$

showing that as long as the sum of the first any number of terms does not reach the value of E, final destruction of the structure does not take place. It clearly suggests that the safety of the structure built with plastic-ductile material is very high against earthquake shocks.

On the other side, when the constructive material is elastic, the structure collapses under synchronous vibration even by far smaller shocks. The structure is given a kinetic energy $T_0=L_0$ by shock K_0 and vibrates with the maximum velocity of $\dot{y}_0$ and at the next worst moment another shock K_0' is given to the structure, then the kinetic energy T_0' of the structure and the velocity $\dot{y}_0'$ at that moment is given by the following relation:

$$m(\dot{y}_0'-\dot{y}_0)=K_0' \qquad m\dot{y}_0'=K_0'+m\dot{y}_0$$

$$\therefore\quad \dot{y}_0'>\dot{y}_0 \qquad T_0'=\frac{K_0'}{2m}+K_0'\dot{y}_0$$

The kinetic energy T_0' changes to the potential energy L_0' which is larger than L_0.

If the third shock K_0'' is given to the structure at the next worst moment, then the kinetic energy T_0'' of the structure and another velocity $\dot{y}_0''$ may be given in the following relation:

$$m(\dot{y}_0''-\dot{y}_0')=K_0''$$

$$m\dot{y}_0''=K_0''+m\dot{y}_0'$$

$$\therefore\quad \dot{y}_0''>\dot{y}_0'$$

$$T_0''=\frac{K_0''}{2m}+K_0'\dot{y}_0'$$

where T_0'' is larger than T_0', and so the maximum velocity of the vibration of the structure becomes larger with the action of shock even if the direction of the shocks is alternating. It inevitably causes the destruction of the structure.

Thus the author explains that the supreme

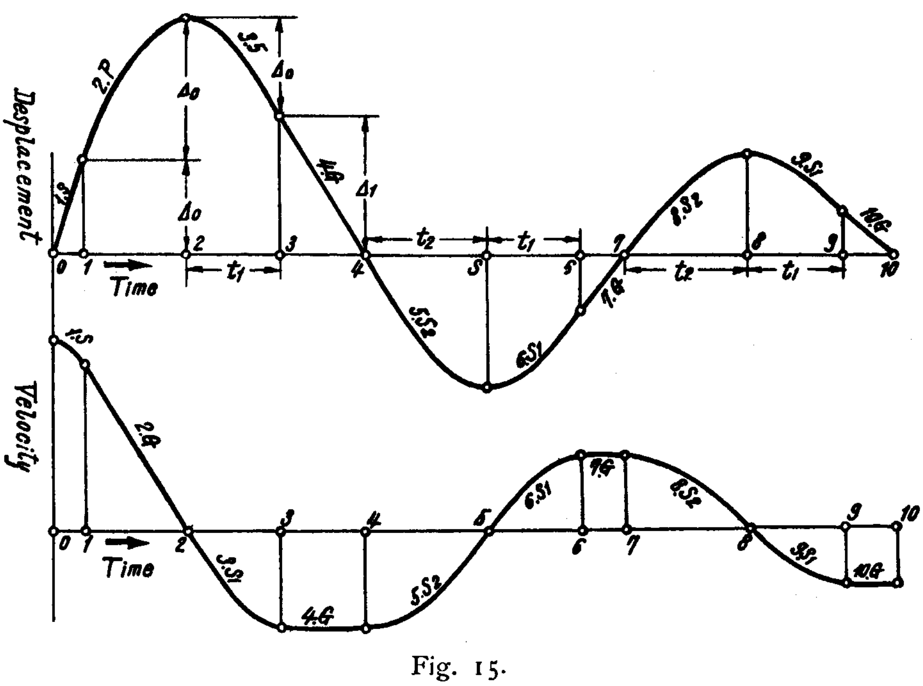


Fig. 15.

Vibration of the system having the load-deformation curve of fig. 16 given by initial velocity. S means sine curve, P parabola G straight line. Numbers in the graph correspond to the line of load-deformation curve of the same number.

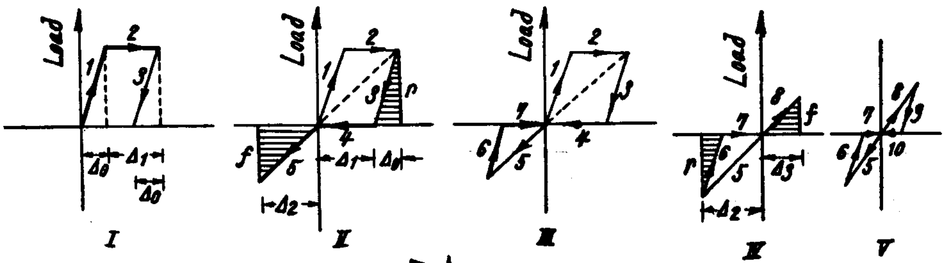


Fig. 16.

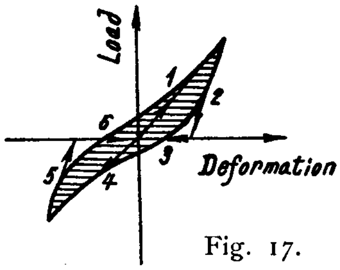


Fig. 17.

earthquake-proof quality is given to the structure by the plastic-ductility of the constructive material. If the structure is statically indeterminate and before its final plastic state elasto-plastic property is given by the yielding of some part of the structure, then we can consider the non-periodicity of the vibration and the high dissipation of energy before its final plastic state. These facts seem to the author to explain clearly the supreme earthquake-proof quality of steel structure.

To explain this quality the author show in fig. 15 the damped vibration of the system having the load-deformation diagram as fig. 16. The load-deformation diagram shown in fig. 16 seems to have some resemblance to that of the revetted connection of steel frame shown with hatched lines in fig. 17 which is got by the author's experimental study published in the Memoirs of the College of Engineering Kyoto Imperial University Vol. VIII, No. 4.

V. Potential energy as the measure of Earthquake-resisting quality.

I have mentioned that according to our present knowledge we must consider the period of natural vibration of the structure to be very near to that of earthquake in the worst case, and the most harmful waves of eartnquake are one or two, five at most and the synchronous vibration of the structure should not be feared, taking into account the plastic deformation of the structure. The author also talked about the possibility of calculating the earthquake-resisting quality upon some assumption of the velocity of the earthquake giving a sliding scale to the quake-factor according to the period of natural vibration of the structure, or calculating the maximum deformation determined by the given initial velocity.

Upon these considerations it can be said that potential energy that can be stored in the structure before its downfall gives the measure of earthquake-resisting quality.

So the earthquake-proof quality of the structure depends not only upon that the methode of construction but also largely upon the property of constructive material ductile or brittle.

Steel is the best, wood the next and concrete or brick or stone are not good for earthquake-resisting construction. Reinforced concrete may be considered as good constructive material provided that the good cooperation of the steel is to be expected, and its plastic-ductility is the most important thing to study from the quake-resisting stand point.

Now let us examine damage done by the great earthquake 1923 in Tokyo and Yokohama. See table 4 and 5.

From these tables we may conclude that the steel structure is the best, wooden houses the next and reinforced concrete buildings are a little worse than wooden.

We are now to examine the types of failure of structures by earthquake shocks, one the destruction of structure by the excess of stress, and the other, the falling down of the structure by the excess of deformation. As to the brick, stone or reinforced concrete building which, as a whole being brittle, collapses by the excess of stress, one of the causes lies in the fact that houses are rigid, with a rather short period of natural vibration and are sensible to the large acceleration of the earthquake. On the other hand the steel and wooden and reinforced concrete buildings of good design that are plastic-ductile as a whold fall down in the worst case by the excess of deformation that makes the structure unstable.

Concerning such building as steel-skeleton with brick facing, it must be noticed that the deformability of the facing cannot follow the deformability of the skeleton and the obvious damage to the facings are done while the skeleton is quite all right. And some of the steel building with brick facing suffered very serious damage by the buckling of columns caused by the excess of deformation.

Besides these cases of damage done directly to the structure there are other types of earthquake damage. One is the sliding of the ground

Table 4.

		No. of Bldgs.	not damaged		facing and partitions slightly damaged		facing and partitions considerably damaged		facing and partitions seriously damaged		damage done to steel skeleton	
Steel-Structure	steel-skeleton with brick facing	27	10	37%	7	26%	0	0%	9	33%	1	3.7%
	steel-skeleton with reinforced concrete facing	12	10	83%	2	17%	0	0%	0	0%	0	0%
	steel-skeleton with light facing	35	30	86%	3	9%	2	5%	0	0%	0	0%
	total	74	50	67%	12	17%	2	3%	9	12%	1	1.35%
Reinforced Concrete Building	Factories	60	42	70%	8	13.4%	3	5.0%	4	6.7%	3	5.0%
	Office Build.	165	129	78%	25	15.2%	8	3.6%	2	1.2%	1	0.6%
	Shops	62	47	75.7%	76	11.3%	6	9.7%	0	0%	2	3.2%
	Dwelling Houses	52	45	86.6%	2	3.8%	4	7.7%	0	0%	1	1.9%
	Schools	21	15	71.5%	5	23.8%	1	4.7%	0	0%	0	0%
	Storage	124	97	78.7%	9	7.3%	15	12.1%	3	2.4%	0	0%
	Auditoriums			60%	1	20%	0	0%	1	2.0%	0	0%
	Public Build.	19	3	68.4%	5	26.3%	0	0%	1	5.3%	0	0%
	Others	73	63	86.3%	3	4.1%	5	6.8%	2	2.8%	0	0%
	total	592	452	78%	69	11.7%	43	7.3%	11	1.8%	7	1.2%
			not damaged		slightly damaged		seri usly damaged		half collapsed		collapsed	

from the Report of Imperial Earthquake Investigation Commitee Vol. 100 (C) (D).

Table 5.

	total number	not damaged or slightly damaged		half collapsed		collapsed or unrestorably damaged	
Brick Buildings	160	35	21.9%	27	16.9%	98	61.3%
Steel Buildings	45	22	48.9%	18	40.0%	5	11.1%
Reinforced concrete Build.	89	48	53.9%	14	15.7%	27	30.0%
Wooden Houses						about	30%

which causes the building on it to incline. We know some examples of these types of failure at Yokohama 1923. And the other type of failure is overturning which is not rare in the case of little edifices but is quite rare among houses.

VI. Plasticity introduced into the calculation of the structure.

From these considerations made by the author, we must introduce the idea of plastic deformation into the calculation of structure especially for computing the resistance of the structure against earthquakes. It means in ordinary cases making the computation more complicated but in some cases it makes the computation rather simpler. We can consider some statically indeterminate structure of higher order, as being one of lower order in extreme cases as being a statically determinate structure by the introduction of the plastic deformation as in the former example.

In fig. 18 we show one story of a many-storied building frame. We can determine the maximum resistance to the shearing force caused by the earthquake as $\sum \frac{M_0 + M_u}{h}$.

The summation made for all columns, M_0 and M_u means the maximum bending moment at the top and the foot of the column determined by cross-section of the column or the connection to the girder independently from ordinary elastic-statics. And the limit of the deformation of this story is given by the horizontal displacement that causes any one of the columns to buckle, and so the potential energy storable in this story can be determined.

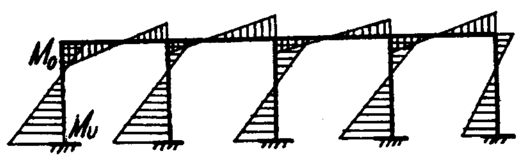


Fig. 18.

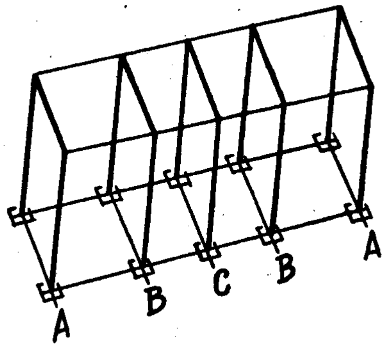


Fig. 19.

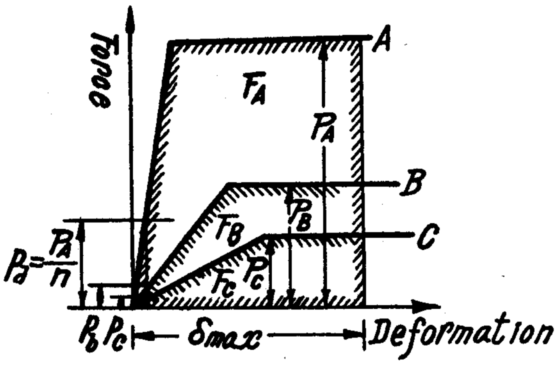


Fig. 20.

I may mention concerning this problem that if the first yielding may happen in the connection then the limit of the bending moment applied to the top and foot of the column is given by the strength of connection making the column safer from destruction. So the quake-resisting design must take into account what part of the structure is the best part to yield first. In our wooden construction of the Buddist temple the block named "Masu" is inserted between the top of the column and the girder, which is compressed in the direction perpendicular to the fibre of the wood with high plastic ductility and the first yielding takes place at that part. This seems to explain partly the high earthquake-resisting feature of the Buddihist temple.

It is apparent that such a self-helping or stress balancing feature is also to be taken into account for a group of structures built as one body. A building shown in fig. 19 is constructed of three kinds of frame A, B and C, and suppose horizontal force is applied to the structure. If we confine our consideration to the elastic domain, then the distributed force to each frame is proportional to $pa:pb:pc$ in fig. 20. This proportion varies with the flexibility of the frame, and so it is very difficult to determine the distributing coefficient of the horizontal forces in this sense

In our present calculation of the building, the distribution-coefficient is said to be determined upon the basis of elastic deformation, but it is quite uncertain, for we cannot determine exactly the flexibility of each structure especially for a multi-storied structure.

But if we introduce the plastic-ductile property into this problem, then the resistance of each frame is simply Pa, Pb and Pc which are the ultimate resistance of each structure.

The resistance of the structure as a whole can be determined by the summation of the potential energy storable in each structure before its fall down as $Fa+Fb+Fc$ in fig. 20. The author is in the opinion that the distribution-coefficient at the present stage gives the ratio $Pa:Pb:Pc$ and is determined from the constructive design of each frame. The clever determination of the distribution-coefficient may economize the good quake-resisting construction. The distribution-coefficient in elastic domain is not so accurate but by the plastic-ductility of the frame the structure designed under these assumed distribution-coefficients may resist earthquake with determined ratio.

VII. Conclution.

We Japanese building engineers are now designing the buildings to resist 0.1 g of earthquake's acceleration taking the allowable stress as about 1/3 of the ultimate stress. But to rationalize the constructive design the author proposes to calculate the structure safely to resist 0.4 g earthquake at least, and this quake-factor may be decreased with the prolonged period of natural vibration of the structure to enable us to design skyscrapers upon the basis of the author's "velocity—potential energy" theory. The constructive material chosen must be plastic-ductile material such as steel or good reinforced concrete or wood. It is necessary that the plastic ductile properties of the reinforced concrete member and at the same time the true ultimate resistance of reinforced concrete must be fully studied in the near future.